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Neutrosophic Bipolar Vague Set and its Application to Neutrosophic Bipolar Vague Graphs

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Abstract: A bipolar model is a significant model wherein positive data revels the liked object, while negative data speaks the disliked object. The principle reason for analysing the vague graphs is to demonstrate the stability of few properties in a graph, characterized or to be characterized in using vagueness. In this present research article, the new concept of neutrosophic bipolar vague sets are initiated. Further, its application to neutrosophic bipolar vague graphs are introduced. Moreover, some remarkable properties of strong neutrosophic bipolar vague graphs, complete neutrosophic bipolar vague graphs and complement neutrosophic bipolar vague graphs are explored and the proposed ideas are outlined with an appropriate example

Keywords: Neutrosophic bipolar vague set, Neutrosophic bipolar vague graphs, Complete neutrosophic bipolar vague graph, Strong neutrosophic bipolar vague graph.

1. Introduction

Fuzzy set theory richly contains progressive frameworks comprising of data with various degrees of accuracy. Vague sets are first investigated by Gau and Buehrer [30] which is an extension of fuzzy set theory. Various issues in real-life problems have fluctuations, one has to handle these vulnerabilities, vague set is introduced. Vague sets are regarded as a special case of context dependent fuzzy sets and it is applicable in real-time systems consisting of information with multiple levels of precision. So as to deal with the uncertain and conflicting data, the neutrosophic set is presented by the creator Smarandache and studied widely about it [13, 21, 28, 31, 41, 42, 4, 5, 43, 44, 22, 23, 45]. Neutrosophic sets are the more generalized sets, one can manage with uncertain informations in a more successful way with a progressive manner when appeared differently in relation to fuzzy sets. It have the greater adaptability, accuracy and similarity to the framework when contrasted with past existing fuzzy models. The neutrosophic set has three completely independent parts, which are truth-membership degree, indeterminacy-membership degree and falsity-membership degree with the sum of these values lies between $\mathbf{0}$ and $\mathbf{3}$; therefore, it is applied to many different areas, such as algebra [32, 33] and decision-making problems (see [46] and references therein).

Bipolar fuzzy sets are extension of fuzzy sets whose membership degree ranges from $[-\mathbf{1},\mathbf{1}]$. The membership degree $(\mathbf{0},\mathbf{1}]$ represents that an object satisfies a certain property whereas the membership degree $[-\mathbf{1},\mathbf{0})$ represents that the element satisfies the implicit counter-property. The positive information indicates that the consideration to be possible and negative information indicates that the consideration is granted to be impossible. Notable that bipolar fuzzy sets and vague sets appear to be comparative, but they are completely different sets. Even though both sets handle with incomplete data, they will not adapt the indeterminate or inconsistent information which appears in many domains like decision support systems. Many researchers pay attention to the development of neutrosophic and bipolar neutrosophic graphs [39, 40]. For example, in [17], the authors studied neutrosophic soft topological K-algebras. In [48], complex neutrosophic graphs are developed. Bipolar single valued neutrosophic graphs are established in [25]. Bipolar neutrosophic sets and its application to incidence graphs are discussed in [15]. In [16], bipolar neutrosophic graphs are established.

Recently, a variety of decision making problems are based on two-sided bipolar judgements on a positive side and a negative side. Nowadays bipolar fuzzy sets are playing a substantial role in chemistry, economics, computer science, engineering, medicine and decision making problems (for more details see [27, 28, 31, 34, 38, 46] and references therein). Akram [6, 8] introduced bipolar fuzzy graphs and discuss its various properties and several new concepts on bipolar neutrosophic graphs and bipolar neutrosophic hypergraphs have been studied in [7] and references therein. In [4], he established the certain notions including strong neutrosophic soft graphs and complete neutrosophic soft graphs. The authors [3] first introduces the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set to improve the reasonability of decision making in reality. It is remarkable that the Definition 2.6 in [37] has a flaw and it not defined in a proper manner. We focussed on to redefine that definition in aa proper way and explained with an example and also we applied to neutrosophic bipolar vague graphs. Motivation of the mentioned works as earlier [10], we mainly contribute the definition of neutrosophic bipolar vague set is redefined. In addition, it is applied to neutrosophic bipolar vague graphs and strong neutrosophic bipolar vague graphs. The developed results will find an application in NBVGs and also in decision making. The objectives in this work as follows:

- Newly defined the neutrosophic bipolar vague set
- Introduce the operations like union and intersection with example in section 2.
- In section 3, neutrosophic bipolar vague graphs are developed with an example. Further, the concepts of neutrosophic bipolar vague subgraph, adjacency, path, connectedness and degree of neutrosophic bipolar vague graph are evolved.
- Further we presented some remarkable properties of strong neutrosophic bipolar vague graphs in section 5, followed by a remark by comparing other types of bipolar graphs. The obtained results will improve the existing result [37].

2. Preliminaries

Definition 2.1 *[5] A vague set A on a non empty set X is a pair (T_A, F_A), where $T_A: X \to [0,1]$ and $F_A: X \to [0,1]$ are true membership and false membership functions, respectively, such that*

$$0 \le T_A(x) + F_A(y) \le 1 \text{ for any } x \in X.$$

Let X and Y be two non-empty sets. A vague relation R of X to Y is a vague set R on $X \times Y$ that is $R = (T_R, F_R)$, where $T_R: X \times Y \to [0,1], F_R: X \times Y \to [0,1]$ which satisfies the condition:

$$0 \leq T_R(x,y) + F_R(x,y) \leq 1 \text{ for any } x \in X.$$

Let $G = (V,E)$ be a graph. A pair $G = (J,K)$ is called a vague graph on G^* or a vague graph where $J = (T_J, F_J)$ is a vague set on V and $K = (T_K, F_K)$ is a vague set on $E \subseteq V \times V$ such that for each $xy \in E$,

$$T_K(xy) \leq (T_J(x) \wedge T_J(y)) \text{ and } F_K(xy) \geq (T_J(x) \vee F_J(y)).$$

Definition 2.2 *[4] A Neutrosophic set A is contained in another neutrosophic set B, (i.e) $A \subseteq B$ if $\forall x \in X, T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$and $F_A(x) \geq F_B(x)$.*

Definition 2.3 *[27, 30] Let X be a space of points (objects), with a generic elements in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership-function $F_A(x)$.*

For each point x in X, $T_A(x), F_A(x), I_A(x) \in [0,1]$, $A = \{\langle x, T_A(x), F_A(x), I_A(x)\rangle\}$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.4 *[24] A neutrosophic graph is defined as a pair $G^* = (V,E)$ where*

(i) $V = \{v_1, v_2, .., v_n\}$ such that $T_1 = V \to [0,1]$, $I_1 = V \to [0,1]$ and $F_1 = V \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

(ii) $E \subseteq V \times V$ where $T_2 = E \to [0,1]$, $I_2 = E \to [0,1]$ and $F_2 = E \to [0,1]$ are such that

$$T_2(uv) \leq \{T_1(u) \wedge T_1(v)\},$$
$$I_2(uv) \leq \{I_1(u) \wedge I_1(v)\},$$
$$F_2(uv) \leq \{F_1(u) \vee F_1(v)\},$$
$$\text{and } 0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3, \forall uv \in E.$$

Definition 2.5 *[46] A bipolar neutrosophic set A in X is defined as an object of the form*

$A = \{< x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x) >: x \in X\}$, where $T^P, I^P, F^P: X \to [0,1]$ and $T^N, I^N, F^N: X \to [-1,0]$ The Positive membership degree $T^P(x), I^P(x), F^P(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^N(x), I^N(x), F^N(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Definition 2.6 *[6] Let X be a non-empty set. Then we call* $A = \{\langle x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)\rangle, x \in X\}$a bipolar single valued neutrosophic relation on X such that $T_A^P(x,y) \in [0,1], I_A^P(x,y) \in [0,1], F_A^P(x,y) \in [0,1]$ and $T_A^N(x,y) \in [-1,0], I_A^N(x,y) \in [-1,0], F_A^N(x,y) \in [-1,0]$.

Definition 2.7 *[6] Let $A = (T_P^A, I_P^A, F_P^A, T_N^A, I_N^A, F_N^A)$ and $B = (T_P^B, I_P^B, F_P^B, T_N^B, I_N^B, F_N^B)$ be bipolar single valued neutrosophic set on X. If $B = (T_P^B, I_P^B, F_P^B, T_N^B, I_N^B, F_N^B)$ is a bipolar single valued neutrosophic relation on $A = (T_P^A, I_P^A, F_P^A, T_N^A, I_N^A, F_N^A)$ then*

$$T_B^P(xy) \leq (T_A^P(x) \wedge T_A^P(y)), T_B^N(xy) \geq (T_A^N(x) \vee T_A^N(y))$$
$$I_B^P(xy) \geq (I_A^P(x) \vee I_A^P(y)), I_B^N(xy) \leq (I_A^N(x) \wedge I_A^N(y))$$
$$F_B^P(xy) \geq (F_A^P(x) \vee F_A^P(y)), F_B^N(xy) \leq (F_A^N(x) \wedge F_A^N(y))$$

A bipolar single valued neutrosophic relation B on X is called symmetric if $T_B^P(xy) = T_B^P(yx), I_B^P(xy) = I_B^P(yx), F_B^P(xy) = F_B^P(yx)$ and $T_B^N(xy) = T_B^N(yx), I_B^N(xy) = I_B^N(yx), F_B^N(xy) = F_B^N(yx)$ for all $xy \in X$.

Definition 2.8 *[20] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as* $A_{NV} = \{\langle x, \hat{T}_{A_{NV}}(x), \hat{I}_{A_{NV}}(x), \hat{F}_{A_{NV}}(x)\rangle, x \in X\}$ whose truth-membership, indeterminacy membership and falsity-membership function is defined as $\hat{T}_{A_{NV}}(x) = [\hat{T}^-(x), \hat{T}^+(x)], [\hat{I}^-(x), \hat{I}^+(x)], [\hat{F}^-(x), \hat{F}^+(x)]$, where $T^+(x) = 1 - F^-(x), F^+(x) = 1 - T^-(x)$, and $0 \leq T^-(x) + I^-(x) + F^-(x) \leq 2$.

Definition 2.9 *[20] The complement of NVS A_{NV} is denoted by A_{NV}^c and it is defined by*

$$\hat{T}_{A_{NV}}^c(x) = [1 - T^+(x), 1 - T^-(x)],$$
$$\hat{I}_{A_{NV}}^c(x) = [1 - I^+(x), 1 - I^-(x)],$$
$$\hat{F}_{A_{NV}}^c(x) = [1 - F^+(x), 1 - F^-(x)],$$

Definition 2.10 *[20] Let A_{NV} and B_{NV} be two NVSs of the universe U. If for all $u_i \in U, \hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i), \hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i), \hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$* then the NVS A_{NV} are included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$ where $1 \leq i \leq n$.

Definition 2.11 *[20] The union of two NVSs A_{NV} and B_{NV} is a NVSs, C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NV} and B_{NV} by*

$$\hat{T}_{C_{NV}}(x) = [(\hat{T}_{A_{NV}}^-(x) \vee \hat{T}_{B_{NV}}^-(x)), (\hat{T}_{A_{NV}}^+(x) \vee \hat{T}_{B_{NV}}^+(x))]$$
$$\hat{I}_{C_{NV}}(x) = [(\hat{I}_{A_{NV}}^-(x) \wedge \hat{I}_{B_{NV}}^-(x)), (\hat{I}_{A_{NV}}^+(x) \wedge \hat{I}_{B_{NV}}^+(x))]$$
$$\hat{F}_{C_{NV}}(x) = [(\hat{F}_{A_{NV}}^-(x) \wedge \hat{F}_{B_{NV}}^-(x)), (\hat{F}_{A_{NV}}^+(x) \wedge \hat{F}_{B_{NV}}^+(x))]$$

Definition 2.12 *[20] The intersection of two NVSs A_{NV} and B_{NV} is a NVSs C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NV} and B_{NV} by*

$$\hat{T}_{C_{NV}}(x) = [(\hat{T}_{A_{NV}}^-(x) \wedge \hat{T}_{B_{NV}}^-(x)), (\hat{T}_{A_{NV}}^+(x) \wedge \hat{T}_{B_{NV}}^+(x))]$$
$$\hat{I}_{C_{NV}}(x) = [(\hat{I}_{A_{NV}}^-(x) \vee \hat{I}_{B_{NV}}^-(x)), (\hat{I}_{A_{NV}}^+(x) \vee \hat{I}_{B_{NV}}^+(x))]$$
$$\hat{F}_{C_{NV}}(x) = [(\hat{F}_{A_{NV}}^-(x) \vee \hat{F}_{B_{NV}}^-(x)), (\hat{F}_{A_{NV}}^+(x) \vee \hat{F}_{B_{NV}}^+(x))]$$

Definition 2.13 *[39] Let $G^* = (V, E)$ be a graph. A pair $G = (J, K)$ is called a neutrosophic vague graph (NVG) on G^* or a neutrosophic graph where $J = (\hat{T}_J, \hat{I}_J, \hat{F}_J)$ is a neutrosophic vague set on V and $K = (\hat{T}_K, \hat{I}_K, \hat{F}_K)$ is a neutrosophic vague set $E \subseteq V \times V$ where*

(1)$V = \{v_1, v_2, \ldots, v_n\}$ such that $T_J^-: V \to [0,1], I_J^-: V \to [0,1], F_J^-: V \to [0,1]$ which satisfies the condition $F_J^- = [1 - T_J^+]$

$T_J^+: V \to [0,1], I_J^+: V \to [0,1], F_J^+: V \to [0,1]$ which satisfies the condition $F_J^+ = [1 - T_1^-]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in V$, and

$$0 \leq T_J^-(v_i) + I_J^-(v_i) + F_J^-(v_i) \leq 2.$$
$$0 \leq T_J^+(v_i) + I_J^+(v_i) + F_J^+(v_i) \leq 2.$$

(2) $E \subseteq V \times V$ where

$$T_K^-: V \times V \to [0,1], I_K^-: V \times V \to [0,1], F_K^-: V \times V \to [0,1]$$
$$T_K^+: V \times V \to [0,1], I_K^+: V \times V \to [0,1], F_K^+: V \times V \to [0,1]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in E$ respectively and such that

$$0 \leq T_K^-(v_i) + I_K^-(v_i) + F_K^-(v_i) \leq 2.$$
$$0 \leq T_K^+(v_i) + I_K^+(v_i) + F_K^+(v_i) \leq 2.$$

such that

$$T_K^-(xy) \leq \{T_J^-(x) \wedge T_J^-(y)\}$$
$$I_K^-(xy) \leq \{I_J^-(x) \wedge I_J^-(y)\}$$
$$F_K^-(xy) \leq \{F_J^-(x) \vee F_J^-(y)\},$$

similarly

$$T_K^+(xy) \leq \{T_J^+(x) \wedge T_J^+(y)\}$$
$$I_K^+(xy) \leq \{I_J^+(x) \wedge I_J^+(y)\}$$
$$F_K^+(xy) \leq \{F_J^+(x) \vee F_J^+(y)\}.$$

Example 2.14 *Consider a neutrosophic vague graph* $G = (J, K)$ *such that* $J = \{a, b, c\}$ *and* $K = \{ab, bc, ca\}$ *defined by*

$$\hat{a} = T[0.5,0.6], I[0.4,0.3], F[0.4,0.5], \hat{b} = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6],$$
$$\hat{c} = T[0.4,0.4], I[0.5,0.3], F[0.6,0.6]$$
$$a^- = (0.5,0.4,0.4), b^- = (0.4,0.7,0.4), c^- = (0.4,0.5,0.6)$$
$$a^+ = (0.6,0.3,0.5), b^+ = (0.6,0.3,0.6), c^+ = (0.4,0.3,0.6)$$

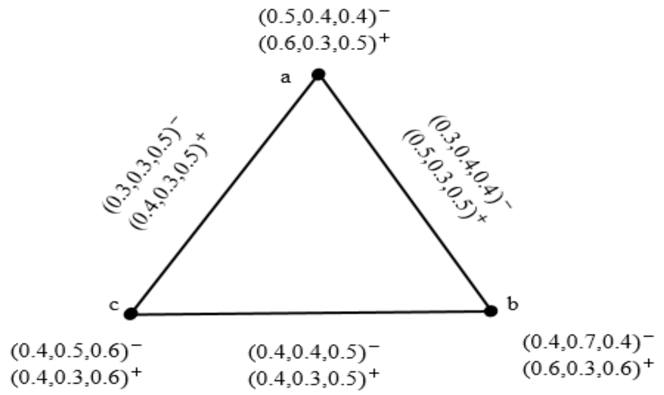

Figure 1 neutrosophic vague graph

3. Neutrosophic Bipolar Vague Set

In this section, the definition of NBVS, complement of NBVS, operations like union, intersection are elaborated with an example.

Definition 3.1 *In a universe of discourse* X, *the neutrosophic bipolar vague set (NBVS), denoted as* A_{NBVS} *represented as,*

$$A_{NBV} = \{\langle x, \hat{T}_{A_{NBV}}^P(x), \hat{I}_{A_{NBV}}^P(x), \hat{F}_{A_{NBV}}^P(x), \hat{T}_{A_{NBV}}^N(x), \hat{I}_{A_{NBV}}^N(x), \hat{F}_{A_{NBV}}^N(x)\rangle, x \in X\}$$

whose truth-membership, indeterminacy membership and falsity-membership function is expanded as

$$\hat{T}^P_{A_{NBV}}(x) = [(T^-)^P(x), (T^+)^P(x)], \hat{I}^P_{A_{NBV}}(x) = [(I^-)^P(x), (I^+)^P(x)], \hat{F}^P_{A_{NBV}}(x) = [(F^-)^P(x), (F^+)^P(x)],$$

where $(T^+)^P(x) = 1 - (F^-)^P(x), (F^+)^P(x) = 1 - (T^-)^P(x)$, and provided that,

$$0 \le (T^-)^P(x) + (I^-)^P(x) + (F^-)^P(x) \le 2.$$

Also

$$\hat{T}^N_{A_{NBV}}(x) = [(T^-)^N(x), (T^+)^N(x)], \hat{I}^N_{A_{NBV}}(x) = [(I^-)^N(x), (I^+)^N(x)], \hat{F}^N_{A_{NBV}}(x) = [(F^-)^N(x), (F^+)^N(x)],$$

where $(T^+)^N(x) = -1 - (F^-)^N(x), (F^+)^N(x) = -1 - (T^-)^N(x)$,

and provided that,

$$0 \ge (T^-)^N(x) + (I^-)^N(x) + (F^-)^N(x) \ge -2.$$

Example 3.2 *Let* $U = \{x_1, x_2, x_3\}$ *be a set of universe we define the NBV set* A_{NBV} *as follows*

$$A_{NBV} = \{\frac{x_1}{[0.3,0.6]^P, [0.5,0.5]^P, [0.4,0.7]^P, [-0.3,-0.5]^N, [-0.4,-0.4]^N, [-0.5,-0.7]^N},$$

$$\frac{x_2}{[0.4,0.6]^P, [0.4,0.6]^P, [0.4,0.6]^P, [-0.4,-0.4]^N, [-0.5,-0.5]^N, [-0.6,-0.6]^N},$$

$$\frac{x_3}{[0.3,0.7]^P, [0.6,0.4]^P, [0.3,0.7]^P, [-0.4,-0.6]^N, [-0.5,-0.6]^N, [-0.4,-0.6]^N}\}$$

Definition 3.3 *IN NBVS, the complement of* A^c_{NBV} *be expanded as,*

$$(\hat{T}^c_{A_{NBV}}(x))^P = \{(1 - T^+(x))^P, (1 - T^-(x))^P\}, (\hat{T}^c_{A_{NBV}}(x))^N = \{(-1 - T^+(x))^N, (-1 - T^-(x))^N\}$$

$$(\hat{I}^c_{A_{NBV}}(x))^P = \{(1 - I^+(x))^P, (1 - I^-(x))^P\}, (\hat{I}^c_{A_{NBV}}(x))^N = \{(-1 - I^+(x))^N, (-1 - I^-(x))^N\}$$

$$(\hat{F}^c_{A_{NBV}}(x))^P = \{(1 - F^+(x))^P, (1 - F^-(x))^P\}, (\hat{F}^c_{A_{NBV}}(x))^N = \{(-1 - F^+(x))^N, (-1 - F^-(x))^N\}$$

Example 3.4 *Considering above example we have*

$$A_{NBV} = \{\frac{x_1}{[0.7,0.4]^P, [0.5,0.5]^P, [0.6,0.3]^P, [-0.7,-0.5]^N, [-0.6,-0.6]^N, [-0.5,-0.3]^N},$$

$$\frac{x_2}{[0.6,0.4]^P, [0.6,0.4]^P, [0.6,0.4]^P, [-0.6,-0.6]^N, [-0.5,-0.5]^N, [-0.4,-0.4]^N},$$

$$\frac{x_3}{[0.7,0.3]^P, [0.4,0.6]^P, [0.7,0.3]^P, [-0.6,-0.4]^N, [-0.5,-0.4]^N, [-0.6,-0.4]^N}\}$$

Definition 3.5 *Two NBVSs* A_{NBV} *and* B_{NBV} *of the universe* U *are said to be equal, if for all* $u_i \in U$,

$$(\hat{T}_{A_{NBV}})^P(u_i) = (\hat{T}_{B_{NBV}})^P(u_i), (\hat{I}_{A_{NBV}})^P(u_i) = (\hat{I}_{B_{NBV}})^P(u_i), (\hat{F}_{A_{NBV}})^P(u_i) = (\hat{F}_{B_{NBV}})^P(u_i)$$

and

$$(\hat{T}_{A_{NBV}})^N(u_i) = (\hat{T}_{B_{NBV}})^N(u_i), (\hat{I}_{A_{NBV}})^N(u_i) = (\hat{I}_{B_{NBV}})^N(u_i), (\hat{F}_{A_{NBV}})^N(u_i) = (\hat{F}_{B_{NBV}})^N(u_i).$$

Definition 3.6 *In the Universe* U*, two NBVSs,* A_{NBV}*,* B_{NBV} *be given as,*

$$(\hat{T}_{A_{NBV}})^P(u_i) \le (\hat{T}_{B_{NBV}})^P(u_i), (\hat{I}_{A_{NBV}})^P(u_i) \ge (\hat{I}_{B_{NBV}})^P(u_i), (\hat{F}_{A_{NBV}})^P(u_i) \ge (\hat{F}_{B_{NBV}})^P(u_i)$$

and

$$(\hat{T}_{A_{NBV}})^N(u_i) \ge (\hat{T}_{B_{NBV}})^N(u_i), (\hat{I}_{A_{NBV}})^N(u_i) \le (\hat{I}_{B_{NBV}})^N(u_i), (\hat{F}_{A_{NBV}})^N(u_i) \le (\hat{F}_{B_{NBV}})^N(u_i)$$

then the NBVS $(A_{NBV})^P$ are included by $(B_{NBV})^P$, denoted by $(A_{NBV})^P \subseteq (B_{NBV})^P$ where $1 \le i \le n$ and $(A_{NBV})^N$ are included by $(B_{NBV})^N$, denoted by $(A_{NBV})^N \subseteq (B_{NBV})^N$ where $1 \le i \le n$.

Definition 3.7 *The union of two NVSs A_{NBV} and B_{NBV} is a NBVSs, C_{NBV}, written as $C_{NBV} = A_{NBV} \cup B_{NBV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NBV} and B_{NBV} by*

$$(\hat{T}_{C_{NBV}})^P(x) = [((T^-_{A_{NBV}})^P(x) \vee (T^-_{B_{NBV}})^P(x)), ((T^+_{A_{NBV}})^P(x) \vee (T^+_{B_{NBV}})^P(x))]$$
$$(\hat{I}_{C_{NBV}})^P(x) = [((I^-_{A_{NBV}})^P(x) \wedge (I^-_{B_{NBV}})^P(x)), ((I^+_{A_{NBV}})^P(x) \wedge (I^+_{B_{NBV}})^P(x))]$$
$$(\hat{F}_{C_{NBV}})^P(x) = [((F^-_{A_{NBV}})^P(x) \wedge (F^-_{B_{NBV}})^P(x)), ((F^+_{A_{NBV}})^P(x) \wedge (F^+_{B_{NBV}})^P(x))], \text{and}$$
$$(\hat{T}_{C_{NBV}})^N(x) = [((T^-_{A_{NBV}})^N(x) \wedge (T^-_{B_{NBV}})^N(x)), ((T^+_{A_{NBV}})^N(x) \wedge (T^+_{B_{NBV}})^N(x))]$$
$$(\hat{I}_{C_{NBV}})^N(x) = [((I^-_{A_{NBV}})^N(x) \vee (I^-_{B_{NBV}})^N(x)), ((I^+_{A_{NBV}})^N(x) \vee (I^+_{B_{NBV}})^N(x))]$$
$$(\hat{F}_{C_{NBV}})^N(x) = [((F^-_{A_{NBV}})^N(x) \vee (F^-_{B_{NBV}})^N(x)), ((F^+_{A_{NBV}})^N(x) \vee (F^+_{B_{NBV}})^N(x))]$$

Definition 3.8 *The intersection of two NVSs A_{NBV} and B_{NBV} is a NBVSs C_{NBV}, written as $C_{NBV} = A_{NBV} \cap B_{NBV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NBV} and B_{NBV} by*

$$(\hat{T}_{C_{NBV}})^P(x) = [((T^-_{A_{NBV}})^P(x) \wedge (T^-_{B_{NBV}})^P(x)), ((T^+_{A_{NBV}})^P(x) \wedge (T^+_{B_{NBV}})^P(x))]$$
$$(\hat{I}_{C_{NBV}})^P(x) = [((I^-_{A_{NBV}})^P(x) \vee (I^-_{B_{NBV}})^P(x)), ((I^+_{A_{NBV}})^P(x) \vee (I^+_{B_{NBV}})^P(x))]$$
$$(\hat{F}_{C_{NBV}})^P(x) = [((F^-_{A_{NBV}})^P(x) \vee (F^-_{B_{NBV}})^P(x)), ((F^+_{A_{NBV}})^P(x) \vee (F^+_{B_{NBV}})^P(x))], \text{and}$$
$$(\hat{T}_{C_{NBV}})^N(x) = [((T^-_{A_{NBV}})^N(x) \vee (T^-_{B_{NBV}})^N(x)), ((T^+_{A_{NBV}})^N(x) \vee (T^+_{B_{NBV}})^N(x))]$$
$$(\hat{I}_{C_{NBV}})^N(x) = [((I^-_{A_{NBV}})^N(x) \wedge (I^-_{B_{NBV}})^N(x)), ((I^+_{A_{NBV}})^N(x) \wedge (I^+_{B_{NBV}})^N(x))]$$
$$(\hat{F}_{C_{NBV}})^N(x) = [((F^-_{A_{NBV}})^N(x) \wedge (F^-_{B_{NBV}})^N(x)), ((F^+_{A_{NBV}})^N(x) \wedge (F^+_{B_{NBV}})^N(x))]$$

Definition 3.9 *Let U be a set of universe and let A_{NBV} and B_{NBV} be NBVSs, then the union $A_{NBV} \cap B_{NBV}$ is defined as follows:*

$$A_{NBV} = \{\frac{x_1}{[0.3,0.6]^P, [0.6,0.6]^P, [0.4,0.7]^P, [-0.4,-0.7]^N, [-0.6,-0.6]^N, [-0.3,-0.6]^N},$$
$$\frac{x_2}{[0.4,0.6]^P, [0.6,0.4]^P, [0.4,0.6]^P, [-0.5,-0.5]^N, [-0.7,-0.3]^N, [-0.5,-0.5]^N},$$
$$\frac{x_3}{[0.7,0.8]^P, [0.6,0.6]^P, [0.2,0.3]^P, [-0.5,-0.4]^N, [-0.5,-0.5]^N, [-0.6,-0.5]^N}\}$$

$$B_{NBV} = \{\frac{x_1}{[0.2,0.8]^P, [0.5,0.4]^P, [0.2,0.8]^P, [-0.5,-0.7]^N, [-0.7,-0.7]^N, [-0.3,-0.5]^N},$$
$$\frac{x_2}{[0.3,0.8]^P, [0.6,0.5]^P, [0.2,0.7]^P, [-0.5,-0.6]^N, [-0.4,-0.3]^N, [-0.4,-0.5]^N},$$
$$\frac{x_3}{[0.2,0.5]^P, [0.5,0.2]^P, [0.5,0.8]^P, [-0.5,-0.5]^N, [-0.4,-0.3]^N, [-0.5,-0.5]^N}\}$$

$$A_{NBV} \cap B_{NBV} = H_{NBV}$$
$$= \{\frac{x_1}{[0.2,0.6]^P, [0.6,0.6]^P, [0.4,0.8]^P, [-0.4,-0.7]^N, [-0.7,-0.7]^N, [-0.3,-0.6]^N},$$
$$\frac{x_2}{[0.3,0.6]^P, [0.6,0.5]^P, [0.4,0.7]^P, [-0.5,-0.5]^N, [-0.7,-0.3]^N, [-0.5,-0.5]^N},$$
$$\frac{x_3}{[0.2,0.5]^P, [0.6,0.6]^P, [0.5,0.8]^P, [-0.5,-0.4]^N, [-0.5,-0.5]^N, [-0.6,-0.5]^N}\}$$

4 Neutrosophic Bipolar Vague graphs

In this section, neutrosophic bipolar vague graphs are defined. The concepts of neutrosophic bipolar vague subgraph, adjacency, path, connectedness and degree of neutrosophic bipolar vague graph are discussed.

Definition 4.1 *In a crisp graph $G^* = (V, E)$. A pair $G = (J, K)$ is called a neutrosophic bipolar vague graph (NBVG) on G^* or a neutrosophic bipolar vague graph where J is a neutrosophic bipolar vague set and K is a neutrosophic bipolar vague relation in G^* such that $J^P = ((\hat{T}_J)^P, (\hat{I}_J)^P, (\hat{F}_J)^P), J^N = ((\hat{T}_J)^N, (\hat{I}_J)^N, (\hat{F}_J)^N)$ is a neutrosophic bipolar vague set on V and $K^P = ((\hat{T}_K)^P, (\hat{I}_K)^P, (\hat{F}_K)^P), K^N = ((\hat{T}_K)^N, (\hat{I}_K)^N, (\hat{F}_K^N))$ is a neutrosophic Bipolar vague set $E \subseteq V \times V$ where*

(1) $V = \{v_1, v_2, \dots, v_n\}$ such that

$$(T_J^-)^P: V \to [0,1], (I_J^-)^P: V \to [0,1], (F_J^-)^P: V \to [0,1]$$

which satisfies the condition $(F_J^-)^P = [1 - (T_J^+)^P]$

$$(T_J^+)^P: V \to [0,1], (I_J^+)^P: V \to [0,1], (F_J^+)^P: V \to [0,1]$$

which satisfies the condition $(F_J^+)^P = [1 - (T_J^-)^P]$, and

$$(T_J^-)^N: V \to [-1,0], (I_J^-)^N: V \to [-1,0], (F_J^-)^N: V \to [-1,0]$$

which satisfies the condition $(F_J^-)^N = [-1 - (T_J^+)^N]$

$(T_J^+)^N: V \to [-1,0], (I_J^+)^N: V \to [-1,0], (F_J^+)^N: V \to [-1,0]$ which satisfies the condition $(F_J^+)^N = [-1 - (T_J^-)^N]$ denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in V$, and

$$0 \le (T_J^-)^P(v_i) + (I_J^-)^P(v_i) + (F_J^-)^P(v_i) \le 2$$
$$0 \le (T_J^+)^P(v_i) + (I_J^+)^P(v_i) + (F_J^+)^P(v_i) \le 2$$
$$0 \ge (T_J^-)^N(v_i) + (I_J^-)^N(v_i) + (F_J^-)^N(v_i) \ge -2$$
$$0 \le (T_J^+)^N(v_i) + (I_J^+)^N(v_i) + (F_J^+)^N(v_i) \ge -2.$$

(2) $E \subseteq V \times V$ where

$$(T_K^-)^P: V \times V \to [0,1], (I_K^-)^P: V \times V \to [0,1], (F_K^-)^P: V \times V \to [0,1]$$
$$(T_K^+)^P: V \times V \to [0,1], (I_K^+)^P: V \times V \to [0,1], (F_K^+)P: V \times V \to [0,1] \text{and}$$
$$(T_K^-)^N: V \times V \to [-1,0], (I_K^-)^N: V \times V \to [-1,0], (F_K^-)^N: V \times V \to [-1,0]$$
$$(T_K^+)^N: V \times V \to [-1,0], (I_K^+)^N: V \times V \to [-1,0], (F_K^+)^N: V \times V \to [-1,0]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in E$ respectively and such that

$$0 \le (T_K^-)^P(v_i, v_j) + (I_K^-)^P(v_i, v_j) + (F_K^-)^P(v_i, v_j) \le 2$$
$$0 \le (T_K^+)^P(v_i, v_j) + (I_K^+)^P(v_i, v_j) + (F_K^+)^P(v_i, v_j) \le 2$$
$$0 \ge (T_K^-)^N(v_i, v_j) + (I_K^-)^N(v_i, v_j) + (F_K^-)^N(v_i, v_j) \ge -2$$
$$0 \ge (T_K^+)^N(v_i, v_j) + (I_K^+)^N(v_i, v_j) + (F_K^+)^N(v_i, v_j) \ge -2,$$

such that

$$(T_K^-)^P(xy) \le \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\}$$
$$(I_K^-)^P(xy) \le \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\}$$
$$(F_K^-)^P(xy) \le \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\}$$
$$(T_K^+)^P(xy) \le \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\}$$
$$(I_K^+)^P(xy) \le \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\}$$

$$(F_K^+)^P(xy) \le \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\},$$

and

$$(T_K^-)^N(xy) \ge \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\}$$
$$(I_K^-)^N(xy) \ge \{(I_J^-)^N(x) \vee (I_K^-)^N(y)\}$$
$$(F_K^-)^N(xy) \ge \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\},$$
$$(T_K^+)^N(xy) \ge \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\}$$
$$(I_K^+)^N(xy) \ge \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\}$$
$$(F_K^+)^N(xy) \ge \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\}.$$

Example 4.2 *Consider a neutrosophic bipolar vague graph* $G = (J, K)$ *such that* $J = \{a, b, c\}$ *and* $K = \{ab, bc, ca\}$ *defined by*

$$(\hat{a})^P = T[0.5,0.6], I[0.4,0.3], F[0.4,0.5],$$
$$(\hat{b})^P = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6],$$
$$(\hat{c})^P = T[0.4,0.4], I[0.5,0.3], F[0.6,0.6]$$
$$(a^-)^P = (0.5,0.4,0.4), (b^-)^P = (0.4,0.7,0.4), (c^-)^P = (0.4,0.5,0.6)$$

$$(a^+)^P = (0.6,0.3,0.5), (b^+)^P = (0.6,0.3,0.6), (c^+)^P = (0.4,0.3,0.6)$$
$$(\hat{a})^N = T[-0.6,-0.5], I[-0.3,-0.4], F[-0.5,-0.4],$$
$$(\hat{b})^N = T[-0.6,-0.4], I[-0.7,-0.3], F[-0.6,-0.4],$$
$$(\hat{c})^N = T[-0.4,-0.4], I[-0.3,-0.5], F[-0.6,-0.6]$$
$$(a^-)^N = (-0.6,-0.3,-0.5), (b^-)^N = (-0.6,-0.7,-0.6), (c^-)^N = (-0.4,-0.3,-0.6)$$
$$(a^+)^N = (-0.5,-0.4,-0.4), (b^+)^P = (-0.4,-0.3,-0.4), (c^+)^P = (-0.4,-0.5,-0.6)$$

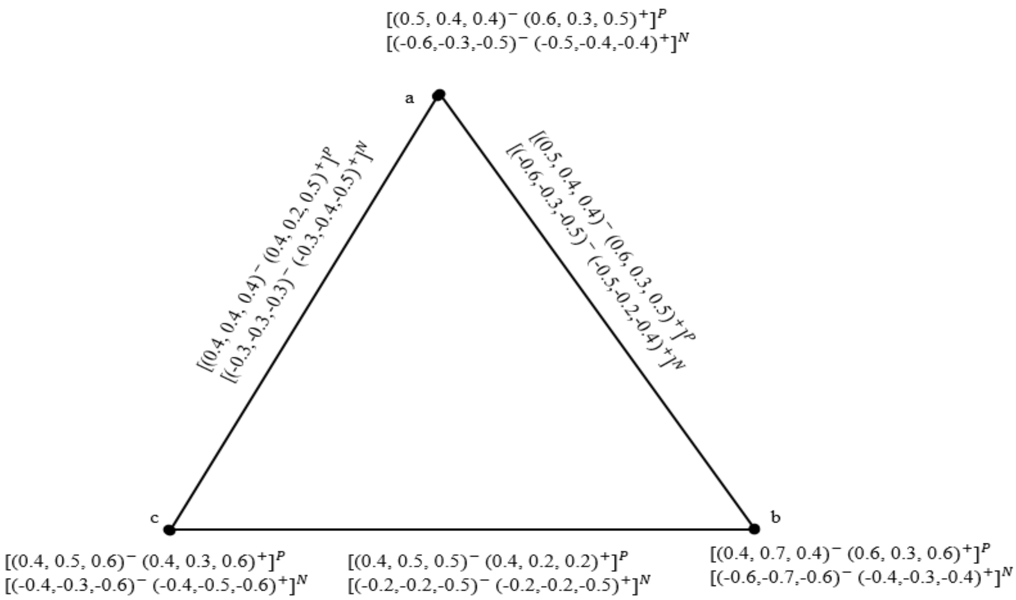

Figure 2 NEUTROSOPHIC BIPOLAR VAGUE GRAPH

Definition 4.3 *A neutrosophic bipolar vague graph* $H = (J'(x), K'(x))$ *is said to be a neutrosophic bipolar vague subgraph of the NVG* $G = (J, K)$ *if* $J'(x) \subseteq J(x)$ *and* $K'(xy) \subseteq K'(xy)$, *in other words, if*

$$(\hat{T}_J')^P(x) \leq (\hat{T}_J)^P(x)$$
$$(\hat{I}_J')^P(x) \leq (\hat{I}_J)^P(x)$$
$$(\hat{F}_J')^P(x) \leq (\hat{F}_J)^P(x)\ \forall x \in V$$
$$(\hat{T}_K')^P(xy) \leq (\hat{T}_K)^P(xy)$$
$$(\hat{I}_K')^P(xy) \leq (\hat{I}_K)^P(xy)$$
$$(\hat{F}_K')^P(xy) \leq (\hat{F}_K)^P(xy), \forall xy \in E.$$

Also,

$$(\hat{T}_J')^N(x) \geq (\hat{T}_J)^N(x)$$
$$(\hat{I}_J')^N(x) \geq (\hat{I}_J)^N(x)$$
$$(\hat{F}_J')^N(x) \geq (\hat{F}_J)^N(x), \forall x \in V$$

and

$$(\hat{T}_K')^N(xy) \geq (\hat{T}_K)^N(xy)$$
$$(\hat{I}_K')^N(xy) \geq (\hat{I}_K)^N(xy)$$
$$(\hat{F}_K')^N(xy) \geq (\hat{F}_K)^N(xy), \forall xy \in E.$$

Definition 4.4 *The two vertices are said to be adjacent in a neutrosophic bipolar vague graph* $G = (J, K)$ *if*

$$(T_K^-)^P(xy) = \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\}$$
$$(I_K^-)^P(xy) = \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\}$$
$$(F_K^-)^P(xy) = \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\},$$
$$(T_K^+)^P(xy) = \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\}$$
$$(I_K^+)^P(xy) = \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\}$$
$$(F_K^+)^P(xy) = \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\},$$
$$(T_K^-)^N(xy) = \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\}$$
$$(I_K^-)^N(xy) = \{(I_J^-)^N(x) \vee (I_K^-)^N(y)\}$$
$$(F_K^-)^N(xy) = \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\},$$
$$(T_K^+)^N(xy) = \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\}$$
$$(I_K^+)^N(xy) = \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\}$$
$$(F_K^+)^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\},$$

Here, x is the neighbour of y and vice versa, also (xy) is incident at x and y.

Definition 4.5 *In a neutrosophic bipolar vague graph* $G = (J, K)$*, a path* ρ *is meant to be a sequence of different points* $x_0, x_1, \ldots, x_n$ *such an extent that*

$$(T_K^-)^P(x_{i-1}, x_1) > 0, (I_K^-)^P(x_{i-1}, x_1) > 0, (F_K^-)^P(x_{i-1}, x_1) > 0,$$
$$(T_K^+)^P(x_{i-1}, x_1) > 0, (I_K^+)^P(x_{i-1}, x_1) > 0, (F_K^+)^P(x_{i-1}, x_1) > 0,$$

and

$$(T_K^-)^N(x_{i-1}, x_1) < 0, (I_K^-)^N(x_{i-1}, x_1) < 0, (F_K^-)^N(x_{i-1}, x_1) < 0,$$
$$(T_K^+)^N(x_{i-1}, x_1) < 0, (I_K^+)^N(x_{i-1}, x_1) < 0, (F_K^+)^N(x_{i-1}, x_1) < 0,$$

for every i lies between 0 and 1. $n \leq 1$ is known as the path length.. A single vertex x_i can represent as a path.

Definition 4.6 *A neutrosophic bipolar vague graph* $G = (J, K)$*, if every pair of vertices has at least one neutrosophic bipolar vague path between them is known as connected, otherwise it is disconnected.*

Definition 4.7 *A vertex $x_i \in V$ of neutrosophic bipolar vague graph $G = (J, K)$ is said to be isolated vertex if there is no effective edge incident at x_i.*

Definition 4.8 *A vertex in a neutrosophic bipolar vague graph $G = (J, K)$ having exactly one neighbours is called a pendent vertex. Otherwise, it is called non-pendent vertex. An edge in a neutrosophic bipolar vague graph incident with a pendent vertex is called a pendent edge other words it is called non-pendent edge. A vertex in a neutrosophic bipolar vague graph adjacent to the pendent vertex is called an support of the pendent edge.*

Definition 4.9 *A neutrosophic bipolar vague graph $G = (J, K)$ that has neither self loops nor parallel edge is called simple neutrosophic bipolar vague graph.*

Definition 4.10 *Let $G = (J, K)$ be a neutrosophic bipolar vague graph. Then the degree of a vertex $x \in G$ is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex x denoted by*

$$(d(x))^P = ([(d_{T_J}^-)P(x), (d_{T_J}^+)^P(x)], [(d_{I_J}^-)^P(x), (d_{I_J}^+)^P(x)], [(d_{F_J}^-)^P(x), (d_{F_J}^+)^P(x)])$$

$$(d(x))^N = ([(d_{T_J}^-)N(x), (d_{T_J}^+)^N(x)], [(d_{I_J}^-)^N(x), (d_{I_J}^+)^N(x)], [(d_{F_J}^-)^N(x), (d_{F_J}^+)^N(x)])$$

where $(d_{T_J}^-)^P(x) = \sum_{x \neq y} (T_K^-)^P(xy)$,$(d_{T_J}^+)^P(x) = \sum_{x \neq y} (T_K^+)^P(xy)$ denotes the positive degree of truth membership vertex, $(d_{I_J}^-)^P(x) = \sum_{x \neq y} (I_K^-)^P(xy)$, $(d_{I_J}^+)^P(x) = \sum_{x \neq y} (I_K^+)^P(xy)$ denotes the positive degree of indeterminacy membership vertex, $(d_{F_J}^-)^P(x) = \sum_{x \neq y} (F_K^-)^P(xy)$, $(d_{F_J}^+)^P(x) = \sum_{x \neq y} (F_K^+)^P(xy)$ denotes the positive degree of falsity membership vertex for all $x, y \in J$.

Similarly, $(d_{T_J}^-)^N(x) = \sum_{x \neq y} (T_K^-)^N(xy)$, $(d_{T_J}^+)^N(x) = \sum_{x \neq y} (T_K^+)^N(xy)$ denotes the negative degree of truth membership vertex, $(d_{I_J}^-)^N(x) = \sum_{x \neq y} (I_K^-)^N(xy)$, $(d_{I_J}^+)^N(x) = \sum_{x \neq y} (I_K^+)^N(xy)$ denotes the negative degree of indeterminacy membership vertex, $(d_{F_J}^-)^N(x) = \sum_{x \neq y} (F_K^-)^N(xy)$, $(d_{F_J}^+)^N(x) = \sum_{x \neq y} (F_K^+)^N(xy)$ denotes the negative degree of falsity membership vertex for all $x, y \in J$.

Definition 4.11 *A neutrosophic bipolar vague graph $G = (J, K)$ is called constant if degree of each vertex is $A = (A_1, A_2, A_3)$ that is $d(x) = (A_1, A_2, A_3)$ for all $x \in V$.*

5 Strong Neutrosophic Bipolar Vague Graphs

In this section, we presented some remarkable properties of strong neutrosophic bipolar vague graphs and a remark is provided by comparing other types of bipolar graphs. Finally conclusion is given.

Definition 5.1 *A neutrosophic bipolar vague graph $G = (J, K)$ of $G^* = (V, E)$ is called strong neutrosophic bipolar vague graph if*

$$(T_K^-)^P(xy) = \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\}$$
$$(I_K^-)^P(xy) = \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\}$$
$$(F_K^-)^P(xy) = \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\},$$
$$(T_K^+)^P(xy) = \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\}$$
$$(I_K^+)^P(xy) = \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\}$$
$$(F_K^+)^P(xy) = \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\},$$
$$(T_K^-)^N(xy) = \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\}$$
$$(I_K^-)^N(xy) = \{(I_J^-)^N(x) \vee (I_K^-)^N(y)\}$$

$$(F_K^-)^N(xy) = \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\},$$
$$(T_K^+)^N(xy) = \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\}$$
$$(I_K^+)^N(xy) = \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\}$$
$$(F_K^+)^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\}, \forall((xy) \in K)$$

Definition 5.2 *The complement of neutrosophic bipolar vague graph $G = (J, K)$ on G^* is a neutrosophic bipolar vague graph G^c where*

- $(J^c)^P(x) = (J)^P(x)$
- $(T_J^{-c})^P(x) = (T_J^-)^P(x), (I_J^{-c})^P(x) = (I_J^-)^P(x), (F_J^{-c})^P(x) = (F_J^-)^P(x)$ for all $x \in V$.
- $(T_J^{+c})^P(x) = (T_J^+)^P(x), (I_J^{+c})^P(x) = (I_J^+)^P(x), (F_J^{+c})^P(x) = (F_J^+)^P(x)$ for all $x \in V$.
- $(T_K^{-c})^P(xy) = \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\} - (T_K^-)^P(xy)$, $(I_K^{-c})^P(xy) = \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\} - (I_K^-)^P(xy)$

$(F_K^{-c})^P(xy) = \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\} - (F_K^-)^P(xy)$ for all $(xy) \in E$

- $(T_K^{+c})^P(xy) = \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\} - (T_K^+)^P(xy)$, $(I_K^{+c})^P(xy) = \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\} - (I_K^+)^P(xy)$

$(F_K^{+c})^P(xy) = \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\} - (F_K^+)^P(xy)$ for all $(xy) \in E$

- $(J^c)^N(x) = (J)^N(x)$
- $(T_J^{-c})^N(x) = (T_J^-)^N(x), (I_J^{-c})^N(x) = (I_J^-)^N(x), (F_J^{-c})^N(x) = (F_J^-)^N(x)$ for all $x \in V$.
- $(T_J^{+c})^N(x) = (T_J^+)^N(x), (I_J^{+c})^N(x) = (I_J^+)^N(x), (F_J^{+c})^N(x) = (F_J^+)^N(x)$ for all $x \in V$.
- $(T_K^{-c})^N(xy) = \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\} - (T_K^-)^N(xy)$

$$(I_K^{-c})^N(xy) = \{(I_J^-)^N(x) \vee (I_J^-)^N(y)\} - (I_K^-)^N(xy)$$
$$(F_K^{-c})^N(xy) = \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\} - (F_K^-)^N(xy) \text{ for all } (xy) \in E$$

- $(T_K^{+c})^N(xy) = \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\} - (T_K^+)^N(xy)$

$$(I_K^{+c})^N(xy) = \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\} - (I_K^+)^N(xy)$$
$$(F_K^{+c})^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\} - (F_K^+)^N(xy) \text{ for all } (xy) \in E$$

Remark 5.3 *If $G = (J, K)$ is a neutrosophic bipolar vague graph on G^* then from above definition, it follows that G^{c^c} is given by the neutrosophic bipolar vague graph $G^{c^c} = (J^{c^c}, K^{c^c})$ on G^* where*

- $((J^c)^c)^P(x) = (J(x))^P$
- $((T_J^{-c})^c)^P(x) = (T_J^-)^P(x), ((I_J^{-c})^c)^P(x) = (I_J^-)^P(x), ((F_J^{-c})^c)^P(x) = (F_J^-)^P(x)$ for all $x \in V$.
- $((T_J^{+c})^c)^P(x) = (T_J^+)^P(x), ((I_J^{+c})^c)^P(x) = (I_J^+)^P(x), ((F_J^{+c})^c)^P(x) = (F_J^+)^P(x)$ for all $x \in V$.
- $((T_K^{-c})^c)^P(xy) = \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\} - (T_K^-)^P(xy)$

$$((I_K^{-c})^c)^P(xy) = \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\} - (I_K^-)^P(xy)$$
$$((F_K^{-c})^c)^P(xy) = \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\} - (F_K^-)^P(xy) \text{ for all } (xy) \in E$$

- $((T_K^{+c})^c)^P(xy) = \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\} - (T_K^+)^P(xy)$

$$((I_K^{+c})^c)^P(xy) = \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\} - (I_K^+)^P(xy)$$
$$((F_K^{+c})^c)^P(xy) = \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\} - (F_K^+)^P(xy) \text{ for all } (xy) \in E$$

- $((J^c)^c)^N(x) = (J(x))^N$
- $((T_J^{-c})^c)^N(x) = (T_J^-)^N(x), ((I_J^{-c})^c)^N(x) = (I_J^-)^N(x), ((F_J^{-c})^c)^N(x) = (F_J^-)^N(x)$ for all $x \in V$.

- $((T_J^{+^c})^c)^N(x) = (T_J^+)^N(x), ((I_J^{+^c})^c)^N(x) = (I_J^+)^N(x), ((F_J^{+^c})^c)^N(x) = (F_J^+)^N(x)$ for all $x \in V$.
- $((T_K^{-^c})^c)^N(xy) = \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\} - (T_K^-)^N(xy)$

$$((I_K^{-^c})^c)^N(xy) = \{(I_J^-)^N(x) \vee (I_J^-)^N(y)\} - (I_K^-)^N(xy)$$

$$((F_K^{-^c})^c)^N(xy) = \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\} - (F_K^-)^N(xy) \text{ for all } (xy) \in E$$

- $((T_K^{+^c})^c)^N(xy) = \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\} - (T_K^+)^N(xy)$

$$((I_K^{+^c})^N)^N(xy) = \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\} - (I_K^+)^N(xy)$$

$$((F_K^{+^c})^c)^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\} - (F_K^+)^N(xy) \text{ for all } (xy) \in E.$$

for any neutrosophic bipolar vague graph G,G^c is strong neutrosophic bipolar vague graph and $G \subseteq G^c$.

Definition 5.4 *Suppose G^c is the complement of neutrosophic bipolar vague graph G. In a strong neutrosophic bipolar vague graph G, $G \cong G^c$ then it is called self-complementary.*

Proposition 5.5 *Let $G = (J, K)$ be a strong neutrosophic bipolar vague graph if*

$$(T_K^-)^P(xy) = \{(T_J^-)^P(x) \wedge (T_J^-)^P(y)\}$$
$$(I_K^-)^P(xy) = \{(I_J^-)^P(x) \wedge (I_J^-)^P(y)\}$$
$$(F_K^-)^P(xy) = \{(F_J^-)^P(x) \vee (F_J^-)^P(y)\},$$
$$(T_K^+)^P(xy) = \{(T_J^+)^P(x) \wedge (T_J^+)^P(y)\}$$
$$(I_K^+)^P(xy) = \{(I_J^+)^P(x) \wedge (I_J^+)^P(y)\}$$
$$(F_K^+)^P(xy) = \{(F_J^+)^P(x) \vee (F_J^+)^P(y)\},$$
$$(T_K^-)^N(xy) = \{(T_J^-)^N(x) \vee (T_J^-)^N(y)\}$$
$$(I_K^-)^N(xy) = \{(I_J^-)^N(x) \vee (I_K^-)^N(y)\}$$
$$(F_K^-)^N(xy) = \{(F_J^-)^N(x) \wedge (F_J^-)^N(y)\},$$
$$(T_K^+)^N(xy) = \{(T_J^+)^N(x) \vee (T_J^+)^N(y)\}$$
$$(I_K^+)^N(xy) = \{(I_J^+)^N(x) \vee (I_J^+)^N(y)\}$$
$$(F_K^+)^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\}, \forall((xy) \in K)$$

Then G is self complementary.

Proof. Let $G = (J, K)$ be a strong neutrosophic bipolar vague graph such that

$$(\hat{T}_K)^P(xy) = \frac{1}{2}[(\hat{T}_J)^P(x) \wedge (\hat{T}_J)^P(y)]$$

$$(\hat{I}_K)^P(xy) = \frac{1}{2}[(\hat{I}_J)^P(x) \wedge (\hat{I}_J)^P(y)]$$

$$(\hat{F}_K)^P(xy) = \frac{1}{2}[(\hat{F}_J)^P(x) \vee (\hat{F}_J)^P(y)],$$

and

$$(\hat{T}_K)^N(xy) = \frac{1}{2}[(\hat{T}_J)^N(x) \vee (\hat{T}_J)^N(y)]$$

$$(\hat{I}_K)^N(xy) = \frac{1}{2}[(\hat{I}_J)^N(x) \vee (\hat{I}_J)^N(y)]$$

$$(\hat{F}_K)^N(xy) = \frac{1}{2}[(\hat{F}_J)^N(x) \wedge (\hat{F}_J)^N(y)]$$

for all $xy \in J$ then $G \approx G^{c^c}$, implies G is self complementary. Hence proved

Proposition 5.6 *Assume that, G is a self complementary neutrosophic bipolar vague graph then*

$$\sum_{x\neq y} (\hat{T}_K)^P(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{T}_J)^P(x) \wedge (\hat{T}_J)^P(y)\}$$
$$\sum_{x\neq y} (\hat{I}_K)^P(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{I}_J)^P(x) \wedge (\hat{I}_J)^P(y)\}$$
$$\sum_{x\neq y} (\hat{F}_K)^P(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{F}_J)^P(x) \vee (\hat{F}_J)^P(y)\}$$
$$\sum_{x\neq y} (\hat{T}_K)^N(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{T}_J)^N(x) \vee (\hat{T}_J)^N(y)\}$$
$$\sum_{x\neq y} (\hat{I}_K)^N(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{I}_J)^N(x) \vee (\hat{I}_J)^N(y)\}$$
$$\sum_{x\neq y} (\hat{F}_K)^N(xy) = \frac{1}{2}\sum_{x\neq y} \{(\hat{F}_J)^N(x) \wedge (\hat{F}_J)^N(y)\}$$

Proof. Suppose that G be an self complementary neutrosophic bipolar vague graph, by its definition, we have isomorphism $f: J_1 \to J_2$ satisfy

$$(\hat{T}^c_{J_1})^P(f(x)) = (\hat{T}_{J_1})^P(f(x)) = (\hat{T}_{J_1})^P(x)$$
$$(\hat{I}^c_{J_1})^P(f(x)) = (\hat{I}_{J_1})^P(f(x)) = (\hat{I}_{J_1})^P(x)$$
$$(\hat{F}^c_{J_1})^P(f(x)) = (\hat{F}_{J_1})^P(f(x)) = (\hat{F}_{J_1})^P(x)$$

and

$$(\hat{T}^c_{K_1})^P(f(x), f(y)) = (\hat{T}_{K_1})^P(f(x), f(y)) = (\hat{T}_{K_1})^P(xy)$$
$$(\hat{I}^c_{K_1})^P(f(x), f(y)) = (\hat{I}_{K_1})^P(f(x), f(y)) = (\hat{I}_{K_1})^P(xy)$$
$$(\hat{F}^c_{K_1})^P(f(x), f(y)) = (\hat{F}_{K_1})^P(f(x), f(y)) = (\hat{F}_{K_1})^P(xy)$$

we have $(\hat{T}^c_{K_1})^P(f(x), f(y)) = ((\hat{T}^c_{J_1})^P(x) \wedge (\hat{T}^c_{J_1})^P(y)) - (\hat{T}_{K_1})^P(f(x), f(y))$.

i.e, $(\hat{T}_{K_1})^P(xy) = ((\hat{T}^c_{J_1})^P(x) \wedge (\hat{T}^c_{J_1})^P(y)) - (\hat{T}_{K_1})^P(f(x), f(y))$.

$(\hat{T}_{K_1})^P(xy) = ((\hat{T}^c_{J_1})^P(x) \wedge (\hat{T}^c_{J_1})^P(y)) - (\hat{T}_{K_1})^P(xy)$, hence

$$\sum_{x\neq y} (\hat{T}_{K_1})^P(xy) + \sum_{x\neq y} (\hat{T}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{T}_{J_1})^P(x) \wedge (\hat{T}_{J_1})^P(y)).$$

Similarly, $\sum_{x\neq y} (\hat{I}_{K_1})^P(xy) + \sum_{x\neq y} (\hat{I}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{I}_{J_1})^P(x) \wedge (\hat{I}_{J_1})^P(y))$

$$\sum_{x\neq y} (\hat{F}_{K_1})^P(xy) + \sum_{x\neq y} (\hat{F}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{F}_{J_1})^P(x) \vee (\hat{F}_{J_1})^P(y))$$
$$2\sum_{x\neq y} (\hat{T}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{T}_{J_1})^P(x) \wedge (\hat{T}_{J_1})^P(y))$$
$$2\sum_{x\neq y} (\hat{I}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{I}_{J_1})^P(x) \wedge (\hat{I}_{J_1})^P(y))$$
$$2\sum_{x\neq y} (\hat{F}_{K_1})^P(xy) = \sum_{x\neq y} ((\hat{F}_{J_1})^P(x) \vee (\hat{F}_{J_1})^P(y))$$

Similarly one can prove for the negative condition, from the equation of the proposition (5.5) holds.

Proposition 5.7 *Suppose G_1 and G_2 is neutrosophic bipolar vague graph which is strong, $\overline{G_1} \approx \overline{G_2}$(isomorphism)*

Proof. Assume that G_1 and G_2 are isomorphic there exist a bijective map $f: J_1 \to J_2$ satisfying,

$$(\hat{T}_{J_1})^P(x) = (\hat{T}_{J_2})^P(f(x)),$$
$$(\hat{I}_{J_1})^P(x) = (\hat{I}_{J_2})^P(f(x)),$$
$$(\hat{F}_{J_1})^P(x) = (\hat{F}_{J_2})^P(f(x)), \text{for all} x \in J_1$$
$$(\hat{T}_{J_1})^N(x) = (\hat{T}_{J_2})^N(f(x)),$$
$$(\hat{I}_{J_1})^N(x) = (\hat{I}_{J_2})^N(f(x)),$$
$$(\hat{F}_{J_1})^N(x) = (\hat{F}_{J_2})^N(f(x)), \text{for all} x \in J_1$$

and

$$(\hat{T}_{K_1})^P(xy) = (\hat{T}_{K_2})^P(f(x), f(y))$$
$$(\hat{I}_{K_1})^P(xy) = (\hat{I}_{K_2})^P(f(x), f(y))$$
$$(\hat{F}_{K_1})^P(xy) = (\hat{F}_{K_2})^P(f(x), f(y)) \forall xy \in K_1$$
$$(\hat{T}_{K_1})^N(xy) = (\hat{T}_{K_2})^N(f(x), f(y))$$
$$(\hat{I}_{K_1})^N(xy) = (\hat{I}_{K_2})^N(f(x), f(y))$$
$$(\hat{F}_{K_1})^N(xy) = (\hat{F}_{K_2})^N(f(x), f(y)) \forall xy \in K_1$$

by definition (5.2) we have

$$(T^c_{K_1})^P(xy) = ((T_{J_1})^P(x) \wedge (T_{J_1})^P(y)) - (T_{K_1})^P(xy)$$
$$= ((T_{J_2})^P f(x) \wedge (T_{J_2})^P f(y)) - (T_{K_2})^P(f(x)f(y))$$
$$= (T^c_{K_2})^P(f(x)f(y))$$
$$(I^c_{K_1})^P(xy) = ((I_{J_1})^P(x) \wedge (I_{J_1})^P(y)) - (I_{K_1})^P(xy)$$
$$= ((I_{J_2})^P f(x) \wedge (I_{J_2})^P f(y)) - (I_{K_2})^P(f(x)f(y))$$
$$= (I^c_{K_2})^P(f(x)f(y))$$
$$(F^c_{K_1})^P(xy) = ((F_{J_1})^P(x) \vee (F_{J_1})^P(y)) - (F_{K_1})^P(xy)$$
$$= ((F_{J_2})^P f(x) \vee (F_{J_2})^P f(y)) - (F_{K_2})^P(f(x)f(y))$$
$$= (F^c_{K_2})^P(f(x)f(y))$$

Hence $G_1^c \approx G_2^c$ for all $(xy) \in K_1$

Definition 5.8 *A neutrosophic bipolar vague graph $G = (J, K)$ is complete if*

$$(T^-_K)^P(xy) = \{(T^-_J)^P(x) \wedge (T^-_J)^P(y)\}$$
$$(I^-_K)^P(xy) = \{(I^-_J)^P(x) \wedge (I^-_J)^P(y)\}$$
$$(F^-_K)^P(xy) = \{(F^-_J)^P(x) \vee (F^-_J)^P(y)\},$$
$$(T^+_K)^P(xy) = \{(T^+_J)^P(x) \wedge (T^+_J)^P(y)\}$$
$$(I^+_K)^P(xy) = \{(I^+_J)^P(x) \wedge (I^+_J)^P(y)\}$$
$$(F^+_K)^P(xy) = \{(F^+_J)^P(x) \vee (F^+_J)^P(y)\},$$
$$(T^-_K)^N(xy) = \{(T^-_J)^N(x) \vee (T^-_J)^N(y)\}$$
$$(I^-_K)^N(xy) = \{(I^-_J)^N(x) \vee (I^-_K)^N(y)\}$$
$$(F^-_K)^N(xy) = \{(F^-_J)^N(x) \wedge (F^-_J)^N(y)\},$$
$$(T^+_K)^N(xy) = \{(T^+_J)^N(x) \vee (T^+_J)^N(y)\}$$
$$(I^+_K)^N(xy) = \{(I^+_J)^N(x) \vee (I^+_J)^N(y)\}$$

$$(F_K^+)^N(xy) = \{(F_J^+)^N(x) \wedge (F_J^+)^N(y)\}, \forall((xy) \in J)$$

Remark 5.9 The complement of NBVGs are NBVGs provided the graph is strong. According to [9], the complement of Single-Valued Neutrosophic Graph (SVNG) is not a SVNG. By the same idea, we implement the definition for NBVGs to obtain the proposed concepts. For other type of bipolar graphs, the complement of Bipolar Fuzzy Graph (BFG) is BFG [6]. The complement of Bipolar Fuzzy Soft Graph (BFSG) and Bipolar Neutrosophic Graph (BNG) are BFSG and BNG, [14, 16] respectively, provided if the graph is strong. The complement of complete bipolar SVNG is bipolar SVFG [25].

Conclusion

This present work characterised the new concept of neutrosophic bipolar vague sets and its application to NBVGs are introduced. Moreover, some remarkable properties of strong NBVGs, complete NBVGs and complement NBVGs have been investigated and the proposed concepts are illustrated with the examples. The obtained results are extended to interval neutrosophic bipolar vague sets. Further we can extend to investigate the domination number, regular and isomorphic properties of the proposed graph.

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