British Journal of Pharmaceutical and Medical Research Vol.04, Issue 04, Pg.2014 - 2021, July-August 2019

Available Online at http://www.bjpmr.org

BRITISH JOURNAL OF PHARMACEUTICAL AND MEDICAL RESEARCH

Cross Ref DOI : https://doi.org/10.24942/bjpmr.2019.567 Volume 04, Issue 04 July-August 2019

ISSN:2456-9836
ICV: 60.37

Research Article

"Mathematical Modeling In The Convivence Of Species"

Ortiz. L[a], Ferreira. R[b], Sánchez. S[a], Guerra. A[b], Z. Ribeiro[b] , M. Lacor[b]t, A. I. Ruiz[b]

[a]Facultad de Matemática y Computación, Universidad de Oriente.
[b]Universidade do Estado do Amazonas.
[c]Universidade Federal da Amazonas.

ARTICLE INFO

Article History:
Received on 11th July, 2019
Peer Reviewed on 26th July, 2019
Revised on 17th August, 2019
Published on 29th August, 2019

Keywords:
Ecology, Prey, Predator, Stability

ABSTRACT

In this work the different forms of coexistence in the open nature are indicated, emphasizing in the case of a pair of species where one is the prey and the other the predator. A model is presented and the direction field of the trajectories is studied, but the cyclical form of these trajectories is later seen computationally; the stability of the equilibrium positions is studied, transforming the system into the normal form to arrive at the conclusions of the future behavior of the studied species

Corresponding Author: *Ortiz. L, Facultad de Matemática y Computación, Universidad de Oriente.*

INTRODUCTION:

Ecology is the science that studies living beings and their interactions with the environment in which they live. This science is of utmost importance because the results of their studies provide data that reveal whether animals and ecosystems are in perfect harmony. At a time when deforestation and the extinction of several species are under way, the work of ecologists is of the utmost importance.

The problem of coexistence between different species in an open ecological space is addressed in [3], and [4], where the different types of coexistence and one can favor or hinder the development of the other. Many types of biological systems have been modeled mathematically with the purpose of realizing a better study of the natural interaction that exists between different species; in particular the prey-predator model has a relevant position due to the applicability not only of biology where it practically governs the coexistence of different species in open space, but also because it can be applied in other areas including economics. Here, in addition to the highly publicized Lorka-Volterra models, we will analyze lesser known ones in addition to their qualitative study.

The model was discovered independently by Lotka and Volterra, and for this reason it is known as a model Lotka-Volterra or model predator-prey that describes the evolution of prey and predators very well when they are located in an isolated ecosystem. Nevertheless, we have to clarify that two distinct populations in the same environment have several ways of surviving, for example:

- Mutual competence, that is to say compete for the same food source, tend to cause the extinction of a population of them, and the other tends to take advantage of the maximum capacity of environmental resources.
- Interdependence, that is to say the two populations provide some food resources, live peacefully among them, and tend to a state of equilibrium.
- The law of the jungle, is to say a population survives depending on the abundance of natural resources, called prey; however, the other population lives depending on the populations of prey, called the predator. The two elements are composed by the prey-predator model.
- The parasitic life, where one species feed on the other without killing it, but which by all means shaves its quality of life.

[8] refers to the mathematical modeling of several processes between them, dealing with the Prey-Predator model, which includes the possibility of system integration that simulates this interaction between two species. In [9] the interaction of different species is treated in an open medium, indicating in particular a model for the coexistence between a prey and a predator. In addition, it draws a parallel in the economy coming to some conclusions of the process. The prey-predator model has been extensively treated using different techniques, here it may be included,[6]. Another focus on the Lootka-Volterra model is presented in [1]. In the master's dissertation [7] a very exhaustive study of the prey-predator model is made.

The treatment that we will make in this case corresponds with other models presented in the researches of diseases, especially the case of sicklemia, quite treated and with a large number of already developed models; we will only mention some of these works, in [10] and [11], the qualitative study of different models in autonomous and non-autonomous form of the formation of polymers in the blood is treated. Following these ideas from these previous works here is simulated the interaction between two species being simplified the referred system to arrive at conclusions of this process of coexistence in the open nature.

In nature the most frequent is the competence between different species in the struggle for survival, appearing here the prey-predator model developed by Lotka 1924; Volterra, 1926; Gause, 1934; Kostitzin, 1939. [2].Drawing on the work of Lotka, the models that consider the population classified by age groups have been developed, in order to solve the limitations of the models that treat all the individuals of the population identically. One of the most commonly used classical mathematical models is the dynamic system consisting of two elements (usually two species of animals) interacting in such a way that one (predator) species feeds on the other (prey). A typical example is the system consisting of foxes and rabbits, but it can be transferred without loss of generality to any other context, for example, that formed by sellers and buyers applicable to the Economy.

Foxes feed on rabbits and grass rabbits that we assume will never run out. When there are many rabbits, the population of foxes will increase since food is abundant, but there will come a time when the rabbit population will decline as foxes are abundant. By not having the foxes, enough food their population will decrease, which will again favor the rabbit population. That is to say, if they produce cycles of growth and decrease of both to the populations. Is there a mathematical model that explains this periodic behavior?

On the other hand, in the second decade of the 20th century the Italian biologist Umberto D'Ancona studied and compiled data on catches of fish of some types in the Mediterranean, on the one hand, seals (sharks, rachis, etc.), and other fish that were eaten by the previous ones (sardines, anchovies, etc.), in other words, one prey (the edible fish) and the other predator (seals).One of the first reasons he thought was related to the First World War. In fact, at that time the first great war developed and this forced less boats to go fishing, and therefore, by reducing the intensity of fishing, this caused an increase in the number of predatory fish (seals). However, this argument had a problem and it was also that the number of edible fish had increased. In fact, if the intensity of fishing is small, then this fact benefits the predators more than the prey. The pertinent question was why?

Briefly, two questions were raised:

- •. How to explain the cyclical behavior of the evolution of two populations, where one species feed on the other?
- •. Why does a low catch intensity favor predator more than prey?

A detailed study of these types of systems is analyzed in the authors' work [5], which characterizes the behavior of the Lotka-Volterra systems under the hypothesis that the prey grows exponentially in the absence of predators and the predator disappears in absence of prey, studying the behavior of the trajectories in an environment of the equilibrium positions, one can perceive the existence of closed orbits due to the periodicity of the solutions.

Among the models of interaction between species the classic prey-predator model can be highlighted, whose mathematical formulation is composed of Malthusian models and the law of mass action. The analogy can be easily observed in epidemiological models. The prey-predator model also known as the Lotka-Volterra model has also been the starting point for the development of new techniques and mathematical theories. Predation is a very fundamental type of interaction in nature, where predators catch prey for their food. We can imagine that this relationship is beneficial only to the predator, but from the ecological point of view this is important to regulate the population density of both prey and predator. Predators remove individuals from the population, consuming them; the ease of catching the prey depends greatly on the size relationship between the prey population and the predator. The greater the population of prey, the greater the possibility of its capture. Predation occurs when an organism kills and feeds on beings of another species; the animal that killed it is called a predator, which already fed on the prey. Predators are usually found in smaller quantities and have characteristics that favor prey capture; among these characteristics, we can mention the sharp claws, speed and agility.

FORMULATION OF THE MODEL.

The prey-predator model that simulates the interaction between two species where one (prey) has food in abundance and the second (predator) is exclusively fed to the prey population. Let's admit that during the process, the medium should not change.

They are:

- $x = x(t)$ density of the prey population at the instant t.
- $y = y(t)$ density of the predator population at the instant t.

In this model it is assumed that prey grow exponentially in the absence of predators and that the mortality rate of predators in the absence of prey is proportional to their population at each instant.

Let's assume that the meeting of the two species is random, so the greater the number of haste the easier it will be to find them and the more predators the more food will be needed.

The possible encounter was modeled by the term linear bi xy , then the Prey-Predator system simplified by the previous impositions, is given by,

$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -cy + dxy \end{cases} \quad (1)$$

The equilibrium positions of the system (1) are the points: $(0,0)$; $(\frac{c}{d}, \frac{a}{b})$. The objective is now to analyze the behavior of the trajectories in a neighborhood of these points and to have more clarity of the future behavior of the trajectories of the system.The pattern in the variations of the population sizes can be repeated, when the conditions remain constant, the process continues in ecological cycle, reason why the trajectories will be periodic.The system (1) is a nonlinear system, but it can be integrated separating the variables, but this will not offer the information that we are looking for, so we will make a qualitative study of the trajectories.

The trajectories of the Prey-Predator model have the following characteristics in the different regions of the first quadrant of the plane:

Region I: $\left(\frac{dx}{dt} > 0, \frac{dy}{dt} > 0\right)$: When the population of prey increases in size, the population of predators will also become larger because of having a larger food base, with a certain delay in time;

Region II: $\left(\frac{dx}{dt} < 0, \frac{dy}{dt} > 0\right)$: The increasing demand of food reduces the population of the prey and the predators have their growth intact;

Region III: $\left(\frac{dx}{dt} < 0, \frac{dy}{dt} < 0\right)$: Food is scarce for predators and as a consequence there is a reduction in size;

Region IV: $\left(\frac{dx}{dt} > 0, \frac{dy}{dt} < 0\right)$: The reduction of predators favors the population of prey that slowly begins to grow.

As the model only has real sense in the first quadrant, we express the continuation of the velocity field of the Prey-Predator model in each of the regions indicated above in the first quadrant of the Cartesian plane:

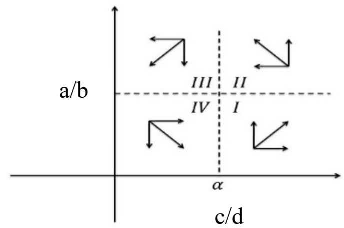

Fig. 1: Speed field of the Prey-Predator model.

It can be seen that in a neighborhood of the equilibrium position $(\frac{c}{d}, \frac{a}{b})$ gives the idea of a rotation around that point, indicated by the phase velocities of the system; so the process continues in a cyclic way, this causes the trajectories to be periodic or spiral that approach periodic trajectories.

The graph of the phase paths in a neighborhood of the point considering different initial conditions is indicated below from a concrete example, which are closed paths containing within it the equilibrium position indicated above; this corroborates what we had indicated earlier when analyzing the possible behavior of the trajectories of the system that models the process.

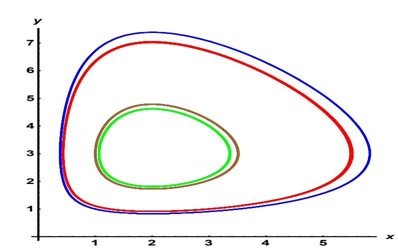

Fig. 2: Graphic of the example of the predator-predator model.

Here we limit ourselves to perform only a graphical analysis of the possible behavior of the trajectories of the system that models the Prey-Predator process, the analytical treatment for different systems will be presented below.

QUALITATIVE ANALYSIS.

To analyze the behavior of the trajectories of the system (1) at the point $P_1(0,0)$, we will determine the eigenvalues of the matrix of the linear part of the system,

$$\begin{cases} \dfrac{dx}{dt} = ax \\ \dfrac{dy}{dt} = -cy \end{cases}$$

The characteristic equation has the form,

$$\begin{vmatrix} a-\lambda & 0 \\ 0 & -c-\lambda \end{vmatrix} = 0 \Leftrightarrow (a-\lambda)(-c-\lambda) = 0,$$

That is to say, one has the proper values, $\lambda_1 = a$ and $\lambda_2 = -c$, therefore by the first approximation method it is concluded that the point $P_1(0,0)$ is an unstable equilibrium position because it has a positive eigenvalue.

To do the analysis on the spot, $P_2(\frac{c}{d}, \frac{a}{b})$ you must move the coordinate source to the point P_2, making use of the following coordinate transformation,

$$\begin{cases} x = x_1 + \dfrac{c}{d} \\ y = y_1 + \dfrac{a}{b} \end{cases} \qquad (2)$$

By deriving the transformation (2) taking into account the system (1) the system is obtained,

$$\begin{cases} x_1' = -\dfrac{bc}{d} y_1 - bx_1 y_1 \\ y_1' = \dfrac{da}{b} x_1 + dx_1 y_1 \end{cases} \qquad (3)$$

At where,

The characteristic equation of the system (3) has the form,

$$\begin{vmatrix} -\lambda & -\dfrac{bc}{d} \\ \dfrac{ad}{b} & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + ac = 0$$

That is to say, one has the pure imaginary own values, $\lambda_1 = \sqrt{ac}i$ and $\lambda_2 = -\sqrt{ac}i$, therefore by the first approximation method no conclusion can be given regarding the behavior of the trajectories, since the eigenvalues have a real zero part; but in a neighborhood of the point $P_2(\frac{c}{d}, \frac{a}{b})$ it can be said that the solutions are oscillating.

NORMAL FORM.

Because this is a doubtful case, it will be necessary to take the system to a more simplified form, which in this case will be the normal form, in order to arrive at some conclusion regarding the growth of the populations of the species.

Theorem1: The non-degenerate linear transformation,

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} b\sqrt{aci} & bc \\ ad & d\sqrt{aci} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \qquad (4)$$

reduces the system (3) to the system,

$$\begin{cases} x_2' = i\sqrt{ac}x_2 + X_2(x_2, y_2) \\ y_2' = -i\sqrt{ac}y_2 + Y_2(x_2, y_2) \end{cases} \qquad (5)$$

$$X_2(x_2,y_2) = \left(\frac{abd}{2} + \frac{bd\sqrt{aci}}{2}\right)x_2^2 - \left(\frac{bd\sqrt{ac}}{2} + \frac{bcd}{2}\right)x_2 y_2 + \left(\frac{bcd}{2} + \frac{bcd\sqrt{aci}}{2a}\right)y_2^2$$

$$Y_2(x_2,y_2) = \left(\frac{abd}{2} + \frac{abd\sqrt{aci}}{2c}\right)x_2^2 - \left(ad^2 - d\sqrt{aci}\right)x_2 y_2 + \left(\frac{bcd}{2} + \frac{d^2\sqrt{aci}}{2}\right)y_2^2$$

Demonstration: By deriving the transformation (4) along the trajectories of the system (3) we arrive at the system (5).

Theorem 2: The transformation of coordinates,

$$\begin{cases} x_2 = x_3 + h_1(x_3, y_3) \\ y_2 = y_3 + h_2(x_3, y_3) \end{cases}$$

(6)

reduces the system (5) to the normal form,

$$\begin{cases} x_3' = i\sqrt{ac}x_3 + x_3 P(x_3 y_3) \\ y_3' = -i\sqrt{ac}y_3 + y_3 \overline{P}(x_3 y_3) \end{cases} \qquad (7)$$

At where $y_3 = \overline{x_3}$.

Demonstration: By deriving the transformation (6) along the trajectories of systems (5) and (7), we obtain the following system,

$$\begin{cases} (p_1 - p_2 - 1)i\sqrt{ac}h_1 + x_3 P = X_2(x_3 + h_1, y_3 + h_2) - \dfrac{\partial h_1}{\partial x_3}(x_3 P) - \dfrac{\partial h_1}{\partial y_3}(y_3 \overline{P}) \\ (p_1 - p_2 + 1)i\sqrt{ac}h_2 + y_3 \overline{P} = Y_2(x_3 + h_1, y_3 + h_2) - \dfrac{\partial h_2}{\partial x_3}(x_3 P) - \dfrac{\partial h_2}{\partial y_3}(y_3 \overline{P}) \end{cases} \quad \textbf{(8)}$$

The system (8) allows determining the series h_1, h_2 and P, because all other series are known, in addition, as the coefficients of P and $\overline{P}$ are resonant their terms are such that $p_1 = p_2 + 1$ and $p_2 = p_1 + 1$ however the h_1 and h_2 are non-resonant, so we conclude that,

$$h_1(x_3, y_3) = \frac{1}{4}\left[(abd + bd\sqrt{aci})x_3^2 - (bcd + bd\sqrt{ac})x_3 y_3\right] + (bcd + \frac{bcd\sqrt{aci}}{a})y_3^2 + \ldots$$

$$h_2(x_3, y_3) = \frac{1}{4}\left[(abd + \frac{abd\sqrt{aci}}{c})x_3^2 - (bd^2 - d\sqrt{aci})x_3 y_3\right] + (bcd + \frac{d^2\sqrt{aci}}{a})y_3^2 + \ldots$$

$$x_3 P(x_3 y_3) = \frac{1}{2}\left[(-bcd - bd\sqrt{ac} + abd + \frac{abd\sqrt{aci}}{c})x_3^2 y_3\right] + \ldots$$

$$y_3 \overline{P}(x_3 y_3) = \frac{1}{2}\left[(-bcd - bd\sqrt{ac} + abd - \frac{abd\sqrt{aci}}{c})x_3 y_3^2\right] + \ldots$$

Theorem3: For the equilibrium position,

$P_2(\frac{c}{d}, \frac{a}{b})$

of the system (3) is asymptotically stable is sufficient that,

$a < c + \sqrt{ac}$.

Demonstration: Let the Liapunov function be positive

,

$V(x_3, y_3) = x_3 y_3$.

The derivative of $V(x_3, y_3)$ along the trajectories of the system (7) is given by the following expression,

$$\frac{dV}{dt}(x_3(t), y_3(t)) = bd(a - c - \sqrt{ac})x_3^2 y_3^2 + \ldots$$

This indicates that the condition $a < c + \sqrt{ac}$, the derivative is negative, thus proving the theorem.

CONCLUSION: -

The point $P_1(0,0)$ is a position of unstable equilibrium, which ensures that under these conditions the species do not disappear.

- Theorems 1 and 2 allow to simplify the system to give conclusions regarding the behavior of the species.
- If the condition is satisfied $a < c + \sqrt{ac}$ to the equilibrium position P_2 is asymptotically stable, this ensures that whenever this condition is maintained, populations of haste and predators will remain oscillating near these values.
- If $a > c + \sqrt{ac}$, then to the equilibrium position P_2 is unstable and therefore cannot guarantee any respect to future populations of haste and predators.

CASE OF PREY COMPETITION.

You can give competition between the prey, either for food or space, then you can present the situation that when there is a lot of fight there is a fight between them, this situation must be contemplated in the model, so will appear a new term that will influence the coexistence , taking the system,

$$\begin{cases} \frac{dx}{dt} = ax - bxy - cx^2 \\ \frac{dy}{dt} = -dy + exy \end{cases} \quad (9)$$

Note: It is evident that in the case of predators the feeding competence is significant in the model, so it is reflected in the linear part of the unknown function that represents the species.

These new conditions in the process make for the system there are three equilibrium positions, which are the points, $P_1(0,0)$, $P_2(\frac{d}{e}, \frac{ae-cd}{be})$ and $P_3(\frac{a}{c}, 0)$ so that the point P_2 is in the first quadrant it is necessary that $ae > cd$; the point P_3 to have the characteristics that the predator disappears is not very interesting, so we will not refer to it.

The analysis of the trajectories in a neighborhood of the point $P_1(0,0)$ is done by the method of the first approximation, which coincides with the previous one, where it was concluded that it is an unstable point. To do the analysis at the point $P_2(\frac{d}{e}, \frac{ae-cd}{be})$ it is necessary to transfer the origin of coordinates to this point, thus obtaining the system,

$$\begin{cases} x_1' = -\frac{cd}{e}x_1 - dy_1 - bx_1y_1 - cx_1^2 \\ y_1' = \frac{ae-cd}{b}x_1 + ex_1y_1 \end{cases} \quad (10)$$

In this case the first approximation system has the form,

$$\begin{cases} x_1' = -\frac{cd}{e}x_1 - dy_1 \\ y_1' = \frac{ae-cd}{b}x_1 \end{cases} \quad (11)$$

And the characteristic equation of the system (11) has the form,

$$\begin{vmatrix} -\frac{cd}{e} - \lambda & -d \\ \frac{ae-cd}{b} & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + \frac{cd}{e}\lambda + \frac{d(ae-cd)}{b} = 0$$

Note: It is evident that the eigenvalues of the matrix have a real negative part and therefore constitute a position of stable equilibrium, this means that the ordered pairs formed by the prey and the predators will be maintained from a given moment in a neighborhood of the point P_2 this ensures that populations will always remain close to values, $x = \frac{d}{e}$ and $y = \frac{ae-cd}{be}$.

BIBLIOGRAPHY.

1) Altair Santos de Oliveira Sobrinho, Camila Fogaça de Oliveira, Carolina Massae Kita, Érica Regina Takano Natti, Paulo Laerte Natti. "Modelagem Matemática e Estabilidade de Sistemas Predador-Presa"Universidade Estadual de Londrina, Londrina. (2016).
2) Antonio I Ruiz Chaveco, Sandy Sánchez Dominguez, Adolfo García. "Mathematical Modeling of Polemerization of Hemoglobin S". Ed. Lab Lambert. 2015.
3) Colin R. Townsend, Michel Begon e John L. Harper "Fundamentos em Ecologia" 3ra Edição. Porto Alegre, (2010).
4) Dajoz, Roger "Principios de Ecologia" 7ma, Edição, Porto Alegre. (2005).
5) López, J. M., & Blé, G. G. *Modelo Depredador - Presa*. (2008). Revista de Ciencias Básicas UJAT, 7 (2), 25 - 34.
6) Luiz A. D. Rodrigues ; Diomar C. Mistro; Carina L. Andrade. "Sistema Presa-Predador com Duas Escalas de Crescimento: Presa Rápida-Predador Lento" Biomatemática. IMECC. UNICAMP. (2010).
7) Oliveira, C.F. "Modelagem Matemática do Crescimento Populacional: Um olhar à luz da Socioepistemologia, Dissertação de Mestrado em Ensino de Ciências e Educação Matemática. Editora Universidade Estadual de Londrina, Londrina, (2011)
8) Rodney Carlos Bassanezi. *"Modelagem Matemática*". São Paulo.2004.
9) Runjie Wu "El modelo presa depredador y sus aplicaciones a la Economía". 2014.
10) Sánchez, Sandy; Ruiz, Antonio I.; Fernández, Adolfo. "Un modelo de los procesos moleculares de la polimerización y cristalización de la Hemoglobina S", Rev. Ciencias Matemáticas. 26 (2012), No. 1, pp 53 -- 57.

11) Sánchez, S., Fernández, G. A. A., Ruiz. A. I., & Carvalho, E. F." Modelo de la sicklemia con coeficientes periódicos en la función de polimerización".Ciencia e Tecnica Vitivinicola Journal, (2016).

How To Cite This Article:
Ortiz. L, Ferreira. R, Sánchez. S, Guerra. A, Z. Ribeiro , M. Lacor T, A. I. Ruiz ***Mathematical Modeling In The Convivence Of Species*** **Br J Pharm Med Res , Vol.04, Issue 04, Pg.2014 - 2021, July - August 2019. ISSN:2456-9836 Cross Ref DOI : https://doi.org/10.24942/bjpmr.2019.567**

Source of Support: Nil **Conflict of Interest:** None declared