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Neutrosophic $\mathcal{N}$-structures over UP-algebras

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Abstract: The notions of (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic $\mathcal{N}$-structures to be (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras are provided. Relations between (special) neutrosophic $\mathcal{N}$-UP-subalgebras (resp., (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, (special) neutrosophic $\mathcal{N}$-strongly UP-ideals) and their level subsets are considered.

Keywords: UP-algebra; (special) neutrosophic $\mathcal{N}$-UP-subalgebra; (special) neutrosophic $\mathcal{N}$-near UP-filter; (special) neutrosophic $\mathcal{N}$-UP-filter; (special) neutrosophic $\mathcal{N}$-UP-ideal; (special) neutrosophic $\mathcal{N}$-strongly UP-ideal

1. Introduction

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [14], BCI-algebras [15], BCH-algebras [11], KU-algebras [28], SU-algebras [21] and others. They are strongly connected with logic. For example, BCI-algebras were introduced by Iséki [15] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [14, 15] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

The branch of the logical algebra, UP-algebras was introduced by Iampan [12] in 2017, and it is known that the class of KU-algebras [28] is a proper subclass of the class of UP-algebras. It have been examined by several researchers, for example, Somjanta et al. [32] introduced the notion of fuzzy sets in UP-algebras, the notion of intuitionistic fuzzy sets in UP-algebras was introduced by Kesorn et al. [22], Kaijae et al. [20] introduced the notions of anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras of UP-algebras, the notion of Q-fuzzy sets in UP-algebras was introduced by Tanamoon et al. [37], etc.

Neutrosophy provides a foundation for a whole family of new mathematical theories with the generalization of both classical and fuzzy counterparts. In a neutrosophic set, an element has three associated defining functions such as truth membership function (T), indeterminate membership function (I) and false membership function (F) defined on a universe of discourse X. These three functions are independent completely. The concept of neutrosophic logics was first introduced by Smarandache [31] in 1999. Jun et al. [16] introduced a new function, called a negative-valued function, and constructed $\mathcal{N}$ -structures in 2009. Khan et al. [23] discussed neutrosophic $\mathcal{N}$ -structures and their applications in semigroups in 2017. Jun et al. [17, 33] considered neutrosophic $\mathcal{N}$ -structures applied to BCK/BCI-algebras and neutrosophic commutative $\mathcal{N}$ -ideals in BCK-algebras in 2017. Jun et al. [19] studied neutrosophic positive implicative $\mathcal{N}$ -ideals in BCK-algebras in 2018. Abdel-Baset and his colleagues applied the notion of neutrosophic set theory in the new fields (see [1, 2, 3, 4, 5, 6, 27]). Jun and his colleagues applied the notion of neutrosophic set theory in BCK/BCI-algebras (see [8, 18, 24, 26, 35, 36]).

The remaining part of the paper is structured as follows: Section 2 gives some definitions and properties of UP-algebras. Section 3 introduces the notions of neutrosophic $\mathcal{N}$ -UP-subalgebras, neutrosophic $\mathcal{N}$ -near UP-filters, neutrosophic $\mathcal{N}$ -UP-filters, neutrosophic $\mathcal{N}$ -UP-ideals, and neutrosophic $\mathcal{N}$ -strongly UP-ideals of UP-algebras, and a level subset of a neutrosophic $\mathcal{N}$ -structure is proved in Section 4. Section 5 introduces the notions of special neutrosophic $\mathcal{N}$ -UP-subalgebras, special neutrosophic $\mathcal{N}$ -near UP-filters, special neutrosophic $\mathcal{N}$ -UP-filters, special neutrosophic $\mathcal{N}$ -UP-ideals, and special neutrosophic $\mathcal{N}$ -strongly UP-ideals of UP-algebras, and a level subset of a neutrosophic $\mathcal{N}$ -structure of special type is proved in Section 6. This paper has been finalized with that result.

2. Basic results on UP-algebras

Before we begin our study, we will give the definition of a UP-algebra.

Definition 2.1 [12] An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* where X is a nonempty set, $\cdot$ is a binary operation on X , and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1) $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,

(UP-2) $(\forall x \in X)(0 \cdot x = x)$,

(UP-3) $(\forall x \in X)(x \cdot 0 = 0)$, and

(UP-4) $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [12], we know that the notion of UP-algebras is a generalization of KU-algebras (see [28]).

Example 2.2 [30] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$ where $\mathcal{P}(X)$ means the power set of X . Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$ where A^C means the complement of a subset A . Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the generalized power UP-algebra of type 1 with respect to Ω . Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the generalized power UP-algebra of type 2 with respect to Ω . In particular, $(\mathcal{P}(X), \cdot, \varnothing)$ is a UP-algebra and we shall call it the power UP-algebra of type 1, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the power UP-algebra of type 2.

Example 2.3 [9] Let $\mathbf{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$(\forall x, y \in \mathbf{N})\left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases}\right) \text{ and } (\forall x, y \in \mathbf{N})\left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases}\right).$$

Then $(\mathbf{N},\circ,0)$ and $(\mathbf{N},\bullet,0)$ are UP-algebras.

Example 2.4 [25] Let $X=\{0,1,2,3,4,5\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	0	0	2	3	2	5
2	0	1	0	3	1	5
3	0	1	2	0	4	5
4	0	0	0	3	0	5
5	0	0	2	0	2	0

Then $(X,\cdot,0)$ is a UP-algebra.

For more examples of UP-algebras, see [7, 13, 29, 30].

The following proposition is very important for the study of UP-algebras.

Proposition 2.5 [12, 13] In a UP-algebra $X=(X,\cdot,0)$, the following properties hold:

1. $(\forall x\in X)(x\cdot x=0)$,
2. $(\forall x,y,z\in X)(x\cdot y=0, y\cdot z=0\Rightarrow x\cdot z=0)$,
3. $(\forall x,y,z\in X)(x\cdot y=0\Rightarrow (z\cdot x)\cdot(z\cdot y)=0)$,
4. $(\forall x,y,z\in X)(x\cdot y=0\Rightarrow (y\cdot z)\cdot(x\cdot z)=0)$,
5. $(\forall x,y\in X)(x\cdot(y\cdot x)=0)$,
6. $(\forall x,y\in X)((y\cdot x)\cdot x=0\Leftrightarrow x=y\cdot x)$,
7. $(\forall x,y\in X)(x\cdot(y\cdot y)=0)$,
8. $(\forall a,x,y,z\in X)((x\cdot(y\cdot z))\cdot(x\cdot((a\cdot y)\cdot(a\cdot z)))=0)$,
9. $(\forall a,x,y,z\in X)((((a\cdot x)\cdot(a\cdot y))\cdot z)\cdot((x\cdot y)\cdot z)=0)$,
10. $(\forall x,y,z\in X)(((x\cdot y)\cdot z)\cdot(y\cdot z)=0)$,
11. $(\forall x,y,z\in X)(x\cdot y=0\Rightarrow x\cdot(z\cdot y)=0)$,
12. $(\forall x,y,z\in X)(((x\cdot y)\cdot z)\cdot(x\cdot(y\cdot z))=0)$, and
13. $(\forall a,x,y,z\in X)(((x\cdot y)\cdot z)\cdot(y\cdot(a\cdot z))=0)$.

On a UP-algebra $X=(X,\cdot,0)$, we define a binary relation $\le$ on X [12] as follows: $(\forall x,y\in X)(x\le y\Leftrightarrow x\cdot y=0)$.

Definition 2.6 [10, 12, 32] A nonempty subset S of a UP-algebra $(X,\cdot,0)$ is called

1. a *UP-subalgebra* of X if $(\forall x,y\in S)(x\cdot y\in S)$.
2. a *near UP-filter* of X if
   (a) the constant 0 of X is in S, and
   (b) $(\forall x,y\in X)(y\in S\Rightarrow x\cdot y\in S)$.
3. a *UP-filter* of X if
   (a) the constant 0 of X is in S, and
   (b) $(\forall x,y\in X)(x\cdot y\in S, x\in S\Rightarrow y\in S)$.
4. a *UP-ideal* of X if
   (a) the constant 0 of X is in S, and
   (b) $(\forall x,y,z\in X)(x\cdot(y\cdot z)\in S, y\in S\Rightarrow x\cdot z\in S)$.
5. a *strongly UP-ideal* of X if

(a) the constant 0 of X is in S, and
(b) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [10] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strongly UP-ideals. Moreover, they also proved that a UP-algebra X is the only one strongly UP-ideal of itself.

Theorem 2.7 *Let* $\mathcal{N}$ *be a nonempty family of near UP-filters of a UP-algebra* $X = (X, \cdot, 0)$. *Then* $\bigcap \mathcal{N}$ *and* $\bigcup \mathcal{N}$ *are near UP-filters of* X.

Proof. Clearly, $0 \in N$ for all $N \in \mathcal{N}$. Then $0 \in \bigcap \mathcal{N}$. Let $x \in X$ and $y \in \bigcap \mathcal{N}$. Then $y \in N$ for all $N \in \mathcal{N}$. Since N is a near UP-filter of X, we have $x \cdot y \in N$ for all $N \in \mathcal{N}$ and so $x \cdot y \in \bigcap \mathcal{N}$. Hence, $\bigcap \mathcal{N}$ is a near UP-filter of X. Since $\bigcap \mathcal{N} \subseteq \bigcup \mathcal{N}$, we have $0 \in \bigcup \mathcal{N}$. Let $x \in X$ and $y \in \bigcup \mathcal{N}$. Then $y \in N$ for some $N \in \mathcal{N}$. Since N is a near UP-filter of X, we have $x \cdot y \in N \subseteq \bigcup \mathcal{N}$. Hence, $\bigcup \mathcal{N}$ is a near UP-filter of X.

3. Neutrosophic $\mathcal{N}$-structures

We denote the family of all functions from a nonempty set X to the closed interval $[-1, 0]$ of the real line by $\mathcal{F}(X, [-1, 0])$. An element of $\mathcal{F}(X, [-1, 0])$ is called a *negative-valued function* from X to $[-1, 0]$ (briefly, $\mathcal{N}$-function on X). An ordered pair (X, f) of X and an $\mathcal{N}$-function f on X is called an $\mathcal{N}$*-structure*.

A *neutrosophic* $\mathcal{N}$*-structure* over a nonempty universe of discourse X [23] is defined to be the structure

$$X_N = \{(x, T_N(x), I_N(x), F_N(x)) \mid x \in X\}$$

where T_N, I_N and F_N are $\mathcal{N}$-functions on X which are called the *negative truth membership function*, the *negative indeterminacy membership function* and the *negative falsity membership function* on X, respectively.

For the sake of simplicity, we will use the notation X_N or $X_N = (X, T_N, I_N, F_N)$ instead of the neutrosophic $\mathcal{N}$-structure [16].

Definition 3.1 Let X_N be a neutrosophic $\mathcal{N}$-structure over a nonempty set X. The neutrosophic $\mathcal{N}$-structure $\overline{X}_N = (X, \overline{T}_N, \overline{I}_N, \overline{F}_N)$ defined by

$$(\forall x \in X)\begin{pmatrix} \overline{T}_N(x) & = -1 - T_N(x) \\ \overline{I}_N(x) & = -1 - I_N(x) \\ \overline{F}_N(x) & = -1 - F_N(x) \end{pmatrix} \tag{3.1}$$

is called the *complement* of X_N in X.

Remark 3.2 *For all neutrosophic* $\mathcal{N}$*-structure* X_N *over a nonempty set* X, *we have* $X_N = \overline{\overline{X}}_N$.

Lemma 3.3 [33] *Let* f *be an* $\mathcal{N}$*-function on a nonempty set* X. *Then the following statements hold:*

1. $(\forall x, y \in X)(-1-\max\{f(x), f(y)\} = \min\{-1-f(x), -1-f(y)\})$, and
2. $(\forall x, y \in X)(-1-\min\{f(x), f(y)\} = \max\{-1-f(x), -1-f(y)\})$.

The following lemmas are easily proved

Lemma 3.4 *Let f be an N -function on a nonempty set X. Then the following statements hold:*

1. $(\forall x, y, z \in X)(\overline{f}(x) \geq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \max\{f(y), f(z)\})$,
2. $(\forall x, y, z \in X)(\overline{f}(x) \leq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \max\{f(y), f(z)\})$,
3. $(\forall x, y, z \in X)(\overline{f}(x) \geq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \min\{f(y), f(z)\})$, and
4. $(\forall x, y, z \in X)(\overline{f}(x) \leq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \min\{f(y), f(z)\})$.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Now, we introduce the notions of neutrosophic N -UP-subalgebras, neutrosophic N -near UP-filters, neutrosophic N -UP-filters, neutrosophic N -UP-ideals, and neutrosophic N -strongly UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 3.5 A neutrosophic N -structure X_N over X is called a *neutrosophic N -UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(T_N(x \cdot y) \leq \max\{T_N(x), T_N(y)\}), \tag{3.2}$$

$$(\forall x, y \in X)(I_N(x \cdot y) \geq \min\{I_N(x), I_N(y)\}), \tag{3.3}$$

$$(\forall x, y \in X)(F_N(x \cdot y) \leq \max\{F_N(x), F_N(y)\}). \tag{3.4}$$

Example 3.6 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	4
3	0	0	2	0	4
4	0	0	0	0	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.8,\ I_N(0) = -0.3,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.6,\ I_N(1) = -0.7,\ F_N(1) = -0.8,$$
$$T_N(2) = -0.4,\ I_N(2) = -0.8,\ F_N(2) = -0.7,$$
$$T_N(3) = -0.1,\ I_N(3) = -0.5,\ F_N(3) = -0.5,$$
$$T_N(4) = -0.2,\ I_N(4) = -0.9,\ F_N(4) = -0.3.$$

Hence, X_N is a neutrosophic N -UP-subalgebra of X.

Definition 3.7 A neutrosophic N -structure X_N over X is called a *neutrosophic* N *-near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(T_N(0) \leq T_N(x)), \tag{3.5}$$

$$(\forall x \in X)(I_N(0) \geq I_N(x)), \tag{3.6}$$

$$(\forall x \in X)(F_N(0) \leq F_N(x)), \tag{3.7}$$

$$(\forall x, y \in X)(T_N(x \cdot y) \leq T_N(y)), \tag{3.8}$$

$$(\forall x, y \in X)(I_N(x \cdot y) \geq I_N(y)), \tag{3.9}$$

$$(\forall x, y \in X)(F_N(x \cdot y) \leq F_N(y)). \tag{3.10}$$

Example 3.8 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	2
2	0	1	0	3	1
3	0	1	2	0	4
4	0	0	0	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.8,\ I_N(0) = -0.3,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.6,\ I_N(1) = -0.7,\ F_N(1) = -0.6,$$
$$T_N(2) = -0.8,\ I_N(2) = -0.8,\ F_N(2) = -0.7,$$
$$T_N(3) = -0.1,\ I_N(3) = -0.5,\ F_N(3) = -0.5,$$
$$T_N(4) = -0.3,\ I_N(4) = -0.8,\ F_N(4) = -0.3.$$

Hence, X_N is a neutrosophic N -near UP-filter of X.

Definition 3.9 A neutrosophic N -structure X_N over X is called a *neutrosophic* N *-UP-filter* of X if it satisfies the following conditions: (3.5), (3.6), (3.7), and

$$(\forall x, y \in X)(T_N(y) \leq \max\{T_N(x \cdot y), T_N(x)\}), \tag{3.11}$$

$$(\forall x, y \in X)(I_N(y) \geq \min\{I_N(x \cdot y), I_N(x)\}), \tag{3.12}$$

$$(\forall x, y \in X)(F_N(y) \leq \max\{F_N(x \cdot y), F_N(x)\}). \tag{3.13}$$

Example 3.10 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	0	0
2	0	1	0	0	4
3	0	1	2	0	4
4	0	1	2	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.9,\ I_N(0) = -0.2,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.5,\ I_N(1) = -0.8,\ F_N(1) = -0.6,$$
$$T_N(2) = -0.2,\ I_N(2) = -0.6,\ F_N(2) = -0.3,$$
$$T_N(3) = -0.6,\ I_N(3) = -0.3,\ F_N(3) = -0.7,$$

$$T_N(4) = -0.7,\ I_N(4) = -0.3,\ F_N(4) = -0.8.$$

Hence, X_N is a neutrosophic N -UP-filter of X.

Definition 3.11 A neutrosophic N -structure X_N over X is called a *neutrosophic* N *-UP-ideal* of X if it satisfies the following conditions: (3.5), (3.6), (3.7), and

$$(\forall x, y, z \in X)(T_N(x \cdot z) \leq \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}), \tag{3.14}$$

$$(\forall x, y, z \in X)(I_N(x \cdot z) \geq \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}), \tag{3.15}$$

$$(\forall x, y, z \in X)(F_N(x \cdot z) \leq \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}). \tag{3.16}$$

Example 3.12 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

·	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	4
3	0	0	2	0	4
4	0	1	2	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.8,\ I_N(0) = -0.3,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.5,\ I_N(1) = -0.6,\ F_N(1) = -0.8,$$
$$T_N(2) = -0.4,\ I_N(2) = -0.8,\ F_N(2) = -0.7,$$
$$T_N(3) = -0.1,\ I_N(3) = -0.7,\ F_N(3) = -0.5,$$
$$T_N(4) = -0.2,\ I_N(4) = -0.8,\ F_N(4) = -0.3.$$

Hence, X_N is a neutrosophic N -UP-ideal of X.

Definition 3.13 A neutrosophic N -structure X_N over X is called a *neutrosophic* N *-strongly UP-ideal* of X if it satisfies the following conditions: (3.5), (3.6), (3.7), and

$$(\forall x, y, z \in X)(T_N(x) \leq \max\{T_N((z \cdot y) \cdot (z \cdot x)), T_N(y)\}), \tag{3.17}$$

$$(\forall x, y, z \in X)(I_N(x) \geq \min\{I_N((z \cdot y) \cdot (z \cdot x)), I_N(y)\}), \tag{3.18}$$

$$(\forall x, y, z \in X)(F_N(x) \leq \max\{F_N((z \cdot y) \cdot (z \cdot x)), F_N(y)\}). \tag{3.19}$$

Example 3.14 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

·	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	2
2	0	1	0	3	1
3	0	1	2	0	4
4	0	0	0	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$(\forall x \in X)\begin{pmatrix} T_N(x) & = -1 \\ I_N(x) & = -0.3 \\ F_N(x) & = -0.7 \end{pmatrix}.$$

Hence, X_N is neutrosophic N -strongly UP-ideal of X.

Definition 3.15 A neutrosophic N -structure X_N over X is said to be *constant* if X_N is a constant function from X to $[-1,0]^3$. That is, T_N, I_N, and F_N are constant functions from X to $[-1,0]$.

Theorem 3.16 Every neutrosophic N -UP-subalgebra of X satisfies the conditions (3.5), (3.6), and (3.7).

Proof. Assume that X_N is a neutrosophic N -UP-subalgebra of X. Then for all $x \in X$, by Proposition 2.5 (1), (3.2), (3.3), and (3.4), we have

$$T_N(0) = T_N(x \cdot x) \leq \max\{T_N(x), T_N(x)\} = T_N(x),$$
$$I_N(0) = I_N(x \cdot x) \geq \min\{I_N(x), I_N(x)\} = I_N(x),$$
$$F_N(0) = F_N(x \cdot x) \leq \max\{F_N(x), F_N(x)\} = F_N(x).$$

Hence, X_N satisfies the conditions (3.5), (3.6), and (3.7).

Theorem 3.17 A neutrosophic N -structure X_N over X is constant if and only if it is a neutrosophic N -strongly UP-ideal of X.

Proof. Assume that X_N is constant. Then for all $x \in X$, $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$ and so $T_N(0) \leq T_N(x), I_N(0) \geq I_N(x)$, and $F_N(0) \leq F_N(x)$. Next, for all $x, y, z \in X$,

$$T_N(x) = T_N(0) = \max\{T_N(0), T_N(0)\} = \max\{T_N((z \cdot y) \cdot (z \cdot x)), T_N(y)\},$$
$$I_N(x) = I_N(0) = \min\{I_N(0), I_N(0)\} = \min\{I_N((z \cdot y) \cdot (z \cdot x)), I_N(y)\},$$
$$F_N(x) = F_N(0) = \max\{F_N(0), F_N(0)\} = \max\{F_N((z \cdot y) \cdot (z \cdot x)), F_N(y)\}.$$

Hence, X_N is a neutrosophic N -strongly UP-ideal of X.

Conversely, assume that X_N is a neutrosophic N -strongly UP-ideal of X. For any $x \in X$, by Proposition 2.5 (1), (UP-2), (UP-3), (3.17), (3.18), and (3.19), we have

$$\begin{aligned} T_N(x) &\leq \max\{T_N((x \cdot 0) \cdot (x \cdot x)), T_N(0)\} = \max\{T_N(0 \cdot (x \cdot x)), T_N(0)\} = \max\{T_N(x \cdot x), T_N(0)\} \\ &= \max\{T_N(0), T_N(0)\} = T_N(0), \end{aligned}$$
$$\begin{aligned} I_N(x) &\geq \min\{I_N((x \cdot 0) \cdot (x \cdot x)), I_N(0)\} = \min\{I_N(0 \cdot (x \cdot x)), I_N(0)\} = \min\{I_N(x \cdot x), I_N(0)\} \\ &= \min\{I_N(0), I_N(0)\} = I_N(0), \end{aligned}$$
$$\begin{aligned} F_N(x) &\leq \max\{F_N((x \cdot 0) \cdot (x \cdot x)), F_N(0)\} = \max\{F_N(0 \cdot (x \cdot x)), F_N(0)\} = \max\{F_N(x \cdot x), F_N(0)\} \\ &= \max\{F_N(0), F_N(0)\} = F_N(0). \end{aligned}$$

Thus $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$ for all $x \in X$. Hence, X_N is constant.

Theorem 3.18 Every neutrosophic N -strongly UP-ideal of X is a neutrosophic N -UP-ideal.

Proof. Assume that X_N is a neutrosophic N -strong UP-ideal of X. Then X_N satisfies the conditions (3.5), (3.6), and (3.7). By Theorem 3.17, we have X_N is constant. Then for all $x \in X$, $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$. By Proposition 2.5 (5), (UP-3), (3.5), (3.6), (3.7), (3.17), (3.18), and (3.19), we have

$$\begin{aligned} T_N(x \cdot z) &= \max\{T_N((z \cdot y) \cdot (z \cdot (x \cdot z))), T_N(y)\} = \max\{T_N((z \cdot y) \cdot 0), T_N(y)\} = \max\{T_N(0), T_N(y)\} = T_N(y) \\ &\leq \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}, \end{aligned}$$
$$\begin{aligned} I_N(x \cdot z) &= \min\{I_N((z \cdot y) \cdot (z \cdot (x \cdot z))), I_N(y)\} = \min\{I_N((z \cdot y) \cdot 0), I_N(y)\} = \min\{I_N(0), I_N(y)\} = I_N(y) \\ &\geq \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}, \end{aligned}$$
$$\begin{aligned} F_N(x \cdot z) &= \max\{F_N((z \cdot y) \cdot (z \cdot (x \cdot z))), F_N(y)\} = \max\{F_N((z \cdot y) \cdot 0), F_N(y)\} = \max\{F_N(0), F_N(y)\} = F_N(y) \\ &\leq \max F_N(x \cdot (y \cdot z)), F_N(y). \end{aligned}$$

Hence, X_N is a neutrosophic N -UP-ideal of X .

The following example show that the converse of Theorem 3.18 is not true.

Example 3.19 Let $X = \{0,1,2,3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $(X,\cdot,0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.6,\ I_N(0) = -0.1,\ F_N(0) = -0.7,$$
$$T_N(1) = -0.4,\ I_N(1) = -0.5,\ F_N(1) = -0.5,$$
$$T_N(2) = -0.3,\ I_N(2) = -0.4,\ F_N(2) = -0.4,$$
$$T_N(3) = -0.2,\ I_N(3) = -0.4,\ F_N(3) = -0.3.$$

Hence, X_N is a neutrosophic N -UP-ideal of X . Since X_N is not constant, it follows from Theorem 3.17 that it is not a neutrosophic N -strongly UP-ideal of X .

Theorem 3.20 Every neutrosophic N -UP-ideal of X is a neutrosophic N -UP-filter.

Proof. Assume that X_N is a neutrosophic N -UP-ideal of X . Then X_N satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$. By (UP-2), (3.14), (3.15), and (3.16), we have

$$T_N(y) = T_N(0\cdot y) \le \max\{T_N(0\cdot(x\cdot y)), T_N(x)\} = \max\{T_N(x\cdot y), T_N(x)\},$$
$$I_N(y) = I_N(0\cdot y) \ge \min\{I_N(0\cdot(x\cdot y)), I_N(x)\} = \min\{I_N(x\cdot y), I_N(x)\},$$
$$F_N(y) = F_N(0\cdot y) \le \max\{F_N(0\cdot(x\cdot y)), F_N(x)\} = \max\{F_N(x\cdot y), F_N(x)\}.$$

Hence, X_N is a neutrosophic N -UP-filter of X .

The following example show that the converse of Theorem 3.20 is not true.

Example 3.21 Let $X = \{0,1,2,3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Then $(X,\cdot,0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.7,\ I_N(0) = -0.1,\ F_N(0) = -0.9,$$
$$T_N(1) = -0.6,\ I_N(1) = -0.5,\ F_N(1) = -0.8,$$
$$T_N(2) = -0.3,\ I_N(2) = -0.4,\ F_N(2) = -0.5,$$
$$T_N(3) = -0.3,\ I_N(3) = -0.4,\ F_N(3) = -0.5.$$

Hence, X_N is a neutrosophic N -UP-filter of X . Since $F_N(2\cdot 3) = -0.3 > -0.8 = \max\{F_N(2\cdot(1\cdot 3)), F_N(1)\}$, we have X_N is not a neutrosophic N -UP-ideal of X .

Theorem 3.22 Every neutrosophic N -UP-filter of X is a neutrosophic N -near UP-filter.

Proof. Assume that X_N is a neutrosophic N -UP-filter. Then X_N satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$. By Proposition 2.5 (5), (3.5), (3.6), (3.7), (3.11), (3.12), and (3.13), we have

$$T_N(x \cdot y) \leq \max\{T_N(y \cdot (x \cdot y)), T_N(y)\} = \max\{T_N(0), T_N(y)\} = T_N(y),$$
$$I_N(x \cdot y) \geq \min\{I_N(y \cdot (x \cdot y)), I_N(y)\} = \min\{I_N(0), I_N(y)\} = I_N(y),$$
$$F_N(x \cdot y) \leq \max\{F_N(y \cdot (x \cdot y)), F_N(y)\} = \max\{F_N(0), F_N(y)\} = F_N(y).$$

Hence, X_N is a neutrosophic N -near UP-filter of X .

The following example show that the converse of Theorem 3.22 is not true.

Example 3.23 Let $X = \{0,1,2,3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	1	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.9,\ I_N(0) = -0.3,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.5,\ I_N(1) = -0.7,\ F_N(1) = -0.7,$$
$$T_N(2) = -0.2,\ I_N(2) = -0.8,\ F_N(2) = -0.6,$$
$$T_N(3) = -0.1,\ I_N(3) = -0.5,\ F_N(3) = -0.3.$$

Hence, X_N is a neutrosophic N -near UP-filter of X . Since $I_N(2) = -0.8 < -0.7$ $= \min\{I_N(1 \cdot 2), I_N(1)\}$, we have X_N is not a neutrosophic N -UP-filter of X .

Theorem 3.24 Every neutrosophic N -near UP-filter of X is a neutrosophic N -UP-subalgebra.

Proof. Assume that X_N is a neutrosophic N -near UP-filter of X . Then for all $x, y \in X$, by (3.8), (3.9), and (3.10), we have

$$T_N(x \cdot y) \leq T_N(y) \leq \max\{T_N(x), T_N(y)\},$$
$$I_N(x \cdot y) \geq I_N(y) \geq \min\{I_N(x), I_N(y)\},$$
$$F_N(x \cdot y) \leq F_N(y) \leq \max\{F_N(x), F_N(y)\}.$$

Hence, X_N is a neutrosophic N -UP-subalgebra of X .

The following example show that the converse of Theorem 3.24 is not true.

Example 3.25 Let $X = \{0,1,2,3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	2
2	0	0	0	2
3	0	0	0	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.8,\ I_N(0) = -0.3,\ F_N(0) = -0.8,$$
$$T_N(1) = -0.6,\ I_N(1) = -0.6,\ F_N(1) = -0.8,$$
$$T_N(2) = -0.4,\ I_N(2) = -0.5,\ F_N(2) = -0.7,$$
$$T_N(3) = -0.1,\ I_N(3) = -0.7,\ F_N(3) = -0.5.$$

Hence, X_N is a neutrosophic N -UP-subalgebra of X. Since $I_N(1\cdot 2) = -0.6 < -0.5 = I_N(2)$, we have X_N is not a neutrosophic N -near UP-filter of X.

By Theorems 3.18, 3.20, 3.22, and 3.24 and Examples 3.19, 3.21, 3.23, and 3.25, we have that the notion of neutrosophic N -UP-subalgebras is a generalization of neutrosophic N -near UP-filters, neutrosophic N -near UP-filters is a generalization of neutrosophic N -UP-filters, neutrosophic N -UP-filters is a generalization of neutrosophic N -UP-ideals, and neutrosophic N -UP-ideals is a generalization of neutrosophic N -strongly UP-ideals. Moreover, by Theorem 3.17, we obtain that neutrosophic N -strongly UP-ideals and constant neutrosophic N -structures coincide.

Theorem 3.26 If X_N is a neutrosophic N -UP-subalgebra of X satisfying the following condition:

$$(\forall x, y \in X)\left(x\cdot y \neq 0 \Rightarrow \begin{cases} T_N(x) \le T_N(y) \\ I_N(x) \ge I_N(y) \\ F_N(x) \le F_N(y) \end{cases} \right), \tag{3.20}$$

then X_N is a neutrosophic N -near UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -UP-subalgebra of X satisfying the condition (3.20). By Theorem 3.16, we have X_N satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$.

Case 1: $x\cdot y = 0$. Then, by (3.5), (3.6), and (3.7), we have

$$T_N(x\cdot y) = T_N(0) \le T_N(y),\ I_N(x\cdot y) = I_N(0) \ge I_N(y),\ F_N(x\cdot y) = F_N(0) \le F_N(y).$$

Case 2: $x\cdot y \neq 0$. Then, by (3.2), (3.3), (3.4), and (3.20), we have

$$T_N(x\cdot y) \le \max\{T_N(x), T_N(y)\} = T_N(y),\ I_N(x\cdot y) \ge \min\{I_N(x), I_N(y)\} = I_N(y),$$
$$F_N(x\cdot y) \le \max\{F_N(x), F_N(y)\} = F_N(y).$$

Hence, X_N is a neutrosophic N -near UP-filter of X.

Theorem 3.27 If X_N is a neutrosophic N -near UP-filter of X satisfying the following condition:

$$T_N = I_N = F_N, \tag{3.21}$$

then X_N is a neutrosophic N -UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -near UP-filter of X satisfying the condition (3.21). Then X_N satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$. Then, by (3.8), (3.9), and (3.21), we have

$$\max\{T_N(x\cdot y), T_N(x)\} = \max\{I_N(x\cdot y), T_N(x)\} \ge \max\{I_N(y), T_N(x)\} = \max\{T_N(y), T_N(x)\} \ge T_N(y),$$
$$\min\{I_N(x\cdot y), I_N(x)\} = \min\{T_N(x\cdot y), I_N(x)\} \le \min\{T_N(y), I_N(x)\} = \min\{I_N(y), I_N(x)\} \le I_N(y),$$
$$\max\{F_N(x\cdot y), F_N(x)\} = \max\{I_N(x\cdot y), F_N(x)\} \ge \max\{I_N(y), F_N(x)\} = \max\{F_N(y), F_N(x)\} \ge F_N(y).$$

Hence, X_N is a neutrosophic N -UP-filter of X.

Theorem 3.28 If X_N is a neutrosophic N -UP-filter of X satisfying the following condition:

$$\left(\forall x, y, z \in X\right)\begin{pmatrix} T_N(y \cdot (x \cdot z)) & = T_N(x \cdot (y \cdot z)) \\ I_N(y \cdot (x \cdot z)) & = I_N(x \cdot (y \cdot z)) \\ F_N(y \cdot (x \cdot z)) & = F_N(x \cdot (y \cdot z)) \end{pmatrix}, \tag{3.22}$$

then X_N is a neutrosophic N -UP-ideal of X.

Proof. Assume that X_N is a neutrosophic N -UP-filter of X satisfying the condition (3.22). Then X_N satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y, z \in X$. Then, by (3.11), (3.12), (3.13), and (3.22), we have

$$T_N(x \cdot z) \leq \max\{T_N(y \cdot (x \cdot z)), T_N(y)\} = \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}, T_N$$
$$I_N(x \cdot z) \geq \min\{I_N(y \cdot (x \cdot z)), I_N(y)\} = \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}, I_N$$
$$F_N(x \cdot z) \leq \max\{F_N(y \cdot (x \cdot z)), F_N(y)\} = \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}. F_N$$

Hence, X_N is a neutrosophic N -UP-ideal of X.

Theorem 3.29 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$\left(\forall x, y, z \in X\right)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(z) \leq \max\{T_N(x), T_N(y)\} \\ I_N(z) \geq \min\{I_N(x), I_N(y)\} \\ F_N(z) \leq \max\{F_N(x), F_N(y)\} \end{cases} \right), \tag{3.23}$$

then X_N is a neutrosophic N -UP-subalgebra of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (3.23). Let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.23) that

$$T_N(x \cdot y) \leq \max\{T_N(x), T_N(y)\},\ I_N(x \cdot y) \geq \min\{I_N(x), I_N(y)\},\ F_N(x \cdot y) \leq \max\{F_N(x), F_N(y)\}.$$

Hence, X_N is a neutrosophic N -UP-subalgebra of X.

Theorem 3.30 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$\left(\forall x, y, z \in X\right)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(z) \leq T_N(y) \\ I_N(z) \geq I_N(y) \\ F_N(z) \leq F_N(y) \end{cases} \right), \tag{3.24}$$

then X_N is a neutrosophic N -near UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (3.24). Let $x \in X$. By (UP-2) and Proposition 2.5 (1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leq x \cdot x$. It follows from (3.24) that $T_N(0) \leq T_N(x), I_N(0) \geq I_N(x)$, and $F_N(0) \leq F_N(x)$. Next, let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.24) that $T_N(x \cdot y) \leq T_N(y), I_N(x \cdot y) \geq I_N(y)$, and $F_N(x \cdot y) \leq F_N(y)$. Hence, X_N is a neutrosophic N -near UP-filter of X.

Theorem 3.31 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$\left(\forall x, y, z \in X\right)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(y) \leq \max\{T_N(z), T_N(x)\} \\ I_N(y) \geq \min\{I_N(z), I_N(x)\} \\ F_N(y) \leq \max\{F_N(z), F_N(x)\} \end{cases} \right), \tag{3.25}$$

then X_N is a neutrosophic N -UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (3.25). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (3.25) that

$$T_N(0) \leq \max\{T_N(x), T_N(x)\} = T_N(x),\ I_N(0) \geq \min\{I_N(x), I_N(x)\} = I_N(x),$$
$$F_N(0) \leq \max\{F_N(x), F_N(x)\} = F_N(x).$$

Next, let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.25) that

$$T_N(y) \leq \max\{T_N(x \cdot y), T_N(x)\},\ I_N(y) \geq \min\{I_N(x \cdot y), I_N(x)\},\ F_N(y) \leq \max\{F_N(x \cdot y), F_N(x)\}.$$

Hence, X_N is a neutrosophic N -UP-filter of X.

Theorem 3.32 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \left\{ \begin{array}{l} T_N(x \cdot z) \leq \max\{T_N(a), T_N(y)\} \\ I_N(x \cdot z) \geq \min\{I_N(a), I_N(y)\} \\ F_N(x \cdot z) \leq \max\{F_N(a), F_N(y)\} \end{array} \right. \right), \tag{3.26}$$

then X_N is a neutrosophic N -UP-ideal of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (3.26). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (3.26) and (UP-2) that

$$T_N(0) = T_N(0 \cdot 0) \leq \max\{T_N(x), T_N(x)\} = T_N(x),\ I_N(0) = I_N(0 \cdot 0) \geq \min\{I_N(x), I_N(x)\} = I_N(x),$$
$$F_N(0) = F_N(0 \cdot 0) \leq \max\{F_N(x), F_N(x)\} = F_N(x).$$

Next, let $x, y, z \in X$. By Proposition 2.5 (1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (3.26) that

$$T_N(x \cdot z) \leq \max\{T_N(x \cdot (y \cdot z)), T_N(y)\},\ I_N(x \cdot z) \geq \min\{I_N(x \cdot (y \cdot z)), I_N(y)\},$$
$$F_N(x \cdot z) \leq \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}.$$

Hence, X_N is a neutrosophic N -UP-ideal of X.

For any fixed numbers $\alpha^-, \alpha^+, \beta^-, \beta^+, \gamma^-, \gamma^+ \in [-1, 0]$ such that $\alpha^- < \alpha^+, \beta^- < \beta^+, \gamma^- < \gamma^+$ and a nonempty subset G of X, a neutrosophic N -structure $X_N^G[^{\alpha^-, \beta^+, \gamma^-}_{\alpha^+, \beta^-, \gamma^+}] = (X, T_N^G[^{\alpha^-}_{\alpha^+}], I_N^G[^{\beta^+}_{\beta^-}], F_N^G[^{\gamma^-}_{\gamma^+}])$ over X where $T_N^G[^{\alpha^-}_{\alpha^+}], I_N^G[^{\beta^+}_{\beta^-}]$, and $F_N^G[^{\gamma^-}_{\gamma^+}]$ are N -functions on X which are given as follows:

$$T_N^G[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases} \quad I_N^G[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise,} \end{cases} \quad F_N^G[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise.} \end{cases}$$

Lemma 3.33 If the constant 0 of X is in a nonempty subset G of X, then a neutrosophic N - structure $X_N^G[^{\alpha^-, \beta^+, \gamma^-}_{\alpha^+, \beta^-, \gamma^+}]$ over X satisfies the conditions (3.5), (3.6), and (3.7).

Proof. If $0 \in G$, then $T_N^G[^{\alpha^-}_{\alpha^+}](0) = \alpha^-, I_N^G[^{\beta^+}_{\beta^-}](0) = \beta^+, F_N^G[^{\gamma^-}_{\gamma^+}](0) = \gamma^-$. Thus

$$(\forall x \in X)\left(\begin{array}{l} T_N^G[^{\alpha^-}_{\alpha^+}](0) = \alpha^- \leq T_N^G[^{\alpha^-}_{\alpha^+}](x) \\ I_N^G[^{\beta^+}_{\beta^-}](0) = \beta^+ \geq I_N^G[^{\beta^+}_{\beta^-}](x) \\ F_N^G[^{\gamma^-}_{\gamma^+}](0) = \gamma^- \leq F_N^G[^{\gamma^-}_{\gamma^+}](x) \end{array} \right).$$

Hence, $X_N^G[^{\alpha^-, \beta^+, \gamma^-}_{\alpha^+, \beta^-, \gamma^+}]$ satisfies the conditions (3.5), (3.6), and (3.7).

Lemma 3.34 If a neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X satisfies the condition (3.5) (resp., (3.6), (3.7)), then the constant 0 of X is in a nonempty subset G of X.

Proof. Assume that the neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X satisfies the condition (3.5). Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](0) \leq T_N^G[{}^{\alpha^-}_{\alpha^+}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus $T_N^G[{}^{\alpha^-}_{\alpha^+}](g) = \alpha^-$, so $T_N^G[{}^{\alpha^-}_{\alpha^+}](0) \leq T_N^G[{}^{\alpha^-}_{\alpha^+}](g) = \alpha^- \leq T_N^G[{}^{\alpha^-}_{\alpha^+}](0)$, that is, $T_N^G[{}^{\alpha^-}_{\alpha^+}](0) = \alpha^-$. Hence, $0 \in G$.

Theorem 3.35 A neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X is a neutrosophic N -UP-subalgebra of X if and only if a nonempty subset G of X is a UP-subalgebra of X.

Proof. Assume that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-subalgebra of X. Let $x, y \in G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](x) = \alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](y)$. Thus, by (3.2), we have

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x \cdot y) \leq \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \leq T_N^G[{}^{\alpha^-}_{\alpha^+}](x \cdot y)$$

and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x) = \alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](y),\ I_N^G[{}^{\beta^+}_{\beta^-}](x) = \beta^+ = I_N^G[{}^{\beta^+}_{\beta^-}](y),\ F_N^G[{}^{\gamma^-}_{\gamma^+}](x) = \gamma^- = F_N^G[{}^{\gamma^-}_{\gamma^+}](y).$$

Thus

$$\max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^-,\ \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x), I_N^G[{}^{\beta^+}_{\beta^-}](y)\} = \beta^+,\ \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\} = \gamma^-.$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-, I_N^G[{}^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^-$. Hence,

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^- \leq \alpha^- = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\},\ I_N^G[{}^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+ \geq \beta^+ = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x), I_N^G[{}^{\beta^+}_{\beta^-}](y)\},$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^- \leq \gamma^- = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x) = \alpha^+ \text{ or } T_N^G[{}^{\alpha^-}_{\alpha^+}](y) = \alpha^+,\ I_N^G[{}^{\beta^+}_{\beta^-}](x) = \beta^- \text{ or } I_N^G[{}^{\beta^+}_{\beta^-}](y) = \beta^-,\ F_N^G[{}^{\gamma^-}_{\gamma^+}](x) = \gamma^+ \text{ or } F_N^G[{}^{\gamma^-}_{\gamma^+}](y) = \gamma^+.$$

Thus

$$\max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^+,\ \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x), I_N^G[{}^{\beta^+}_{\beta^-}](y)\} = \beta^-,\ \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\} = \gamma^+.$$

Therefore,

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)\leq \alpha^+ = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\},\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y)\geq \beta^- = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x), I_N^G[{}^{\beta^+}_{\beta^-}](y)\},$$

$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y)\leq \gamma^+ = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\}$. Hence, $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-subalgebra of X.

Theorem 3.36 A neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X is a neutrosophic N -near UP-filter of X if and only if a nonempty subset G of X is a near UP-filter of X.

Proof. Assume that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is neutrosophic N -near UP-filter of X. Since $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (3.5), it follows from Lemma 3.34 that $0\in G$. Next, let $x\in X$ and $y\in G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](y)=\alpha^-$. Thus, by (3.8), we have $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)\leq T_N^G[{}^{\alpha^-}_{\alpha^+}](y)=\alpha^-\leq T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)$

and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)=\alpha^-$. Thus $x\cdot y\in G.$ Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0\in G$, it follows from Lemma 3.33 that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x,y\in X$.

Case 1: $y\in G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](y)=\alpha^-, I_N^G[{}^{\beta^+}_{\beta^-}](y)=\beta^+$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](y)=\gamma^-$. Since G is a near UP-filter of X, we have $x\cdot y\in G$ and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)=\alpha^-, I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y)=\beta^+$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y)=\gamma^-$. Thus

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)=\alpha^-\leq\alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](y),\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y)=\beta^+\geq\beta^+ = I_N^G[{}^{\beta^+}_{\beta^-}](y),$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y)=\gamma^-\leq\gamma^- = F_N^G[{}^{\gamma^-}_{\gamma^+}](y).$$

Case 2: $y\notin G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](y)=\alpha^+, I_N^G[{}^{\beta^+}_{\beta^-}](y)=\beta^-$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](y)=\gamma^+$. Thus

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)\leq\alpha^+ = T_N^G[{}^{\alpha^-}_{\alpha^+}](y),\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y)\geq\beta^- = I_N^G[{}^{\beta^+}_{\beta^-}](y),\ F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y)\leq\gamma^+ = F_N^G[{}^{\gamma^-}_{\gamma^+}](y).$$

Hence, $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -near UP-filter of X.

Theorem 3.37 A neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X is a neutrosophic N -UP-filter of X if and only if a nonempty subset G of X is a UP-filter of X.

Proof. Assume that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-filter of X. Since $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (3.5), it follows from Lemma 3.34 that $0\in G$. Next, let $x,y\in X$ be such that $x\cdot y\in G$ and $x\in G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y)=\alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](x)$. Thus, by (3.11), we have

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](y) \le \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y), T_N^G[{}^{\alpha^-}_{\alpha^+}](x)\} = \alpha^- \le T_N^G[{}^{\alpha^-}_{\alpha^+}](y)$$

and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](y) = \alpha^-$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 3.33 that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$.

Case 1: $x\cdot y \in G$ and $x \in G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y) = \alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](x),\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y) = \beta^+ = I_N^G[{}^{\beta^+}_{\beta^-}](x),\ F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y) = \gamma^- = F_N^G[{}^{\gamma^-}_{\gamma^+}](x).$$

Since G is a UP-filter of X, we have $y \in G$ and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](y) = \alpha^-, I_N^G[{}^{\beta^+}_{\beta^-}](y) = \beta^+$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](y) = \gamma^-$. Thus

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](y) = \alpha^- \le \alpha^- = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y), T_N^G[{}^{\alpha^-}_{\alpha^+}](x)\},\ I_N^G[{}^{\beta^+}_{\beta^-}](y) = \beta^+ \ge \beta^+ = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y), I_N^G[{}^{\beta^+}_{\beta^-}](x)\},$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](y) = \gamma^- \le \gamma^- = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y), F_N^G[{}^{\gamma^-}_{\gamma^+}](x)\}.$$

Case 2: $x\cdot y \notin G$ or $x \notin G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y) = \alpha^+ \text{ or } T_N^G[{}^{\alpha^-}_{\alpha^+}](x) = \alpha^+,\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y) = \beta^- \text{ or } I_N^G[{}^{\beta^+}_{\beta^-}](x) = \beta^-,$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y) = \gamma^+ \text{ or } F_N^G[{}^{\gamma^-}_{\gamma^+}](x) = \gamma^+.$$

Thus

$$\max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y), T_N^G[{}^{\alpha^-}_{\alpha^+}](x)\} = \alpha^+,\ \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y), I_N^G[{}^{\beta^+}_{\beta^-}](x)\} = \beta^-,\ \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y), F_N^G[{}^{\gamma^-}_{\gamma^+}](x)\} = \gamma^+.$$

Therefore,

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](y) \le \alpha^+ = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot y), T_N^G[{}^{\alpha^-}_{\alpha^+}](x)\},\ I_N^G[{}^{\beta^+}_{\beta^-}](y) \ge \beta^- = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot y), I_N^G[{}^{\beta^+}_{\beta^-}](x)\},$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](y) \le \gamma^+ = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot y), F_N^G[{}^{\gamma^-}_{\gamma^+}](x)\}.$$

Hence, $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-filter of X.

Theorem 3.38 A neutrosophic N -structure $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X is a neutrosophic N -UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.

Proof. Assume that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-ideal of X. Since $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (3.5), it follows from Lemma 3.34 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x\cdot(y\cdot z) \in G$ and $y \in G$. Then $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)) = \alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](y)$. Thus, by (3.17), we have

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z) \le \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \le T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z)$$

and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^-$. Thus $x\cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 3.33 that $X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y, z \in X$.

Case 1: $x\cdot(y\cdot z) \in G$ and $y \in G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)) = \alpha^- = T_N^G[{}^{\alpha^-}_{\alpha^+}](y),\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)) = \beta^+ = I_N^G[{}^{\beta^+}_{\beta^-}](y),\ F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)) = \gamma^- = F_N^G[{}^{\gamma^-}_{\gamma^+}](y).$$

Thus

$$\max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^-,\ \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), I_N^G[{}^{\beta^+}_{\beta^-}](y)\} = \beta^+,$$

$$\max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\} = \gamma^-.$$

Since G is a UP-ideal of X, we have $x\cdot z \in G$ and so $T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^-, I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot z) = \beta^+$, and $F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot z) = \gamma^-$. Thus

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^- \le \alpha^- = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\},$$

$$I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot z) = \beta^+ \ge \beta^+ = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), I_N^G[{}^{\beta^+}_{\beta^-}](y)\},$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot z) = \gamma^- \le \gamma^- = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x\cdot(y\cdot z) \notin G$ or $y \notin G$. Then

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)) = \alpha^+ \text{ or } T_N^G[{}^{\alpha^-}_{\alpha^+}](y) = \alpha^+,\ I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)) = \beta^- \text{ or } I_N^G[{}^{\beta^+}_{\beta^-}](y) = \beta^-,$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)) = \gamma^+ \text{ or } F_N^G[{}^{\gamma^-}_{\gamma^+}](y) = \gamma^+.$$

Thus

$$\max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\} = \alpha^+,\ \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), I_N^G[{}^{\beta^+}_{\beta^-}](y)\} = \beta^-,$$

$$\max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\} = \gamma^+.$$

Therefore,

$$T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot z) \le \alpha^+ = \max\{T_N^G[{}^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), T_N^G[{}^{\alpha^-}_{\alpha^+}](y)\},$$

$$I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot z) \ge \beta^- = \min\{I_N^G[{}^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), I_N^G[{}^{\beta^+}_{\beta^-}](y)\},$$

$$F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot z) \le \gamma^+ = \max\{F_N^G[{}^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), F_N^G[{}^{\gamma^-}_{\gamma^+}](y)\}.$$

Hence, $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -UP-ideal of X .

Theorem 3.39 A neutrosophic N -structure $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ over X is a neutrosophic N -strongly UP-ideal of X if and only if a nonempty subset G of X is a strongly UP-ideal of X .

Proof. Assume that $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -strongly UP-ideal of X . By Theorem 3.17, we have $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is constant, that is, $T_N^G[^{\alpha^-}_{\alpha^+}]$ is constant. Since G is nonempty, we have $T_N^G[^{\alpha^-}_{\alpha^+}](x) = \alpha^-$ for all $x \in X$. Thus $G = X$. Hence, G is a strongly UP-ideal of X .

Conversely, assume that G is a strongly UP-ideal of X . Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} T_N^G[^{\alpha^-}_{\alpha^+}](x) & = \alpha^- \\ I_N^G[^{\beta^+}_{\beta^-}](x) & = \beta^+ \\ F_N^G[^{\gamma^-}_{\gamma^+}](x) & = \gamma^- \end{pmatrix}.$$

Thus $T_N^G[^{\alpha^-}_{\alpha^+}], I_N^G[^{\beta^+}_{\beta^-}]$, and $F_N^G[^{\gamma^-}_{\gamma^+}]$ are constant, that is, $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is constant. By Theorem 3.17, we have $X_N^G[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a neutrosophic N -strongly UP-ideal of X .

4. Level subsets of a neutrosophic N -structure

In this section, we discuss the relationships among neutrosophic N -UP-subalgebras (resp., neutrosophic N -near UP-filters, neutrosophic N -UP-filters, neutrosophic N -UP-ideals, neutrosophic N -strongly UP-ideals) of UP-algebras and their level subsets.

Definition 4.1 [34] Let f be an N -function on a nonempty set X . For any $t \in [-1,0]$, the sets

$$U(f;t) = \{x \in X \mid f(x) \geq t\},\ L(f;t) = \{x \in X \mid f(x) \leq t\},\ E(f;t) = \{x \in X \mid f(x) = t\}$$

are called an *upper* t *-level subset*, a *lower* t *-level subset*, and an *equal* t *-level subset* of f , respectively.

Theorem 4.2 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-subalgebra of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, the sets $L(T_N;\alpha), U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-subalgebras of X if $L(T_N;\alpha), U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a neutrosophic N -UP-subalgebra of X . Let $\alpha,\beta,\gamma \in [-1,0]$ be such that $L(T_N;\alpha), U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x, y \in L(T_N;\alpha)$. Then $T_N(x) \leq \alpha$ and $T_N(y) \leq \alpha$, so α is an upper bound of $\{T_N(x), T_N(y)\}$. By (3.2), we have $T_N(x \cdot y) \leq \max\{T_N(x), T_N(y)\} \leq \alpha$. Thus $x \cdot y \in L(T_N;\alpha)$.

Let $x, y \in U(I_N;\beta)$. Then $I_N(x) \geq \beta$ and $I_N(y) \geq \beta$, so β is a lower bound of $\{I_N(x), I_N(y)\}$. By (3.3), we have $I_N(x \cdot y) \geq \min\{I_N(x), I_N(y)\} \geq \beta$. Thus $x \cdot y \in U(I_N;\beta)$.

Let $x, y \in L(F_N;\gamma)$. Then $F_N(x) \leq \gamma$ and $F_N(y) \leq \gamma$, so γ is an upper bound of $\{F_N(x), F_N(y)\}$. By (3.4), we have $F_N(x \cdot y) \leq \max\{F_N(x), F_N(y)\} \leq \gamma$. Thus $x \cdot y \in L(F_N;\gamma)$.

Hence, $L(T_N;\alpha), U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-subalgebras of X .

Conversely, assume that for all $\alpha,\beta,\gamma\in[-1,0]$, the sets $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-subalgebras of X if $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x,y\in X$. Then $T_N(x),T_N(y)\in[-1,0]$. Choose $\alpha=\max\{T_N(x),T_N(y)\}$. Thus $T_N(x)\leq\alpha$ and $T_N(y)\leq\alpha$, so $x,y\in L(T_N;\alpha)\neq\emptyset$. By assumption, we have $L(T_N;\alpha)$ is a UP-subalgebra of X and so $x\cdot y\in L(T_N;\alpha)$. Thus $T_N(x\cdot y)\leq\alpha=\max\{T_N(x),T_N(y)\}$.

Let $x,y\in X$. Then $I_N(x),I_N(y)\in[-1,0]$. Choose $\beta=\min\{I_N(x),I_N(y)\}$. Thus $I_N(x)\geq\beta$ and $I_N(y)\geq\beta$, so $x,y\in U(I_N;\beta)\neq\emptyset$. By assumption, we have $U(I_N;\beta)$ is a UP-subalgebra of X and so $x\cdot y\in U(I_N;\beta)$. Thus $I_N(x\cdot y)\geq\beta=\min\{I_N(x),I_N(y)\}$.

Let $x,y\in X$. Then $F_N(x),F_N(y)\in[-1,0]$. Choose $\gamma=\max\{F_N(x),F_N(y)\}$. Thus $F_N(x)\leq\gamma$ and $F_N(y)\leq\gamma$, so $x,y\in L(F_N;\gamma)\neq\emptyset$. By assumption, we have $L(F_N;\gamma)$ is a UP-subalgebra of X and so $x\cdot y\in L(F_N;\gamma)$. Thus $F_N(x\cdot y)\leq\gamma=\max\{F_N(x),F_N(y)\}$.

Therefore, X_N is a neutrosophic N -UP-subalgebra of X.

Theorem 4.3 A neutrosophic N -structure X_N over X is a neutrosophic N -near UP-filter of X if and only if for all $\alpha,\beta,\gamma\in[-1,0]$, the sets $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are near UP-filters of X if $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a neutrosophic N -near UP-filter of X. Let $\alpha,\beta,\gamma\in[-1,0]$ be such that $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x\in L(T_N;\alpha)$. Then $T_N(x)\leq\alpha$. By (3.5), we have $T_N(0)\leq T_N(x)\leq\alpha$. Thus $0\in L(T_N;\alpha)$. Next, let $x\in X$ and $y\in L(T_N;\alpha)$. Then $T_N(y)\leq\alpha$. By (3.8), we have $T_N(x\cdot y)\leq T_N(y)\leq\alpha$. Thus $x\cdot y\in L(T_N;\alpha)$.

Let $x\in U(I_N;\beta)$. Then $I_N(x)\geq\beta$. By (3.6), we have $I_N(0)\geq I_N(x)\geq\beta$. Thus $0\in U(I_N;\beta)$. Next, let $x\in X$ and $y\in U(I_N;\beta)$. Then $I_N(y)\geq\beta$. By (3.9), we have $I_N(x\cdot y)\geq I_N(y)\geq\beta$. Thus $x\cdot y\in U(I_N;\beta)$.

Let $x\in L(F_N;\gamma)$. Then $F_N(x)\leq\gamma$. By (3.7), we have $F_N(0)\leq F_N(x)\leq\gamma$. Thus $0\in L(F_N;\gamma)$. Next, let $x\in X$ and $y\in L(F_N;\gamma)$. Then $F_N(y)\leq\gamma$. By (3.10), we have $F_N(x\cdot y)\leq F_N(y)\leq\gamma$. Thus $x\cdot y\in L(F_N;\gamma)$.

Hence, $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are near UP-filters of X.

Conversely, assume that for all $\alpha,\beta,\gamma\in[-1,0]$, the sets $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are near UP-filters of X if $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x\in X$. Then $T_N(x)\in[-1,0]$. Choose $\alpha=T_N(x)$. Thus $T_N(x)\leq\alpha$, so $x\in L(T_N;\alpha)\neq\emptyset$. By assumption, we have $L(T_N;\alpha)$ is a near UP-filter of X and so $0\in L(T_N;\alpha)$. Thus $T_N(0)\leq\alpha=T_N(x)$. Next, let $x,y\in X$. Then $T_N(y)\in[-1,0]$. Choose $\alpha=T_N(y)$. Thus $T_N(y)\leq\alpha$, so $y\in L(T_N;\alpha)\neq\emptyset$. By assumption, we have $L(T_N;\alpha)$ is a near UP-filter of X and so $x\cdot y\in L(T_N;\alpha)$. Thus $T_N(x\cdot y)\leq\alpha=T_N(y)$.

Let $x\in X$. Then $I_N(x)\in[-1,0]$. Choose $\beta=I_N(x)$. Thus $I_N(x)\geq\beta$, so $x\in U(I_N;\beta)\neq\emptyset$. By assumption, we have $U(I_N;\beta)$ is a near UP-filter of X and so $0\in U(I_N;\beta)$. Thus $I_N(0)\geq\beta=I_N(x)$. Next, let $x,y\in X$. Then $I_N(y)\in[-1,0]$. Choose $\beta=I_N(y)$. Thus $I_N(y)\geq\beta$, so $y\in U(I_N;\beta)\neq\emptyset$. By assumption, we have $U(I_N;\beta)$ is a near UP-filter of X and so $x\cdot y\in U(I_N;\beta)$. Thus $I_N(x\cdot y)\geq\beta=I_N(y)$.

Let $x\in X$. Then $F_N(x)\in[-1,0]$. Choose $\gamma=F_N(x)$. Thus $F_N(x)\leq\gamma$, so $x\in L(F_N;\gamma)\neq\emptyset$. By assumption, we have $L(F_N;\gamma)$ is a near UP-filter of X and so $0\in L(F_N;\gamma)$. Thus

$F_N(0) \leq \gamma = F_N(x)$. Next, let $x, y \in X$. Then $F_N(y) \in [-1, 0]$. Choose $\gamma = F_N(y)$. Thus $F_N(y) \leq \gamma$, so $y \in L(F_N; \gamma) \neq \varnothing$. By assumption, we have $L(F_N; \gamma)$ is a near UP-filter of X and so $x \cdot y \in L(F_N; \gamma)$. Thus $F_N(x \cdot y) \leq \gamma = F_N(y)$.

Therefore, X_N is a neutrosophic N -near UP-filter of X .

Theorem 4.4 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [-1, 0]$, the sets $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-filters of X if $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Proof. Assume that X_N is a neutrosophic N -UP-filter of X . Let $\alpha, \beta, \gamma \in [-1, 0]$ be such that $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Let $x \in L(T_N; \alpha)$. Then $T_N(x) \leq \alpha$. By (3.5), we have $T_N(0) \leq T_N(x) \leq \alpha$. Thus $0 \in L(T_N; \alpha)$. Next, let $x, y \in X$ be such that $x \cdot y \in L(T_N; \alpha)$ and $x \in L(T_N; \alpha)$. Then $T_N(x \cdot y) \leq \alpha$ and $T_N(x) \leq \alpha$, so α is an upper bound of $\{T_N(x \cdot y), T_N(x)\}$. By (3.11), we have $T_N(y) \leq \max\{T_N(x \cdot y), T_N(x)\} \leq \alpha$. Thus $y \in L(T_N; \alpha)$.

Let $x \in U(I_N; \beta)$. Then $I_N(x) \geq \beta$. By (3.5), we have $I_N(0) \geq I_N(x) \geq \beta$. Thus $0 \in U(I_N; \beta)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(I_N; \beta)$ and $x \in U(I_N; \beta)$. Then $I_N(x \cdot y) \geq \beta$ and $I_N(x) \geq \beta$, so β is a lower bound of $\{I_N(x \cdot y), I_N(x)\}$. By (3.12), we have $I_N(y) \geq \min\{I_N(x \cdot y), I_N(x)\} \geq \beta$ Thus $y \in U(I_N; \beta)$.

Let $x \in L(F_N; \gamma)$. Then $F_N(x) \leq \gamma$. By (3.5), we have $F_N(0) \leq F_N(x) \leq \gamma$. Thus $0 \in L(F_N; \gamma)$. Next, let $x, y \in X$ be such that $x \cdot y \in L(F_N; \gamma)$ and $x \in L(F_N; \gamma)$. Next, let $x, y \in L(F_N; \gamma)$ and $x \in L(F_N; \gamma)$. Then $F_N(x \cdot y) \leq \gamma$ and $F_N(x) \leq \gamma$, so γ is an upper bound of $\{F_N(x \cdot y), F_N(x)\}$. By (3.13), we have $F_N(y) \leq \max\{F_N(x \cdot y), F_N(x)\} \leq \gamma$. Thus $y \in L(F_N; \gamma)$.

Hence, $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-filters of X .

Conversely, assume that for all $\alpha, \beta, \gamma \in [-1, 0]$, the sets $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-filters of X if $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Let $x \in X$. Then $T_N(x) \in [-1, 0]$. Choose $\alpha = T_N(x)$. Thus $T_N(x) \leq \alpha$, so $x \in L(T_N; \alpha) \neq \varnothing$. By assumption, we have $L(T_N; \alpha)$ is a UP-filter of X and so $0 \in L(T_N; \alpha)$. Thus $T_N(0) \leq \alpha = T_N(x)$. Next, let $x, y \in X$. Then $T_N(x \cdot y), T_N(x) \in [-1, 0]$. Choose $\alpha = \max\{T_N(x \cdot y), T_N(x)\}$. Thus $T_N(x \cdot y) \leq \alpha$ and $T_N(x) \leq \alpha$, so $x \cdot y, x \in L(T_N; \alpha) \neq \varnothing$. By assumption, we have $L(T_N; \alpha)$ is a UP-filter of X and so $y \in L(T_N; \alpha)$. Thus $T_N(y) \leq \alpha = \max\{T_N(x \cdot y), T_N(x)\}$.

Let $x \in X$. Then $I_N(x) \in [-1, 0]$. Choose $\beta = I_N(x)$. Thus $I_N(x) \geq \beta$, so $x \in U(I_N; \beta) \neq \varnothing$. By assumption, we have $U(I_N; \beta)$ is a UP-filter of X and so $0 \in U(I_N; \beta)$. Thus $I_N(0) \geq \beta = I_N(x)$. Next, let $x, y \in X$. Then $I_N(x \cdot y), I_N(x) \in [-1, 0]$. Choose $\beta = \min\{I_N(x \cdot y), I_N(x)\}$. Thus $I_N(x \cdot y) \geq \beta$ and $I_N(x) \geq \beta$, so $x \cdot y, x \in U(I_N; \beta) \neq \varnothing$. By assumption, we have $U(I_N; \beta)$ is a UP-filter of X and so $y \in U(I_N; \beta)$. Thus $I_N(y) \geq \beta = \min\{I_N(x \cdot y), I_N(x)\}$.

Let $x \in X$. Then $F_N(x) \in [-1, 0]$. Choose $\gamma = F_N(x)$. Thus $F_N(x) \leq \gamma$, so $x \in L(F_N; \gamma) \neq \varnothing$. By assumption, we have $L(F_N; \gamma)$ is a UP-filter of X and so $0 \in L(F_N; \gamma)$. Thus $F_N(0) \leq \gamma = F_N(x)$. Next, let $x, y \in X$. Then $F_N(x \cdot y), F_N(x) \in [-1, 0]$. Choose $\gamma = \max\{F_N(x \cdot y), F_N(x)\}$. Thus $F_N(x \cdot y) \leq \gamma$ and $F_N(x) \leq \gamma$, so $x \cdot y, x \in L(F_N; \gamma) \neq \varnothing$. By assumption, we have $L(F_N; \gamma)$ is a UP-filter of X and so $y \in L(F_N; \gamma)$. Thus $F_N(y) \leq \gamma = \max\{F_N(x \cdot y), F_N(x)\}$.

Therefore, X_N is a neutrosophic N -UP-filter of X .

Theorem 4.5 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-ideal of X if and only if for all $\alpha,\beta,\gamma\in[-1,0]$, the sets $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-ideals of X if $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a neutrosophic N -UP-ideal of X. Let $\alpha,\beta,\gamma\in[-1,0]$ be such that $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x\in L(T_N;\alpha)$. Then $T_N(x)\le\alpha$. By (3.5), we have $T_N(0)\le T_N(x)\le\alpha$. Thus $0\in L(T_N;\alpha)$. Next, let $x,y,z\in X$ be such that $x\cdot(y\cdot z)\in L(T_N;\alpha)$ and $y\in L(T_N;\alpha)$. Then $T_N(x\cdot(y\cdot z))\le\alpha$ and $T_N(y)\le\alpha$, so α is an upper bound of $\{T_N(x\cdot(y\cdot z)),T_N(y)\}$. By (3.14), we have $T_N(x\cdot z)\le\max\{T_N(x\cdot(y\cdot z)),T_N(y)\}\le\alpha$. Thus $x\cdot z\in L(T_N;\alpha)$.

Let $x\in U(I_N;\alpha)$. Then $I_N(x)\ge\beta$. By (3.5), we have $I_N(0)\ge I_N(x)\ge\beta$. Thus $0\in U(I_N;\beta)$. Next, let $x,y,z\in X$ be such that $x\cdot(y\cdot z)\in U(I_N;\beta)$ and $y\in U(I_N;\beta)$. Then $I_N(x\cdot(y\cdot z))\ge\beta$ and $I_N(y)\ge\beta$, so β is a lower bound of $\{I_N(x\cdot(y\cdot z)),I_N(y)\}$. By (3.15), we have $I_N(x\cdot z)\ge\min\{I_N(x\cdot(y\cdot z)),I_N(y)\}\ge\beta$. Thus $x\cdot z\in U(I_N;\beta)$.

Let $x\in L(F_N;\gamma)$. Then $F_N(x)\le\gamma$. By (3.5), we have $F_N(0)\le F_N(x)\le\gamma$. Thus $0\in L(F_N;\gamma)$. Next, let $x,y,z\in X$ be such that $x\cdot(y\cdot z)\in L(F_N;\gamma)$ and $y\in L(F_N;\gamma)$. Then $F_N(x\cdot(y\cdot z))\le\gamma$ and $F_N(y)\le\gamma$, so γ is an upper bound of $\{F_N(x\cdot(y\cdot z)),F_N(y)\}$. By (3.16), we have $F_N(x\cdot z)\le\max\{F_N(x\cdot(y\cdot z)),F_N(y)\}\le\gamma$. Thus $x\cdot z\in L(F_N;\gamma)$.

Hence, $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha,\beta,\gamma\in[-1,0]$, the sets $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are UP-ideals of X if $L(T_N;\alpha),U(I_N;\beta)$, and $L(F_N;\gamma)$ are nonempty.

Let $x\in X$. Then $T_N(x)\in[-1,0]$. Choose $\alpha=T_N(x)$. Thus $T_N(x)\le\alpha$, so $x\in L(T_N;\alpha)\ne\varnothing$. By assumption, we have $L(T_N;\alpha)$ is a UP-ideal of X and so $0\in L(T_N;\alpha)$. Thus $T_N(0)\le\alpha=T_N(x)$. Next, let $x,y,z\in X$. Then $T_N(x\cdot(y\cdot z)),T_N(y)\in[-1,0]$. Choose $\alpha=\max\{T_N(x\cdot(y\cdot z)),T_N(y)\}$. Thus $T_N(x\cdot(y\cdot z))\le\alpha$ and $T_N(y)\le\alpha$, so $x\cdot(y\cdot z),y\in L(T_N;\alpha)\ne\varnothing$. By assumption, we have $L(T_N;\alpha)$ is a UP-ideal of X and so $x\cdot z\in L(T_N;\alpha)$. Thus $T_N(x\cdot z)\le\alpha=\max\{T_N(x\cdot(y\cdot z)),T_N(y)\}$.

Let $x\in X$. Then $I_N(x)\in[-1,0]$. Choose $\beta=I_N(x)$. Thus $I_N(x)\ge\beta$, so $x\in U(I_N;\beta)\ne\varnothing$. By assumption, we have $U(I_N;\beta)$ is a UP-ideal of X and so $0\in U(I_N;\beta)$. Thus $I_N(0)\ge\beta=I_N(x)$. Next, let $x,y,z\in X$. Then $I_N(x\cdot(y\cdot z)),I_N(y)\in[-1,0]$. Choose $\beta=\min\{I_N(x\cdot(y\cdot z)),I_N(y)\}$. Thus $I_N(x\cdot(y\cdot z))\ge\beta$ and $I_N(y)\ge\beta$, so $x\cdot(y\cdot z),y\in U(I_N;\beta)\ne\varnothing$. By assumption, we have $U(I_N;\beta)$ is a UP-ideal of X and so $x\cdot z\in U(I_N;\beta)$. Thus $I_N(x\cdot z)\ge\beta=\min\{I_N(x\cdot(y\cdot z)),I_N(y)\}$.

Let $x\in X$. Then $F_N(x)\in[-1,0]$. Choose $\gamma=F_N(x)$. Thus $F_N(x)\le\gamma$, so $x\in L(F_N;\gamma)\ne\varnothing$. By assumption, we have $L(F_N;\gamma)$ is a UP-ideal of X and so $0\in L(F_N;\gamma)$. Thus $F_N(0)\le\gamma=F_N(x)$. Next, let $x,y,z\in X$. Then $F_N(x\cdot(y\cdot z)),F_N(y)\in[-1,0]$. Choose $\gamma=\max\{F_N(x\cdot(y\cdot z)),F_N(y)\}$. Thus $F_N(x\cdot(y\cdot z))\le\gamma$ and $F_N(y)\le\gamma$, so $x\cdot(y\cdot z),y\in L(F_N;\gamma)\ne\varnothing$. By assumption, we have $L(F_N;\gamma)$ is a UP-ideal of X and so $x\cdot z\in L(F_N;\gamma)$. Thus $F_N(x\cdot z)\le\gamma=\max\{F_N(x\cdot(y\cdot z)),F_N(y)\}$.

Therefore, X_N is a neutrosophic N -UP-ideal of X.

Theorem 4.6 A neutrosophic N -structure X_N over X is a neutrosophic N -strongly UP-ideal of X if and only if the sets $E(T_N;T_N(0)),E(I_N;I_N(0))$, and $E(F_N;F_N(0))$ are strongly UP-ideals of X .

Proof. Assume that X_N is a neutrosophic N -strongly UP-ideal of X. By Theorem 3.17, we have X_N is constant, that is, T_N, I_N, and F_N are constant. Thus

$$(\forall x \in X)\begin{pmatrix} T_N(x) & = T_N(0) \\ I_N(x) & = I_N(0) \\ F_N(x) & = F_N(0) \end{pmatrix}.$$

Hence, $E(T_N;T_N(0)) = X, E(I_N;I_N(0)) = X$, and $E(F_N;F_N(0)) = X$ and so $E(T_N;T_N(0)), E(I_N;I_N(0))$, and $E(F_N;F_N(0))$ are strongly UP-ideals of X.

Conversely, assume that $E(T_N;T_N(0)), E(I_N;I_N(0))$, and $E(F_N;F_N(0))$ are strongly UP-ideals of X. Then $E(T_N;T_N(0)) = X, E(I_N;I_N(0)) = X$, $E(F_N;F_N(0)) = X$ and so

$$(\forall x \in X)\begin{pmatrix} T_N(x) & = T_N(0) \\ I_N(x) & = I_N(0) \\ F_N(x) & = F_N(0) \end{pmatrix}.$$

Thus T_N, I_N, and F_N are constant, that is X_N is constant. By Theorem 3.17, we have X_N is a neutrosophic N -strongly UP-ideal of X.

5. Neutrosophic N -structures of special type

In this section, we introduce the notions of special neutrosophic N -UP-subalgebras, special neutrosophic N -near UP-filters, special neutrosophic N -UP-filters, special neutrosophic N -UP-ideals, and special neutrosophic N -strongly UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 5.1 A neutrosophic N -structure X_N over X is called a *special neutrosophic* N *-UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(T_N(x \cdot y) \geq \min\{T_N(x), T_N(y)\}), \tag{5.1}$$

$$(\forall x, y \in X)(I_N(x \cdot y) \leq \max\{I_N(x), I_N(y)\}), \tag{5.2}$$

$$(\forall x, y \in X)(F_N(x \cdot y) \geq \min\{F_N(x), F_N(y)\}). \tag{5.3}$$

Example 5.2 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	0	0
3	0	1	2	0	0
4	0	1	2	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$T_N(0) = -0.2,\ I_N(0) = -0.9,\ F_N(0) = -0.2,$

$T_N(1) = -0.4,\ I_N(1) = -0.8,\ F_N(1) = -0.4,$

$T_N(2) = -0.8,\ I_N(2) = -0.7,\ F_N(2) = -0.6,$

$$T_N(3) = -0.3,\ I_N(3) = -0.5,\ F_N(3) = -0.7,$$
$$T_N(4) = -0.8,\ I_N(4) = -0.3,\ F_N(4) = -0.8.$$

Hence, X_N is a special neutrosophic N -UP-subalgebra of X.

Definition 5.3 A neutrosophic N -structure X_N over X is called a *special neutrosophic* N *-near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(T_N(0) \geq T_N(x)), \tag{5.4}$$
$$(\forall x \in X)(I_N(0) \leq I_N(x)), \tag{5.5}$$
$$(\forall x \in X)(F_N(0) \geq F_N(x)), \tag{5.6}$$
$$(\forall x, y \in X)(T_N(x \cdot y) \geq T_N(y)), \tag{5.7}$$
$$(\forall x, y \in X)(I_N(x \cdot y) \leq I_N(y)), \tag{5.8}$$
$$(\forall x, y \in X)(F_N(x \cdot y) \geq F_N(y)). \tag{5.9}$$

Example 5.4 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	0
2	0	2	0	3	0
3	0	2	2	0	0
4	0	2	2	3	0

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.2,\ I_N(0) = -0.8,\ F_N(0) = -0.3,$$
$$T_N(1) = -0.5,\ I_N(1) = -0.5,\ F_N(1) = -0.7,$$
$$T_N(2) = -0.4,\ I_N(2) = -0.7,\ F_N(2) = -0.4,$$
$$T_N(3) = -0.3,\ I_N(3) = -0.4,\ F_N(3) = -0.6,$$
$$T_N(4) = -0.8,\ I_N(4) = -0.2,\ F_N(4) = -0.8.$$

Hence, X_N is a special neutrosophic N -near UP-filter of X.

Definition 5.5 A neutrosophic N -structure X_N over X is called a *special neutrosophic* N *-UP-filter* of X if it satisfies the following conditions: (5.4), (5.5), (5.6), and

$$(\forall x, y \in X)(T_N(y) \geq \min\{T_N(x \cdot y), T_N(x)\}), \tag{5.10}$$
$$(\forall x, y \in X)(I_N(y) \leq \max\{I_N(x \cdot y), I_N(x)\}), \tag{5.11}$$
$$(\forall x, y \in X)(F_N(y) \geq \min\{F_N(x \cdot y), F_N(x)\}). \tag{5.12}$$

Example 5.6 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	3	0
3	0	1	2	0	0
4	0	1	2	3	0

Then $(X,\cdot,0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.2,\ I_N(0) = -0.8,\ F_N(0) = -0.2,$$
$$T_N(1) = -0.8,\ I_N(1) = -0.5,\ F_N(1) = -0.8,$$
$$T_N(2) = -0.6,\ I_N(2) = -0.4,\ F_N(2) = -0.5,$$
$$T_N(3) = -0.7,\ I_N(3) = -0.6,\ F_N(3) = -0.7,$$
$$T_N(4) = -0.5,\ I_N(4) = -0.7,\ F_N(4) = -0.4.$$

Hence, X_N is a special neutrosophic N -UP-filter of X .

Definition 5.7 A neutrosophic N -structure X_N over X is called a *special neutrosophic* N *-UP-ideal* of X if it satisfies the following conditions: (5.4), (5.5), (5.6), and

$$(\forall x,y,z\in X)(T_N(x\cdot z)\geq \min\{T_N(x\cdot(y\cdot z)),T_N(y)\}), \tag{5.13}$$
$$(\forall x,y,z\in X)(I_N(x\cdot z)\leq \max\{I_N(x\cdot(y\cdot z)),I_N(y)\}), \tag{5.14}$$
$$(\forall x,y,z\in X)(F_N(x\cdot z)\geq \min\{F_N(x\cdot(y\cdot z)),F_N(y)\}). \tag{5.15}$$

Example 5.8 Let $X=\{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	0	4
2	0	0	0	0	0
3	0	3	2	0	4
4	0	3	2	0	0

Then $(X,\cdot,0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$T_N(0) = -0.3,\ I_N(0) = -0.8,\ F_N(0) = -0.2,$$
$$T_N(1) = -0.6,\ I_N(1) = -0.6,\ F_N(1) = -0.3,$$
$$T_N(2) = -0.8,\ I_N(2) = -0.4,\ F_N(2) = -0.8,$$
$$T_N(3) = -0.6,\ I_N(3) = -0.6,\ F_N(3) = -0.3,$$
$$T_N(4) = -0.7,\ I_N(4) = -0.5,\ F_N(4) = -0.7.$$

Hence, X_N is a special neutrosophic N -UP-ideal of X .

Definition 5.9 A neutrosophic N -structure X_N over X is called a *special neutrosophic* N *-strongly UP-ideal* of X if it satisfies the following conditions: (5.4), (5.5), (5.6), and

$$(\forall x,y,z\in X)(T_N(x)\geq \min\{T_N((z\cdot y)\cdot(z\cdot x)),T_N(y)\}), \tag{5.16}$$
$$(\forall x,y,z\in X)(I_N(x)\leq \max\{I_N((z\cdot y)\cdot(z\cdot x)),I_N(y)\}), \tag{5.17}$$
$$(\forall x,y,z\in X)(F_N(x)\geq \min\{F_N((z\cdot y)\cdot(z\cdot x)),F_N(y)\}). \tag{5.18}$$

Example 5.10 Let $X=\{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	0	4
3	0	1	2	0	4
4	0	4	2	3	0

Then $(X,\cdot,0)$ is a UP-algebra. We define a neutrosophic N -structure X_N over X as follows:

$$\left(\forall x \in X\right)\begin{pmatrix} T_N(x) & = -0.5 \\ I_N(x) & = -1 \\ F_N(x) & = -0.3 \end{pmatrix}.$$

Hence, X_N is a special neutrosophic N -strongly UP-ideal X.

Theorem 5.11 Every special neutrosophic N -UP-subalgebra of X satisfies the conditions (5.4), (5.5), and (5.6).

Proof. Assume that X_N is a special neutrosophic N -UP-subalgebra of X. Then for all $x \in X$, by Proposition 2.5 (1), (5.1), (5.2), and (5.3), we have

$$T_N(0) = T_N(x \cdot x) \geq \min\{T_N(x), T_N(x)\} = T_N(x), \ \ I_N(0) = I_N(x \cdot x) \leq \max\{I_N(x), I_N(x)\} = I_N(x),$$
$$F_N(0) = F_N(x \cdot x) \geq \min\{F_N(x), F_N(x)\} = F_N(x).$$

Hence, X_N satisfies the conditions (5.4), (5.5), and (5.6).

By Lemma 3.4 (1) and (4), we have the following five theorems.

Theorem 5.12 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-subalgebra of X if and only if $\overline{X}_N$ is a special neutrosophic N -UP-subalgebra of X.

Theorem 5.13 A neutrosophic N -structure X_N over X is a neutrosophic N -near UP-filter of X if and only if $\overline{X}_N$ is a special neutrosophic N -near UP-filter of X.

Theorem 5.14 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-filter of X if and only if $\overline{X}_N$ is a special neutrosophic N -UP-filter of X.

Theorem 5.15 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-ideal of X if and only if $\overline{X}_N$ is a special neutrosophic N -UP-ideal of X.

Theorem 5.16 A neutrosophic N -structure X_N over X is a neutrosophic N -strongly UP-ideal of X if and only if $\overline{X}_N$ is a special neutrosophic N -strongly UP-ideal of X.

Theorem 5.17 A neutrosophic N -structure X_N over X is constant if and only if it is a special neutrosophic N -strongly UP-ideal of X.

Proof. It is straightforward by Remark 3.2 and Theorems 3.17 and 5.16.

By Remark Remark 3.2 and Theorems 5.12, 5.13, 5.14, 5.15, and 5.16, we have that the notion of special neutrosophic N -UP-subalgebras is a generalization of special neutrosophic N -near UP-filters, special neutrosophic N -near UP-filters is a generalization of special neutrosophic N -UP-filters, special neutrosophic N -UP-filters is a generalization of special neutrosophic N -UP-ideals, and special neutrosophic N -UP-ideals is a generalization of special neutrosophic N -strongly UP-ideals. Moreover, by Theorem 5.17, we obtain that special neutrosophic N -strongly UP-ideals and constant neutrosophic N -structures coincide.

Theorem 5.18 If X_N is a special neutrosophic N -UP-subalgebra of X satisfying the following condition:

$$\left(\forall x, y \in X\right)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} T_N(x) \geq T_N(y) \\ I_N(x) \leq I_N(y) \\ F_N(x) \geq F_N(y) \end{cases} \right), \tag{5.19}$$

then X_N is a special neutrosophic N -near UP-filter of X.

Proof. Assume that X_N is a special neutrosophic N -UP-subalgebra of X satisfying the condition (5.19). By Theorem 5.11, we have X_N satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then, by (5.4), (5.5), and (5.6), we have

$$T_N(x \cdot y) = T_N(0) \geq T_N(y),\ I_N(x \cdot y) = I_N(0) \leq I_N(y),\ F_N(x \cdot y) = F_N(0) \geq F_N(y).$$

Case 2: $x \cdot y \neq 0$. Then, by (5.1), (5.2), (5.3), and (5.19), we have

$$T_N(x \cdot y) \geq \min\{T_N(x), T_N(y)\} = T_N(y),\ I_N(x \cdot y) \leq \max\{I_N(x), I_N(y)\} = I_N(y),$$
$$F_N(x \cdot y) \geq \min\{F_N(x), F_N(y)\} = F_N(y).$$

Hence, X_N is a special neutrosophic N -near UP-filter of X .

Theorem 5.19 If X_N is a special neutrosophic N -near UP-filter of X satisfying the condition (3.21), then X_N is a special neutrosophic N -UP-filter of X .

Proof. Assume that X_N is a special neutrosophic N -near UP-filter of X satisfying the condition (3.21). Then X_N satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y, z \in X$. By (5.7), (5.8), and (3.21), we have

$$\min\{T_N(x \cdot y), T_N(x)\} = \min\{I_N(x \cdot y), T_N(x)\} \leq \min\{I_N(y), T_N(x)\} = \min\{T_N(y), T_N(x)\} \leq T_N(y),$$
$$\max\{I_N(x \cdot y), I_N(x)\} = \max\{T_N(x \cdot y), I_N(x)\} \geq \max\{T_N(y), I_N(x)\} = \max\{I_N(y), I_N(x)\} \geq I_N(y),$$
$$\min\{F_N(x \cdot y), F_N(x)\} = \min\{I_N(x \cdot y), F_N(x)\} \leq \min\{I_N(y), F_N(x)\} = \min\{F_N(y), F_N(x)\} \leq F_N(y).$$

Hence, X_N is a special neutrosophic N -UP-filter of X .

Theorem 5.20 If X_N is a special neutrosophic N -UP-filter of X satisfying the condition (3.22), then X_N is a special neutrosophic N -UP-ideal of X .

Proof. Assume that X_N is a special neutrosophic N -UP-filter of X satisfying the condition (3.22). Then X_N satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y, z \in X$. By (5.10), (5.11), (5.12), and (3.22), we have

$$T_N(x \cdot z) \geq \min\{T_N(y \cdot (x \cdot z)), T_N(y)\} = \min\{T_N(x \cdot (y \cdot z)), T_N(y)\},$$
$$I_N(x \cdot z) \leq \max\{I_N(y \cdot (x \cdot z)), I_N(y)\} = \max\{I_N(x \cdot (y \cdot z)), I_N(y)\},$$
$$F_N(x \cdot z) \geq \min\{F_N(y \cdot (x \cdot z)), F_N(y)\} = \min\{F_N(x \cdot (y \cdot z)), F_N(y)\}.$$

Hence, X_N is a special neutrosophic N -UP-ideal of X .

Theorem 5.21 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(z) \geq \min\{T_N(x), T_N(y)\} \\ I_N(z) \leq \max\{I_N(x), I_N(y)\} \\ F_N(z) \geq \min\{F_N(x), F_N(y)\} \end{cases} \right), \tag{5.20}$$

then X_N is a special neutrosophic N -UP-subalgebra of X .

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (5.20). Let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (5.20) that

$$T_N(x \cdot y) \geq \min\{T_N(x), T_N(y)\},\ I_N(x \cdot y) \leq \max\{I_N(x), I_N(y)\},\ F_N(x \cdot y) \geq \min\{F_N(x), F_N(y)\}.$$

Hence, X_N is a special neutrosophic N -UP-subalgebra of X .

Theorem 5.22 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(z) \geq T_N(y) \\ I_N(z) \leq I_N(y) \\ F_N(z) \geq F_N(y) \end{cases} \right), \tag{5.21}$$

then X_N is a special neutrosophic N -near UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (5.21). Let $x \in X$. By (UP-2) and Proposition 2.5 (1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leq x \cdot x$. It follows from (5.21) that $T_N(0) \geq T_N(x), I_N(0) \leq I_N(x)$, and $F_N(0) \geq F_N(x)$. Next, let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (5.21) that $T_N(x \cdot y) \geq T_N(y), I_N(x \cdot y) \leq I_N(y)$, and $F_N(x \cdot y) \geq F_N(y)$. Hence, X_N is a special neutrosophic N -near UP-filter of X.

Theorem 5.23 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} T_N(y) \geq \min\{T_N(z), T_N(x)\} \\ I_N(y) \leq \max\{I_N(z), I_N(x)\} \\ F_N(y) \geq \min\{F_N(z), F_N(x)\} \end{cases} \right), \tag{5.22}$$

then X_N is a special neutrosophic N -UP-filter of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (5.22). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (5.22) that

$$T_N(0) \geq \min\{T_N(x), T_N(x)\} = T_N(x),\ I_N(0) \leq \max\{I_N(x), I_N(x)\} = I_N(x),\ F_N(0) \geq \min\{F_N(x), F_N(x)\} = F_N(x).$$

Next, let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (5.22) that

$$T_N(y) \geq \min\{T_N(x \cdot y), T_N(x)\},\ I_N(y) \leq \max\{I_N(x \cdot y), I_N(x)\},\ F_N(y) \geq \min\{F_N(x \cdot y), F_N(x)\}.$$

Hence, X_N is a special neutrosophic N -UP-filter of X.

Theorem 5.24 If X_N is a neutrosophic N -structure over X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} T_N(x \cdot z) \geq \min\{T_N(a), T_N(y)\} \\ I_N(x \cdot z) \leq \max\{I_N(a), I_N(y)\} \\ F_N(x \cdot z) \geq \min\{F_N(a), F_N(y)\} \end{cases} \right), \tag{5.23}$$

then X_N is a special neutrosophic N -UP-ideal of X.

Proof. Assume that X_N is a neutrosophic N -structure over X satisfying the condition (5.23). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (5.23) and (UP-2) that

$$T_N(0) = T_N(0 \cdot 0) \geq \min\{T_N(x), T_N(x)\} = T_N(x),\ I_N(0) = I_N(0 \cdot 0) \leq \max\{I_N(x), I_N(x)\} = I_N(x),$$
$$F_N(0) = F_N(0 \cdot 0) \geq \min\{F_N(x), F_N(x)\} = F_N(x).$$

Next, let $x, y, z \in X$. By Proposition 2.5 (1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (5.23) that

$$T_N(x \cdot z) \geq \min\{T_N(x \cdot (y \cdot z)), T_N(y)\},\ I_N(x \cdot z) \leq \max\{I_N(x \cdot (y \cdot z)), I_N(y)\},$$
$$F_N(x \cdot z) \geq \min\{F_N(x \cdot (y \cdot z)), F_N(y)\}.$$

Hence, X_N is a special neutrosophic N -UP-ideal of X.

For any fixed numbers $\alpha^-,\alpha^+,\beta^-,\beta^+,\gamma^-,\gamma^+ \in [-1,0]$ such that $\alpha^- < \alpha^+, \beta^- < \beta^+, \gamma^- < \gamma^+$ and a nonempty subset G of X, a neutrosophic N-structure ${}^G X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}] = (X, {}^G T_N[{}^{\alpha^+}_{\alpha^-}], {}^G I_N[{}^{\beta^-}_{\beta^+}], {}^G F_N[{}^{\gamma^+}_{\gamma^-}])$ over X where ${}^G T_N[{}^{\alpha^+}_{\alpha^-}], {}^G I_N[{}^{\beta^-}_{\beta^+}]$, and ${}^G F_N[{}^{\gamma^+}_{\gamma^-}]$ are N-functions on X which are given as follows:

$$ {}^G T_N[{}^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise,} \end{cases} \quad {}^G I_N[{}^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases} \quad {}^G F_N[{}^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise.} \end{cases} $$

Lemma 5.25 Let $\alpha^-,\alpha^+,\beta^-,\beta^+,\gamma^-,\gamma^+ \in [-1,0]$. Then the following statements hold:

1. $\overline{X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = {}^G X_N[{}^{-1-\alpha^-,-1-\beta^+,-1-\gamma^-}_{-1-\alpha^+,-1-\beta^-,-1-\gamma^+}]$, and
2. $\overline{{}^G X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = X_N^G[{}^{-1-\alpha^+,-1-\beta^-,-1-\gamma^+}_{-1-\alpha^-,-1-\beta^+,-1-\gamma^-}]$.

Proof. 1. Let $\overline{X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]}$ be a neutrosophic N-structure over X. Then $\overline{X_N^G[{}^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = (X, \overline{T_N^G[{}^{\alpha^-}_{\alpha^+}]}, \overline{I_N^G[{}^{\beta^+}_{\beta^-}]}, \overline{F_N^G[{}^{\gamma^-}_{\gamma^+}]})$. Since

$$ T_N^G[{}^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases} \quad I_N^G[{}^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise,} \end{cases} \quad F_N^G[{}^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise} \end{cases}, $$

we have

$$ \overline{T_N^G[{}^{\alpha^-}_{\alpha^+}]}(x) = \begin{cases} -1-\alpha^- & \text{if } x \in G, \\ -1-\alpha^+ & \text{otherwise} \end{cases} = {}^G T_N[{}^{-1-\alpha^-}_{-1-\alpha^+}](x), \quad \overline{I_N^G[{}^{\beta^+}_{\beta^-}]}(x) = \begin{cases} -1-\beta^+ & \text{if } x \in G, \\ -1-\beta^- & \text{otherwise} \end{cases} = {}^G I_N[{}^{-1-\beta^+}_{-1-\beta^-}](x), $$

$$ \overline{F_N^G[{}^{\gamma^-}_{\gamma^+}]}(x) = \begin{cases} -1-\gamma^- & \text{if } x \in G, \\ -1-\gamma^+ & \text{otherwise} \end{cases} = {}^G F_N[{}^{-1-\gamma^-}_{-1-\gamma^+}](x). $$

Hence, $(X, {}^G T_N[{}^{-1-\alpha^-}_{-1-\alpha^+}], {}^G I_N[{}^{-1-\beta^+}_{-1-\beta^-}], {}^G F_N[{}^{-1-\gamma^-}_{-1-\gamma^+}]) = {}^G X_N[{}^{-1-\alpha^-,-1-\beta^+,-1-\gamma^-}_{-1-\alpha^+,-1-\beta^-,-1-\gamma^+}]$.

2. Let $\overline{{}^G X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]}$ be a neutrosophic N-structure over X. Then $\overline{{}^G X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = (X, \overline{{}^G T_N[{}^{\alpha^+}_{\alpha^-}]}, \overline{{}^G I_N[{}^{\beta^-}_{\beta^+}]}, \overline{{}^G F_N[{}^{\gamma^+}_{\gamma^-}]})$. Since

$$ {}^G T_N[{}^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise,} \end{cases} \quad {}^G I_N[{}^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases} \quad {}^G F_N[{}^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise} \end{cases}, $$

we have

$$ \overline{{}^G T_N[{}^{\alpha^+}_{\alpha^-}]}(x) = \begin{cases} -1-\alpha^+ & \text{if } x \in G, \\ -1-\alpha^- & \text{otherwise} \end{cases} = T_N^G[{}^{-1-\alpha^+}_{-1-\alpha^-}](x), \quad \overline{{}^G I_N[{}^{\beta^-}_{\beta^+}]}(x) = \begin{cases} -1-\beta^- & \text{if } x \in G, \\ -1-\beta^+ & \text{otherwise} \end{cases} = I_N^G[{}^{-1-\beta^-}_{-1-\beta^+}](x), $$

$$ \overline{{}^G F_N[{}^{\gamma^+}_{\gamma^-}]}(x) = \begin{cases} -1-\gamma^+ & \text{if } x \in G, \\ -1-\gamma^- & \text{otherwise} \end{cases} = F_N^G[{}^{-1-\gamma^+}_{-1-\gamma^-}](x). $$

Hence, $(X, T_N^G[^{-1-\alpha^+}_{-1-\alpha^-}], I_N^G[^{-1-\beta^-}_{-1-\beta^+}], F_N^G[^{-1-\gamma^+}_{-1-\gamma^-}]) = X_N^G[^{-1-\alpha^+,-1-\beta^-,-1-\gamma^+}_{-1-\alpha^-,-1-\beta^+,-1-\gamma^-}]$.

Lemma 5.26 If the constant 0 of X is in a nonempty subset G of X, then a neutrosophic N -structure ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X satisfies the conditions (5.4), (5.5), and (5.6).

Proof. If $0 \in G$, then ${}^G T_N[^{\alpha^+}_{\alpha^-}](0) = \alpha^+, {}^G I_N[^{\beta^-}_{\beta^+}](0) = \beta^-$, and ${}^G F_N[^{\gamma^+}_{\gamma^-}](0) = \gamma^+$. Thus

$$(\forall x \in X)\begin{pmatrix} {}^G T_N[^{\alpha^+}_{\alpha^-}](0) = \alpha^+ \geq {}^G T_N[^{\alpha^+}_{\alpha^-}](x) \\ {}^G I_N[^{\beta^-}_{\beta^+}](0) = \beta^- \leq {}^G I_N[^{\beta^-}_{\beta^+}](x) \\ {}^G F_N[^{\gamma^+}_{\gamma^-}](0) = \gamma^+ \geq {}^G F_N[^{\gamma^+}_{\gamma^-}](x) \end{pmatrix}.$$

Hence, ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (5.4), (5.5), and (5.6).

Lemma 5.27 If a neutrosophic N -structure ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X satisfies the condition (5.4) (resp., (5.5), (5.6)), then the constant 0 of X is in a nonempty subset G of X

Proof. Assume that a neutrosophic N -structure ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X satisfies the condition (5.4). Then ${}^G T_N[^{\alpha^+}_{\alpha^-}](0) \geq {}^G T_N[^{\alpha^+}_{\alpha^-}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus ${}^G T_N[^{\alpha^+}_{\alpha^-}](g) = \alpha^+$, so ${}^G T_N[^{\alpha^+}_{\alpha^-}](0) \geq {}^G T_N[^{\alpha^+}_{\alpha^-}](g) = \alpha^+$, that is, ${}^G T_N[^{\alpha^+}_{\alpha^-}](0) = \alpha^+$. Hence, $0 \in G$.

Theorem 5.28 A neutrosophic N -structure ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X is a special neutrosophic N - UP-subalgebra of X if and only if a nonempty subset G of X is a UP-subalgebra of X.

Proof. Assume that ${}^G X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-subalgebra of X. Let $x, y \in G$. Then ${}^G T_N[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ = {}^G T_N[^{\alpha^+}_{\alpha^-}](y)$. Thus

$${}^G T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \min\{{}^G T_N[^{\alpha^+}_{\alpha^-}](x), {}^G T_N[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq {}^G T_N[^{\alpha^+}_{\alpha^-}](x \cdot y)$$

and so ${}^G T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$${}^G T_N[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ = {}^G T_N[^{\alpha^+}_{\alpha^-}](y),\ {}^G I_N[^{\beta^-}_{\beta^+}](x) = \beta^- = {}^G I_N[^{\beta^-}_{\beta^+}](y),\ {}^G F_N[^{\gamma^+}_{\gamma^-}](x) = \gamma^+ = {}^G F_N[^{\gamma^+}_{\gamma^-}](y).$$

Thus

$$\min\{{}^G T_N[^{\alpha^+}_{\alpha^-}](x), {}^G T_N[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+,\ \max\{{}^G I_N[^{\beta^-}_{\beta^+}](x), {}^G I_N[^{\beta^-}_{\beta^+}](y)\} = \beta^-,\ \min\{{}^G F_N[^{\gamma^+}_{\gamma^-}](x), {}^G F_N[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^+.$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+, {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^-$, and ${}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+$. Hence,

$$ {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ \geq \alpha^+ = \min\{{}^{G}T_N[^{\alpha^+}_{\alpha^-}](x), {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\}, $$

$$ {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- \leq \beta^- = \max\{{}^{G}I_N[^{\beta^-}_{\beta^+}](x), {}^{G}I_N[^{\beta^-}_{\beta^+}](y)\}, $$

$$ {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ \geq \gamma^+ = \min\{{}^{G}F_N[^{\gamma^+}_{\gamma^-}](x), {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}. $$

Case 2: $x \notin G$ or $y \notin G$. Then

$$ {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x) = \alpha^- \text{ or } {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^-, \ {}^{G}I_N[^{\beta^-}_{\beta^+}](x) = \beta^+ \text{ or } {}^{G}I_N[^{\beta^-}_{\beta^+}](y) = \beta^+, $$

$$ {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x) = \gamma^- \text{ or } {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y) = \gamma^-. $$

Thus

$$ \min\{{}^{G}T_N[^{\alpha^+}_{\alpha^-}](x), {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-, \ \max\{{}^{G}I_N[^{\beta^-}_{\beta^+}](x), {}^{G}I_N[^{\beta^-}_{\beta^+}](y)\} = \beta^+, \ \min\{{}^{G}F_N[^{\gamma^+}_{\gamma^-}](x), {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-. $$

Therefore,

$$ {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \alpha^- = \min\{{}^{G}T_N[^{\alpha^+}_{\alpha^-}](x), {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\}, $$

$$ {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) \leq \beta^+ = \max\{{}^{G}I_N[^{\beta^-}_{\beta^+}](x), {}^{G}I_N[^{\beta^-}_{\beta^+}](y)\}, $$

$$ {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) \geq \gamma^- = \min\{{}^{G}F_N[^{\gamma^+}_{\gamma^-}](x), {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}. $$

Hence, ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-subalgebra of X.

Theorem 5.29 A neutrosophic N -structure ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X is a special neutrosophic N -near UP-filter of X if and only if a nonempty subset G of X is a near UP-filter of X.

Proof. Assume that ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -near UP-filter of X. Since ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (5.4), it follows from Lemma 5.27 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus, by (5.7), we have

$$ {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) $$

and so ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 5.26 that ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+, {}^{G}I_N[^{\beta^-}_{\beta^+}](y) = \beta^-$, and ${}^{G}F_N[^{\gamma^+}_{\gamma^-}](y) = \gamma^+$. Since G is a near UP-filter of X, we have $x \cdot y \in G$ and so ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+, {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^-$, and ${}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+$. Thus

$${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ \geq \alpha^+ = {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y),\ {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- \leq \beta^- = {}^{G}I_N[^{\beta^-}_{\beta^+}](y),$$

$${}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ \geq \gamma^+ = {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y).$$

Case 2: $y \notin G$. Then ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^-, {}^{G}I_N[^{\beta^-}_{\beta^+}](y) = \beta^+$, and ${}^{G}F_N[^{\gamma^+}_{\gamma^-}](y) = \gamma^-$. Thus

$${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \alpha^- = {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y),\ {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) \leq \beta^+ = {}^{G}I_N[^{\beta^-}_{\beta^+}](y),\ {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) \geq \gamma^- = {}^{G}F_N[^{\gamma^+}_{\gamma^-}](y).$$

Hence, ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -near UP-filter of X.

Theorem 5.30 A neutrosophic N -structure ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X is a special neutrosophic N -UP-filter of X if and only if a nonempty subset G of X is a UP-filter of X.

Proof. Assume that ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-filter of X. Since ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (5.4), it follows from Lemma 5.27 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ = {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x)$. Thus, by (5.10), we have

$${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) \geq \min\{{}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y), {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x)\} = \alpha^+ \geq {}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)$$

and so ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 5.26 that ${}^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$${}^{G}T_N[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ = {}^{G}T_N[^{\alpha^+}_{\alpha^-}](x),\ {}^{G}I_N[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- = {}^{G}I_N[^{\beta^-}_{\beta^+}](x),\ {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ = {}^{G}F_N[^{\gamma^+}_{\gamma^-}](x).$$

Since G is a UP-filter of X, we have $y \in G$ and so ${}^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+, {}^{G}I_N[^{\beta^-}_{\beta^+}](y) = \beta^-$, and ${}^{G}F_N[^{\gamma^+}_{\gamma^-}](y) = \gamma^+$. Thus

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq \alpha^+ = \min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot y), ^{G}T_N[^{\alpha^+}_{\alpha^-}](x)\},$$

$$^{G}I_N[^{\beta^-}_{\beta^+}](y) = \beta^- \leq \beta^- = \max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot y), ^{G}I_N[^{\beta^-}_{\beta^+}](x)\},$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](y) = \gamma^+ \geq \gamma^+ = \min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot y), ^{G}F_N[^{\gamma^+}_{\gamma^-}](x)\}.$$

Case 2: $x\cdot y \notin G$ or $x \notin G$. Then

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot y) = \alpha^- \text{ or } ^{G}T_N[^{\alpha^+}_{\alpha^-}](x) = \alpha^-,\ ^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot y) = \beta^+ \text{ or } ^{G}I_N[^{\beta^-}_{\beta^+}](x) = \beta^+,$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot y) = \gamma^- \text{ or } ^{G}F_N[^{\gamma^+}_{\gamma^-}](x) = \gamma^-.$$

Thus

$$\min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot y), ^{G}T_N[^{\alpha^+}_{\alpha^-}](x)\} = \alpha^-,\ \max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot y), ^{G}I_N[^{\beta^-}_{\beta^+}](x)\} = \beta^+,$$

$$\min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot y), ^{G}F_N[^{\gamma^+}_{\gamma^-}](x)\} = \gamma^-.$$

Therefore,

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x) \geq \alpha^- = \min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot y), ^{G}T_N[^{\alpha^+}_{\alpha^-}](x)\},$$

$$^{G}I_N[^{\beta^-}_{\beta^+}](x) \leq \beta^+ = \max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot y), ^{G}I_N[^{\beta^-}_{\beta^+}](x)\},$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x) \geq \gamma^- = \min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot y), ^{G}F_N[^{\gamma^+}_{\gamma^-}](x)\}.$$

Hence, $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-filter of X.

Theorem 5.31 A neutrosophic N -structure $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X is a special neutrosophic N -UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.

Proof. Assume that $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-ideal of X. Since $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (5.4), it follows from Lemma 5.27, that $0 \in G$. Next, let $x, y, z \in X$ be such that $x\cdot(y\cdot z) \in G$ and $y \in G$. Then $^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) = \alpha^+ = ^{G}T_N[^{\alpha^+}_{\alpha^-}](y)$. Thus, by (5.13), we have

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z) \geq \min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)), ^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq ^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z)$$

and so $^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z) = \alpha^+$. Thus $x\cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 5.26 that $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y, z \in X$.

Case 1: $x\cdot(y\cdot z) \in G$ and $y \in G$. Then

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z))=\alpha^+={}^{G}T_N[^{\alpha^+}_{\alpha^-}](y),\ {}^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z))=\beta^-={}^{G}I_N[^{\beta^-}_{\beta^+}](y),$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z))=\gamma+={}^{G}F_N[^{\gamma^+}_{\gamma^-}](y).$$

Thus

$$\min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)),{}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\}=\alpha^+,\ \max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)),{}^{G}I_N[^{\beta^-}_{\beta^+}](y)\}=\beta^-,$$

$$\min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z)),{}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}=\gamma^+.$$

Since G is a UP-ideal of X, we have $x\cdot z\in G$ and so $^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z)=\alpha^+,{}^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot z)=\beta^-$, and $^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot z)=\gamma^+$. Thus

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z)=\alpha^+\geq\alpha^+=\min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)),{}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\},$$

$$^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot z)=\beta^-\leq\beta^-=\max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)),{}^{G}I_N[^{\beta^-}_{\beta^+}](y)\},$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot z)=\gamma^+\geq\gamma^+=\min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z)),{}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}.$$

Case 2: $x\cdot(y\cdot z)\not\in G$ or $y\not\in G$. Then

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z))=\alpha^-\text{ or }{}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)=\alpha^-,\ {}^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z))=\beta^+\text{ or }{}^{G}I_N[^{\beta^-}_{\beta^+}](y)=\beta^+,$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z))=\gamma^-\text{ or }{}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)=\gamma^-.$$

Thus

$$\min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)),{}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\}=\alpha^-,\ \max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)),{}^{G}I_N[^{\beta^-}_{\beta^+}](y)\}=\beta^+,$$

$$\min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z)),{}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}=\gamma^-.$$

Therefore,

$$^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot z)\geq\alpha^-=\min\{^{G}T_N[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)),{}^{G}T_N[^{\alpha^+}_{\alpha^-}](y)\},$$

$$^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot z)\leq\beta^+=\max\{^{G}I_N[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)),{}^{G}I_N[^{\beta^-}_{\beta^+}](y)\},$$

$$^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot z)\geq\gamma^-=\min\{^{G}F_N[^{\gamma^+}_{\gamma^-}](x\cdot(y\cdot z)),{}^{G}F_N[^{\gamma^+}_{\gamma^-}](y)\}.$$

Hence, $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -UP-ideal of X.

Theorem 5.32 A neutrosophic N -structure $^{G}X_N[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ over X is a special neutrosophic N -strongly UP-ideal of X if and only if a nonempty subset G of X is a strongly UP-ideal of X.

Proof. Assume that ${}^{G}X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -strongly UP-ideal of X. By Theorem 5.17, we have ${}^{G}T_N[{}^{\alpha^+}_{\alpha^-}]$ is constant, that is, ${}^{G}T_N[{}^{\alpha^+}_{\alpha^-}]$ is constant. Since G is nonempty, we have ${}^{G}T_N[{}^{\alpha^+}_{\alpha^-}](x) = \alpha^+$ for all $x \in X$. Thus $G = X$. Hence, G is a strongly UP-ideal of X.

Conversely, assume that G is a strongly UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} {}^{G}T_N[{}^{\alpha^+}_{\alpha^-}](x) & = \alpha^+ \\ {}^{G}I_N[{}^{\beta^-}_{\beta^+}](x) & = \beta^- \\ {}^{G}F_N[{}^{\gamma^+}_{\gamma^-}](x) & = \gamma^+ \end{pmatrix}.$$

Thus ${}^{G}T_N[{}^{\alpha^+}_{\alpha^-}], {}^{G}I_N[{}^{\beta^-}_{\beta^+}]$, and ${}^{G}F_N[{}^{\gamma^+}_{\gamma^-}]$ are constant, that is, ${}^{G}X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is constant. By Theorem 5.17, we have ${}^{G}X_N[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic N -strongly UP-ideal of X.

6. Level subset of a neutrosophic N -structure of special type

In the last section of this paper, we discuss the relationships among special neutrosophic N -UP-subalgebras (resp., special neutrosophic N -near UP-filters, special neutrosophic N -UP-filters, special neutrosophic N -UP-ideals, special neutrosophic N -strongly UP-ideals) of UP-algebras and their level subsets.

Theorem 6.1 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-subalgebra of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, the sets $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-subalgebras of X if $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a special neutrosophic N -UP-subalgebra of X. Let $\alpha,\beta,\gamma \in [-1,0]$ be such that $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x,y \in U(T_N;\alpha)$. Then $T_N(x) \geq \alpha$ and $T_N(y) \geq \alpha$, so α is a lower bound of $\{T_N(x), T_N(y)\}$. By (5.1), we have $T_N(x \cdot y) \geq \min\{T_N(x), T_N(y)\} \geq \alpha$. Thus $x \cdot y \in U(T_N;\alpha)$.

Let $x,y \in L(I_N;\beta)$. Then $I_N(x) \leq \beta$ and $I_N(y) \leq \beta$, so β is an upper bound of $\{I_N(x), I_N(y)\}$. By (5.2), we have $I_N(x \cdot y) \leq \max\{I_N(x), I_N(y)\} \leq \beta$. Thus $x \cdot y \in L(I_N;\beta)$.

Let $x,y \in U(F_N;\gamma)$. Then $F_N(x) \geq \gamma$ and $F_N(y) \geq \gamma$, so γ is a lower bound of $\{F_N(x), F_N(y)\}$. By (5.3), we have $F_N(x \cdot y) \geq \min\{F_N(x), F_N(y)\} \geq \gamma$. Thus $x \cdot y \in U(F_N;\gamma)$.

Hence, $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-subalgebras of X.

Conversely, assume that for all $\alpha,\beta,\gamma \in [-1,0]$, the set $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-subalgebras if $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x,y \in X$. Then $T_N(x), T_N(y) \in [-1,0]$ Choose $\alpha = \min\{T_N(x), T_N(y)\}$. Thus $T_N(x) \geq \alpha$ and $T_N(y) \geq \alpha$, so $x,y \in U(T_N;\alpha) \neq \varnothing$. By assumption, we have $U(T_N;\alpha)$ is a UP-subalgebra of X and so $x,y \in U(T_N;\alpha)$. Thus $T_N(x \cdot y) \geq \alpha = \min\{T_N(x), T_N(y)\}$.

Let $x, y \in X$. Then $I_N(x), I_N(y) \in [-1,0]$ Choose $\beta = \max\{I_N(x), I_N(y)\}$. Thus $I_N(x) \leq \beta$ and $I_N(y) \leq \beta$, so $x, y \in L(I_N; \beta) \neq \varnothing$. By assumption, we have $L(I_N; \beta)$ is a UP-subalgebra of X and so $x, y \in L(I_N; \beta)$. Thus $I_N(x \cdot y) \leq \beta = \max\{I_N(x), I_N(y)\}$.

Let $x, y \in X$. Then $F_N(x), F_N(y) \in [-1,0]$. Choose $\gamma = \min\{F_N(x), F_N(y)\}$. Thus $F_N(x) \geq \gamma$ and $F_N(y) \geq \gamma$, so $x, y \in U(F_N; \gamma) \neq \varnothing$. By assumption, we have $U(F_N; \gamma)$ is a UP-subalgebra of X and so $x, y \in U(F_N; \gamma)$. Thus $F_N(x \cdot y) \leq \gamma = \min\{F_N(x), F_N(y)\}$.

Therefore, X_N is a special neutrosophic N -UP-subalgebra of X .

Theorem 6.2 A neutrosophic N -structure X_N over X is a special neutrosophic N -near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, the sets $U(T_N; \alpha), L(I_N; \beta)$, and $U(F_N; \gamma)$ are near UP-filters of X if $U(T_N; \alpha), L(I_N; \beta)$, and $U(F_N; \gamma)$ are nonempty.

Proof. Assume that X_N is a special neutrosophic N -near UP-filter of X . Let $\alpha, \beta, \gamma \in [-1,0]$ be such that $U(T_N; \alpha), L(I_N; \beta)$, and $U(F_N; \gamma)$ are nonempty.

Let $x \in U(T_N; \alpha)$. Then $T_N(x) \geq \alpha$. By (5.4), we have $T_N(0) \geq T_N(x) \geq \alpha$. Thus $0 \in U(T_N; \alpha)$. Next, let $y \in U(T_N; \alpha)$. Then $T_N(y) \geq \alpha$. By (5.7), we have $T_N(x \cdot y) \geq T_N(y) \geq \alpha$. Thus $x \cdot y \in U(T_N; \alpha)$.

Let $x \in L(I_N; \beta)$. Then $I_N(x) \leq \beta$. By (5.5), we have $I_N(0) \leq I_N(x) \leq \beta$. Thus $0 \in L(I_N; \beta)$. Next, let $y \in L(I_N; \beta)$. Then $I_N(y) \leq \beta$. By (5.8), we have $I_N(x \cdot y) \leq I_N(y) \leq \beta$. Thus $x \cdot y \in L(I_N; \beta)$.

Let $x \in U(F_N; \gamma)$. Then $F_N(x) \geq \gamma$. By (5.6), we have $F_N(0) \geq F_N(x) \geq \gamma$. Thus $0 \in U(F_N; \gamma)$. Next, $y \in U(F_N; \gamma)$. Then $F_N(y) \geq \gamma$. By (5.9), we have $F_N(x \cdot y) \geq F_N(y) \geq \gamma$. Thus $x \cdot y \in U(F_N; \gamma)$.

Hence, $U(T_N; \alpha)$, $L(I_N; \beta)$, and $U(F_N; \gamma)$ are near UP-filters of X .

Conversely, assume that for all $\alpha, \beta, \gamma \in [-1,0]$, the set $U(T_N; \alpha), L(I_N; \beta)$, and $U(F_N; \gamma)$ are near UP-filters if $U(T_N; \alpha), L(I_N; \beta)$, and $U(F_N; \gamma)$ are nonempty.

Let $x \in X$. Then $T_N(0) \in [-1,0]$. Choose $\alpha = T_N(x)$. Thus $T_N(x) \geq \alpha$, so $x \in L(T_N; \alpha) \neq \varnothing$. By assumption, we have $U(T_N; \alpha)$ is a near UP-filter of X and so $0 \in U(T_N; \alpha)$. Thus $T_N(0) \geq \alpha = T_N(x)$. Next, let $y \in X$. Then $T_N(y) \in [-1,0]$. Choose $\alpha = T_N(y)$. Thus $T_N(y) \geq \alpha$, so $y \in U(T_N; \alpha) \neq \varnothing$. By assumption, we have $U(T_N; \alpha)$ is a near UP-filter of X , and so $x \cdot y \in U(T_N; \alpha)$. Thus $T_N(x \cdot y) \geq \alpha = T_N(y)$.

Let $x \in X$. Then $I_N(0) \in [-1,0]$. Choose $\beta = I_N(x)$. Thus $I_N(x) \leq \beta$, so $x \in L(I_N; \beta) \neq \varnothing$. By assumption, we have $L(I_N; \beta)$ is a near UP-filter of X and so $0 \in L(I_N; \beta)$. Thus $I_N(0) \leq \beta = I_N(x)$. Next, let $y \in X$. Then $I_N(y) \in [-1,0]$. Choose $\beta = I_N(y)$. Thus $I_N(y) \leq \beta$, so $y \in L(I_N; \beta) \neq \varnothing$. By assumption, we have $L(I_N; \beta)$ is a near UP-filter of X , and so $x \cdot y \in L(I_N; \beta)$. Thus $I_N(x \cdot y) \leq \beta = I_N(y)$.

Let $x \in X$. Then $F_N(0) \in [-1,0]$. Choose $\gamma = F_N(x)$. Thus $F_N(x) \geq \gamma$, so $x \in U(F_N; \gamma) \neq \varnothing$. By assumption, we have $U(F_N; \gamma)$ is a near UP-filter of X and so $0 \in U(F_N; \gamma)$. Thus $F_N(0) \geq \gamma = F_N(x)$. Next, let $y \in X$. Then $F_N(y) \in [-1,0]$. Choose $\gamma = F_N(y)$. Thus $F_N(y) \geq \gamma$, so $y \in U(F_N; \gamma) \neq \varnothing$. By assumption, we have $U(F_N; \gamma)$ is a near UP-filter of X , and so $x \cdot y \in U(F_N; \gamma)$. Thus $F_N(x \cdot y) \geq \gamma = F_N(y)$.

Therefore, X_N is a special neutrosophic N -near UP-filter of X .

Theorem 6.3 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, the sets $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-filters of X if $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a special neutrosophic N -UP-filter of X. Let $\alpha,\beta,\gamma \in [-1,0]$ be such that $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x \in U(T_N;\alpha)$. Then $T_N(x) \geq \alpha$. By (5.4), we have $T_N(0) \geq T_N(x) \geq \alpha$. Thus $0 \in U(T_N;\alpha)$. Next, let $x \cdot y \in U(T_N;\alpha)$ and $x \in U(T_N;\alpha)$. Then $T_N(x \cdot y) \geq \alpha$ and $T_N(x) \leq \alpha$, so α is a lower bound of $\{T_N(x \cdot y), T_N(x)\}$. By (5.10), we have $T_N(y) \geq \min\{T_N(x \cdot y), T_N(x)\} \geq \alpha$. Thus $y \in U(T_N;\alpha)$.

Let $x \in L(I_N;\beta)$. Then $I_N(x) \leq \beta$. By (5.5), we have $I_N(0) \leq I_N(x) \leq \beta$. Thus $0 \in L(I_N;\beta)$. Next, let $x \cdot y \in L(I_N;\beta)$ and $x \in L(I_N;\beta)$. Then $I_N(x \cdot y) \leq \beta$ and $I_N(x) \leq \beta$, so β is an upper bound of $\{I_N(x \cdot y), I_N(x)\}$. By (5.11), we have $I_N(y) \leq \max\{I_N(x \cdot y), I_N(x)\} \leq \beta$. Thus $y \in L(I_N;\beta)$.

Let $x \in U(F_N;\gamma)$. Then $F_N(x) \geq \gamma$. By (5.6), we have $F_N(0) \geq F_N(x) \geq \gamma$. Thus $0 \in U(F_N;\gamma)$. Next, let $x \cdot y \in U(F_N;\gamma)$ and $x \in U(F_N;\gamma)$. Then $F_N(x \cdot y) \geq \gamma$ and $F_N(x) \geq \gamma$, so γ is a lower bound of $\{F_N(x \cdot y), F_N(x)\}$. By (5.12), we have $F_N(y) \geq \min\{F_N(x \cdot y), F_N(x)\} \geq \gamma$. Thus $y \in U(F_N;\gamma)$.

Hence, $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-filters of X.

Conversely, assume that for all $\alpha,\beta,\gamma \in [-1,0]$, the set $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-filters if $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x \in X$. Then $T_N(x) \in [-1,0]$. Choose $\alpha = T_N(x)$. Thus $T_N(x) \geq \alpha$, so $x \in U(T_N;\alpha) \neq \varnothing$. By assumption, we have $U(T_N;\alpha)$ is a UP-filter of X and so $0 \in U(T_N;\alpha)$. Thus $T_N(0) \geq \alpha = T_N(x)$. Next, let $x, y \in X$. Then $T_N(x \cdot y), T_N(x) \in [-1,0]$. Choose $\alpha = \min\{T_N(x \cdot y), T_N(x)\}$. Thus $T_N(x \cdot y) \geq \alpha$ and $T_N(x) \geq \alpha$, so $x \cdot y, x \in U(T_N;\alpha) \neq \varnothing$. By assumption, we have $U(T_N;\alpha)$ is a UP-filter of X and so $y \in U(T_N;\alpha)$. Thus $T_N(y) \geq \alpha = \min\{T_N(x \cdot y), T_N(x)\}$.

Let $x \in X$. Then $I_N(x) \in [-1,0]$. Choose $\beta = I_N(x)$. Thus $I_N(x) \leq \beta$, so $x \in L(I_N;\beta) \neq \varnothing$. By assumption, we have $L(I_N;\beta)$ is a UP-filter of X and so $0 \in L(I_N;\beta)$. Thus $I_N(0) \leq \beta = I_N(x)$. Next, let $x, y \in X$. Then $I_N(x \cdot y), I_N(x) \in [-1,0]$. Choose $\beta = \max\{I_N(x \cdot y), I_N(x)\}$. Thus $I_N(x \cdot y) \leq \beta$ and $I_N(x) \leq \beta$, so $x \cdot y, x \in L(I_N;\beta) \neq \varnothing$. By assumption, we have $L(I_N;\beta)$ is a UP-filter of X and so $y \in L(I_N;\beta)$. Thus $I_N(y) \leq \beta = \max\{I_N(x \cdot y), I_N(x)\}$.

Let $x \in X$. Then $F_N(x) \in [-1,0]$. Choose $\gamma = F_N(x)$. Thus $F_N(x) \leq \gamma$, so $x \in U(F_N;\gamma) \neq \varnothing$. By assumption, we have $U(F_N;\gamma)$ is a UP-filter of X and so $0 \in U(F_N;\gamma)$. Thus $F_N(0) \geq \gamma = F_N(x)$. Next, let $x, y \in X$. Then $F_N(x \cdot y), F_N(x) \in [-1,0]$. Choose $\gamma = \min\{F_N(x \cdot y), F_N(x)\}$. Thus $F_N(x \cdot y) \geq \gamma$ and $F_N(x) \geq \gamma$, so $x \cdot y, x \in U(F_N;\gamma) \neq \varnothing$. By assumption, we have $U(F_N;\gamma)$ is a UP-filter of X and so $y \in U(F_N;\gamma)$. Thus $F_N(y) \geq \gamma = \min\{F_N(x \cdot y), F_N(x)\}$.

Therefore, X_N is a special neutrosophic N -UP-filter of X.

Theorem 6.4 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-ideals of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, the sets $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-ideals of X if $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Proof. Assume that X_N is a special neutrosophic N -UP-ideal of X. Let $\alpha,\beta,\gamma \in [-1,0]$ be such that $U(T_N;\alpha), L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x \in U(T_N;\alpha)$. Then $T_N(x) \geq \alpha$. By (5.4), we have $T_N(0) \geq T_N(x) \geq \alpha$. Thus $0 \in U(T_N;\alpha)$. Next, let $x \cdot (y \cdot z) \in U(T_N;\alpha)$ and $y \in U(T_N;\alpha)$. Then $T_N(x \cdot (y \cdot z)) \geq \alpha$ and $T_N(y) \geq \alpha$, so α is a lower bound of $\{T_N(x\cdot(y\cdot z)),T_N(y)\}$. By (5.13), we have $T_N(x\cdot z)\geq\min\{T_N(x\cdot(y\cdot z)),T_N(y)\}\geq\alpha$. Thus $x\cdot z\in U(T_N;\alpha)$.

Let $x\in L(I_N;\beta)$. Then $I_N(x)\leq\beta$. By (5.5), we have $I_N(0)\leq I_N(x)\leq\beta$. Thus $0\in L(I_N;\beta)$. Next, let $x\cdot(y\cdot z)\in L(I_N;\beta)$ and $y\in L(I_N;\beta)$. Then $I_N(x\cdot(y\cdot z))\leq\beta$ and $I_N(y)\leq\beta$, so β is an upper bound of $\{I_N(x\cdot(y\cdot z)),I_N(y)\}$. By (5.14), we have $I_N(x\cdot z)\leq\max\{I_N(x\cdot(y\cdot z)),I_N(y)\}\leq\beta$. Thus $x\cdot z\in L(I_N;\beta)$.

Let $x\in U(F_N;\gamma)$. Then $F_N(x)\geq\gamma$. By (5.6), we have $F_N(0)\geq F_N(x)\geq\gamma$. Thus $0\in U(F_N;\gamma)$. Next, let $x\cdot(y\cdot z)\in U(F_N;\gamma)$ and $y\in U(F_N;\gamma)$. Then $F_N(x\cdot(y\cdot z))\geq\gamma$ and $F_N(y)\geq\gamma$, so γ is a lower bound of $\{F_N(x\cdot(y\cdot z)),F_N(y)\}$. By (5.15), we have $F_N(x\cdot z)\geq\min\{F_N(x\cdot(y\cdot z)),F_N(y)\}\geq\gamma$. Thus $x\cdot z\in U(F_N;\gamma)$.

Hence, $U(T_N;\alpha),L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha,\beta,\gamma\in[-1,0]$, the set $U(T_N;\alpha),L(I_N;\beta)$, and $U(F_N;\gamma)$ are UP-ideals if $U(T_N;\alpha),L(I_N;\beta)$, and $U(F_N;\gamma)$ are nonempty.

Let $x\in X$. Then $T_N(x)\in[-1,0]$. Choose $\alpha=T_N(x)$. Thus $T_N(x)\geq\alpha$, so $x\in U(T_N;\alpha)\neq\varnothing$. By assumption, we have $U(T_N;\alpha)$ is a UP-ideal of X and so $0\in U(T_N;\alpha)$. Thus $T_N(0)\geq\alpha=T_N(x)$. Next, let $x,y,z\in X$. Then $T_N(x\cdot(y\cdot z)),T_N(y)\in[-1,0]$. Choose $\alpha=\min\{T_N(x\cdot(y\cdot z)),T_N(y)\}$. Thus $T_N(x\cdot(y\cdot z))\geq\alpha$ and $T_N(y)\geq\alpha$, so $x\cdot(y\cdot z),y\in U(T_N;\alpha)\neq\varnothing$. By assumption, we have $U(T_N;\alpha)$ is a UP-ideal of X and so $x\cdot z\in U(T_N;\alpha)$. Thus $T_N(x\cdot z)\geq\alpha=\min\{T_N(x\cdot(y\cdot z)),T_N(y)\}$.

Let $x\in X$. Then $I_N(x)\in[-1,0]$. Choose $\beta=I_N(x)$. Thus $I_N(x)\leq\beta$, so $x\in L(I_N;\beta)\neq\varnothing$. By assumption, we have $L(I_N;\beta)$ is a UP-ideal of X and so $0\in L(I_N;\beta)$. Thus $I_N(0)\leq\beta=I_N(x)$. Next, let $x,y,z\in X$. Then $I_N(x\cdot(y\cdot z)),I_N(y)\in[-1,0]$. Choose $\beta=\max\{I_N(x\cdot(y\cdot z)),I_N(y)\}$. Thus $I_N(x\cdot(y\cdot z))\leq\beta$ and $I_N(y)\leq\beta$, so $x\cdot(y\cdot z),y\in L(I_N;\beta)\neq\varnothing$. By assumption, we have $L(I_N;\beta)$ is a UP-ideal of X and so $x\cdot z\in L(I_N;\beta)$. Thus $I_N(x\cdot z)\leq\beta=\max\{I_N(x\cdot(y\cdot z)),I_N(y)\}$.

Let $x\in X$. Then $F_N(x)\in[-1,0]$. Choose $\gamma=F_N(x)$. Thus $F_N(x)\geq\gamma$, so $x\in U(F_N;\gamma)\neq\varnothing$. By assumption, we have $U(F_N;\gamma)$ is a UP-ideal of X and so $0\in U(F_N;\gamma)$. Thus $F_N(0)\geq\gamma=F_N(x)$. Next, let $x,y,z\in X$. Then $F_N(x\cdot(y\cdot z)),F_N(y)\in[-1,0]$. Choose $\gamma=\min\{F_N(x\cdot(y\cdot z)),F_N(y)\}$. Thus $F_N(x\cdot(y\cdot z))\geq\gamma$ and $F_N(y)\geq\gamma$, so $x\cdot(y\cdot z),y\in U(F_N;\gamma)\neq\varnothing$. By assumption, we have $U(F_N;\gamma)$ is a UP-ideal of X and so $x\cdot z\in U(F_N;\gamma)$. Thus $F_N(x\cdot z)\geq\gamma=\min\{F_N(x\cdot(y\cdot z)),F_N(y)\}$.

Therefore, X_N is a special neutrosophic N -UP-ideal of X.

Definition 6.5 Let X_N be a neutrosophic N -structure over X. For $\alpha,\beta,\gamma\in[-1,0]$, the sets

$$ULU_{X_N}(\alpha,\beta,\gamma)=\{x\in X\mid T_N\geq\alpha,I_N\leq\beta,F_N\geq\gamma\},$$

$$LUL_{X_N}(\alpha,\beta,\gamma)=\{x\in X\mid T_N\leq\alpha,I_N\geq\beta,F_N\leq\gamma\},$$

$$E_{X_N}(\alpha,\beta,\gamma)=\{x\in X\mid T_N=\alpha,I_N=\beta,F_N=\gamma\}$$

are called a *ULU-(α,β,γ)-level subset*, an *LUL-(α,β,γ)-level subset*, and an *E-(α,β,γ)-level subset* of X_N, respectively. Then we see that

$$ULU_{X_N}(\alpha,\beta,\gamma)=U(T_N;\alpha)\cap L(I_N;\beta)\cap U(F_N;\gamma),$$

$$LUL_{X_N}(\alpha,\beta,\gamma)=L(T_N;\alpha)\cap U(I_N;\beta)\cap L(F_N;\gamma),$$

$$E_{X_N}(\alpha,\beta,\gamma) = E(T_N;\alpha) \cap E(I_N;\beta) \cap E(F_N;\gamma).$$

Corollary 6.6 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-subalgebra of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $LUL_{X_N}(\alpha,\beta,\gamma)$ is a UP-subalgebra of X where $LUL_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 4.2.

Corollary 6.7 A neutrosophic N -structure X_N over X is a neutrosophic N -near UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $LUL_{X_N}(\alpha,\beta,\gamma)$ is a near UP-filter of X where $LUL_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 4.3.

Corollary 6.8 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $LUL_{X_N}(\alpha,\beta,\gamma)$ is a UP-filter of X where $LUL_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 4.4.

Corollary 6.9 A neutrosophic N -structure X_N over X is a neutrosophic N -UP-ideal of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $LUL_{X_N}(\alpha,\beta,\gamma)$ is a UP-ideal of X where $LUL_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 4.5.

Corollary 6.10 A neutrosophic N -structure X_N over X is a neutrosophic N -strongly UP-ideal of X if and only if $E(T_N, T_N(0)) = X, E(I_N, I_N(0)) = X$, and $E(F_N, F_N(0)) = X$.

Proof. It is straightforward by Theorem 4.6.

Corollary 6.11 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-subalgebra of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $ULU_{X_N}(\alpha,\beta,\gamma)$ is a UP-subalgebra of X where $ULU_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 6.1.

Corollary 6.12 A neutrosophic N -structure X_N over X is a special neutrosophic N -near UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [-1,0]$, $ULU_{X_N}(\alpha,\beta,\gamma)$ is a near UP-filter of X where $ULU_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 6.2.

Corollary 6.13 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-filter of X if and only if for all $\alpha,\beta,\gamma\in[-1,0]$, $ULU_{X_N}(\alpha,\beta,\gamma)$ is a UP-filter of X where $ULU_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 6.3.

Corollary 6.14 A neutrosophic N -structure X_N over X is a special neutrosophic N -UP-ideal of X if and only if for all $\alpha,\beta,\gamma\in[-1,0]$, $ULU_{X_N}(\alpha,\beta,\gamma)$ is a UP-ideal of X where $ULU_{X_N}(\alpha,\beta,\gamma)$ is nonempty.

Proof. It is straightforward by Theorem 6.4.

7. Conclusions

In this paper, we have introduced the notions of (special) neutrosophic N -UP-subalgebras, (special) neutrosophic N -near UP-filters, (special) neutrosophic N -UP-filters, (special) neutrosophic N -UP-ideals, and (special) neutrosophic N -strongly UP-ideals of UP-algebras and investigated some of their important properties. Then we have that the notion of (special) neutrosophic N -UP-subalgebras is a generalization of (special) neutrosophic N -near UP-filters, (special) neutrosophic N -near UP-filters is a generalization of (special) neutrosophic N -UP-filters, (special) neutrosophic N -UP-filters is a generalization of (special) neutrosophic N -UP-ideals, and (special) neutrosophic N -UP-ideals is a generalization of (special) neutrosophic N -strongly UP-ideals. Moreover, we obtain that (special) neutrosophic N -strongly UP-ideals and constant neutrosophic N -structures coincide.

In our future study, we will apply these notion/results to other type of neutrosophic N -structures in UP-algebras. Also, we will study the soft set theory/cubic set theory of such neutrosophic N -structures.

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