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Domination Number in Neutrosophic Soft Graphs

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Abstract: The soft set theory is a mathematical tool to represent uncertainty, imprecise, and vagueness is often employed in solving decision making problem. It has been widely used to identify irrelevant parameters and make reduction set of parameters for decision making in order to bring out the optimal choices. This manuscript is designed with the concept of neutrosophic soft graph structures. We introduce the domination number of neutrosophic soft graphs and elaborate them with suitable examples by using strength of path and strength of connectedness. Moreover, some remarkable properties of independent domination number, strong neighborhood domination, weights of a dominated graph and strong perfect domination of neutrosophic soft graph is investigated and the proposed concepts are described with suitable examples.

Keywords: Domination Number, Neutrosophic graphs, Strong neighborhood domination, Strong perfect domination, Soft graph.

1 Introduction

Fuzzy graph theory was introduced by Azriel Rosenfied in 1975. Still it is very young, it has been growing very fast and has crucial applications in various domain. Fuzzy set was introduced by Zadeh [8] whose basic components is only a membership function. The generalization of Zadeh's fuzzy set, called intuitionistic fuzzy set was introduced by atanassov [16] which is characterized by a membership function and a non membership function. According to Atanassov, the sum of membership degree and a non membership degree does not exceed one. A. Somasundaram and S. Somasundaram [33] presented more concept of independent domination, connected domination in fuzzy graphs, R. Parvathi and G. Thamilzhendhi [23] introduced domination in intuitionistic fuzzy graphs and discussed some of its properties.

The soft graphs represents need any addition information about the data such as the probability in statistic or possibility value in fuzzy graphs and give the accurate value. The theory use parameterization as its main vehicle in developing theory and its applications. The crucial model of parameter reduction and decision making is developing fascinating in dealing with uncertainties that making problems in soft set theory are interesting field. Molodtsov [25] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Molodtsov's soft sets give us new technique for dealing with uncertainty from the view point of parameters. It has been revealed that soft sets have potential applications in several fields. In [7], author studied the fuzzy soft graphs. Operations of fuzzy soft graphs are studied in [8]. Recently, Akram M [9] introduced an idea about neutrosophic soft graphs and its application. Recently, the author Smarandache [29, 30, 13, 14, 31, 32, 17, 18, 19, 20, 35] introduced and studied extensively about neutrosophic set and it receives applications in many domains. The neutrosophic set has three completely independent parts, which are truth-membership degree, indeterminacy-membership degree and falsity-membership degree with the sum of these values lies between 0 and 3. Akram [9] established the certain notions including neutrosophic soft graphs, strong neutrosophic soft graphs, complete neutrosophic soft graphs. Motivation of the above, we introduced the concept of domination number in neutrosophic fuzzy soft graphs, strong neighborhood domination and strong perfect domination in neutrosophic fuzzy soft graphs. The major contribution of this work as follows:

- The domination set of neutrosophic soft graphs is established by using the concept of strength of a path, strength of connectedness and strong arc.
- The necessary and sufficient condition for the minimum domination set of neutrosophic soft graph is investigated.
- Some properties of independent domination number of neutrosophic soft graph are obtained and the proposed concepts are described with suitable examples.
- Further we presented a remarkable properties of independent domination number, strong neighborhood domination and strong perfect domination of neutrosophic soft graph.

2 Preliminaries

Definition 2.1 [30] A Neutrosophic set A is contained in another neutrosophic set B, (i.e) $A \subseteq C$ if $\forall x \in X, T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$.

Definition 2.2 [35] Let X be a space of points (objects), with a generic elements in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership-function $F_A(x)$.
For each point x in X, $T_A(x), F_A(x), I_A(x) \in [0,1]$.

$$A = \{x, T_A(x), F_A(x), I_A(x)\} \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Definition 2.3 [17, 18] A neutrosophic graph is defined as a pair $G^* = (V, E)$ where
(i) $V = \{v_1, v_2, .., v_n\}$ such that $T_1 = V \to [0,1]$, $I_1 = V \to [0,1]$ and $F_1 = V \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function,respectively and

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

(ii) $E \subset V \times V$ where $T_2 = E \to [0,1]$,$I_2 = E \to [0,1]$ and $F_2 = E \to [0,1]$ are such that

$$T_2(uv) \leq \min\{T_1(u), T_1(v)\},$$
$$I_2(uv) \leq \min\{I_1(u), I_1(v)\},$$
$$F_2(uv) \leq \max\{F_1(u), F_1(v)\},$$
$$\text{and } 0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3, \forall uv \in E$$

Definition 2.4 Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. (J, A) is said to be neutrosophic soft subset of (G, B) if $A \subset B$, if $T_{J(e)}(x) \leq T_{G(e)}(x), I_{J(e)}(x) \leq I_{G(e)}(x)$ and $F_{J(e)}(x) \geq F_{G(e)}(x)$ for all $e \in M, x \in U$.

Definition 2.5 Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. The union of two neutrosophic soft sets (H, A) and (G, B) is neutrosophic soft set $(K, C) = (H, A) \cup (G, B)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are defined by $T_{K(e)}(x) = T_{H(e)}(x), \text{if} e \in A - B, T_{G(e)}(x), \text{if} e \in B - A, \max(T_{H(e)}(x), T_{G(e)}(x) \text{if} e \in A \cap B$.

Definition 2.6 Let U be an initial universe and P be the set of all parameters. $\rho(U)$ denotes the set of all neutrosophic sets of U. Let A be a subset of P. A pair (J, A) is called a neutrosophic soft set over U. Let $\rho(V)$ denotes the set of all neutrosophic sets of V and $\rho(E)$ denotes the set of all neutrosophic sets of E.

Definition 2.7 [9] A neutrosophic soft graph $G = (G^*, J, K, A)$ is an ordered four tuple, if it satisfies the following conditions:

(i)$G^* = (V, E)$ is a simple graph,

(ii)A is a non-empty set of parameters ,

(iii)(J, A) is a neutrosophic soft set over V,

(iv)(K, A) is a neutrosophic soft set over E,

(v)$(J(e), K(e))$ is a neutrosophic graph of G^*, then

$$T_{K(e)}(xy) \leq \{T_{J(e)}(x) \wedge T_{J(e)}(y)\},$$
$$I_{K(e)}(xy) \leq \{I_{J(e)}(x) \wedge I_{J(e)}(y)\},$$
$$F_{K(e)}(xy) \leq \{F_{J(e)}(x) \vee F_{J(e)}(y)\},$$

such that

$$0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 3 \text{ for all } e \in A \text{ and } x, y \in V.$$

The neutrosophic graph (J_e, K_e) is denoted by $H(e)$ for convenience. A neutrosophic soft graph is a parametrized family of neutrosophic graphs. The class of all neutrosophic soft graphs is denoted by $NS(G^*)$. Note that $T_{K(e)}(xy) = I_{K(e)}(xy) = 0$ and $F_{K(e)}(xy) = 1 \forall xy \in V \times V - E, e \notin A$.

Definition 2.8 [9] Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of G^*. Then G_1 is a neutrosophic subgraph of G_2 if

(i)$A \subseteq B$.

(ii)$H_1(e)$ is a partial subgraph of $H_2(e)$ for all $e \in A$.

3 MAIN RESULT

Definition 3.1 Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. Then the degree of a vertex $u \in G$ is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex u denoted by $d(u) = (d_{TJ(e)}(u), d_{IJ(e)}(u), d_{FJ(e)}(u))$ where

$d_{TJ(e)}(u) = \sum_{e \in A} (\sum_{u \notin v \in V} T_{K(e)}(u, v))$ called the degree of truth membership vertex

$d_{IJ(e)}(u) = \sum_{e \in A} (\sum_{u \notin v \in V} I_{K(e)}(u, v))$ called the degree of indeterminacy membership vertex

$d_{FJ(e)}(u) = \sum_{e \in A} (\sum_{u \notin v \in V} F_{K(e)}(u, v))$ called the degree of falsity membership vertex for all $e \in A, u, v \in V$.

Definition 3.2 Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. Then the total degree of a vertex $u \in G$ is defined by $td(u) = (td_{TJ(e)}(u), td_{IJ(e)}(u), td_{FJ(e)}(u))$ where

$td_{TJ(e)}(u) = \sum_{e \in A} (\sum_{u \notin v \in V} T_{K(e)}(u, v) + T_{J(e)}(u, v)$ called the degree of truth membership vertex

$td_{IJ(e)}(u) = \sum_{e \in A} \left(\sum_{u \notin v \in V} I_{K(e)}(u,v) + I_{J(e)}(u,v) \right)$ called the degree of indeterminacy membership vertex

$td_{FJ(e)}(u) = \sum_{e \in A} \left(\sum_{u \notin v \in V} F_{K(e)}(u,v) + F_{J(e)}(u,v) \right)$ called the degree of falsity membership vertex for all $e \in A, u, v \in V$.

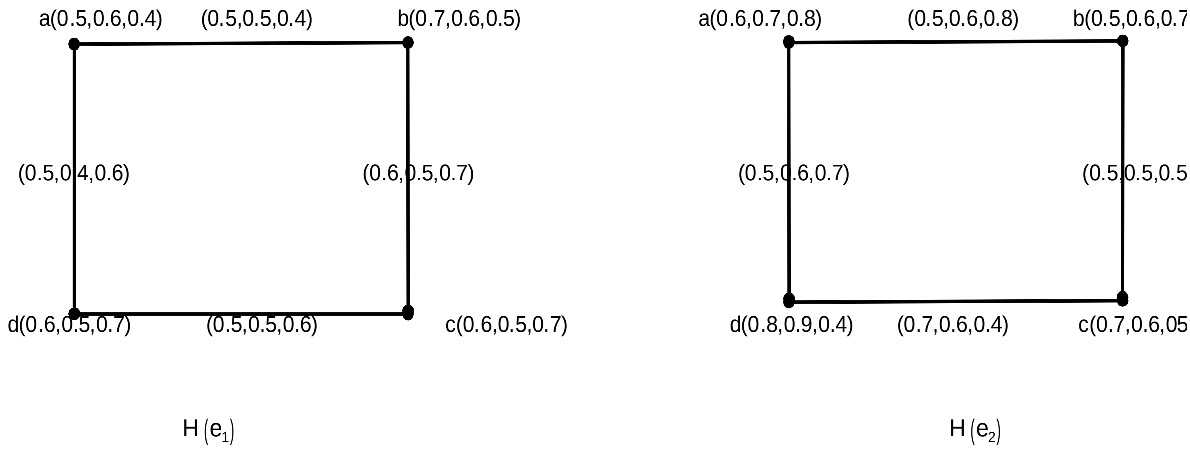

Figure 1

Example 3.3 Consider a simple graph $G^* = (V, E)$ such that $V = \{a, b, c, d\}$ and $E = \{(ab), (bc), (cd), (ad)\}$. Let $A = (J, A)$ be a neutrosophic soft over V with the approximation function $J: A \to \rho(V)$ defined by

$$J(e_1) = a(0.5,0.6,0.4), b(0.7,0.6,0.5), c(0.6,0.5,0.7), d(0.6,0.5,0.7)$$

$$J(e_2) = a(0.6,0.7,0.8), b(0.5,0.6,0.7), c(0.7,0.6,0.5), d(0.8,0.9,0.4)$$

Let (K, A) be a neutrosophic soft over E with neutrosophic approximation function $K: A \to \rho(E)$ defined by

$$K(e_1) = ab(0.5,0.5,0.4), bc(0.6,0.5,0.7), cd(0.5,0.5,0.6), ad(0.5,0.4,0.6)$$

$$K(e_2) = ab(0.5,0.6,0.8), bc(0.5,0.5,0.5), cd(0.7,0.6,0.4), ad(0.5,0.6,0.7)$$

Clearly, $H(e_1) = (J(e_1), K(e_1))$ and $H(e_2) = (J(e_2), K(e_2))$ are neutrosophic graphs corresponding to the parameters e_1 and e_2 respectively as shown in Figure 1.

For the graph $H(e_1)$ degree of vertices as follows, $deg(a) = (1,0.9,1.0), deg(b) = (1.1,1.0,1.1), deg(c) = (1.1,1.0,1.3), deg(d) = (1.0,0.9,1.2)$

For the graph $H(e_2)$ degree of vertices as follows, $deg(a) = (1.0,1.2,1.5)$, $deg(b) = (1.0,1.1,1.3), deg(c) = (1.2,1.1,0.9), deg(d) = (1.2,1.2,1.1)$

Definition 3.4 A simple graph G is said to be a regular if each vertices has a same degree for all $e \in A$, $x, y \in V$. Let $G^* = (V, E)$ be a neutrosophic graph then G is said to be a regular neutrosophic graph if $H(e)$ is a regular graph for all $e \in A$, if $H(e)$ is a regularr neutrosophic graph of degree r for all $e \in A$, then G is a $r-$ regular fuzzy graph. Let $G^* = (V, E)$ be a neutrosophic graph then G is said to be a totally regular neutrosophic graph if $H(e)$ is a totally regular graph for all $e \in A$, if $H(e)$ is a totally regular neutrosophic graph of degree r for all $e \in A$, then G is a $r-$totally regular neutrosophic fuzzy graph.

Example 3.5 Consider a simple graph $G^* = (V, E)$ such that $V = \{a, b, c, d\}$ and

$E = \{(ab), (bc), (cd), (ad)\}$. Let $A = \{e_1, e_2\}$. Let (J, A) be a neutrosophic soft over V with its approximation function $J = A \to \rho(V)$ defined by

$$J(e_1) = a(0.4,0.3,0.3), b(0.3,0.3,0.4), c(0.4,0.4,0.4), d(0.5,0.5,0.5)$$

$$J(e_2) = a(0.5,0.4,0.4), b(0.4,0.4,0.5), c(0.5,0.5,0.5), d(0.6,0.6,0.6).$$

Let (K, A) be a neutrosophic soft over E with neutrosophic approximation function $K: A \to \rho(E)$ defined by

$$K(e_1) = ab(0.2,0.2,0.2), bc(0.1,0.1,0.1), cd(0.2,0.2,0.2), ad(0.1,0.1,0.1)$$

$$K(e_2) = ab(0.2,0.2,0.2), bc(0.3,0.3,0.3), cd(0.2,0.2,0.2), ad(0.3,0.3,0.3).$$

Obviously, $H(e_1) = (F(e_1), K(e_1))$ and $H(e_2) = (F(e_2), K(e_2))$ are neutrosophic graphs corresponding to the parameters e_1 and e_2 respectively as shown in Figure 2

For the graph $H(e_1)$ degree of vertices as follows, $deg(a) = (0.3,0.3,0.3)$, $deg(b) = (0.3,0.3,0.3), deg(c) = (0.3,0.3,0.3), deg(d) = (0.3,0.3,0.3)$

For the graph $H(e_2)$ degree of vertices as follows, $deg(a) = (0.5,0.5,0.5)$, $deg(b) = (0.5,0.5,0.5), deg(c) = (0.5,0.5,0.5), deg(d) = (0.5,0.5,0.5)$

Here, $H(e_1)$ and $H(e_2)$ all the vertices degree are same so neutrosophic soft graph G is regular neutrosophic graph.

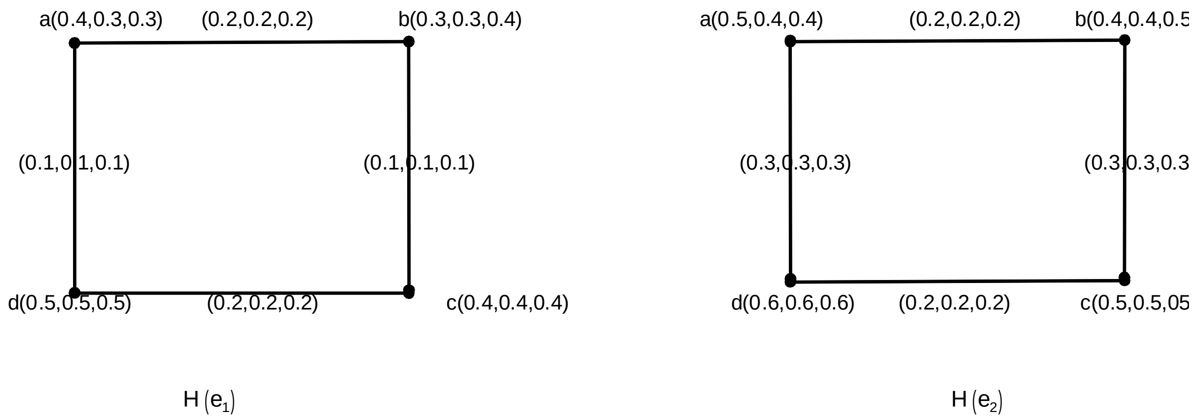

Figure 2

Definition 3.6 A graph $G^* = (V, E)$ is said to be a totally regular neutrosophic graph if each vertex has a same total degree for all $e \in A$, $u, v \in V$.

Example 3.7 Consider a simple graph $G^* = (V, E)$ such that $V = \{a, b, c, d, i, j, k\}$ and

$E = \{(ab), (bc), (cd), (ad), (ij), (jk), (kj)\}$. Let $A = \{e_1, e_2\}$ parameter set. Let (J, A) be a neutrosophic soft over V with its approximation function $J = A \to \rho(V)$ defined by

$$J(e_1) = \{a(0.5,0.6,0.4), b(0.4,0.7,0.6), c(0.4,0.6,0.7), d(0.5,0.5,0.5)\}$$

$$J(e_2) = \{i(0.6,0.7,0.5), j(0.5,0.7,0.9), k(0.6,0.6,0.7)\}$$

Let (K, A) be a neutrosophic soft over E with neutrosophic approximation function $K: A \to \rho(E)$ defined by

$$K(e_1) = ab(0.4,0.3,0.5), bc(0.4,0.3,0.3), cd(0.5,0.4,0.3), ad(0.3,0.4,0.5)$$
$$K(e_2) = ij(0.5,0.5,0.4), jk(0.6,0.5,0.4), ik(0.4,0.5,0.6),$$

clearly, $H(e_1) = (J(e_1), K(e_1))$ and $H(e_2) = (J(e_2), K(e_2))$ are neutrosophic graphs corresponding to the parameters e_1 and e_2 respectively as shown in Figure 3. For the graph $H(e_1)$ total degree of vertices as follows,

$tdeg(a) = (1.2,1.3,1.4)$, $tdeg(b) = (1.2,1.3,1.4)$, $tdeg(c) = (1.2,1.3,1.4)$, $tdeg(d) = (1.2,1.3,1.4)$

For the graph $H(e_2)$ degree of vertices as follow, $tdeg(i) = (1.5,1.6,1.5)$, $tdeg(j) = (1.5,1.6,1.5)$, $tdeg(k) = (1.5,1.6,1.5)$

Here $H(e_1)$ and $H(e_2)$ all the vertices total degrees are same so neutrosophic soft graph G is totally regular neutrosophic soft graph.

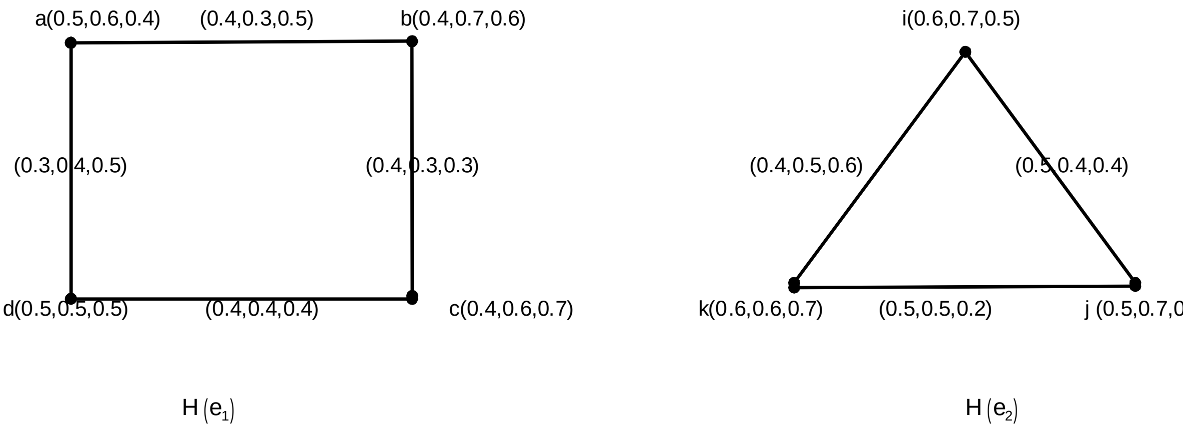

Figure 3

Definition 3.8 The order of a neutrosophic soft graph G is

$$Ord(G) = \sum_{e_i \in A} (\sum_{x\in V} T_{J(e_i)}(e_i)(x), \sum_{x\in V} I_{F(e_i)}(e_i)(x), \sum_{x\in V} F_{J(e_i)}(e_i)(x)).$$

Definition 3.9 The size of a neutrosophic soft graph G is

$$S(G) = \sum_{e_i \in A}(\sum_{xy\in V} T_{K_{ei}}(e_i)(xy), \sum_{xy\in V} I_{K_{ei}}(e_i)(xy), \sum_{xy\in V} F_{K_{ei}}(e_i)(xy),$$

Example 3.10 In example Figure 1, we consider the order of neutrosophic soft graph is

$$Ord(G) = \sum_{e_i\in A} (\sum_{x\in V} T_{J(e_i)}(e_i)(x), \sum_{x\in V} I_{F(e_i)}(e_i)(x), \sum_{x\in V} F_{J(e_i)}(e_i)(x)).$$

$$Ord(G) = (5.0,5.0,4.7). \text{ Similarly } S(G) = (4.3,4.2,4.7)$$

Definition 3.11 Let $G = (G^*, J, K, A)$ be an neutrosophic soft graph. then cardinality of G is defined to be

$$|\mathbf{G}| = \sum_{\mathbf{e}\in \mathbf{A}} |\sum_{\mathbf{v_i}\in \mathbf{V}} \frac{\mathbf{1} + \mathbf{T_{J(e)}(x)} + \mathbf{I_{J(e)}(x)} - \mathbf{F_{J(e)}(x)}}{\mathbf{2}} + |\sum_{\mathbf{v_i},\mathbf{v_j}\in \mathbf{V}} \frac{\mathbf{1} + \mathbf{T_{J(e)}(xy)} + \mathbf{I_{J(e)}(xy)} - \mathbf{F_{J(e)}(xy)}}{\mathbf{2}}|$$

Example 3.12 Consider the above Figure 3, here $H(e_1)$ and $H(e_2)$ are neutrosophic soft graph of G corresponding to the parameter e_1, the cardinality is $G = 5.60$ and corresponding to the parameter e_2, the cardinality is $G = 4.60$

Definition 3.13 Let $GG = (G^*, J, K, A)$ be an neutrosophic soft graph, then vertex cardinality of G is defined to be

$$|V| = \sum_{e \in A} | \sum_{v_i \in V} \frac{1 + T_{J(e)}(x) + I_{J(e)}(x) - F_{J(e)}(x)}{2} |$$

Example 3.14 For the above Figure 3, $H(e_1)$ and $H(e_2)$ are neutrosophic soft graph of G corresponding to the parameter e_1 cardinality is $V = 0.85 + 0.75 + 0.65 + 0.75 = 3.0$ corresponding to the parameter e_2, the cardinality is $V = 2.30$. Then $G(V) = 5.30$

Definition 3.15 Let $G = (G^*, J, K, A)$ be an neutrosophic soft graph, Edge cardinality of E is defined to be

$$|E| = \sum_{e \in A} | \sum_{xy \in E} \frac{1 + T_{K(e)}(xy) + I_{K(e)}(xy) - F_{K(e)}(xy)}{2} |$$

Example 3.16 For the above Figure 3, $H(e_1)$ and $H(e_2)$ are neutrosophic soft graph of G corresponding to the parameter e_1 cardinality is $E = 2.6$ corresponding to the parameter e_2, the cardinality is $E = 2.30$ then $G(E) = 4.90$.

Definition 3.17 The sum of weight of the strong edges incident at v is means to be $d_G(v)$. in neutrosophic soft graph. The minimum $\deg(G)$ is $\delta(G) = \min\{d_g(v)/v \in V, e \in A.\}$

The maximum $\deg(G)$ is $\Delta(G) = \max\{d_g(v)/v \in V, e \in A.\}$

Definition 3.18 Two vertices x and y are said to be neighbors in neutrosophic soft graph if either one of the following conditions hold.

$(1) T_{K(e)}(xy) > 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) > 0,$

$(2) T_{K(e)}(xy) > 0, I_{K(e)}(xy) = 0, F_{K(e)}(xy) > 0,$

$(3) T_{K(e)}(xy) > 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) = 0,$

$(4) T_{K(e)}(xy) = 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) > 0,$ for all $x, y \in V, e \in A$.

Definition 3.19 A path in an neutrosophic is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$, such that either one of the following conditions are satisfied.

$(1) T_{K(e)}(xy) > 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) > 0,$

$(2) T_{K(e)}(xy) > 0, I_{K(e)}(xy) = 0, F_{K(e)}(xy) > 0,$

$(3) T_{K(e)}(xy) > 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) = 0,$

$(4) T_{K(e)}(xy) = 0, I_{K(e)}(xy) > 0, F_{K(e)}(xy) > 0,$ for all $x, y \in V, e \in A$.

Definition 3.20 The length of a path $P = v_1, v_2, \ldots, v_{n+1} (n > 0)$ in Neutrosophic soft graph is n.

Definition 3.21 If v_i, v_j are vertices in G and if they are connected means of a path then the strength of that path is defined as $(\min_{i,j} T_{K(e)}(v_i, v_j), \min_{i,j} I_{K(e)}(v_i, v_j), \max_{i,j} F_{K(e)}(v_i, v_j))$ where $\min_{i,j} T_{K(e)}(v_i, v_j)$ is the $T_{K(e)}$- strength of weakest arc and $\min_{i,j} I_{K(e)}(v_i, v_j)$ is the $I_{K(e)}$- strength of weakest arc and $\max_{i,j} F_{K(e)}(v_i, v_j))$ is the $F_{K(e)}$- strength of strong arc.

Definition 3.22 If $v_i, v_j \in V \subseteq G$, the $T_{K(e)}$ -strength of connectedness between v_i and v_j is $T^{\infty}_{K(e)}(v_i, v_j) = \sup\{T^{K}_{K(e)}(v_i, v_j)/k = 1,2, \ldots n, e \in A\}$ and $I_{K(e)} -$ strength of connectedness between v_i and v_j is $I^{\infty}_{K(e)}(v_i, v_j) = \sup\{I^{k}_{K(e)}(v_i, v_j)/k = 1,2, \ldots n, e \in A\}$ and $F^{\infty}_{K(e)}(v_i, v_j) = \inf\{F^{k}_{K(e)}(v_i, v_j)/k = 1,2, \ldots n, e \in A\}$.

If u, v are connected by means of path of length k then $T^{k}_{K(e)}(v_i, v_j)$ is defined as

$\sup\{T_{K(e)}(u, v_1) \wedge T_{K(e)}(v_1, v_2) \wedge T_{K(e)}(v_1, v_3) \ldots, T_{K(e)}(v_{k-1}, v_k)/u, v, v_1, \ldots v_{k-1}, v \in V\},$

$I^k_{k(e)}(v_i, v_j)$ is defined as

$\sup\{I_{K(e)}(u, v_1) \wedge I_{K(e)}(v_1, v_2) \wedge I_{K(e)}(v_1, v_3)\ldots, I_{K(e)}(v_{k-1}, v_k)/u, v, v_1, \ldots v_{k-1}, v \in V\}$ and

$F^k_{K(e)}(v_i, v_j)$ is defined as

$\inf\{F_{K(e)}(u, v_1) \vee F_{K(e)}(v_1, v_2) \vee F_{K(e)}(v_1, v_3)\ldots, F_{K(e)}(v_{k-1}, v_k)/u, v, v_1, \ldots v_{k-1}, v \in V\}, e \in A$.

Definition 3.23 Two vertices that are joined by a path is called connected neutrosophic soft graph.

Definition 3.24 Let u be a vertex in an neutrosophic soft graph $G^* = (V, E)$, then $N(u) = \{v: v \in V\}$ and (u, v) is a strong arc is called neighborhood of u.

Definition 3.25 A vertex $u \in V$ of an neutrosophic soft graph $G = (V, E)$ is said to be an isolated vertex if $T_{K(e)}(u, v) = 0$, $I_{K(e)}(u, v)$ and $F_{K(e)}(u, v) = 0$, thus an isolated vertex does not dominated any other vertex in G.

Definition 3.26 An arc (u, v) is said to be strong arc, if $T_{K(e)}(u, v) \geq T^{\infty}_{K(e)}(u, v)$ and $I_{K(e)}(u, v) \geq I^{\infty}_{K(e)}(u, v)$ and $F_{K(e)}(u, v) \geq F^{\infty}_{K(e)}(u, v)$.

Definition 3.27 Let $G = (V, E)$ be an neutrosophic soft graph on V. Let $u, v \in V$, we say that u dominates v in G if there exists an strong arc between them.

Note:

1) For any $u, v \in V$, if u dominates v then v dominates u and hence domination is a symmetric relation on V.
2) For any $v \in V, N(v)$ is precisely the set of all vertices in V which are dominated by v.
3) If $T_{K(e)}(u, v) < T^{\infty}_{K(e)}(u, v)$ and $I_{K(e)}(u, v) < I^{\infty}_{K(e)}(u, v)$ and $F_{K(e)}(u, v) < F^{\infty}_{K(e)}(u, v)$, for all $u, v \in V$ and $e \in A$, then the only dominating set of G is V.

Definition 3.28 Given $S \subset V$ is called a dominating set in G if for every vertex $v \in V - S$ there exists a vertex $u \in S$ such that u dominates v. for all $e \in A, u, v \in V$.

Definition 3.29 A dominating set S of an Neutrosophic soft graph is said to be minimal domiating set if no proper subset of S is a dominating set. for all $e \in A, u, v \in V$.

Definition 3.30 Minimum cardinality among all minimal dominating set is called lower domination number of G, and is denoted by $\sum_{e\in A} (d_{NS}(G)) \forall e \in A, u, v \in V$.

Maximum cardinality among all minimal dominating set is called upper domination number of G, and is denoted by $\sum_{e\in A} (D_{NS}(G)) \forall e \in A, u, v \in V$.

Example 3.31 Consider an neutrosophic soft graph $G = (V, E)$, such that $V = \{a, b, c, d\}$ and $E = \{(ab), (bc), (cd), (da), (ac)\}$. Let $A = \{e_1, e_2\}$ be a set of parameters and let neutrosophic soft over V with neutrosophic approximation function $J: A \to \rho(v)$ defined by

$$J(e_1) = a(0.5,0.5,0.6), b(0.5,0.6,0.7), c(0.4,0.3,0.6), d(0.4,0.5,0.7)$$

$$J(e_2) = a(0.6,0.6,0.7), b(0.6,0.7,0.8), c(0.5,0.4,0.7), d(0.5,0.6,0.7)$$

Let (K, A) be a neutrosophic approximation function $K: A \to \rho(E)$ is defined by

$$K(e_1) = ab(0.4,0.5,0.6), bc(0.4,0.3,0.6), cd(0.4,0.3,0.6), ad(0.4,0.5,0.6), bd(0.4,0.5,07)$$

$$K(e_2) = ab(0.5,0.6,0.7), bc(0.5,0.4,0.7), cd(0.5,0.4,0.7), ad(0.5,0.6,0.7), ac(0.5,04,06)$$

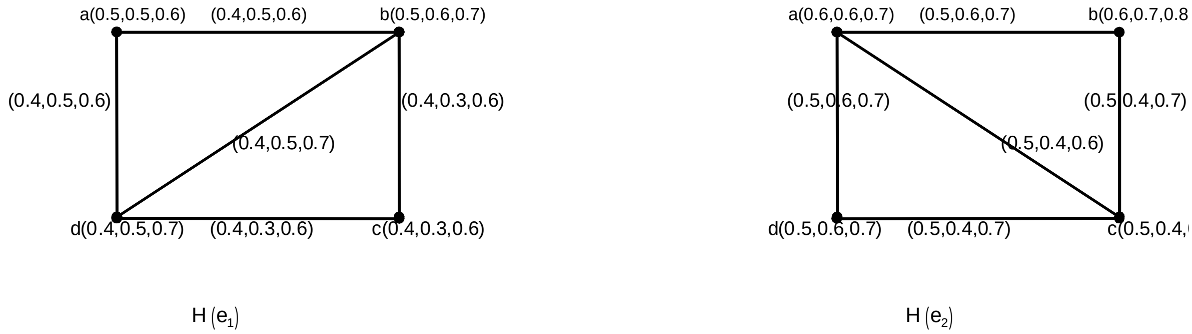

Figure 4

Here, corresponding to the parameter $H(e_1)$, the dominating set is $\{(a,b),(b,c),(c,d),(d,a),(a,b,c),(d,c,a),(b,d,a),(d)\}$

Corresponding to the parameter e_1, the minimum dominating set $\{d\}$.

Corresponding to the parameter e_1, the maximum dominating set $\{a,b\}$.

Corresponding to the parameter e_1, the minimum dominating number 0.6.

Corresponding to the parameter e_1, the maximum dominating number 1.4.

Here, corresponding to the parameter $H(e_2)$, the dominating set is $\{(a,b),(b,c),(c,d),(a,b,c),(d,c,a)\}$

Corresponding to the parameter e_2, the minimum dominating set $\{c,d\}$.

Corresponding to the parameter e_2, the maximum dominating set $\{a,b\}$.

Corresponding to the parameter e_2, the minimum dominating number 1.3.

Corresponding to the parameter e_2, the maximum dominating number 1.5.

For Figure 4, domination number is

$$\sum_{e\in A}(d_{NS}(G)) = 0.6 + 1.3 = 1.9$$

$$\sum_{e\in A}(D_{NS}(G)) = 1.4 + 1.5 = 2.9$$

Definition 3.32 Two vertices in an neutrosophic soft graph, $G = (V,E)$ are said to be independent if there is no strong arc between them.

Definition 3.33 Given $S \subset V$ is said to be independent set of G if $T_{K(e)}(u,v) < T^{\infty}_{K(e)}(u,v)$ and $I_{K(e)}(u,v) < I^{\infty}_{K(e)}(u,v)$ and $F_{K(e)}(u,v) < F^{\infty}_{K(e)}(u,v) \forall e \in A, u,v \in S$.

Definition 3.34 An indepentent set S of G in an neutrosophic soft graph is said to be maximal independent, if for every vertex $v \in V - S$, the set $S \cup \{v\}$ is not independent.

Definition 3.35 The minimum cardinality among all maximal independent set is called lower independence number of G, and it is denoted by $\sum_{e\in A}(i_{NS}(G))$. The maximum cardinality among all maximal independent set is called lower independence number of G, and it is denoted by $\sum_{e\in A}(I_{NS}(G))$.

Example 3.36 Consider an above example for an neutrosophic soft graph $G = (V,E)$, such that $V = \{a,b,c,d\}$ and $E = \{(a,b),(bc),(cd),(da),(ac)\}$. Let $A = \{e_1,e_2\}$ be a set of parameters and let neutrosophic soft over V with neutrosophic approximation function $J: A \to \rho(v)$ defined as follows:

we have corresponding to the parameter e_2 arc (ac) is weakest arc us does not dominated by $\{c\}$ and $\{a\}$.

$$J(e_1) = a(0.5,0.5,0.6), b(0.5,0.6,0.7), c(0.4,0.3,0.6), d(0.4,0.5,0.7)$$

$$J(e_2) = a(0.6,0.6,0.7), b(0.6,0.7,0.8), c(0.5,0.4,0.7), d(0.5,0.6,0.7)$$

Let (K, A) be a neutrosophic approximation function $K: A \to \rho(E)$ is defined by

$$K(e_1) = ab(0.4,0.5,0.6), bc(0.4,0.3,0.6), cd(0.4,0.3,0.6), ad(0.4,0.5,0.6), bd(0.4,0.5,07)$$

$$K(e_2) = ab(0.5,0.6,0.7), bc(0.5,0.4,0.7), cd(0.5,0.4,0.7), ad(0.5,0.6,0.7), ac(0.5,04,06).$$

For the Corresponding to the parameter e_1, the minimum Independent Dominating Det (IDS) is $\{a, c\}$.

For the Corresponding to the parameter e_1, the maximum (IDS) is $\{a, c\}$.

For the Corresponding to the parameter e_1, the minimum independent dominating number is 1.25.

For the Corresponding to the parameter e_1, the maximum independent dominating number is 1.25.

For the Corresponding to the parameter e_2, the minimum (IDS)is $\{c, a\}$.

For the Corresponding to the parameter e_2, the maximum (IDS)is $\{d, b\}$.

For the Corresponding to the parameter e_2, the minimum independent dominating number is 1.35.

For the Corresponding to the parameter e_2, the maximum independent dominating number is 1.45.

Independent domination number is $\sum_{e\in A} (i_{NS}(G)) = 2.60$ and $\sum_{e\in A} (I_{NS}(G)) = 2.70$

Theorem 3.37 A dominating set S of an NSG, $G = (G^*, J, K, A)$ is a minimal dominating set if and only if for each $d \in D$ one of the following conditions holds.

(i) d is not a strong neighbor of any vertex in D.

(ii) There is a vertex $v \in V - \{D\}$ such that $N(u) \cap D = d$.

Proof. Assume that D is a minimal dominating set of $G = (G^*, J, K, A)$. Then for every vertex $d \in D$, $D - \{d\}$ is not a dominating set and hence there exists $v \in V - (D - \{d\})$ which is not dominated by any vertex in $D - \{d\}$. If $v = d$, we get, v is not a strong neighbor of any vertex in D. If $v \neq d$,v is not dominated by $D - \{v\}$, but is dominated by D, then the vertex v is strong neighbor only to d in D. That is, $N(v) \cap D = d$. Conversely, assume that D is a dominating set and for each vertex $d \in D$, one of the two conditions holds, suppose D is not a minimal dominating set, then there exists a vertex $d \in D$, $D - \{d\}$ is a dominating set. Hence d is a strong neighbor to at least one vertex in $D - \{d\}$, the condition one does not hold. If $D - \{d\}$ is a dominating set then every vertex in $V - D$ is a strong neighbor at least one vertex in $D - \{d\}$, the second condition does not hold which contradicts our assumption that at least one of thse conditions holds. So D is a minimal dominating set.

Theorem 3.38 Let G be an NSG without isolated vertices and D is a minimal dominating set. Then $V - D$ is a dominating set of $G = (G^*, J, K, A)$.

Proof.D be a minimal dominating set. Let v be a any vertex of D. Since $G = (G^*, F, K, A)$ has no isolated vertices, there is a vertex $d \in N(v)$. v must be dominated by at least one vertex in $D - v$, that is $D - v$ is a dominating set. By above theorem, it follows that $d \in V - D$. Thus every vertex in D is dominated by at least one vertex in $V - D$, and $V - D$ is a dominating set.

Theorem 3.39 An independent set is a maximal independent set of NSG, $G = (G^*, J, K, A)$ if and only if it is independent and dominating set.

Proof. Let D be a maximal independent set in an NSG, and hence for every vertex $v \in V - D$, the set $D \cup v$ is not in dependent. For every vertex $v \in V - D$,there is a vertex $u \in D$ such that u is a strong neighbor to v. Thus D is a dominating set. Hence D is both dominating and independent set. Conversely, assume D is both independent and dominating. Suppose D is not maximal independent, then there exists a vertex $v \in V - D$, the set $D \cup v$ is independent. If $D \cup v$ is independent then no vertex in D is strong neighbor to v. Hence D cannot be a dominating set, which is contradiction, Hence D is a maximal independent set.

Theorem 3.40 Every maximal independent set in an NSG, $G = (G^*, J, K, A)$ is a minimal dominating set.

Proof. Let S be a maximal independent set in a NSG, by previous theorem, S is a dominating set. Suppose S is not a minimal dominating set, then there exists at least one vertex $v \in S$ for which $S - v$ is a dominating set, But if $S - v$ dominates $V - S - (v)$, then at least one vertex in $S - v$ must be strong neighbor to v. This contradicts the fact that S is an independent st of G. Therefore, must be a minimal dominating set.

4 STRONG NEIGHBORHOOD DOMINATION

Definition 4.1 Let $G = (V, E)$ be a neutrosophic soft graph and $u \in V$. Then $u \in V$ is called a strong neighbour of u if uv is a strong arc. the set of strong neighbor of u is called the strong neighborhood of u and is denoted by $N_s(u)$. The closed strong neighborhood of u is defined as $N_s[u] = N_s(u) \cup u$.for all $u \in V, e \in A$.

Definition 4.2 Let $G. = (V, E)$ be a strong neutrosophic soft graph and $v \in V$.

(i) The strong degree and the strong neighborhood degree of v are defined, respectively

$$d_s(v) = \sum_{e\in A} \Big(\sum_{u\in N_{(s)}(v)} T_{K(e)}(uv), \sum_{u\in N_{(s)}(v)} I_{K(e)}(uv) \sum_{u\in N_{(s)}(v)} F_{K(e)}(uv)\Big)$$

$$d_sN(v) = \sum_{e\in A} \Big(\sum_{u\in N_{(s)}(v)} T_{J(e)}(u), \sum_{u\in N_{(s)}(v)} I_{J(e)}(u) \sum_{u\in N_{(s)}(v)} F_{J(e)}(u)\Big)$$

Definition 4.3 The strong degree cardinality and the strong neighborhood degree cardinality of v are defined by

$$|d_s(v)| = \textstyle\sum_{e\in A} \big(\sum_{u\in N_s(v)} \frac{1+T_{K(e)}(u,v)+I_{K(e)}(u,v)-F_{K(e)}(u,v)}{2}\big)$$

$$|d_sN(v)| = \textstyle\sum_{e\in A} \big(\sum_{u\in N_s(v)} \frac{1+T_{J(e)}(u)+I_{J(e)}(u)-F_{J(e)}(u)}{2}\big)$$

The minimum and maximum strong degree of G are defined, respectively as

$\delta_s(G) = \wedge\, |d_s(v)|\ \forall v \in V$ and

$$\Delta_s(G) = \vee\, |d_s(v)|\ \forall u, v \in V, e \in A.$$

The minimum and maximum strong neighborhood degree of G are defined by

$\delta_sN(G) = \wedge\, |d_sN(v)| \forall v \in V$ and

$$\Delta_sN(G) = \vee\, |d_sN(v)| \forall u, v \in V, e \in A.$$

Example 4.4 Consider a neutrosophic soft graph $G = (V, E)$ in figure we see that

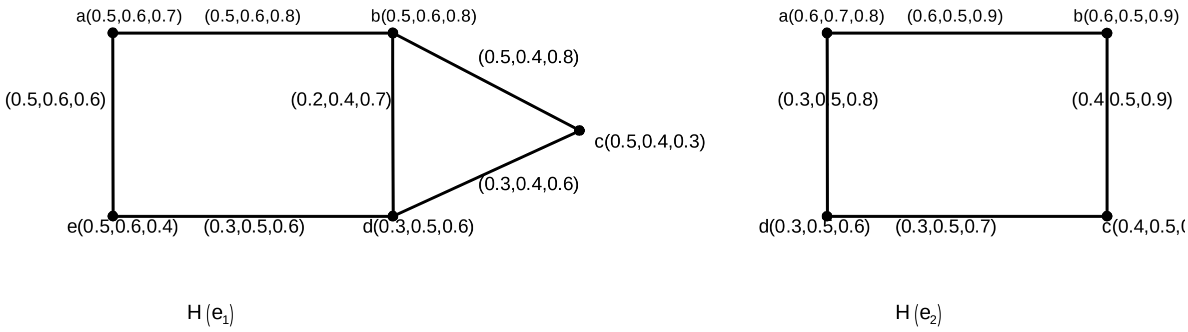

Figure 5

Corresponding to the parameter $H(e_1) = (ab), (bc), (cd), (de)$ are strong arc also for corresponding to the parameter $H(e_2)$ all arcs are strong.

Here for corresponding parameter $H(e_1)$, $d_s(a) = (0.5,0.6,0.8), d_s(b) = (1.0,1.0,1.6)$, $d_s(c) = (0.8,0.8,1.4), d_s(d) = (0.6,0.9,1.2), d_s(e) = (0.3,0.5,0.6)$

$$|d_s(a)| = (0.65), |d_s(b)| = (1.2), |d_s(c)| = (1.1), |d_s(d)| = (1.15), |d_s(e)| = (0.6)$$

Here $\delta_s(G) = 0.6$ and $\Delta_s(G) = 1.2$ and also corresponding to the parameter $H(e_2)$ we get, $d_s(a) = (0.9,1.0,1.7), d_s(b) = (1.0,1.0,1.8), d_s(c) = (0.7,1.0,1.6), d_s(d) = (0.6,1.0,1.5)$

$$|d_s(a)| = (1.1), |d_s(b)| = (1.1), |d_s(c)| = (1.05), |d_s(d)| = (1.05)$$

Here $\delta_s(G) = 1.05$ and $\Delta_s(G) = 1.1$ and also corresponding to the parameter $H(e_1)$ we get,

$$d_sN(a) = (0.5,0.6,0.8), d_sN(b) = (1.0,1.0,1.0), d_sN(c) = (0.8,1.1,1.4), d_sN(d) = (1.0,1.0,0.7), d_sN(e) = (0.3,0.5,0.6)$$

$$|d_sN(a)| = (0.65), |d_sN(b)| = (1.5), |d_sN(c)| = (1.25), |d_sN(d)| = (1.50), |d_sN(e)| = (0.6)$$

Here $\delta_sN(G) = 0.6$ and $\Delta_sN(G) = 1.50$ and also corresponding to the parameter $H(e_2)$, we get

$$d_sN(a) = (0.9,1.0,1.5), d_sN(b) = (1.0,1.2,1.5), d_sN(c) = (0.9,1.0,1.5), d_sN(d) = (1.0,1.2,1.5)$$

$$|d_sN(a)| = (1.2), |d_sN(b)| = (1.35), |d_sN(c)| = (1.2), |d_sN(d)| = (1.35),$$

Here $\delta_sN(G) = 1.2$and $\Delta_sN(G) = 1.35$.

Definition 4.5 The strong size and the strong order of neutrosophic soft graph of G are defined by

$$S_{NS}(G) = \{\textstyle\sum_{e\in A} \sum_{uv\in E} \frac{1+T_{K(e)}(uv)+I_{K(e)}(uv)-F_{K(e)}(uv)}{2} / \text{ uv is a strongarc}\} \text{ and}$$

$$O_{NS}(G) = \{\sum_{e\in A} \sum_{u\in V} \frac{1 + T_{J(e)}(u) + I_{J(e)}(u) - F_{J(e)}(u)}{2} \text{ uv is a strongarc}\}$$

Example 4.6 Consider above Figure 5 neutrosophic soft graph G for a strong arc $H(e_1)$ is

$(ab), (bc), (cd), (de)$ in $H(e_1)$ we get for corresponding parameter $e_1 S_{NS}(e_1) = \frac{1.3+1.1+1.1+1.2}{2} = 2.35$.

Corresponding parameter e_2 all arcs are strong we get

$$S_{NS}(e_2) = \frac{1.2 + 1.0 + 1.1 + 1.0}{2} = 2.15$$

$S_{NS}(G) = 4.5$.

Corresponding parameter $e_1 O_{NS}(e_1) = \frac{1.4+1.3+1.6+1.2+1.7}{2} = 3.6.$

Corresponding parameter e_2 all arcs are strong we get $O_{NS}(e_2) = \frac{1.5+1.2+1.2+1.2}{2} = 2.55.$

$O_{NS}(G) = 6.15.$

Definition 4.7 Let D be a dominating set in a neutrosophic soft graph. The arc weight and the node weight of D are defined as follows, respectively,

$$W_e(D) = \{\sum_{e\in A} \sum_{u\in D, v\in N_S(u)} \frac{1 + \wedge T_{K(e)}(uv) + \wedge I_{K(e)}(uv) - \vee F_{K(e)}(uv)}{2}\}$$

$$W_v(D) = \sum_{e\in A} \sum_{u\in D, v\in N_S(u)} \frac{1 + \wedge T_{J(e)}(u) + \wedge I_{J(e)}(u) - \vee F_{J(e)}(u)}{2}$$

The strong domination number and the strong neighborhood domination number of G are defined as the minimum arc weight and the minimum node weight of dominating sets in G are denoted by $N\delta_S(G)$ and $N\delta_{SN}(G)$ respectively.

Example 4.8 Consider the neutrosophic soft graph G in Figure 5. The dominating set in G are, corresponding to the parameter e_1 sets are

$$D_1 = \{a, d\}, D_2 = \{b, d\}, D_3 = \{b, e\}, D_4 = \{a, b, d\}, D_5 = \{b, d, e\}$$

$$W_e(D_1) = 1.20, W_e(D_2) = 1.1, W_e(D_3) = 1.15, W_e(D_4) = 1.75, W_e(D_5) = 1.70$$

Here corresponding to the parameter e_1 minimum dominating set $N\delta_S(e_1) = \{b, d\}$ and domination number $N\delta_S(e_1) = 1.1$

Similarly, for corresponding to the parameter e_2, the domination sets are

$$D_1 = \{a, b\}, D_2 = \{b, c\}, D_3 = \{c, d\}, D_4 = \{a, c\}, D_5 = \{a, d\}, D_6 = \{b, d\}, D_7 = \{a, b, c\}, D_8 = \{b, d, c\}, D_9 = \{c, d, a\}$$

$$W_e(D_1) = 0.95, W_e(D_2) = 0.95, W_e(D_3) = 0.95, W_e(D_4) = 0.9, W_e(D_5) = 0.95, W_e(D_6) = 1.0, W_e(D_7) = 1.45, W_e(D_8) = 1.45, W_e(D_9) = 1.4$$

Here corresponding to the parameter e_2, the minimum dominating set $N\delta_S(e_2) = \{b, d\}$ and domination number $N\delta_S(e_1) = 0.95$

In addition ,we have Corresponding to the parameter e_1, the dominating set

$$D_1 = \{a, d\}, D_2 = \{b, d\}, D_3 = \{b, e\}, D_4 = \{a, b, d\}, D_5 = \{b, d, e\}$$

$$W_v(D_1) = 1.40, W_v(D_2) = 1.35, W_v(D_3) = 1.2, W_v(D_4) = 1.95, W_v(D_9) = 1.95$$

Here corresponding to the parameter e_1 minimum dominating set $N\delta_{SN}(e_1) = \{b, e\}$ and domination number $N\delta_{SN}(e_1) = 1.2$

similarly, corresponding to the parameter e_2 domination sets are

$$D_1 = \{a, b\}, D_2 = \{b, c\}, D_3 = \{c, d\}, D_4 = \{a, c\}, D_5 = \{a, d\}, D_6 = \{b, d\}, D_7 = \{a, b, c\}, D_8 = \{b, d, c\}, D_9 = \{c, d, a\}$$

$$W_v(D_1) = 1.15, W_v(D_2) = 1.0, W_v(D_3) = 1.0, W_v(D_4) = 1.05, W_v(D_5) = 1.15, W_v(D_6) = 1.1, W_v(D_7) = 1.60, W_v(D_8) = 1.55, W_v(D_9) = 1.60$$

Here corresponding to the parameter e_2 minimum dominating set $N\delta_{SN}(e_2) = D_2, D_3$ and domination number $N\delta_{SN}(e_2) = 1.0.$

5 STRONG PERFECT DOMINATION

In this section, we have define the perfect dominating set and strong perfect domination number of a neutrosophic soft graph using proper condition.

Definition 5.1 Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. A subset D of V is a perfect dominating set (or D^p) in G, if for every node $v \in V - D$, there exists a only one node $u \in D$ such that u dominates v. A set D^p is said to be minimal perfect dominating set if for each $v \in D^p$,$D^p - v$ is not a perfect dominating set in G.

Example 5.2 Consider the neutrosophic soft graph $G = (V, E)$ figure we see that all arcs are strong arc.

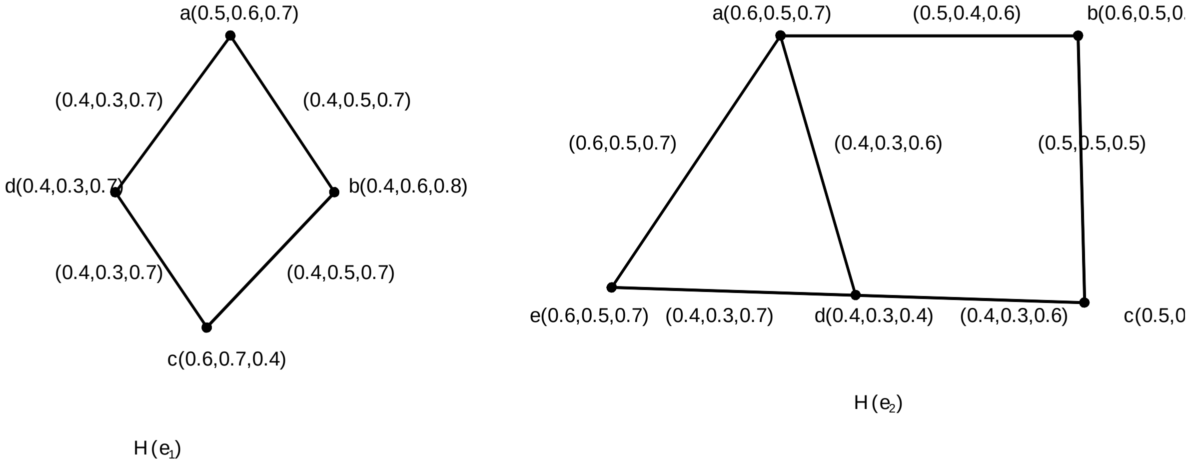

Figure 6

Here corresponding to the parameter e_1, the perfect dominating sets are

$$D_1^P = \{a, b\}, D_2^P = \{b, c\}, D_3^P = \{c, d\}, D_4^P = \{a, d\}$$

Then corresponding to the parameter e_2, the perfect dominating sets are

$$D_1^P = \{a, b\}, D_2^P = \{d, c\}, D_3^P = \{a, d, e\}$$

Proposition 5.3 Any perfect dominating set in neutrosophic soft graph G is a dominating set.

Remark 5.4 The converse of proposition in not correct in general cases. for this consider the neutrosophic soft graph G in figure 6, we see that in $D = \{a, c\}$ is a domination set in G, but it is not a a perfect domination set. Because b and d has two strong neighbors in D.

Definition 5.5 The strong perfect domination number of a neutrosophic soft graph G is defined as the minimum arc weights of perfect dominating sets of G which is denoted by $N\delta_{SP}(G)$.

Example 5.6 Consider the neutrosophic soft graph $G = (G^*, J, K, A)$ in Figure 6

Corresponding to the parameter e_1, the perfect domination sets are,

$D_1^P = \{a, b\}, D_2^P = \{b, c\}, D_3^P = \{c, d\}, D_4^P = \{a, d\}$ in $H(e_1)$ we get

$$W_e(D_1^P) = 1.1, W_e(D_2^P) = 1.1, W_e(D_3^P) = 1.0, W_e(D_4^P) = 1.0$$

Then $N\delta_{SP}(e_1) = 1.0$

Corresponding to the parameter e_2, the perfect domination sets are,

$D_1^P = \{a, b\}, D_2^P = \{d, c\}, D_3^P = \{a, d, e\}$ in $H(e_2)$ we get

$$W_e(D_1^P) = 1.15, W_e(D_2^P) = 1.05, W_e(D_3^P) = 1.50$$

Then $N_{SP}(e_2) = 1.05$, $N\delta_{SP}(G) = 1.05$.

Theorem 5.7 A perfect dominating set D^P of an NSG, $G = (G^*, J, K, A)$ is a minimal perfect dominating set if and only if for each $d \in D^P$ one of the following conditions holds.

(i) $N_s(v) \cap D^P = \{\emptyset\}$ or

(ii) There is a vertex $u \in V - \{D\}$ such that $N_s(u) \cap D^P = \{v\}$.

Proof. Let D^P be a minimal perfect dominating set and $v \in D^P$. Suppose that (i) and (ii) are not established. Then there exists a node $u \in D^P$ such that uv is strong and v has no strong neighbors in $V - D^P$. Therefore $D^P - \{v\}$ is a perfect dominating set in G, which is contradiction by the minimality of D^P.

Conversely, suppose that (i) or (ii) is established and D^P is not a minimal perfect dominating set in G. Then there exists $v \in V - D^P$ such that $D^P - \{v\}$ is a perfect dominating set. Hence v has a strong neighbor in D^P and so (i)is not established. Then there exists is a node $u \in V - D^P$ such that u isa strong neighbor of v and since $D^P - \{v\}$ is a dominating set, then u has a strong neighbor in $D^P - \{v\}$. Therefore $u \in V - D^P$ has two strong neighbors in D^P and so D^P is not a perfect dominating set, that is a contradiction. Then D^P is a minimal perfect dominating set in G.

Corollary 5.8 A dominating set D in a neutrosophic soft graph $G = (V, E)$ is a minimal dominating set if and only if for each node $v \in D$,either

(i) $N_s(v) \cap D^P = \{\emptyset\}$ or.

(ii) There is a vertex $u \in V - \{D\}$ such that $N_s(u) \cap D^P = \{v\}$.

Theorem 5.9 Let G be a neutrosophic soft graph which every its node has at least one strong neighbor. If D^P is a minimal perfect dominating set in G, then $V - D^P$ is a dominating set.

Proof. Suppose that D^P be a minimal perfect dominating set in G and $v \in V - (V - D^P)$. If there is no $u \in V - D^P$ such that $N_S(u) \cap D^P = \{v\}$. Then by above theorem, $N_S(v) \cap D^P = \{\emptyset\}$. Therefore there exists a node in G which has no strong neighbors that is contradiction. This implies that $V - D^P$ is a dominating set.

Corollary 5.10 Let G be a neutrosophic soft graph every node of which has at least one strong neighbor. If D is a minimal dominating set in G, then $V - D$ is a dominating set in G.

Theorem 5.11 Let G be a neutrosophic soft graph every node of which has exactly one strong neighbor. If D^P is a minimal perfect dominating set in G, then $V - D^P$ is a perfect dominating set in G.

Proof. Suppose that D^P is a minimal perfect dominating set in the neutrosophic soft graph G. Then by above theorem $V - D^P$ is a dominating set and since every node in G has exactly one strong neighbor, $V - D^P$ is a perfect dominating set in G.

6 Conclusion

In this work, we derived the domination number of neutrosophic soft graphs and elaborate them with suitable examples by using strength of path and strength of connectedness. Further, we investigate some remarkable properties of independent domination number, strong neighborhood domination and strong perfect domination of neutrosophic soft graph and the proposed concepts are described with suitable examples. Further we can extend to investigate the isomorphic properties of the proposed graph

References

1. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. Symmetry, 11(7), 903.
2. Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems, 1-21.
3. Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108, 210-220.
4. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452.
5. Abdel-Baset, M., Chang, V., Gamal, A., & Smarandache, F. (2019). An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. Computers in Industry, 106, 94-110.
6. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2019). A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems, 43(2), 38.
7. Akram M and Nawaz S, On fuzzy soft graphs, Italian Journal of Pure and Applied Mathematics 34 (2015) 497-514.
8. Akram M and Nawaz S , Operation on soft graphs, Fuzzy Information and Engineering 7(4) (2015) 423-449.
9. Akram M and Sundas Shahzadi, Neutrosophic soft graphs with application, Journal of Intelligent & Fuzzy Systems, vol. 32, no. 1, pp. 841-858, 2017
10. Akram M and Gulfam Shahzadi, Operations on Single-Valued Neutrosophic Graphs, Journal of uncertain systems, Vol.11, No.1, pp. 1-26, 2017.
11. Akram, M., Ishfaq, N., Smarandache, F., & Broumi, S. (2019). Application of Bipolar Neutrosophic sets to Incidence Graphs. Neutrosophic Sets & Systems, 27,(2019), 180-200.
12. Akram, M., Gulzar, H., & Smarandache, F. (2019). Neutrosophic Soft Topological K-Algebras. Neutrosophic Sets and Systems 25 (2019),104-124.
13. Ali M and Smarandache F Complex Neutrosophic Set, Neural Computing and Applications, Vol. 27, no. 01.
14. Ali M, Deli I, Smarandache F, The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition, Journal of Intelligent and Fuzzy Systems, (In press).
15. Bhutani K. R., Rosenfeld A. , Strong arcs in fuzzy graphs, Information Sciences 152 (2003),319 - 322.
16. Bondy J. A and Murthy U.S.R, Graph Theory with Applications, American Elsevier Publishing Co., New York (1976).
17. Broumi S, Smarandache, F., Talea, M., and Bakali, Single Valued Neutrosophic Graphs: Degree,Order and Size, 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE).
18. Broumi S, Talea M, Bakali A, Smarandache F, Single Valued Neutrosophic Graphs. The Journal of New Theory, 2016(10),861-101
19. Broumi S, Florentin Smarandache, Mohamed Talea, Assia Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. Applied Mechanics and Materials, 2016(841)184-191.
20. Broumi S, Florentin Smarandache, Mohamed Talea, Assia Bakali, An Isolated Bipolar Single-Valued Neutrosophic Graphs.Information Systems Design and Intelligent Applications, 2018(672)816-822.
21. Cokayne E. J. and Hedetniee S. T., Towards Theory of Domination in Graphs, Networks 7(1977), 247 - 261.

22. Nagarajan, D., Lathamaheswari, M., Broumi, S., & Kavikumar, J. Blockchain Single and Interval Valued Neutrosophic Graphs. Neutrosophic Sets and Systems, 27,(2019), 23-35.
23. Nagoorgani N, and Jahir Hussain R, Fuzzy independent dominating set, Advances in fuzzy sets and Systems 2(1) (2007), 99-108.
24. Jan, N., Zedam, L., Mahmood, T., Ullah, K., Broumi, S., & Smarandache, F. Constant single valued neutrosophic graphs with applications. Neutrosophic Set and System 27, (2019)., 77-89.
25. Molodtsov D.A , Soft set theory-first results, Computers and Mathematics with Application 37 (1999), pp 19-31.
26. Molodtsov D.A., The Theory of Soft Sets(in Russain), URSS Publishers, Moscow, 2004.
27. Nabeeh, N. A., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Neutrosophic multi-criteria decision making approach for iot-based enterprises. IEEE Access, 7, 59559-59574.
28. Rosenfeld. A Fuzzy Graphs. In Fuzzy Sets and their Applications to Cognitive and Decision Processes. Zadeh LA,FuKS, Shimura M, editors. New York Academic press, 1975. pp 77-95.
29. Smarandache, F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, 1999.
30. Smarandache F, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Inter. J.Pure Appl. Math. 24 (2005) 287-297.
31. Smarandache M, Neutrosophy, Neutrosophic Probability, Set, and Logic, Amer. Res. Press, Rehoboth, USA, 105 pages, 1998; http://fs.gallup.unm.edu/eBookneutrosophics4.pdf(4th edition).
32. Smarandache F, Neutrosophic Graphs, in his book Symbolic Neutrosophic Theory, Europa, Nova.
33. [33] Somasundram A. and Somasundram S., Domination in Fuzzy Graphs - I , pattern Recognition Letters, 19(1998), 787 -791.
34. Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics, 24, (2010), 289-297.
35. Wang, H., Smarandache, F., Zhang, Y., and Sunderraman, R. (2010). Single valued neutrosophic sets. Multispace and Multistructure Vol 4, pp 410-413.
36. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, Vol 8 No 3, 338-353.

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