Досліджуються процеси перерозподілу енергії оптичного сигналу між серцевиною і оболонкою оптичного волокна в місцях їх з'єднання. В аналітичній операторній моделі отримані замкнуті формули залежності тривалості сигналу на стороні приймача від кількості з'єднань і вірогідності переходу енергії із серцевини в оболонку оптичного волокна. Виконано аналіз перерозподілу енергії за рахунок геометричної нестиковки волокон в точках їх з'єднання.

Для синтезу аналітичних моделей використано апарат теорії вірогідності і теорії перетворення Фур'є. В якості міри тривалості сигналу на стороні приймача використовується ефективна тривалість імпульсу.

Чисельний аналіз дозволив зробити висновки, що досліджувані ефекти практично не залежать від початкової форми оптичного сигналу і від варіанту розподілу будівельних довжин вздовж лінії регенераційної ділянки.

Отримані оцінки дозволяють стверджувати, що можливі дисперсійні ефекти, які викликані перерозподілом енергії в точках з'єднання оптичних волокон. Дані ефекти обумовлені різницею групових швидкостей в різних середовищах оптичного волокна (в серцевині і в оболонці). По своєму впливу на форму сигналу вони можуть бути порівняні з добре вивченими ефектами за рахунок матеріальної, хвилевідної та інших видів дисперсії.

На основі виконаних досліджень висунута наукова гіпотеза про існування ефекту дисперсії, що викликана рознімними і нерознімними з'єднаннями оптичних волокон. Обґрунтовано схему лабораторної установки для виконання натурних досліджень передбачуваних ефектів. Особливість даної установки в тому, що точки з'єднання оптичних волокон можуть розташовуватись з практично довільним кроком. Це дозволяє виконати перевірку висунутих гіпотез в лабораторних умовах

Ключові слова: загасання в місці з'єднання, оптичний сигнал, групова швидкість, ефективна тривалість імпульса

UDC 621.391.6
DOI: 10.15587/1729-4061.2019.163777

THEORETICAL STUDY OF THE DISPERSION EFFECTS OCCURRING AT OPTICAL FIBER CONNECTIONS

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1. Introduction

The well-known engineering procedures [1, 2] for calculating maximum length of a regeneration section (RS) of a fiber-optic transmission system (FOTS) take into consideration effects of optical signal attenuation. At the same time, attenuation is taken into consideration both in face-to-face lengths and connections of optical fibers (OF). In addition, these procedures take into consideration effects of optical pulse distortion (including elongation) caused by dispersion. However, the authors' production experience in the telecommunications industry has made it possible to establish an effect being of interest for scientific analysis in some cases. Essence of this effect consists in a contradiction:

– direct reflectometer measurements indicate that attenuation in the line is within acceptable limits;

– RS length calculations for dispersion give satisfactory results as well;

– at the same time, the fiber optic transmission line does not function normally.

It is obvious that the existing procedures do not take into consideration all possible causes of optical signal distortion.

In a number of cases, reduction of number of OF connections resulted in restoration of communication. Thus, the authors' conclusion is as follows: effect of connections on signal form is one of possible unaccounted causes of connection failures. From a physical point of view, this may mean that there is redistribution of energy between the OF core and shell at the connection points. The proposed paper is devoted to theoretical analysis of this phenomenon.

Taking into consideration ever increasing traffic, it can be stated that there is a need for development of fundamentally new fiber optic transmission systems. Such systems are subject to requirements of raised data transfer speed. At the same time, the problem of accounting for such effects in transmission lines which previously could be considered negligibly small is pressing.

2. Literature review and problem statement

Additional information on a possible RS length gives analysis of various components of dispersion expansion of the signal as it propagates along the OF [3, 4].

A considerable number of scientific studies are devoted to studies in the field of signal attenuation at OF connection points. Specifically, models of signal energy distribution in detachable [5] and permanent OF connections were synthesized [6, 7]. However, no consideration of the effect of signal form change in the points of fiber connection was provided in these studies. Instead, a considerable attention is paid to improvement of OF connections in terms of reducing errors at the receiving side [8]. There are papers devoted to metrological aspects of study of attenuation occurring in the OF connection points [9, 10]. These studies also do not take into consideration a phenomenon of signal form distortion caused by redistribution of signal power in an OF connection point. An extensive list of references devoted to the mentioned issues is given in [5–10]. However, these studies do not consider the effect of fiber connection points on the optical signal length and magnitude of introduced dispersion.

Characteristics of attenuation in OF connections are so well established and connection technology is so well developed that basic attenuation parameters and geometric parameters of connections were introduced in recommended international standards [11].

At the same time, it is important to consider distortion of the fine signal structure caused by a set of factors [13] for prospective FOTS where signal multiplex methods are used [12]. It seems obvious that there is signal energy redistribution between the fiber core and outer layers in the OF connection points [14]. The fact that rate of signal propagation in a medium depends on the medium refractive index (more precisely, on the group refractive index) should be taken into consideration. Then, it is easy to assume that the connection points can cause effects of signal distortion similar to dispersion [15, 16].

At the same time, published sources [5–10] do not provide information on the effect of the investigated power redistribution processes on form, and, consequently, duration of the optical signal.

Therefore, studies devoted to theoretical analysis of the effect of energy redistribution factor in OF connection points on evolution of the optical signal form should be considered promising. Specifically, we can speak of an earlier unexplored form of dispersion in FOTS, that is dispersion in OF connections.

3. The aim and objectives of the study

The study objective is to study dispersion effects at the points of connection of optical fibers. This will give an opportunity to more accurately calculate increase in duration of optical pulses in prospective FOTS. In the engineering way, these studies will clarify methods of calculating lengths of regeneration sections.

To achieve this objective, the following tasks were set:

– synthesize corresponding mathematical models of signal transformation in the connection points;

– synthesize analytical models of signal transformation in a group of connection points;

– obtain approximate quantitative estimates of the effect of dispersion effects in OF connections on variation of signal duration at the receiving side;

– develop measurement diagrams for checking the proposed hypotheses.

4. Factors and models of dispersion in optical fiber connection points

The investigated effects are caused by signal energy redistribution between the OF core and shell in the connection points. In this case, the shell and the core can be considered in some way as two independent light conductors [17]. The dispersion effects are caused by difference in the group refractive indices [18, 19] (hence, also by difference in the group velocities) in the OF core and shell.

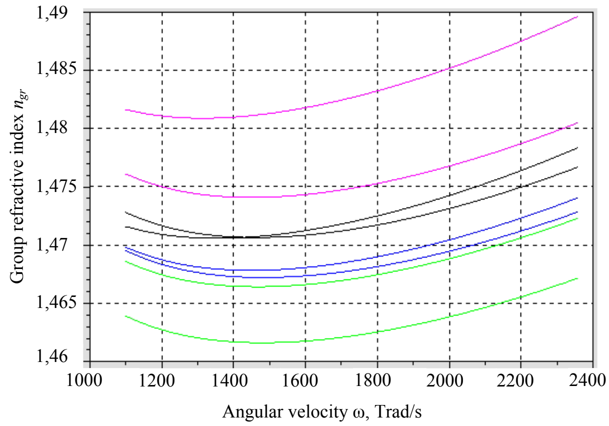
Calculation of values of the group refractive index and the group velocity depending on material and angular frequency by means of the method given in [20, 21] are shown in Fig. 1. Analysis of these graphs makes it possible to make the following approximate calculations. The difference in group velocities ΔV_{gr} per 1 % of difference in $\mathrm{GeO_2}$ concentration is approximately 200 km/s. Assume that the group velocity ΔV_{0gr} corresponds to the core velocity and $\mathrm{GeO_2}$ concentration in the shell material is 1 % less than that of the core material. Then, an approximate estimate of relative advance of the signal component in the shell along the line length, l, is obtained:

$$\tau(l) = l\left(\frac{1}{V_{0gr}} - \frac{1}{V_{0gr} + \Delta V_{gr}}\right) \approx$$
$$\approx \frac{l \cdot \Delta V_{gr}}{V_{0gr}^2} \approx 5 \cdot 10^{-9}\ \mathrm{s} = 5\ \mathrm{ns}. \qquad (1)$$

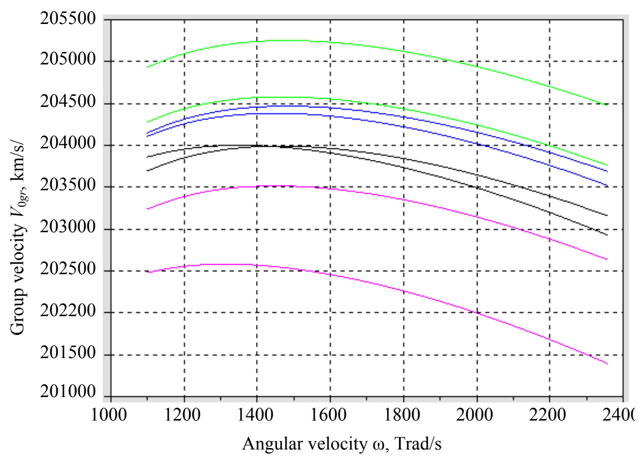
Let us assume that approximately the same amount of signal energy propagates in the core and the shell and the photodetector aperture captures both the area of the core and the area of the shell. Then estimate (1) will result in a very modest rate of information transmission at a distance of 100 km: less than 1 Mbps. This value is in no way consistent with typical linear transmission rate, for example, with STM class equipment [2]: up to 100 Gbps. This is because coarsened parameter estimates were taken here. In fact, a relatively small part of signal energy propagates in the shell. At the same time, one should not neglect the investigated effects for prospective FOTS.

Among the causes of energy redistribution in the OF connection points, the following groups are distinguished: diffusion effects, geometric factors, mounting (structural) factors.

Diffusion effects [5, 22] manifest themselves as a result of mixing materials of the OF coating, shell and core. In what follows, we will limit our consideration to an OF with a stepped profile (Fig. 2).



a



b

Fig. 1. Dependences on angular frequency and concentration of doping additives of germanium dioxide GeO_2 for: group refractive index (*a*); group velocity (*b*). Concentrations: 0 %, 3.1 %, 3.5 %, 4.1 %, 5.8 %, 7.0 %, 7.9 %, 13.5 %. Position of the graphs depends on GeO_2 concentration: the higher the curve, the greater GeO_2 concentration (*a*); the inverse order (*b*)

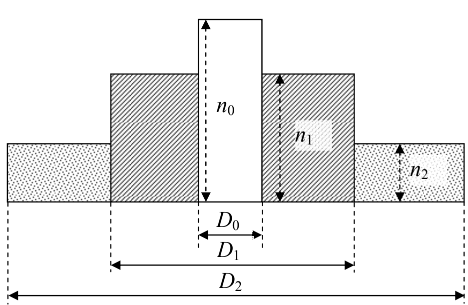


Fig. 2. Profile of the refractive index of the stepped fiber: $D_0<D_1<<D_2$ (diameters of the OF core, shell and coating); $n_0>n_1>>n_2$ (refractive index of material of core, shell and coating, respectively)

Obviously, diffusion creates a point effect where the OF acquires character of the gradient fiber (1) at a small length with a random gradient at that. This group of effects is characteristic for permanent welded joints.

Geometric factors [11] are reduced to a mismatch of directions of the incoming and outgoing OFs (tangential and angular shift) as well as the gap between these OFs. In the first case, there is a discrepancy between the modal spot from the incoming fiber and the outgoing OF core area. In the second case, energy output from the core to the shell is caused not by the absolute spatial coherence of radiation. These factors are characteristic of both detachable and permanent connections.

Mounting (structural) factors [23] bring about additional energy redistribution effects in a case of connecting fibers of different types (including fibers with different characteristic diameters).

Abstracted from the causes of energy redistribution, we shall restrict ourselves to considering the model as an operator (Fig. 3).

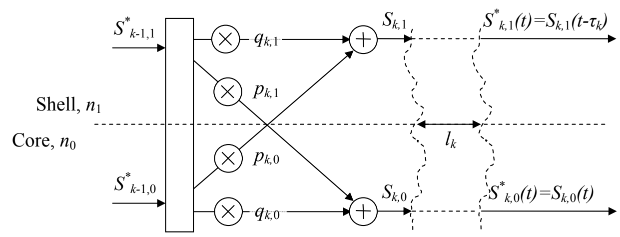


Fig. 3. The operator of transformations in the *k*-th point of OF connection

It is assumed in the operator diagram shown in Fig. 3 that two independent signals $S_0(t)$ and $S_1(t)$ act at the input of the *k*-th connection point, that is the signals in the fiber core and shell, respectively. Since energy parameters will be considered further, corresponding notation is taken in Fig. 3: $S_0(t)=s^2(t)$ is the power distribution function (PDF) in the time domain of the signal in the OF core; $S^*_{k-1,1}(t)=s_1^2(t)$ is the signal PDF in the OF shell.

To estimate duration of a signal of arbitrary form at the receiving side, two characteristics will be used: the effective pulse duration (EPD) T_{eff}^2 with quadratic dimension [24] and the calculated pulse duration (CPD) $T_s=6\sqrt{T_{eff}^2}$ with linear dimension [12]. The first of these characteristics is convenient for analytic studies, and the second one determines a part of the signal duration (possibly theoretically infinite) which contains no less than 90 % of the signal energy. Next, define EPD:

$$T_{eff}^2=\frac{1}{E}\int_{-\infty}^{\infty}\left(t-\bar{t}\right)^2 S(t)\mathrm{d}t,\quad E=\int_{-\infty}^{\infty}S(t)\mathrm{d}t,$$

$$\bar{t}=\int_{-\infty}^{\infty}t\,S(t)\mathrm{d}t, \qquad (2)$$

where E is the signal energy, W; $S(t)$ is the signal PDF, W/s; $\bar{t}$ is the position of the signal energy center in the time domain, s.

Take into consideration that expressions of EPD (2) include normalization to the energy of the signal, E. Then, without loss of generality, one can assume that a signal with unit energy acts at the input of the *k*-th connection point:

$$E_{\Sigma,k-1}=E_{k-1,0}+E_{k-1,1}=\int_{-\infty}^{\infty}S^*_{k-1,0}(t)\mathrm{d}t+\int_{-\infty}^{\infty}S^*_{k-1,1}(t)\mathrm{d}t=1. \qquad (3)$$

Assume also that phenomena of inverse energy scatter at the connection point and output of the part of signal energy from the OF shell to the coating, etc. do not affect the model characteristics as well. They reduce the signal amplitude at the receiving side but do not affect the signal duration. Then shares of p and q of redistributed energy in the connection point can be represented as a probabilistic model:

$$0 \le q_{0,k} \le 1, \quad p_{0,k} = 1 - q_{0,k},$$

$$0 \le q_{1,k} \le 1k, \quad p_{1,k} = 1 - q_{1,k}, \tag{4}$$

and parameters p and q will be further called by corresponding probabilities of energy transition. The "law of energy conservation", $E_{\Sigma,k}=E_{\Sigma,k\text{-}1}$, obviously follows from expressions (3) and (4).

PDF transformation is obtained at the output of the k-th connection point:

$$\begin{cases} S_{k,0} = q_{k,0} \cdot S^*_{k-1,0} + p_{k,1} \cdot S^*_{k-1,1}, \\ S_{k,1} = q_{k,1} \cdot S^*_{k-1,1} + p_{k,0} \cdot S^*_{k-1,0}, \end{cases} \tag{5}$$

and an additional transformation of delay-advance type is done at the output of the k-th face-to-face length (that is, at the input of the k+1-st connection point):

$$\begin{cases} S^*_{k,0}(t) = S_k(t), \\ S^*_{k,1}(t) = S_{k,1}(t-\tau_k). \end{cases} \tag{6}$$

Assume for the last operator that the signal propagates faster in the shell than in the core. That is, $\tau_k>0$ where the value of advance parameter is determined by formula (1) depending on the face-to-face length, l_k, and the difference of group velocities in the OF shell and core.

Thus, operator equations (5) and (6) define an iterative model for redistribution of the signal energy between the OF core and shell. If it is assumed that transition probabilities at the connection points k–1 and k do not depend on each other, then the transition model (5) defines a random process as the Markov chain [25]. From a formal-mathematical point of view, specificity of the problem to be solved is provided by the delay-advance operator (6) as well as the necessity of obtaining the final result in accordance with formulas (2). In the general case, such a task obviously has no analytical solution. Let us consider a number of special cases.

5. Synthesis of the simplest dispersion model

To simplify the model (2)–(6), assume the following. Consider that energy at the OF connection points can pass from the core to the shell but does not return from the shell to the core. This assumption is justified to a certain extent by the fact that the shell diameter and, consequently, the area of the outgoing modal spot is substantially larger than the core diameter. Assume also that the face-to-face lengths of the selected regeneration section are approximately equal to each other. Consequently, the step of relative advance of the signal components, τ, can be assumed to be the same for different face-to-face lengths. In addition, in order to accelerate the algorithm, assume that a signal with PDF completely concentrated in the core exists at the input of the first connection.

Then, a system of operator equations for the k-th face-to-face length is obtained:

$$\begin{cases} S_{k,0} = q \cdot S^*_{k-1,0}, \\ S_{k,1} = q \cdot S^*_{k-1,1} + p \cdot S^*_{k-1,0}, \end{cases} \quad q + p = 1,$$

$$\begin{cases} S_{0,0} = S_0(t), \\ S_{0,1} = 0, \end{cases} \quad \begin{cases} S^*_{1,0} = qS_0(t), \\ S^*_{1,1} = pS_0(t-\tau). \end{cases} \tag{7}$$

It follows from expressions (7) that PDF of the total signal at the output of the second face-to-face length will be:

$$S^*_{\Sigma,2} = S^*_{2,0} + S^*_{2,1} = q^2 S_0(t) + pqS_0(t-\tau) + pS_0(t-2\tau). \tag{8}$$

Suppose that the regeneration section consists of N face-to-face lengths. Then, proceeding from expressions (7) and (8), it is proved by induction that the signal at the output of the N-th face-to-face length (that is, at the input of the photodetector) will have a total PDF:

$$S^*_{\Sigma,N}(t) = q^N S_0(t) + p\sum_{k=1}^{N} q^{N-k} S_0(t-k\tau) =$$

$$= q^N S_0(t) + p\sum_{k=0}^{N-1} q^k S_0[t-(N-k)\tau]. \tag{9}$$

Assume for the signal $s_0(t)$ at the OF input that its energy is equal to one, the energy center is at a zero point and its initial EPD is T_0^2 at the moment of the signal input into the OF:

$$E_0 = \int_{-\infty}^{\infty} s_0^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} S_0(t)\mathrm{d}t = 1,$$

$$\overline{t_0} = \int_{-\infty}^{\infty} tS_0(t)\mathrm{d}t = 0, \quad T_0^2 = \int_{-\infty}^{\infty} t^2 S_0(t)\mathrm{d}t. \tag{10}$$

A formula for the signal energy at the output of the N-th face-to-face length follows from expressions (9) and (10):

$$E_N = q^N \int_{-\infty}^{\infty} S_0(t)\mathrm{d}t + p\sum_{k=0}^{N-1} q^k \times$$

$$\times \int_{-\infty}^{\infty} S_0[t-(N-k)\tau]\mathrm{d}t = q^N + p\sum_{k=0}^{N-1} q^k. \tag{11}$$

There are members of geometric progression in the latter expression under the summation sign. Then:

$$E_N = q^N + p\frac{1-q^N}{1-q} = q^N + 1 - q^N = 1, \tag{12}$$

that is, the "law of energy conservation" is fulfilled in this case. To obtain expression for EPD at the output of the N-th face-to-face length, use the well-known probability-theoretic dependence [25] of the second central moment:

$$T_N^2 = \overline{t_N^2} - \left(\overline{t_N}\right)^2 = \int_{-\infty}^{\infty} t^2 S_N(t)\mathrm{d}t - \left[\int_{-\infty}^{\infty} t\, S_N(t)\mathrm{d}t\right]^2, \tag{13}$$

where $\overline{t_N^2}$ is the value of the second initial signal moment at the output of the N-th face-to-face length, s; $\overline{t_N}$ is displacement of the signal energy center with respect to the zero initial value, s.

Use two table sums for calculations according to formula (13) by the scheme of formula (2) derivation:

$$\sum_{k=0}^{N} kq^k = (1-q)^{-2} \cdot [q + (Nq - N - 1)q^{N+1}], \tag{14}$$

$$\sum_{k=0}^{N} k^2 q^k = (1-q)^{-3} \cdot \{q(1+q) - \\ -q^{N+1}[(N+1)^2 - (2N^2 + 2N - 1)q + N^2 q^2]\}. \tag{15}$$

Using formula (9), determine value of parameter $\overline{t_N}$ in formula (13) immediately replacing the variables. Then, taking into consideration zero position of the signal energy center at the initial moment of time,

$$\overline{t_N} = p\sum_{k=0}^{N-1} q^k \int_{-\infty}^{\infty} (t + N\tau - k\tau) S_0(t)\mathrm{d}t = \\ = p\tau N \sum_{k=0}^{N-1} q^k - p\tau \sum_{k=0}^{N-1} kq^k. \tag{16}$$

Perform similar operations for the initial moment $\overline{t_N^2}$ to obtain an intermediate expression:

$$\overline{t_N^2} = q^N T_0^2 + pT_0^2 \sum_{k=0}^{N-1} q^k + \\ + p\tau^2 N^2 \sum_{k=0}^{N-1} q^k - 2p\tau^2 N \sum_{k=0}^{N-1} kq^k + p\tau^2 \sum_{k=0}^{N-1} k^2 q^k. \tag{17}$$

Applying summation formulas (14) and (15) to expressions (16) and (17) and neglecting the terms of order of smallness less than 1, approximate expressions are obtained:

$$\overline{t_N} \approx \tau N (1 - q^N), \overline{t_N^2} \approx T_0^2 + \tau^2 N^2 (1 - q^N),$$

from which the following is obtained finally taking into consideration formula (13):

$$T_N^2 = T_0^2 + \tau^2 N^2 q^N (1 - q^N). \tag{18}$$

As a result, a relatively simple dependence convenient for analysis is obtained. It includes all parameters of the modeled regeneration section. In two extreme cases, when the signal energy completely remains in the core (q=1) or completely goes into the shell (p=1, q=0), final signal duration will be equal to its initial duration: $T_N^2 = T_0^2$. This corresponds perfectly to the model of races of group velocities in the OF core and shell.

To evaluate dispersion effects in the OF connections, consider a conditional example. Suppose that the regeneration section length is approximately 200 km and the connection points are placed at a distance of 2 km from each other. The number of such points N=100. Assume that probability of energy exit in the shell is small, for example $p \approx 10^{-6}$, that is, $q \approx 0.999999$. Take difference of group velocities $\Delta V_{gr} \sim 4$ km/s in formula (1). In order to obtain value of CPD, neglect the initial effective signal duration in formula (18). Then the increment of its duration will be approximately as follows:

$$\Delta T_p = 6 \cdot \tau \cdot N \cdot \sqrt{q^N (1 - q^N)}. \tag{19}$$

Calculations by formula (19) give approximately $\Delta T_p \approx$ $\approx$1.2 ns. Compare the obtained value with typical rated values of the specific coefficient of dispersion [1]. They have order of 3...20 ps/(nm·km) for different frequencies and different OF types. Take some average value of 10 ps/(nm·km) for calculations. An increment of initial signal duration about 2 ns is obtained at a 200 km distance for a typical narrowband transmission channel with a width of about 100 GHz (this band corresponds to about 1 nm at a carrier frequency of 200 THz) [2].

As can be seen, calculations by formula (19) and estimation using rated data of the optical cable give values of the same order. Of course, in this simplest model, phenomena such as return of energy from the shell to the core at distances of face-to-face lengths, difference between attenuation coefficients in the core and the shell media, etc. were not taken into consideration. However, even from this simplest model, it can be seen that the phenomenon of dispersion in connections, generally speaking, should not be neglected.

6. Estimation of probabilities of energy redistribution in connections

Real estimates of probabilities of energy transition are provided by various schemes of direct measurement [5, 9, 22]. For the problems considered in this paper, it is sufficient to be restricted to approximate estimates. In the simplest case, we shall proceed from the typical value of signal attenuation of an order of 0.1 Db in the OF connection point [11]. In other words, about 1 % of incoming energy dissipates in the given point.

Let us consider a model of energy transition from the core to the shell caused by geometric factors: tangential or angular shift of incoming and outgoing OFs (Fig. 4).

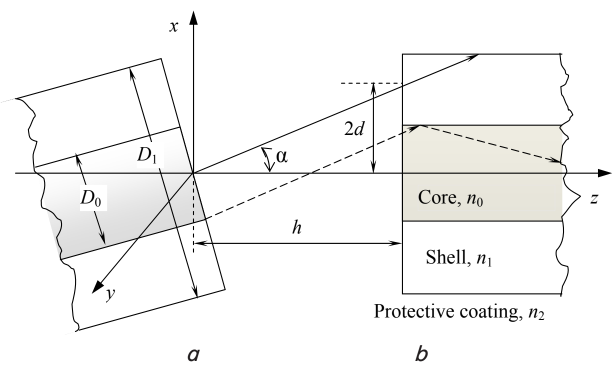


Fig. 4. Angular OF shift in the connection point: ingoing fiber (*a*); outgoing fiber (*b*)

Assuming that the component of energy that propagates further in the shell is a small fraction of these losses (of the order of 10^{-4}), estimates of the CPD increment given in the preceding paragraph will be more or less adequate.

Let us assume that the signal outgoing from the end of the ingoing fiber has an absolute spatial coherence; diameter of the modal spot (parameter D_M in Fig. 5) is equal to the core diameter D_0 and the signal energy in the area of the modal spot is uniformly distributed.

In the model of area mismatch (Fig. 5), portion of the energy passing from the core to the shell will be proportional to the area of mismatch, W_d. This "crescent" shaped area can be calculated as the difference in areas: $W_d = W_0 - 4W_1$ where W_0 is the area of the OF core (with diameter $D_0 = D_M$) and W_1 is the area of the curvilinear trapezoid with characteristic points 0, 1 and 2 in Fig. 5. With notations of Fig. 4 and Fig. 5, equation of the circle O_1 will be $y^2+(x+d)$ 2=r^2 where radius $r=D_0/2$. Equation of the arc 1–2 that envelopes area W_1 follows from the last expression: $y=\sqrt{r^2-(x+d)^2}$. Then expression for the sought area will be:

$$W_d=\pi r^2-4\int_0^{r-d}\sqrt{r^2-(x+d)^2}\mathrm{d}x. \quad (20)$$

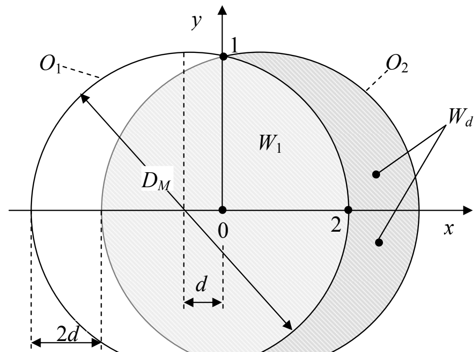


Fig. 5. Mismatch of the modal spot O_1 and the area O_2 of the outgoing fiber core

Using the table integral from expression (20), the following is obtained:

$$W_d=\pi r^2-2\cdot\left[\begin{matrix}(d+x)\sqrt{-d^2-2dx+r^2-x^2}+\\ +r^2\arctan\left(\dfrac{d+x}{\sqrt{r^2-(x+d)^2}}\right)\end{matrix}\right]\Bigg|_0^{r-d}. \quad (21)$$

When calculating value of the arctan(φ) function for the upper limit in formula (21), uncertainty of division-by-zero type is obtained. Perform limit transition for the first quadrant to obtain arctan(φ→∞)→π/2. Then, the following will be obtained after addition of similar terms:

$$W_d=2\cdot\left[d\sqrt{r^2-d^2}+r^2\arctan\left(\frac{d}{\sqrt{r^2-d^2}}\right)\right]. \quad (22)$$

Assume that $r>>d$ in formula (22), then parameter d in the radicands can be neglected. In addition, take into consideration that within arctan(φ→0)→φ. Then $W_d\approx 4dr$ and the probability of energy transfer from the core to the shell in relative terms will be:

$$p=W_d/W_0=\frac{4d}{\pi r}. \quad (23)$$

Make estimate of the shift parameter, d, based on the values of typical angular displacements [12] of the order of $\alpha^o\approx 1^o$ (or $\alpha\approx\pi/180$ in the radial measure). Since $\sin(\alpha)\approx\alpha$ at small angles, simplified expression for the transition probability is obtained in the form: $p\approx 2\pi h\alpha^o/180\pi r$ where h is the distance between incoming and outgoing OF ends (Fig. 4). Assuming that the distance h is approximately equal to the radius r of the OF core and $\alpha^o\approx 1^o$, estimate of the probability value $p\approx 0.01$ is obtained.

The obtained estimate far exceeds the previously made presumption that the fraction of dissipated energy passing from the OF core to the shell is of the order of 10^{-4}. In derivation of formula (23), it was assumed that distribution of energy in a modal spot has an approximately uniform nature. The known models of approximation of energy distribution [5, 22] give the Gaussian distribution function (a two-dimensional normal distribution). Then some portion of the value given by formula (23) will be concentrated in W_d area. Depending on the coefficient of distribution excess (pointedness) [26], this portion can be of an order of 1 % to 0.1 %. Then the probability value $p=10^{-4}\div 105$ which is quite consistent with the estimates given above.

7. Numerical experiment: modeling the dispersion effect in connections

Coefficient $\vartheta=q^N(1-q^N)$ is used in formula (18) to increment the EPD depending on the number of connections, N, and the probability of energy redistribution in the connection point $q=1-p$. Let us consider graphs of dependence of the ϑ function on the number of connections, N (Fig. 6).

Analysis of these graphs shows that for any value of N, there are clearly expressed maxima corresponding to the "worst" version of energy redistribution between the OF core and shell. At N=1000, peak value corresponds to the probability $p=10^{-3.16}$. At N=100, maximum was obtained for the value $p=10^{-2.16}$ and at the value N=10, $p=10^{-1.17}$, respectively.

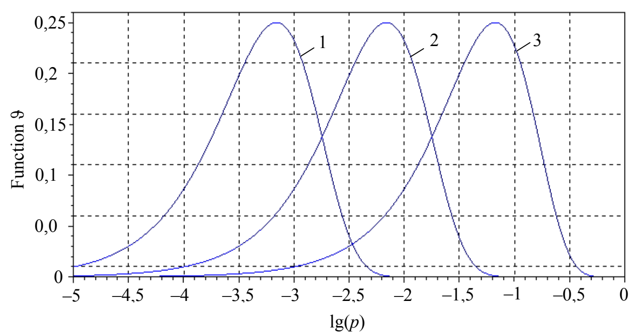


Fig. 6. Graphs of the function $\vartheta=q^N(1-q^N)$ for different values of the number of connections N: at N=1000 (1); at N=100 (2); at N=10 (3)

The formulas for the effective pulse duration (18) and the increment of CPD (19) were obtained with some simplifications. Direct numerical modeling of the operator (7) requires explicit specification of the initial form of optical signals and model parameters.

To visualize effects of dispersion in the OF connections, assume that relative signal advance in the shell relative to the core is of the order of τ=10 ps/km. In this case, according to formula (1), a rather underestimated difference of the group velocities in the OF core and shell of the order of $\Delta V_{gr}\approx 0.4$ km/s is obtained. However, a task of qualitative analysis of the phenomenon is set in this case. Suppose that the line at the regeneration section is 200 km long and the points of connection of the optical fibers are distributed evenly along this length in 2 km increments. In order to visualize the effect of dispersion in connections, probability of energy transition from the OF core to the shell must also be significantly overestimated.

Choose a rectangular pulse of a 200 ps width as a test signal. In this case, the calculated pulse duration will be approximately 346.4 ps (Fig. 7). The parameter CPD does not coincide with the finite signal length. At the same time, this parameter allows one to compare signals of various forms including theoretically infinite "tails".

To visualize the effect of dispersion in connections of optical fibers, a significant magnitude of probability of energy transition $p=0.1$ was taken in calculations according to Fig. 7.

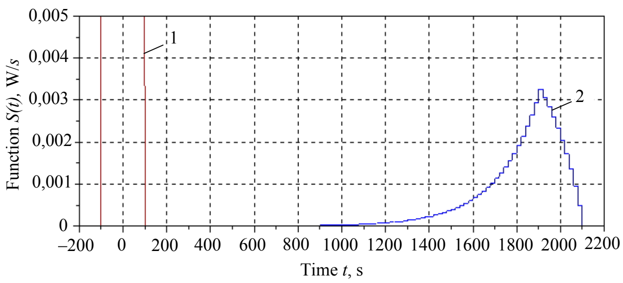


Fig. 7. Dispersion effect on the OF connections for a rectangular pulse: initial form of the PDF signal (1); the PDF signal form at a distance of 200 km (2)

As can be seen from the graphs of Fig. 7, the signal energy center at the OF output is significantly shifted towards the advance of the center of the initial signal in the OF core. At the same time, energy center of the signal (16) shifts to a point of $\overline{t_N} \approx 1{,}819.5$ ps and the CPD (19) is 1189.7 ps. In other words, duration of the signal in the final phase increases approximately threefold in comparison with its initial duration.

As an alternative, consider evolution of a signal with a power distribution function in a form of Gaussian pulse with the same unit energy and with the same initial CPD value (Fig. 8). At the same time, $\overline{t_N} \approx 1{,}709.8$ ps and the calculated pulse duration in the final phase is 1,189.4 ps. An interim conclusion can be drawn: the CPD of the rectangular pulse and the CPD of the Gaussian pulse at the output of the optical fiber are approximately equal in this case.

Similar results are obtained for a number of probability values p (Fig. 9).

Additional calculations give estimates of displacement of the signal energy center and its CPD finite value for the cases presented in Fig. 9 and Table 1.

Data in Table 1 confirm regularity of presence of a pronounced maximum of the function in Fig. 6. Maximum CPD value takes place at probability $p=10^{-2}$. In calculations according to Fig. 6, a close maximum was obtained at $N=100$ for the value $p=10^{-2,16}$ which confirms correctness of approximations calculated by formulas (18) and (19).

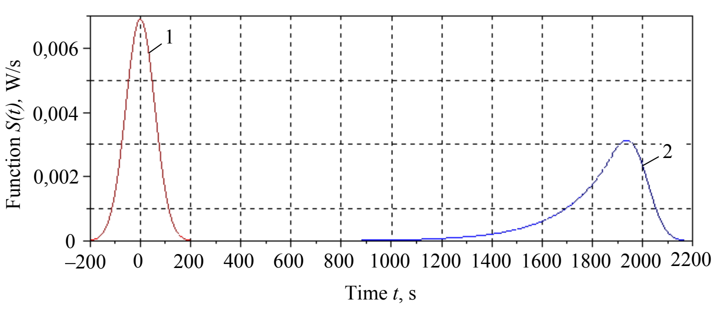


Fig. 8. The effect of dispersion in OF connections for a Gaussian pulse: the initial form of the PDF signal (1); the PDF signal form at a distance of 200 km (2)

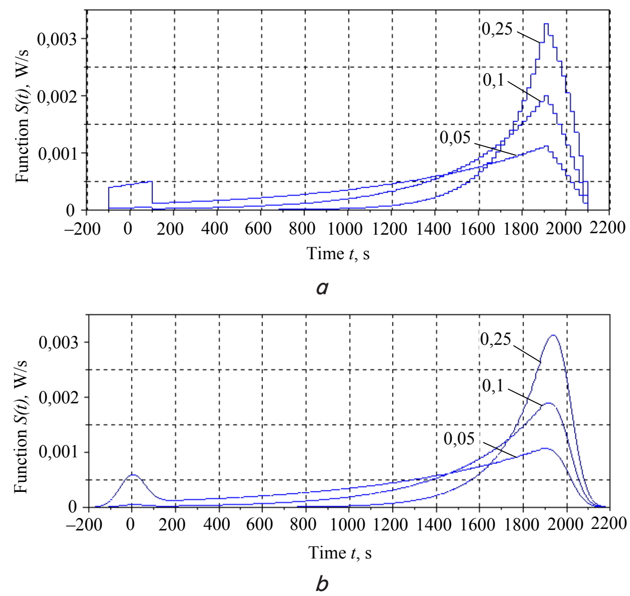


Fig. 9. Dependence of PDF at the final phase of signal propagation for the values of parameter p=0.1; 0.05; 0.25: for a rectangular pulse (*a*); for the Gaussian pulse (*b*)

Table 1

Values of parameters of optical signal distortion at a distance of 200 km from the input point

p	Rectangular pulse		Gaussian pulse		δ CPD, %
	$\overline{t_N}$, ps	CPD, ps	$\overline{t_N}$, ps	CPD, ps	
10^{-1}	1,819.5	1,189.7	1,790.7	1,189.4	0.03
$10^{-1,5}$	1,411.7	3,236.9	1,189.9	3,219.4	0.54
10^{-2}	744.2	4,345.8	733.3	4,327.6	0.42
$10^{-2,5}$	280.0	3,376.6	284.1	3,350.0	0.79
10^{-3}	97.2	2,128.5	96.3	2,114.9	0.64
$10^{-3,5}$	31.1	1,260.0	31.1	1,261.6	0.07
10^{-4}	9.6	776.1	9.9	771.3	0.62
$10^{-4,5}$	2.7	523.0	3.1	520.2	0.54
10^{-5}	0.5	410.7	1.0	408.9	0.44

The most important conclusion for data in Table 1 consists in the fact that relative difference between the CPD (right column of the Table 1) of pulses of a substantially different form but having approximately the same initial duration turns out to be almost the same at the receiving side. In fact, relative difference in no case exceeds 1 %. Additional calculations for pulses of other forms confirm this conclusion.

The above results of numerical modeling in the Scilab environment [27] were obtained for a case rather symbolic from the practical point of view when face-to-face lengths are assumed to be equal along the entire cable line. In fact, the face-to-face lengths in large settlements are usually smaller than those outside settlements. This situation is caused by both complexity of laying cables in the cable ductwork and commercial projects of operators. Indeed, intermediate branch boxes are used as passive switching points to provide traffic to intermediate nodes, corporate clients, etc.

Numerical modeling makes it possible to establish relationship between the plans of distribution of face-to-face lengths and the CPD of optical signals at the receiving side. Let us consider two alternative (and model) cases of planning distribution of face-to-face lengths in a regeneration section (Fig. 10). In the first case, it is assumed that the first face-to-face length is 500 m, the second length is approximately 500+30.3 m, the third length is 500+60.6 m, and so on, with an increment of subsequent lengths of approximately 30.3 m. The last face-to-face length is approximately 3.5 km. In the second case, the plan of distribution of face-to-face lengths is constructed in a reverse order: the first face-to-face length is approximately 3.5 km and then the face-to-face lengths are reduced by about 30.3 m. In both cases, total number of face-to-face lengths is 100 and the total length of the regeneration line is 200 km as in the preceding calculations.

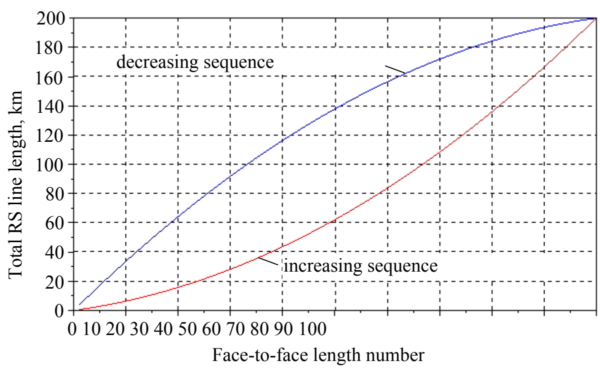


Fig. 10. Plans of distribution of face-to-face lengths of the RS line

The results of modeling the operator (7) for the Gaussian pulse with the previously defined parameters and probability $p=10^{-2}$ for two plans of distribution of face-to-face lengths from Fig. 10 are given in Fig. 11. As is seen, the two graphs obtained were virtually indistinguishable visually. Calculated CPD values in this case turned out to be as follows: 4,343.1 ps for the first plan and 4,354.7 ps for the second plan. These values differ slightly and approximately correspond to the CPD value of 4,327.6 ps for a uniform distribution of face-to-face lengths (Table 1).

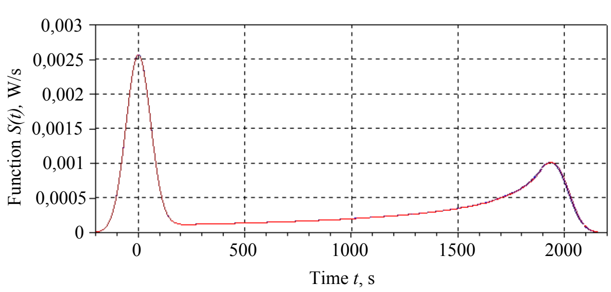


Fig. 11. Evolution of the Gaussian pulse at a distance of 200 km at $p=10^{-2}$ for the plans of distribution of face-to-face lengths from Fig. 10

This result as well as a number of additional calculations have allowed us to make somewhat unexpected but very useful conclusion: effect of dispersion in OF connections is weakly dependent on distribution of face-to-face lengths along the regeneration section line.

8. A metrology diagram for checking results of theoretical studies

The conclusions made concerning independence of the studied effect from the initial form of the optical signal and the plan of distribution of face-to-face lengths have allowed us to propose a metrology diagram for conducting field experiments in laboratory conditions (Fig. 12).

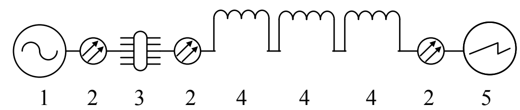


Fig. 12. The diagram of measurements for studying the effect of dispersion in optical fiber connections: generator (source) of an optical signal (1); optical patchcords (2); a set of models of "face-to-face lengths" (3); coils with an optical fiber cable (4); dispersion meter (5)

To detect weakly expressed effects of dispersion on connections, laboratory equipment shown in Fig. 12 must meet certain requirements. The generator 1 must form an optical signal with a carrier frequency corresponding to the near-zero dispersion point for the selected OF type. Bandwidth of emission has to be negligible: about 100 GHz (or about 0.5–1 nm in different frequency bands). These parameter values will minimize influence on the results of experiments with other dispersion components (material, waveguide, polarization). The patchcords 2 are selected from a batch according to the minimum attenuation criterion. Geometric effects which introduce an additional dispersion component are minimized.

The node 3 of the model of face-to-face lengths is fundamental. This node is a set of OF segments with welding points. The first line is just an OF segment of a length of the order of 10 m. The second line has two equal OF segments of 10 meters long each with one weld connection between them, etc. Thus, the last line has N+1 connected segments of a length of 10 m each with N welding points between them. Each of these lines ends with pigtails with sockets coordinated with the sockets of patchcords 2. In this case, length of these lines varies from 10 m (first line) to about 1 km (that is the last line if N=100 is laid).

For modeling the total length of the regeneration line (about 100–300 km), standard coils 4 with optical fiber are used. For example, standard Corning coils contain 63 km of optical fiber of SMF-28e+ type [28]. Thus, to complete the study, it is enough to connect 2–5 coils.

The dispersion meter 5 should be capable of implementing a direct measurement method, that is, dispersion measurement by comparing duration of the input and output optical signals.

9. Discussion of results of modeling the dispersion effect at connections

The analytical and numerical studies performed allow us to make the main conclusion: the detachable and permanent OF connections make an additional contribution to increase of duration of the optical signal at the receiving side. This effect is caused by energy redistribution between the OF core and shell in the connection points and the difference in group velocities in respective media.

This effect with a large number of connections is comparable in its contribution to the studied effects of material, polarization and other dispersion components.

The proposed diagram of the laboratory setup makes it possible to conduct field studies of the proposed effects in a laboratory due to its compactness.

The performed calculations and informal analysis allow us to give fairly simple and partly obvious recommendations for reducing the dispersion effect in connections when designing and installing optical lines. Naturally, the number of connections should be minimized as much as possible. Welding machines should provide thermal conditions that eliminate diffusion of the OF core and shell materials. In this case, only the surface layers (coating and partially shell material) are subjected to welding proper. To reduce geometric effects, high demands should be made to the fiber staplers and connectors of the optical patchcords. Pigtails should be completed with collecting lenses.

A certain disadvantage of this study consists in the absence of direct experiment results to verify the advanced scientific hypothesis. In the case of its metrological confirmation, it is advisable to make appropriate corrections to the methods of calculation of the regenerated section length.

10. Conclusions

1. The synthesized mathematical models in the form of a probabilistic scheme have made it possible to carry out analysis of the phenomenon of the signal energy redistribution between the OF core and shell.

2. Analysis of the synthesized transformation models in a group of connection points has made it possible to draw an important conclusion about the absence of a significant effect of the plan of distribution of face-to-face lengths on duration of the signal at the receiving side. This conclusion has enabled us to propose a metrological diagram for testing the advanced hypotheses in laboratory conditions.

3. Based on an analysis of analytical models and a numerical experiment, quantitative estimates suggest that the dispersion effects in the OF connections can be comparable with chromatic dispersion in a number of cases.

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