

Comparison of Numerical Eye Models and its Representation within a Mechanical Eye Model^{*}

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Abstract: Today the increased computer power admits very detailed numerical modelling of the human eye. Simultaneous new technologies as well as the defined production of aspherical lenses permits the realization of nearly realistic ex vivo mechanical simulation of the natural human eye. Amongst others such ex vivo mechanical eye models are used to verify the optical quality of new intraocular lenses. Measurements of modulation transfer function (MTF) performed by an ex vivo mechanical eye model and numerical analysis of saggital and tangential MTF are presented.

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Keywords: Mechanical eye model, Numerical optical analysis, Intraocular lens, Image quality, MTF

1. INTRODUCTION

The mathematical and numerical characterization of the optical and physiological properties of the human eye has a more than hundred years old history. An overview of different publications separated by multiplicity levels of complexity was given by different authors [Atchison (2005); de Almeida and Cavalho (2007); Navarro (2009)].

In the nineties ex vivo mechanical eye models were developed to measure optical quality criteria after ophthalmic surgeries and to predict the quality of implanted intraocular lenses [Norrby et al. (2007); Drauschke et al. (2013); Díaz and Celestino (2014)]. First modern mechanical eye models are oriented to the ISO 11979–2 standard [International Organization for Standardization ISO Central Secretariat (2003)] for measuring intraocular lens (IOL) quality. The definitions of the ISO eye model are not adaptive for all cases of application. On the one hand new advanced models do not fulfill the requirement of this standard, e.g. using more physiological aspherical cornea lenses [Drauschke et al. (2013)]. On the other hand Norrby had pointed out, that the ISO eye model cannot be adapted to test of aspherical lenses [Norrby (2008)]. Due to this mismatch, the development of non ISO eye models has been strongly motivated.

The quality measurement and test instructions for IOL are defined in the ISO 11979–2 [International Organization for Standardization ISO Central Secretariat (2003)]. The quality definition is based on the measurement of the optical resolution and the MTF in a well defined measurement set-up. However, since new optical design strategies allow the design of aberration free aspherical IOL additional optical quality parameters are used to find applicable mechanical eye models and better accordance

between patients physiological image quality feeling and physical image quality [Pieh et al. (2009); Applegate et al. (2003); Drauschke et al. (2013)].

2. MATERIALS AND METHODS

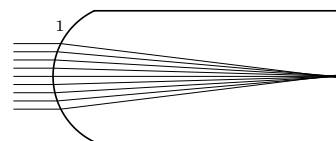
2.1 Numerical eye models

The investigation of the human eye starts with explanation of the optical basics of imaging by Gauss in 1841. As a result many theoretical eye models were developed to characterize physiological aspects based on measured optical properties of the human in vivo eye.

A reduced eye model was presented by Emsley in 1952 [Emsley (1952)] as shown in table 1. Advanced eye models use three spherical interfaces as presented by Helmholtz & Laurance (1909), Gullstrand (1911) and Le Grand & El Hage (1980) [Alpern (1978); Southall (1924); Le Grand and El Hage (1980)]. Optical properties of this eye models are shown in table 2. Schematic eye models with four spherical interfaces were presented by Gullstrand (1911) and Le Grand & El Hage (1980) [Le Grand and El Hage

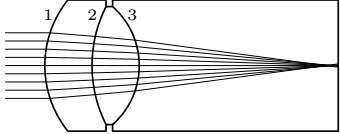
Table 1. Reduced eye model with 1 spherical surface

Surface	Physical parameters		
	Radius in [mm]	Thickness in [mm]	Refractive index
[Emsley (1952)]			
1	5.55	22.22	1.3333



^{*} The project is funded by the *Municipal Department 23 – Economic Affairs, Labour and Statistics – City of Vienna* within the project *Laser & Optics in Applied Life Sciences (LOALiS)*.

Table 2. Reduced eye model with 3 spherical surfaces

Physical parameters			
Surface	Radius in [mm]	Thickness in [mm]	Refractive index
Helmholtz & Laurance (1909) (in [Alpern (1978)])			
			
1	8.00	3.60	1.3330
2	10.00	3.60	1.4500
3	-6.00	15.18	1.3330
Gullstrand (1911) (in [Southall (1924)])			
1	7.80	3.60	1.3360
2	10.00	3.60	1.4130
3	-6.00	16.97	1.3360
[Le Grand and El Hage (1980)]			
1	7.8	3.60	1.3330
2	10.20	3.60	1.4160
3	-6.00	16.70	1.3330

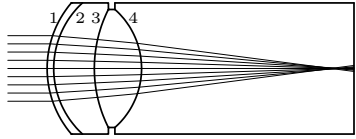
(1980)]. The parameters of that schematic eyes are listed in table 3. Most sophisticated schematic eye models use aspherical interfaces, the graded index structure of the eye lens and include the dispersion of tissue material as presented by Schwiegerling(1995), Liou & Brennan (1997) and Escudero & Navarro (1999) [Schwiegerling (1995); Liou and Brennan (1997); Escudero-Sanz and Navarro (1999)]. Optical parameters of that schematic eye models are listed in table 4.

2.2 The numerical and mechanical eye model set-up

The ex vivo mechanical eye model used in this paper is not conform to the ISO eye model. A numerical and a mechanical eye model based on the measurements of optical properties of the human eye performed by Liou and Brennan was used [Liou and Brennan (1997); Drauschke et al. (2012, 2013)]. The small differences between the mechanical eye model and the eye model by Liou and Brennan are explained in the following. The numerical set-up is performed within ZEMAX©[Radiant Zemax Corporate Offices & Research Center (2013)].

Both models, numerical and mechanical, were built up with aspherical cornea lenses. The numerical eye model consists of homogeneous material with refractive index of 1.376 and Abbe V -number of 61.2 according to Liou & Brennan [Liou and Brennan (1997); Atchison (2005)]. The anterior surface has a spherical radius of $R = 7.77$ mm and an asphericity of $Q = -0.18$. The posterior surface has a

Table 3. Eye model with 4 spherical surfaces

Physical parameters			
Surface	Radius in [mm]	Thickness in [mm]	Refractive index
Gullstrand (1911) (in [Le Grand and El Hage (1980)])			
			
1	7.70	0.50	1.3760
1	6.80	3.10	1.3360
2	10.00	3.6	1.4085
3	-6.00	16.97	1.3360
[Le Grand and El Hage (1980)]			
1	7.80	0.55	1.3771
2	6.50	3.05	1.3374
3	10.20	4.00	1.4200
4	-6.00	12.45	3.3360

radius of $R = 6.4$ mm and an asphericity of $Q = -0.6$. The cornea has a center thickness of $d = 0.5$ mm.

In the mechanical eye model an aspherical meniscus lens is used too. The anterior and posterior surfaces are defined by

$$z = \frac{\frac{r^2}{R}}{1 + \sqrt{1 - (1 + Q) \left(\frac{r}{R}\right)^2}}. \quad (1)$$

Concerning manufacturing constraints and the use of material with different refractive index (Polymethyl methacrylate: PMMA) a lens with same focal distance but different shapes is used as cornea lens. In equation 1, the parameters are defined as follows; $R = 7.77$ mm the spherical radius, $0 \text{ mm} \leq r \leq 6 \text{ mm}$ the radial coordinate and $Q = -0.194053$ the asphericity of the anterior lens surface and $R = 7.186631$ mm the spherical radius, $0 \text{ mm} \leq r \leq 6 \text{ mm}$ the radial coordinate and $Q = -0.019958$ the asphericity of the posterior lens surface, respectively. The center thickness of the lens is $d = 0.7$ mm. A small shift of the principal planes of the cornea lens is compensated within the set-up. The spherical aberration is somewhat smaller than that for the natural cornea lens.

A natural (double) gradient-index (GRIN) eye lens, a spherical IOL, or an aspherical IOL is used in the numerical eye model according to simulations of Drauschke et al [Drauschke et al. (2013)]. The gradient function of the (double) GRIN lens is taken according to Liou & Brennan [Liou and Brennan (1997)]. Within the mechanical eye model a spherical Artisan Aphakia 5/8.5 IOL with 24dpt is used. It consists of PMMA. The IOL can be shifted and tilted for alignment in reference to the optical axis which is defined by the center points of the cornea and the pupil aperture.

Table 4. Eye model with 4 partly aspherical surfaces

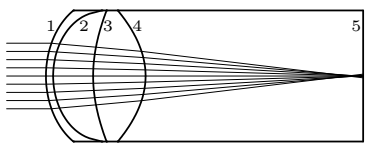
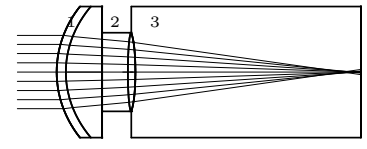
Surface	Physical parameters			Refractive index	
	Radius in [mm]	Asphericity	Thickness in [mm]		
[Schwiegerling (1995)]					
					
1	7.80	0.75	0.55	1.3771	
2	6.50	0.75	3.05	1.3374	
3	11.03	-3.30	4.00	1.4200	
4	-5.72	-1.17	16.60	1.3360	
[Liou and Brennan (1997)]					
1	7.77	-0.18	0.50	1.376	
2	6.40	-0.60	3.16	1.336	
3	12.4	-0.94	1.59	$1.368 + 0.049057 \cdot z - 0.015427 \cdot z^2 - 0.001978 \cdot r^2$	
4	∞	—	2.43	$1.407 - 0.006605 \cdot z^2 - 0.001978 \cdot r^2$	
5	-8.10	0.96	16.26	1.336	
Dispersion	$n(\lambda) = n(0.555 \mu\text{m}) + 0.0512 - 0.1455 \cdot \lambda + 0.0961 \cdot \lambda^2$				
[Escudero-Sanz and Navarro (1999)]					
1	7.72	-0.26	0.55	1.3777	
2	6.50	—	3.05	1.3391	
3	10.2	-3.1316	4.00	1.4222	
4	-6.00	-3.1316	16.3203	1.3377	
Dispersion	Wavelength in [nm]				
	Medium	458	543	589.3	632.8
	Cornea	1.3828	1.3777	1.3760	1.3747
	Aqueous	1.3445	1.3391	1.3374	1.336
	Lens	1.4292	1.4222	1.4200	1.4183
	Vitreous	1.3428	1.3377	1.3360	1.3347

Table 5. Basic optics if the realized mechanical eye model as modified [Liou and Brennan (1997)]

Surface	Physical parameters			Refractive index
	Radius in [mm]	Asphericity	Thickness in [mm]	
				
1	7.77	-0.18	0.70	PMMA
2	7.1866	-0.35	4.2	1.336
3	7.15	—	1.11	PMMA
4	-36.39	—	17.2	1.336

Circular apertures are used for simulating the pupil of the human eye. The aperture is shifted perpendicular to the optical axis in the case of numerical eye model and centered in the case of mechanical eye model. The spherical radius of the retina is -12 mm in the case of numerical eye model. Within the mechanical set-up a transparent flat glass plate is used to simulate the retina.

Photopic (bright) and scotopic (dark) illuminations are used in the numerical eye model. The photopic illumination is used for pupil diameters of 2 mm and the scotopic illumination is used for pupil diameters of 8 mm. A uniform diffuser light source (Lambertian surface) is used to illuminate the mechanical eye model [Metaphase Technologies, Inc. (2011b,a)]. It simulates a photopic illumination situation. The pupil diameter in measurements was 2 mm.

The Abbe *V*-number is 55.2 for all numerical simulations. The aqueous & vitreous humour is simulated in the eye model with sodium chloride solution. The refractive index is manipulated by the mass concentration of the salt.

Sodium chloride solutions with mass concentrations of 1.6% is used for the measurement.

2.3 Measurement set-up

For numerical analysis of the optical quality properties the MTF curves were calculated. Within the mechanical set-up the the MTF are measured. These quality criteria are measured in full mechanical set-up, so a combination of aberrations of the crystalline lens and the cornea are measured.

The slanted edge method is used for the measurement of the modulation transfer function [Rank (2013)]. In order to measure these values a target according to ISO 12233 as shown in figure 1 is used. A slanted edge of 5° is used and scanned with a CCD camera with the objective lens. The measured pixels intensity is plotted as a function of the of the pixel position [International Organization for Standardization (2000)] which leads to the edge spread function (ESF).

The used model eye contains some differences to the ISO model eye:

- an aspherical meniscus lens is used for cornea
- the aqueous humour is not bordered by a plane but by the aspherical posterior surface of the cornea
- sodium chloride solutions of different mass concentrations are used, which have refractive index differences higher than 0.005 in comparison to in situ measurements. The dispersion of the aqueous & vitreous humour solution is taken into account.
- image plane is positioned directly onto the plane artificial retina.

2.4 Quality criteria

To verify the optical imaging quality of the IOL in the mechanical eye model standardized measurement methods are adapted. The evaluation and test instruction procedures for IOLs are defined in the ISO 11979-2 [International Organization for Standardization ISO Central Secretariat (2003)]. As mentioned before significant differences in mechanical set-up are implemented in comparison to ISO standard. The ISO 11979-2 instructions include the measurement of the spatial resolution and MTF of IOLs [Navarro et al. (1993); Grossman and Faaland (1993)]. However, in practice additional quality parameters are used for the specification of the optical quality of IOLs [Pieh et al. (2001); Applegate et al. (2003); Salmon and van de Pol (2006); Pieh et al. (2009); Cheng et al. (2010)]. The

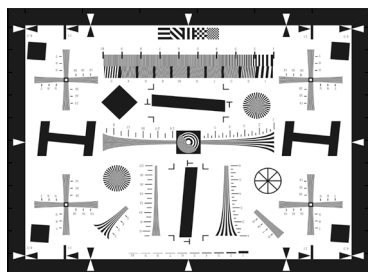


Fig. 1. Test target for optical lenses according to the ISO 12233 standard [International Organization for Standardization (2000)]

usage of additional quality criteria is necessary to explain the image quality in more detail and to verify the causes of bad vision of patients or because ISO test instructions are not valid or critical for quality assessing of the analysed IOL [Norrby (2008)].

The spatial resolution ξ defines typically the ability to distinguish object details of a resolution test chart (e.g. the ISO 12233 resolution test chart defined in [International Organization for Standardization (2000)] and shown in figure 1) after passing an optical imaging system as the presented mechanical eye model. Because this definition the spatial resolution is expressed in terms of line-pairs per mm ($\text{lp} \cdot \text{mm}^{-1}$). Due to its definition unit is well known from image processing the unit is called the spatial frequency too.

A much better quality criterion as simple resolution for the distinction between light and dark regions of an image is the contrast in an image. Contrast $C_{\%}$ can be defined in absolute terms or in terms of percentage (%). It is defined as a mathematical function of the image's maximum I_{\max} and minimum I_{\min} intensities of a test chart

$$C_{\%} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} 100\%. \quad (2)$$

Again a standard resolution test chart has to be used as object. The contrast is measured after passing the full optical imaging system. A more sophisticated quality criterion is the modulation transfer function. It combines measurement information of resolution and contrast in one single quality criterion. Incoherent light sources have to be used. Because the MTF combines only spatial resolution and contrast information aberrations which lead to distortions, these are not recorded by the MTF. One definition of the MTF is given by

$$MTF(\xi) = \frac{1}{\pi} (\phi - \cos \phi \sin \phi) \quad (3)$$

$$\text{with } \phi = \cos^{-1} \left(\frac{\xi}{\xi_c} \right) \quad (4)$$

$$\text{and } \xi_c = \frac{1}{\lambda f / \#} \quad (5)$$

$$f / \# = \frac{\text{focallength}}{\text{clearaperture diameter}}$$

An alternative definition of the measurement process of the MTF can be found by application of image processing technologies. Lets assume an original input image $f(x, y)$ described in Cartesian spatial coordinates (x, y) . It can be analyzed by calculating the convolution of this original input image $f(x, y)$ with a system impuls function $h(x, y) : g(x, y) = h(x, y) * f(x, y)$. The 2-D-convolution is abbreviated by the $*$ -symbol as usually.

In Fourier space a convolution will give the product of the fourier transform functions $F(u, v)$, $H(u, v)$, and $G(u, v)$ of the original input image, the system impuls functions and their convolution, respectively. u and v are the frequency coordinates in fourier space. The MTF can be defined as the absolute value of the fourier transform function of the system impuls function: $MTF = |H(u, v)|$.

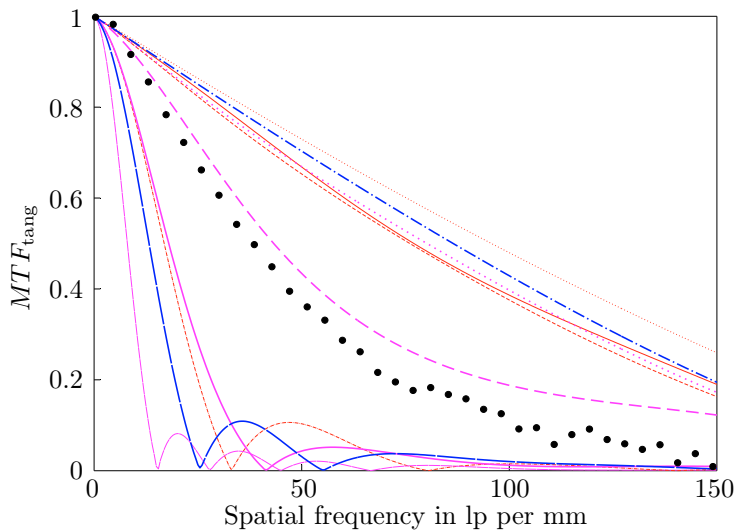


Fig. 2. Comparison of tangential MTF curves of different schematic eyes with iris diameter of 2 mm and photopic illumination according Emsley (thin red full line), Helmholtz & Laurance (thin red dotted line), Gulstrand from Southall (thin red short dashed line), reduced Le Grand and El Hage (thin red dashed-dotted line), Gulstrand from Le Grand and El Hage (thin magenta long dashed line), Le Grand and El Hage (thick magenta full line), Schwierierling (thick magenta dotted line), Liou and Brennan (thick magenta short dashed line), Escudero-Sanz & Navarro (thick blue dashed-dotted line), and mechanical eye model (thick blue long dashed line) and measurement results with mechanical eye modeln (black bullets).

If input image is a simple step function as in case of slanted edge method, it can be defined as $f(x, y) = k(x) \cdot 1$. This definition leads to the edge spread function (ESF) definition

$$ESF(x) = h(x, y) * f(x, y) = h(x, y) * [k(x) \cdot 1] \quad (6)$$

from which the MTF can be calculated by $MTF(u) = |\mathcal{F}[\frac{d}{dx}ESF(x)]| = |\mathcal{F}[\frac{d}{dx}h(x, y) * [k(x) \cdot 1]]|$, (\mathcal{F} describes the Fourier transformation of the following argument) according to typical definitions [Webb (2010); Zhang et al. (2012)].

3. RESULTS

MTF curves for different illumination situations as photopic illumination with 2 mm pupile diameter and scotopic illumination with 8 mm diameter are shown in figures 2 and 3. It is found, that the MTF curves show significant differences in both cases of illumination for the different analyzed schematic eye models. These differences do not correlate with the complexity of the used schematic eye models. The measured MTF curves of the realized mechanical eye model show lower but comparable values as the corresponding schematic eye model by Liou & Brennan. In comparison with the most complex eye model by Liou & Brennan one found that this eye model generates MTF values which fell between the other presented eye models on the curve, in the case of photopic illumination. The results

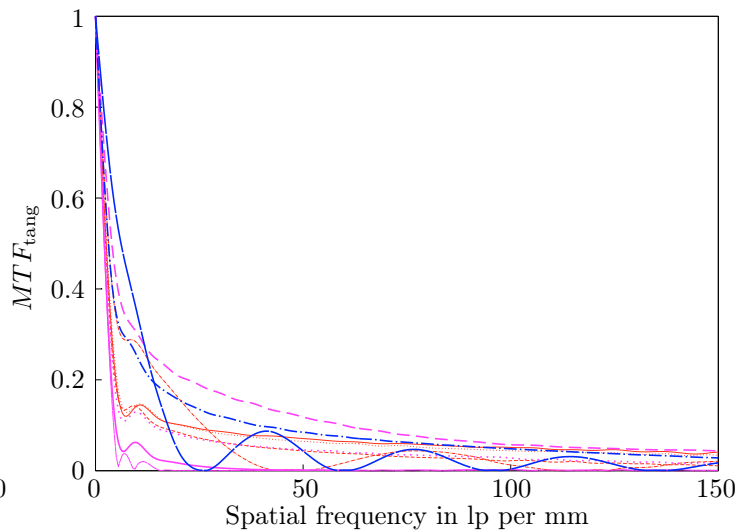


Fig. 3. Comparison of tangential MTF curves of different schematic eyes with iris diameter of 8 mm and scotopic illumination according Emsley (thin red full line), Helmholtz & Laurance (thin red dotted line), Gulstrand from Southall (thin red short dashed line), reduced Le Grand and El Hage (thin red dashed-dotted line), Gulstrand from Le Grand and El Hage (thin magenta long dashed line), Le Grand and El Hage (thick magenta full line), Schwierierling (thick magenta dotted line), Liou and Brennan (thick magenta short dashed line), Escudero-Sanz & Navarro (thick blue dashed-dotted line), and mechanical eye model (thick long dashed line)

showed that one cluster of eye models with MTW curves of typically lower values (Gulstrand model from Le Grand publication, simple Le Grand and El Hage model, full Le Grand and El Hage model and mechanical eye model) and one cluster of models with MTF curves of typically higher values (Emsley model, Helmholtz & Laurence model, Gulstrand model presented by Southall, Schweigerling eye model, and Escudero & Navarro eye model). Both clusters show significantly different oscillations of the MFT curves depending on the spacial frequency. This indicates, that the main difference of the presented different schematic eye models are generated optical aberrations.

Because the larger iris diameter in case of scotopic illumination findings showed larger oscillations in this MTF curves in comparison to photopic illumination. The eye model of Liou & Brennan shows the lowest oscillation. No measurments are performed with mechanical eye model yet. Because of this, in case of scotopic illumination the results showed that for larger spatial frequencies more than 10 lp/mm the Liou & Brennan eye model generates MTF curves of larger values in comparison with all other eye models. Additionally the MTF curves drop much faster in comparison to photopic illumination because of larger generated aberrations caused by the larger iris diameter.

4. DISCUSSION

The optical quality of a set of nine schematic eye models was compared with a mechanical eye model which was designed to evaluate IOLs under realistic conditions. As

quality parameter MTF curves were calculated and measured for photopic illumination. It was shown, that for both cases, photopic and scotopic illumination, variations of the MTF curves were found in comparison with the most physiological schematic eye model of Liou & Brennan. The measurement results of the mechanical eye model are comparable to the schematic eye model by Liou & Brennan. The small differences are caused by manufacturing tolerances. These differences are not foremost caused by the complexity of the schematic eye models. In case of mechanical eye model the differences can be explained by the more basic optical properties of the used IOL in comparison the the more complex human eye lens in the schematic eye model of Liou & Brennan.

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