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SELEN

a program for solving the "Sea Level Equation"

User Manual for version 4.0

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The SELEN logo has been drawn by Daniela Riposati from the INGV Laboratorio Grafica e Immagini.

1 Introduction

The open source program SELEN solves numerically the so-called "Sea Level Equation" (SLE) for a spherical, layered, rotating Earth with viscoelastic rheology. The SLE is an integral equation that was introduced in the 70s to model the sea level variations in response to the melting of late–Pleistocene ice–sheets and refined in the following decades to account for the horizontal migration of the shorelines and the Earth rotational feedback to glacio-hydro-isostatic processes.

SELEN solves the SLE in the spectral domain and computes several quantities of interest for the Glacial Isostatic Adjustement (GIA) problem. These include: synthetic Relative Sea–Level (RSL) curves, present-day sealevel rates at tide–gauges, surface displacements and perturbations to the gravity field, geodetic "fingerprints" and paleo–topography maps. Harmonic coefficients of the solution of the SLE are accessible to the user, so it is possible to obtain further relevant quantities through harmonic synthesis.

The current release of SELEN (version 4.0, also typeset as SELEN^4) considerably improves upon the previous version of the code by implementing a *gravitationally* and *topographically* self-consistent SLE (Peltier, 1994, Mitrovica and Milne, 2003, Lambeck, 2004), instead of the original Farrell and Clark (1976) formulation. SELEN has also been streamlined and simplified to facilitate code readability, portability and reusability.

This document is focused on the practical aspects related to setting up and running SELEN. For a discussion of the physical meaning of the SLE, on the underlying mathematical aspects and on its discretization in a form suitable for numerical solution, we refer the reader to Spada and Melini (2019) and to the accompanying supplement (SM19 and SSM19 hereinafter).

2 System requirements

SELEN can be run in any modern UNIX environment, including Linux and Mac OS X. On Windows systems, SELEN can run if either the Cygwin environment[1] or the Windows Subsystem for Linux[2] are installed. A standard Fortran 90 and the `make` utility are needed to build the executables; SELEN has been successfully tested with both the GNU `gfortran` compiler, which is available free of charge on a wide range of systems, and with the commercial Intel Fortran compiler.

On modern multi-core systems, SELEN can leverage multi-threaded parallelism in order to speed up the most computationally intensive portions of the code. To take advantage of this feature, the OpenMP libraries must be installed and available; these are generally included as a default with all recent versions of Fortran compilers.

The SELEN package includes several scripts to produce graphical outputs from the computed results. To run these scripts, the GMT (Generic Mapping Tools) free software package must be installed (Wessel and Smith, 1998). However, GMT is not strictly required by SELEN at run time, so the user can visualize the results using any other choice of plotting software. The scripts provided with SELEN are compatible with version 4 of GMT; if a newer version is installed, it is likely that slight changes to the scripts are needed due to the new option syntax introduced with GMT version 5 and above. SELEN takes advantage of some subroutines from the SHTOOLS library (Wieczorek and Meschede, 2018) for the numerical evaluation of spherical harmonics; a copy of the Fortran version of SHTOOLS is included in the SELEN distribution package.

The hardware requirements of SELEN are related to the desired grid resolution parameter R and to the maximum harmonic degree l_{max}. For low to medium resolutions ($l_{max} \leq 128$, $R \leq 44$) SELEN can run without effort on a common laptop. Conversely, high-resolution runs might require a multi-processor system with tens of cores and/or large amounts of RAM.

Execution times of SELEN depend on the available computing power and on the adopted configuration for the solution of the SLE. They can range between tens of minutes for a low-resolution setup on a common desktop or laptop to several days for very high-resolution runs. As a rule of thumb, the execution time is proportional to the number of pixels of the spatial grid used to discretize the SLE, to the number of terms in the harmonic expansions, to the number of iterations and to the number of time steps. For high-resolution runs, a considerable amount of time may be needed to perform disk I/O.

3 Conventions used in this document

3.1 Geographical coordinates

In SM19 and SSM19, geographical coordinates are expressed as $\gamma = (\theta, \lambda)$, where θ is colatitude and λ is longitude. For consistency, in this document all geographical coordinates are expressed with the same notation. However, in all SELEN input and output files the geographical coordinates are given in terms of longitude λ and latitude $\pi/2 - \theta$, expressed in decimal degrees. Therefore, when

[1] `http://www.cygwin.com/`

[2] `https://docs.microsoft.com/windows/wsl/install-win10`

we refer to a data file containing *geographical coordinates* $\gamma = (\theta, \lambda)$, it shall be intended that the file lists *longitude* and *latitude* corresponding to γ.

3.2 Time steps

As discussed in SM19, in the solution of the SLE all times t are measured as relative to a reference epoch $t_0 = 0$ in the past, with t_N being the present time. In GIA studies, times are commonly given as relative epochs before present (BP), and all SELEN input and output files follow this convention as well. In this document, relative epochs are indicated with the symbol t^{BP} and are related to time t through $t = t_N - t^{BP}$.

3.3 Typographic notations

In this user guide, data file listings and transcripts of interactive sessions are formatted in fixed-width typeface on a gray background. In interactive session transcripts, the command prompt is represented with a dollar character (`$`); alphanumeric sequences following that character are intended to be typed by the user. Conversely, lines not beginning with `$` represent output printed in response to user commands.

As an example, the following line indicate that the user shall type the `ls` command at the system prompt:

```
$ ls
```

Please beware that the actual system prompt depends on the specific user environment setup and may appear different.

4 Setting up SELEN

SELEN does not require any particular installation procedure. The program can be downloaded from `https://github.com/geodynamics/selen` and it is entirely contained in a directory tree which will be referred to as the SELEN *home directory* in this user manual. The contents of the home directory is described in Table 1. To preserve disk space, some large data files are distributed in compressed form. Before running SELEN for the first time, they should be expanded by typing the following commands:

```
$ cd DATA/
$ gunzip *.gz
$ cd ..
```

To build the SELEN executables, it is generally sufficient to go into the `src/` subdirectory and type `make` at the command prompt:

```
$ cd src
$ make
gfortran -m64 -O -c shtools.f90
gfortran -m64 -O -fopenmp sle-REV14.f90 stop_config.f90 mj.f90 lj.f90 dom.f90 j_index.f90
   config.f90 -o ../sle.exe
gfortran -m64 -O pproc-sle-REV14.f90 stop_config.f90 mj.f90 lj.f90 dom.f90 plmbar_mod.f90
   plmbar.f90 harmo.f90 j_index.f90 config.f90 -o ../ppr.exe
gfortran -m64 -O -fopenmp sha-REV2.f90 sindd.f90 cosdd.f90 plmbar.f90 j_index.f90 plmbar_mod.f90
   stop_config.f90 mj.f90 -o ../sha.exe
```

After a successful build, the executables `sle.exe`, `ppr.exe` and `sha.exe` are made available in the SELEN home directory. The three executables correspond, respectively, to the SLE solver, the SLE post–processor and the program that pre-computes harmonic functions.

The build process is controlled by the `Makefile` in the `src/` subdirectory. The supplied `Makefile` contains directives for building SELEN with the GNU `gfortran` compiler; if the user wants to build SELEN with a different compiler, the `Makefile` should be edited and modified according to the specific compiler setup[3].

[3]Please see `https://www.gnu.org/software/make/` for further details on the `Makefile` syntax.

File	Description
src/	SELEN sources
Makefile	Makefile for building the SELEN executables
sle-REV16.f90	SLE solver program
pproc-sle-REV16.f90	SLE post-processing program
sha-REV2.f90	Program for pre-computing harmonic functions
parameters.inc	Include file with parameters and constants
cosdd.f90, sindd.f90	Trigonometric functions with input in degrees
config.f90	Configuration subroutine
j_index.f90, lj.f90, mj.f90	Utilities for the conversion between j and (l, m)
dom.f90	Function for the evaluation of $\delta_{0,m}$
harmo.f90	Function for computing the 4π-normalised complex spherical harmonics $\mathcal{Y}_{lm}(\theta, \lambda)$
plmbar.f90, plmbar_mod.f90	Functions for computing the fully normalized associated Legendre functions $\bar{P}_{lm}(\cos\theta)$
stop_config.f90	Utility to handle certain error conditions
shtools.f90	Source of the SHTOOLS library
DATA/	SELEN input files
px-Rres.dat	Pixel file for resolution res
px-lat-Rres.dat	Main latitude file for resolution res
topo-Rres.pix	Topography file for resolution res
i6g-Rresr-i.pix	ICE-6G(VM5a) ice model file for resolution res
eb$f$-l1024-vm5a.dat	Surface Green's function files for rheological model VM5a (f=u,g)
gamma-REV-vm5a.dat	Rotational Green's function file for rheological model VM5a (revised theory)
gamma-vm5a.dat	Rotational Green's function file for rheological model VM5a (traditional theory)
PMTF-REV-vm5a.dat	Modified polar motion transfer function file for rheological model VM5a (revised theory)
PMTF-vm5a.dat	Modified polar motion transfer function file for rheological model VM5a (traditional theory)
geodetic-points-GS18.txt	Example of geodetic points file
sealevel-REV4.dat	Example of relative sea-level database containing the Tushingham & Peltier (1993) dataset
GMT_scripts/	GMT scripts for the visualization of SELEN results
make_sle.sh	Launch script for SELEN
config.sle.I6G-R44-L128-I33	Example SELEN configuration file
README.md	A quick overview of SELEN
LICENSE	A copy of the 3-clause BSD license
User_Manual.pdf	This document

Table 1: Contents of the SELEN distribution package.

R	P	δ (deg)	r_{cell} (km)	L_M	File names
30	$34,812$	0.61	68.29	323	`px-R30.dat`, `px-lat-R30.dat`
44	$75,692$	0.42	46.31	476	`px-R44.dat`, `px-lat-R44.dat`
100	$396,012$	0.18	20.25	$1,089$	`px-R100.dat`, `px-lat-R100.dat`

Table 2: Statistics of Tegmark grids included in the SELEN distribution package. R is the grid resolution, P is the corresponding number of pixels, δ is the radius of the equivalent (*i.e.* with the same area) disk on the sphere, r_{cell} is the approximate radius of the equivalent disk on a plane, and L_M is the maximum harmonic degree admissible for the chosen value of R (see also §8.6 of SSM19).

5 Preparing SELEN input files

The numerical solution scheme of the SLE outlined in SSM19 requires a set of input variables, which are read by SELEN from formatted data files. These files shall be prepared and made available to SELEN before its execution; here we briefly discuss their contents and formats. A summary of input files and their correspondence with variables introduced in SSM19 is given in Table 3.

5.1 The Tegmark grid

As discussed in §8.6 of SSM19, in SELEN the spatial discretization of the SLE is accomplished on the equal-area, icosahedron-shaped pixelization of the sphere first introduced by Tegmark (1996) in the realm of cosmological applications. The grid density is controlled by a resolution parameter R; for a given value of R, the number of grid points (which we call *pixels*) is $P = 40R(R-1) + 12$. If P is sufficiently large, each pixel can be approximated with a disc of radius $\sim 2a/\sqrt{P}$, where a is the Earth radius. As discussed by Tegmark (1996), the numerical integration of harmonic functions is accurate as long as the number of pixels satisfies $P \geq l_{max}^2/3$, where l_{max} is the maximum harmonic degree of the integrand functions. Since $P \sim 40R^2$, the approximate condition $R \geq l_{max}/10$ holds true. In configuring SELEN, the user shall check that the resolution R and the maximum harmonic degree l_{max} satisfy this condition in order to ensure accurate results.

The pixels in the Tegmark grid, located at coordinates $\gamma_p = (\theta_p, \lambda_p)$, can be classified in subsets located at the same latitude; these are called *main latitudes* and are defined by $\theta_p =$ constant. Since the Legendre polynomials (LPs) $P_{lm}(\cos\theta)$ depend only on colatitude θ, to speed up computation and reduce its memory footprint SELEN evaluates P_{lm} only at the main latitudes; to obtain the of the LPs at each grid pixel, SELEN needs a correspondence table between each grid pixel and its main latitude.

Hence, the Tegmark grid is completely described by two input files: the *pixel file* and *main latitude file*. These files contain, respectively, the pixels and the main latitudes, sorted by ascending latitude. The pixel file contains four columns: the geographical coordinates (longitude λ_p and latitude $\pi/2 - \theta_p$) of each pixel, a pointer to the main latitude associated to each pixel, and the index of each pixel (which coincides with the line number of the file). The main latitude file contains also four columns: the index of each main latitude (corresponding to the line number of the file), the main latitude, and the range of pixel indexes associated to that main latitude.

The SELEN distribution package includes pre-computed input files describing the Tegmark grids for resolutions $R = 30, 44$ and 100. The corresponding pixel files and main latitude files are named `px-`R`.dat` and `px-lat-R`R`.dat`, respectively, and are stored in the `DATA/` subfolder (see Table 1). These Tegmark grid files may be intended as low-resolution ($R = 30$), mid-resolution ($R = 44$) and high-resolution ($R = 100$) and should cover all common use-cases; statistics for these grids are given in Table 2. If the user needs to run SELEN with a different resolution, suitable input files shall be prepared using the subroutines provided by Tegmark (1996), and following the file format described above.

5.2 Spherical harmonics file

To compute the coefficients of spherical harmonic expansions and to obtain spatial fields through harmonic synthesis, SELEN needs the fully-normalized LPs $\bar{P}_{lm}(\cos\theta_p)$ (referred to as FNALPs in SSM19) and the trigonometric functions $(\sin m\lambda_p, \cos m\lambda_p)$ at each grid pixel γ_p $(p = 1, \ldots, P)$ and for each harmonic degree $0 \leq l \leq l_{max}$ and order $0 \leq m \leq l$. Since the evaluation of these functions is rather time-consuming, they are pre-computed before invoking SELEN and stored in a binary file, which is referred to as *spherical harmonics file*. As we will show in §7, this task is accomplished by the `sha.exe` executable. The resulting spherical harmonics file can be re-used across different SELEN sharing the same values of R and l_{max}. In the following examples, the name of a spherical harmonics file for resolution R and maximum harmonic degree l_{max} follows the convention `sh-R`R`L`l_{max}`.bin`.

5.3 Ice model file

The spatio-temporal distribution of the ice thickness is the basic input of SELEN. Within the numerical solution scheme described in SSM19, the ice distribution shall be discretized into disk-shaped elements whose location and area is defined by the Tegmark grid. At each pixel p, the evolution of the ice thickness is described by a piecewise constant function $I_{p,n} = I(\gamma_p, t_n)$, where $t_n = n\,\Delta t$ defines a discretization of the time axis into $N+1$ equal-length steps Δt, $t_0 = 0$ is a reference time in the past and $0 \leq n \leq N$. The corresponding discretized axis of relative epochs (see §3.2) is defined $t_n^{BP} = t_N - t_n$, with t_N being the present time.

The time discretization of the ice sheet evolution defines the time grid used throughout SELEN; therefore, all output fields will be made available at the same time steps.

The evolution of ice thickness is described in the *ice model file*. For each pixel in the Tegmark grid, the ice model file includes a line containing: the pixel index p, the pixel geographical coordinates (longitude and latitude), the pixel angular half-amplitude (which will be the same for all the pixels), and the ice thickness time-history $I_{p,n}$, with $0 \leq n \leq N+1$, where $n = N+1$ correpsonds to ice thickness at times $t > t_N$ in the future. The ice model file shall include data lines also for pixels where the ice thickness is zero at all the considered times; in that case the corresponding line shall list $I_{p,n} = 0$ for all n.

The SELEN distribution package includes ice model files describing the GIA model ICE-6G(VM5a), originally introduced by Peltier et al. (2015). As discussed in detail in SM19, the spatio-temporal discretization of the SLE in SELEN is such that these files are not an exact reproduction of ICE-6G, but rather a *realization* of it on the Tegmark grid of a given resolution. ICE-6G realizations are included for $R = 30, 44$ and 100, and are named following the convention `i6g-R`R`r-i.pix`.

5.4 Green's function files

As discussed in §7.2 and §7.3 of SSM19, the surface response functions and the rotational response function appearing in the SLE depend on the employed rheological model through the quantities e_l^f, $\beta_{l,k}^f$ and γ_k^f, where k is the time step in the range $0 \leq k \leq N$ and $f = s, u, g$ is the considered field (surface response function for sealevel, vertical displacement and geoid).

SELEN reads the vertical displacement (e_l^u,$\beta_{l,k}^u$) and geoid (e_l^g,$\beta_{l,k}^g$) Green's functions from two data files, which are called the *surface Green's function files*; the sea-level Green's functions are obtained internally by SELEN as $e_l^s = e_l^g - el_l^u$ and $\beta_{l,k}^s = \beta_{l,k}^g - \beta_{l,k}^u$. Each line of the surface Green's function files shall list the harmonic degree l (in ascending order, starting from $l = 0$) in the first column, e_l^f in the second column and $\beta_{l,k}^f$ in the remaining $N+1$ columns. The rotational Green's functions γ_k^f are read from an additional data file, called the *rotational Green's function file*, which in each line shall contain the time step k in the first column (starting from $k = 0$, in ascending order), γ_k^g in the second column and γ_k^u in the third column.

The Green's functions supplied to SELEN must be coherent with the rheological model on which the ice model is based and need to be available on the same time steps defined by the ice model discretization. The SELEN distribution package includes Green's functions computed for viscosity profile VM5a and suitable for using with the provided realization of ICE-6G(VM5a). The Green's functions are stored in files `ebu-l1024-vm5a.dat`, `ebg-l1024-vm5a.dat` and `gamma-REV-vm5a.dat`. These files include harmonic terms up to degree $1,024$. Since SELEN ignores the extra harmonic terms possibly present in these files, they can be employed in any SELEN run configured with $l_{max} \leq 1,024$.

5.5 Topography file

To evaluate the ocean function and to obtain the time-dependent topography, SELEN needs a discretization fo the present-day topography on the Tegmark grid. This variable corresponds to $T_{p,N}$, defined in §7.1 of SSM19 and is stored on a data file called the *topography file*. The topography file contains, for each line, the geographical coordinates of the grid pixel and the elevation of that pixel; the ordering of pixels in the topography files must correspond to that of one pixel file. The user can build topography files by suitably interpolating any digital elevation model on the Tegmark grid of the desired resolution; the SELEN distribution package includes topography files based on the ETOPO1 global relief dataset (Amante and Eakins, 2009) for resolutions $R = 30, 44$ and 100; these files are stored in the `DATA/` folder and are named `topo-R`R`.pix`.

5.6 Geodetic points file

Once the SLE has been solved, SELEN can obtain predictions of present-day rates of sealevel change ($\dot{\mathcal{S}}$), vertical displacement ($\dot{\mathcal{U}}$) and geoid height change ($\dot{\mathcal{G}}$) at any point of interest, like tide-gauge sites. These predictions are obtained by the SELEN post-processor at all the geographical locations contained in a user-supplied data file, which is called the *geodetic points file*.

The first 10 lines of the geodetic points file are skipped by SELEN and can be used to write comments and other information describing the contents of the file. The number of those header lines is controlled by the `NH_TG` parameter in the post-processor source `pproc-sle-REV16.f90` and can be changed according to the user needs.

The rest of the geodetic points file contains one data line for each point of interest. Each data line has eight columns: the geographical coordinates of the site (latitude and longitude), an integer code identifying the site, a six-digit PSMSL code, three columns containing rates of secular sea-level rise at that point and a text label with the site name. In SELEN computations, only the first two columns are used, so the remaining data columns may be filled with dummy values if they do not have any practical meaning in the specific case. However, SELEN reads the third and last columns (site code and name) and reports them in the output file along with predicted rates.

The SELEN package includes an example geodetic point file containing the coordinates of PSMSL sites in the database used by WR Peltier to compute present-day predictions of rates on the basis of the ICE-6G(VM5a) model, integrated with few further TG sites. The file, named `geodetic-points-GS18.txt` and stored in the `DATA` directory, can be used as a template to create custom geodetic points files.

5.7 Relative sealevel database

SELEN can compute synthetic relative sea level (RSL) curves at a set of N_{RSL} user-supplied points γ_i ($1 \leq i \leq N_{RSL}$). For each of those points, SELEN will obtain predictions of relative sea level $RSL_{i,n} = RSL(\gamma_i, t_n)$ for each point of the discretized time axis t_n defined by the ice thickness time-history.

The geographical coordinates of points γ_i are contained in a SELEN input data file which is called the *relative sealevel database*. This file shall contain also observed RSL curves for each point; indeed, SELEN provides GMT scripts to compare synthetic RSL curves obtained through the solution of the SLE with observed RSL data, in order to assess whether a specific GIA model is consistent with RSL observations.

For each point γ_i, the relative sealevel database contains a data line with an integer code which identifies the RSL site, its geographical coordinates (longitude λ_i and latitude $\pi/2 - \theta_i$), the number N_i^{RSL} of RSL observations available at that site, and a text label with the site name. This data line is followed by N_i^{RSL} additional lines, each containing the time of observation t^{BP} in years before present, its associated uncertainty, the observed RSL value at that epoch, and the uncertainty on the observed RSL.

The SELEN distribution package includes a relative sealevel database containing the dataset[4] of Tushingham and Peltier (1993), which despite its age still mantains a key relevance in GIA studies. Of course, any other input dataset can be supplied by the user following the format described above.

5.8 Polar motion transfer function file

For a given GIA model, SELEN can compute the position of the istantaneous pole of rotation $\mathbf{m}(t)$ and its time derivative $\dot{\mathbf{m}}(t)$ for each point $t = t_k$ of the discretized time axis. As discussed in detail in SSM19, obtaining $\mathbf{m}(t)$ and its time derivative require the knowledge of the modified polar motion transfer function (MPMTF), which is completely described by the quantities A'^e, A'^s, A'_i and a_i introduced in §5.5 of SSM19. These variables depend on the adopted rheological profile, and shall be coherent with the Green's function files supplied to SELEN.

The variables describing the MPMTF are read by SELEN from a data file called the *polar motion transfer function file*. This file contains several blocks of data; each block starts with three header lines describing its contents. The first block contains the variables A^e, A^s, A_i and a_i, describing the (un-modified) polar motion trasfer file, and is skipped by SELEN; the second block contains variables describing the MPMTF and it is read by SELEN. After the three header lines, two data lines contain A'^e and A'^s, followed by M' data lines, each one containing i, A'_i and a_i. As discussed in SSM19, M' depends on the choice of rotational theory (*traditional* or *new rotational theory*), so the supplied polar motion transfer function file shall be coherent with the specific choice of rotational theory.

6 Configuring SELEN

In this section we will describe the SELEN configuration procedure by illustrating how the run discussed in SM19 can be set up. In this run, we are going to use SELEN to solve the SLE using a realization of the ICE-6G(VM5a) on a Tegmark grid of resolution $R = 44$ and considering harmonic terms up to degree $l_{max} = 128$.

As discussed in §5.1, when configuring SELEN is important to check that the chosen grid resolution R results in a sufficently dense grid to ensure reliable numerical integration of harmonic functions up to the selected l_{max}. For $R = 44$, numerical integrals

[4] Available at `ftp://ftp.ncdc.noaa.gov/pub/data/paleo/paleocean/relative_sea_level/sealevel.dat` (last accessed June 19, 2019).

File	Example	Variable	Description
Input files			
pixel file	`px-R44.dat`	$\gamma_p = (\theta_p, \lambda_p)$	Geographical coordinates of pixels in the Tegmark grid[a]
main latitude file	`px-lat-R44.dat`	θ_p	Unique latitudes in the Tegmark grid
spherical harmonics file	`sh-R44L128.bin`	$\bar{P}_{lm}(\cos\theta_p)$, $(\cos m\lambda_p, \sin m\lambda_p)$	FNALPs and trigonometric functions
ice model file	`i6g-R44r-i.pix`	$I_{p,n}$	Ice thickness time-history
surface Green's function files	`ebu-l1024-vm5a.dat`	e^u_l, $\beta^u_{l,k}$	Surface GF for sea-level change
	`ebg-l1024-vm5a.dat`	e^g_l, $\beta^g_{l,k}$	Surface GF for geoid height variation
rotation Green's function files	`gamma-REV-vm5a.dat`	$\gamma^{u,g}_k$	Rotation GFs for vertical displacement and geoid height variation
topography file	`topo-R44.pix`	$T_{p,N}$	Present-day topoghraphy
geodetic points file	`geodetic-points-GS18.txt`	$\gamma_{tg} = (\theta_{tg}, \lambda_{tg})$	Geographical coordinates of tide-gauges
relative sealevel database	`sealevel-REV4.dat`	$\gamma_i = (\theta_i, \lambda_i)$, $RSL^{obs}_{i,k}$	Geographical coordinates of RSL sites and observed RSL curves
polar motion transfer function file	`PMTF-REV-vm5a.dat`	A'^e, A'^s, A'_i, a_i	Quantities defining the modified PMTF
Output files			
topography	`top-`*label*`.bin`	$T_{p,n}$	Discretized topography
ocean function	`ofu-`*label*`.bin`	$O_{p,n}$	Discretized ocean function
sea-level change	`shs-`*label*`.bin`	$\mathcal{S}_{j,n}$	SH coefficients of sea-level change
vertical displacement	`shu-`*label*`.bin`	$\mathcal{U}_{j,n}$	SH coefficients of vertical displacement
geoid height variation	`shg-`*label*`.bin`	$\mathcal{G}_{j,n}$	SH coefficients of geoid height variation
sea surface variation	`shn-`*label*`.bin`	$\mathcal{N}_{j,n}$	SH coefficients of sea surface variation
surface loading excitation function	`psi-`*label*`.dat`	$\mathbf{\Psi}^{rig}_n$	Polar motion excitation function for a rigid Earth
ocean-averaged sea-level change	`sav-`*label*`.dat`	$\mathcal{S}^{equ}_n$, $\mathcal{S}^{ofu}_n$, $\mathcal{S}^{ave}_n$	Ocean average of field $\mathcal{S}$

Table 3: Summary of input and output files used in SELEN. For ease of table formatting, in the output file names *label* stands for the character sequence `R44-L128-I6G-VM5a-I33`. Output files ending with the `.bin` suffix are stored in binary format, while those ending in `.dat` are formatted ASCII files.

[a] Please note that the geographical coordinates in SELEN data files are always specified in terms of latitudes instead of co-latitudes.

are stable up to harmonic degree $\sim 10R = 440$, so the chosen paramenters largely ensure the reliability of numerical quadratures (Tegmark, 1996).

The SELEN configuration is specified in a text file which is passed as a command-line argument to the SLE solver and post-processor (`sle.exe` and `ppr.exe`, respectively). An example configuration file is included in SELEN (`config.sle.I6G-R44-L128-I33`) and its contents is listed in Appendix A. While parsing the configuration file, the portion of a line starting with an exclamation mark (‘`!`’) is interpreted by SELEN as a comment and therefore it is ignored. Therefore, lines starting with the exclamation mark are completely skipped. Please be aware that the configuration file cannot contain blank lines. In what follows, we will describe in detail the contents of the the example configuration file.

6.1 Basic parameters

The first block of the configuration file contains the basic parameters for the SELEN run:

```
!
! >>>> Basic parameters ----------------------------------------
!
 44 ! Tegmark resolution
  32 ! LMAX
   52 ! Number of time-steps in the ice history
    9 ! Number of v/e layers
     0.5 ! Ice history time step (kyr)
      2 ! 0 is no rot, 1 classical, 2 revised
       3 ! Number of external iterations
        3 ! Number of internal iterations
```

The first two data lines specify the desired resolution R for the Tegmark grid and the maximum harmonic degree l_{max} for the spherical harmonic expansions ($R = 44$ and $l_{max} = 128$ in this specific example). The following line specifies the number N of time-steps in the ice melting history, which defines the discretization of the time axis in the SELEN solution. In the example above, we are using $N = 52$, which is the number of time–steps in the realization of ICE-6G(VM5a) that is provided with SELEN. The next line defines the number n_v of viscoelastic layers in the rheological profile. In this example, we are using our realization the rheological model VM5a, consistently with the choice of ice model, for which $n_v = 9$ (see SM19 for details). Then, the discretization time-step Δt is declared; the realization of ICE-6G(VM5a) provided with SELEN is given on a grid with $\Delta t = 0.5$ kyr, so that the reference time t_0 corresponds to $t_0^{BP} = N\Delta t = 26$ kyr BP. The following line is used to select the rotational theory that SELEN will apply in the solution of the SLE. This choice is controlled by a switch i_{rot}, with which the user can select to employ the revised rotational theory of Mitrovica et al. (2005) and Mitrovica and Wahr (2011) ($i_{rot} = 2$), the traditional rotational theory of Milne and Mitrovica (1998) and Spada et al. (2011) ($i_{rot} = 1$), or to completely turn off rotational feedback ($i_{rot} = 0$). In this specific case, we are selecting the revised rotational theory by setting $i_{rot} = 2$. The last two data lines of this block define the number of external and internal iterations in the numerical solution of the SLE (n_{ext} and n_{int}, respectively; see SSM19 for details). In the example above, we are setting $n_{ext} = n_{int} = 3$.

6.2 Input files

The second block of the configuration file specifies the names of the input needed by SELEN, which have been described in detail in §5.

```
! >>>> Input files required for the execution ----------------
!
 ./DATA/ ! Directory for input files
!
 px-R44.dat ! pixel file
!
 sh-R44L128.bin ! SH file
!
 ebu-l1024-vm5a.dat ! U Green's Function
 ebg-l1024-vm5a.dat ! G Green's Function
!
 gamma-REV-vm5a.dat ! Rotational GF
!
```

```
 i6g-R44r-i.pix ! Pixelized ice model
!
 topo-R44.pix ! Pixelized present-day topo
!
 ofp-i6g-R44.pix ! Pixelized present-day OF (only for elastic models)
!
 geodetic-points-GS18.txt ! set of points for present-day velocity estimation
!
 sealevel-REV4.dat ! RSL database
!
 PMTF-REV-vm5a.dat ! PMTF
!
```

The first data line in this block specifies the path where SELEN will look for all the following input data files. In this case, we instruct SELEN to look for input files into the `DATA/` folder, that is where the example files provided in the distribution package are stored (see Table 1). The next line indicates the name of the *pixel file*, containing the geographical coordinates of the Tegmark grid pixels for the selected resolution. In the example above we are using the pixel file provided with SELEN for $R = 44$, whose name is `px-R44.dat`.

The next line specifies the name of the *spherical harmonics file*, a binary file in which the precomputed FNALPs and trigonometric functions are stored. This file shall be created before running SELEN with the `sha.exe` program, as we will show later, and stored into the `DATA/` folder. In this example, we choose to name this file `sh-R44L128.bin`, following the format `sh-R`R`L`l_{max}`.bin`.

The following three data lines specify the names of the Green's functions file describing the rheological model of the Earth. As illustrated in §5, these are two *surface Green function files* corresponding vertical deformation U and geoid change G and a *rotational Green's function file* describing the rotational feedback. These files must be coherent with the chosen ice model both in terms of rheological model and time discretization. In this example we are using the Green's functions files provided in the SELEN package and suitable for use with the $R = 44$ realization of ICE-6G(VM5a) on the Tegmark grid, which are `ebu-l1024-vm5a.dat`, `ebg-l1024-vm5a.dat` and `gamma-REV-vm5a.dat`.

The next line contains the name of the *ice model file*, describing the spatial distribution and the temporal evolution of the ice sheets. Here we chose the ice model file corresponding to the $R = 44$ realization of ICE-6G(VM5a), which is provided with the SELEN package in file `i6g-R44r-i.pix` into the `DATA/` folder. The following line specifies the name of the *topography file*, containing a discretization of present-day topography on the Tegmark grid. In the example, we select the interpolation of the ETOPO1 global relief dataset corresponding to $R = 44$, which is included in SELEN in the file `topo-R44.pix`.

After the *topography file*, the configuration file contains a reference to a file with the present-day ocean function. This input file is needed only for elastic models, which will be fully implemented in a future revision of SELEN, and it is ignored by the current version of SELEN.

The following line specifies the name of the *geodetic points file*, containing the geographic coordinates of points where SELEN shall obtain predictions of present-day rates of geodetic observables. The SELEN distribution package includes an example geodetic points file with the coordinates of the 2,145 PSMSL tide gauges for which WR Peltier obtained predictions from the ICE-6G model[5], integrated with a few additional tide gauge locations. Here we choose to use this example file, whose name is `geodetic-points-GS18.txt`.

The next configuration line contains the name of the *relative sealevel database*, containing the location of sites where SELEN will compute synthetic RSL curves, and RSL observations that will be compared with model predictions. Here we specify the realtive sealevel database `sealevel-REV4.dat`, included in the SELEN distribution package, which contains the RSL database by Tushingham & Peltier (1993).

The last data line in this configuration block contains the name of the polar motion transfer function file which, as discussed above, contains the variables needed to evaluate the MPMTF. Here we instruct SELEN to read the file `PMTF-REV-MP-vm5a.dat`, which contains a MPMTF obtained with rheological model VM5a, coherently with the ice model and with the Green's functions.

6.3 Output files

The last block of the configuration files specifies the file names where the SELEN solver will save various output quantities. These include the the time-dependent discretized topography and ocean function; the spectral components of sea-level change, vertical displace-

[5] `http://www.atmosp.physics.utoronto.ca/~peltier/datasets/Ice6G_C_VM5a_O512/drsl.PSMSL.ICE6G_C_VM5a_O512.txt`, last accessed on June 14, 2019.

ment, geoid height variation and sea surface variation; the surface loading excitation function for a rigid Earth and the ocean-averaged sea-level change.

```
!
! >>>>>>> Output files -------------------------------------
!
top-R44-L128-I6G-VM5a-I33.bin
ofu-R44-L128-I6G-VM5a-I33.bin
shs-R44-L128-I6G-VM5a-I33.bin
shu-R44-L128-I6G-VM5a-I33.bin
shg-R44-L128-I6G-VM5a-I33.bin
shn-R44-L128-I6G-VM5a-I33.bin
psi-R44-L128-I6G-VM5a-I33.dat
sav-R44-L128-I6G-VM5a-I33.dat
```

These data files are generally not intended to be accessed by the user, but are instead used as input by the SELEN post-processor in order to obtain further outputs. However, they may be read and processed with user-supplied customized procedures if further analysis are needed. The contents of output files is summarized in Table 3. For the details of the format of these output files, we refer the reader to the SELEN sources.

The configuration file that has been illustrated above is entirely listed for convenience in Appendix A. The file is contained in the SELEN distribution package with the name `config.sle.I6G-R44-L128-I33`, where the label `I6G-R44-L128-I33` summarizes the main configuration parameters of this run (ICE-6G ice model, $R = 44$, $l_{max} = 128$, $n_{ext} = n_{int} = 3$). In what follows, this run will be referred to as the `I6G-R44-L128-I33` run.

7 Running SELEN

In this section we guide the user through the practical steps needed to run SELEN with the configuration discussed in §6. In what follows, it is assumed that the user has downloaded and uncompressed the SELEN distribution package, and that the executables have been built as described in §4. Following these instruction, the user should be able to reproduce all outputs discussed in SM19.

7.1 Building the spherical harmonics file

As discussed in §5, before running SELEN the FNALPs and the trigonometric functions at the grid points must be pre-computed and stored in a binary file. This task is performed by the `sha.exe` program, which is invoked from the SELEN home directory with the following syntax:

```
$ ./sha.exe <RES> <LMAX> <pxfile> <pxlatfile> <outfile>
```

where `<RES>` is the resolution parameter R, `<LMAX>` is the maximum harmonic degree l_{max}, `<pxfile>` and `<pxlatfile>` are the pixel file and the main latitude file corresponding to resolution R, and `<outfile>` is the name of the output binary file that will be created. If `<outfile>` is omitted, the output file will be named according to the convention sh-RRLl_{max}.bin.

For the `I6G-R44-L128-I33` run, we have $R = 44$ and $l_{max} = 128$, while the pixel file and the main latitude file names are `px-R44.dat` and `px-lat-R44.dat`. Therefore we can invoke the `sha.exe` program as follows, accepting the default name `sh-R44L128.bin` for the output file:

```
$ ./sha.exe 44 128 DATA/px-R44.dat DATA/px-lat-R44.dat
 ---- Maximum degree: 128
 ---- Resolution: 44
 ---- Number of pixels: 75692
 ---- Reading pixels data from file: DATA/px-R44.dat
 ---- Number of MAIN pixels: 13334
 ---- Reading pixels data from file: DATA/px-lat-R44.dat
 ---- Reading longitude data
 ---- Opening file: sh-R44L128.bin
 ---- Pre-computing ALFs at latitudes of anchor pixels
 ---- First call to PLMBAR_MOD
 ---- Other calls
       5000 of 13334
      10000 of 13334
```

```
 ---- Writing the Legendre functions on file: sh-R44L128.bin
 ---- Pre-computing TRIG functions at pixels
 ---- Writing the sines and cosines on file: sh-R44L128.bin
 *****************************************************
 - Copy or better move the following file into DATA/
sh-R44L128.bin
 *****************************************************
 Thank you!
```

As suggested in the output messages printed by `sha.exe`, we move the spherical harmonics file into the `DATA/` folder, where we have instructed SELEN to look for all its input files:

```
$ mv sh-R44L128.bin DATA/
```

At this point, all the needed input files have been set up and we can proceed with running SELEN.

7.2 Launching SELEN

SELEN can be conveniently launched through the `make_sle.sh` script, provided in the distribution package and stored in the SELEN home directory. This script performs the following sequence of operations: *i)* launches the SLE solver `sle.exe`; *ii)* executes the SLE post-processor `ppr.exe`; *iii)* creates a directory structure for the SELEN outputs and moves all run output files into the corresponding subfolders; *iv)* copies the relevant GMT visualization scripts from the `GMT_scripts/` folder into the appropriate subfolders of the output directory tree. The output directory tree, whose structure is described in Table 4, is called the *run depot*.

The `make_sle.sh` script can be invoked with the following syntax:

```
$ sh make_sle.sh <RUN_LABEL> <CONFIG_NAME>
```

where <RUN_LABEL> is a text label identifying the run, and <CONFIG_NAME> is the name of the text file containing the SELEN configuration, whose format has been described in detail in §6. The run depot tree will be created by `make_sle.sh` under a directory named `RUN_`*label*`/`, where *label* is the text label passed to `make_sle.sh`. If the <CONFIG_NAME> parameter is omitted, `make_sle.sh` will look for a default configuration file named `config.sle.`*label*.

A configuration file for the `I6G-R44-L128-I33` run is included in the SELEN home directory with the name `config.sle.I6G-R44-L128-`
We can therefore invoke the `make_sle.sh` script with *label* set to `I6G-R44-L128-I33` and without explicitly specifying the name of the configuration file:

```
$ sh make_sle.sh I6G-R44-L128-I33
```

At the end of the execution, all the outputs created by the SELEN post-processor will be organized in a directory tree named `RUN_I6G-R44-L128-I3`
(see Table 4).

A partial trascript of the console output of the `I6G-R44-L128-I33` run is given in Appendix B for user reference. Please note that in the transcript a substantial portion of the SELEN outputs has been omitted. The corresponding portion of the transcript is annotated with the `[......MORE OUTPUT......]` sequence.

8 Examining the SELEN outputs

After a successful SELEN run, the various outputs obtained by the post-processor and some GMT scripts useful for their visualization are organized in a directory tree called the *output depot*. The output depot is stored under a directory named `RUN_`*label*`/`, where *label* is a user-supplied alphanumeric label identifying the SELEN run. The structure of the output depot is summarized in Table 4; in this section we will give an overview of its contents.

8.1 Geodetic fingerprints (FPR/ folder)

The term *GIA fingerprints* is commonly used to indicate present-day *rates of change* of various geodetic quantities. The SELEN post-processor obtains estimates of present-day rates of sea-level change ($\dot{\mathcal{S}}$), vertical displacement ($\dot{\mathcal{U}}$), geoid height variation ($\dot{\mathcal{G}}$) and sea surface variation ($\dot{\mathcal{N}}$); these fields are evauluated on the Tegmark grid and stored in the ASCII files `sdot.pix`, `udot.pix`, `gdot.pix`, `ndot.pix`, respectively. All these files are archived into the `FPR/` folder, along with a GMT script that plots global fingerprint maps using these files. The SELEN post-processor computes also present-day averages of the above fields on the oceans

($< \dot{\mathcal{F}} >^o_N$) and on the whole Earth ($< \dot{\mathcal{F}} >^e_N$). These averages are saved in the file `fps-stats.dat`; its contents for the `I6G-R44-L128-I33` run is listed below:

```
---- Average of S-DOT over the oceans (mm/yr): -8.1017911397982430E-003
---- Average of U-DOT over the oceans (mm/yr): -0.24356858716647581
---- Average of N-DOT over the oceans (mm/yr): -0.25182267294467803
---- Average of G-DOT over the oceans (mm/yr): -5.2008914209203401E-002

---- Average of S-DOT over the earth (mm/yr): -0.19981372069650266
---- Average of U-DOT over the earth (mm/yr): -8.5462482569912145E-008
---- Average of N-DOT over the earth (mm/yr): -0.19981380779493360
---- Average of G-DOT over the earth (mm/yr): -4.9059459805053571E-008
```

To ensure that SELEN is working correctly, the user may want to check that the averages computed by the post-processor for the `I6G-R44-L128-I33` are in agreement with the results listed above, within the limit of numerical roundoffs. As discussed in SM19, by virtue of mass conservation $< \dot{\mathcal{G}} >^e = < \dot{\mathcal{U}} >^e = 0$ and $< \dot{\mathcal{S}} >^e = < \dot{\mathcal{N}} >^e = \dot{c}$, where $c(t)$ is a spatially invariant term introduced by Farrell and Clark (1976) in their formulation of the SLE. The numerical results listed above show that the first condition is satisfied within the limits of floating-point roundoffs, while the present-day value of $\dot{c}$ is estimated as ~ -0.20 mm/yr.

8.2 Relative sea-level (RSL/ folder)

In the `RSL/` folder of the output depot, SELEN will store synthetic realtive sea-level curves computed at all the location contained in the *relative sealevel database*. ASCII file `RSL.DAT` contains relative sea-level $RSL_{i,n}$ for all time steps t_n and all the N_{RSL} sites ($1 \leq i \leq N_{RSL}$). Files named `rsld-`*code*`.dat` and `rslp-`*code*`.dat` contain, respectively, observed and predicted RSL curves at the site labeled by the three-digit integer *code*. Please beware that *code* is in general different from i, since i is the RSL site index in the range $1 \leq i \leq N_{RSL}$ while *code* is a custom site code specified in the relative sealevel database file supplied to SELEN.

The `RSL/` folder contains also two GMT scripts for the comparison of observed RSL data with synthetic predictions. The script `rsl-SINGLE.gmt` draws a plot for a single RSL site, whose code shall be specified inside the script source; the script `rsl-MULTI.gmt` draws a plot for each RSL site in the database. Beware that plotting RSL curves for all sites can be a time consuming process, if the number of sites in the database is large.

8.3 Tide gauges (TGS/ folder)

The `TGS/` folder contains predictions for rates of sea-level change, vertical displacement, geoid height variation and sea surface variation. These rates are evaluated at the sites specified in the *geodetic points database*, and are stored in the data file `tg.dat`.

8.4 Stokes coefficients (STK/ folder)

Present-day rates of change of GIA perturbations to Stokes coefficients of the graviity field are stored in file `stokes.dat` that can be found in the `STK/` folder. For each harmonic degree and order (l, m), the file lists $j = l(l+1)/2 + m + 1$, l, m, the fully normalised cosine ($\dot{\overline{\delta c}}_{lm}$) and sine ($\dot{\overline{\delta s}}_{lm}$) coefficients, and the corresponding modulus $(\dot{\overline{\delta c}}^2_{lm} + \dot{\overline{\delta s}}^2_{lm})^{1/2}$.

8.5 Polar motion (PMT/ folder)

Files `m.dat` in the `PMT/` folder contains the time evolution of the position of the istantaneous pole of rotation $\mathbf{m}(t) = m_1(t) + i m_2(t)$. For each time step the file lists t_n, the corresponding relative epoch t_N^{BP}, the cartesian components $m_1(t_n)$ and $m_2(t_n)$, followed by $\arg(\mathbf{m}(t_n))$ and $|\mathbf{m}(t_n)|$. The file `m.dot` lists $\dot{\mathbf{m}}(t) = \dot{m}_1(t) + i\dot{m}_2(t)$ with the same format.

8.6 Ocean function (OFU/ folder)

Folder `OFU/` contains files describing the time evolution of the ocean function. For each time step t_n, pixels γ_p of the Tegmark grid are partitioned between *wet* pixels (for which $O(\gamma_p) = 1$) and *dry* pixels (for which $C(\gamma_p) = 1 - O(\gamma_p) = 1$). Pixels belonging to the two subsets are stored in files `ocean.`t_n^{BP}`.dat` and `continent.`t_n^{BP}`.dat`, respectively. Furthermore, for each time step SELEN identifies pixels where ice is grounded above or below sealevel and stores them into files `ice_grounded_above.`t_n^{BP}`.dat` and `ice_grounded_below.`t_n^{BP}`.dat`, respectively. An included GMT script may be used to obtain maps of the ocean function.

8.7 Paleo-topography (TOP/ folder)

Files in the TOP/ folder describe the time evolution of topography obtained through the solution of the SLE. For each time step t_n, a file topo.t_n^{BP}.dat contains a Tegmark grid discretization of paleo-topography at time $t_n = t_N - t_n^{BP}$. A GMT script in this folder can be used to draw a set of global paleo-topography maps for all the time steps; beware that this may be a rather time-consuming process, especially if a high-resolution grid has been employed. Regional maps can be easily obtained by suitably modifying the provided script.

8.8 Ice thickness (ICE/ folder)

Folder ICE/ contains a set of data files describing the ice melting history of the chosen deglaciation model. These files are directly obtained from the *ice model file* supplied to SELEN as input, and are provided in this folder fir user's convenience. For each time step t_n, a file ice.t_n^{BP}.dat contains the discretized spatial distribution of ice thickness at relative epoch t_n^{BP} on the Tegmark grid. A GMT script is also provided to obtain maps that visualize the ice distribution.

File	Variable	Description
RUN_*label*/		Main directory for outputs of run identified by *label*
BIN/		Output files from the SELEN solver (see Table 3)
CON/		Configuration file config.sle.*label*
FPR/		Geodetic fingerprints folder
$f$dot.pix	$\dot{\mathcal{F}}_{p,N}$	Present-day rates of discretized fields $\mathcal{F} = \mathcal{S},\mathcal{U},\mathcal{G},\mathcal{N}$ ($f =$s,u,g,n).
fps-stats.dat	$< \dot{\mathcal{F}} >^{o}_{N}, < \dot{\mathcal{F}} >^{e}_{N}$	Average of present-day rates $\dot{\mathcal{F}} = \dot{\mathcal{S}},\dot{\mathcal{U}},\dot{\mathcal{G}},\dot{\mathcal{N}}$ over the oceans and over the Earth
fps-sle.gmt		GMT script to obtain global maps of fingerprints
ICE/		Ice thickness folder
ice.t_n^{BP}.dat	$I_{p,n}$	Ice thickness at t_n^{BP} kyr BP
ice-REV3-nn52.gmt		GMT script to obtain plots of ice thickness
OFU/		Ocean function folder
ocean.t_n^{BP}.dat	$O_{p,n}$	Pixels $\gamma_p = (\theta_p, \lambda_p)$ where $O_{p,n} = 1$
continent.t_n^{BP}.dat	$C_{p,n} = 1 - O_{p,n}$	Pixels $\gamma_p = (\theta_p, \lambda_p)$ where $C_{p,n} = 1$
ice_floating.t_n^{BP}.dat		Pixels with floating ice at t_n^{BP} kyr BP
ice_grounded_above.t_n^{BP}.dat		Pixels with grounded ice (above SL) at t_n^{BP} kyr BP
ice_grounded_below.t_n^{BP}.dat		Pixels with grounded ice (below SL) at t_n^{BP} kyr BP
of-REV4-nn52.gmt		GMT script to obtain plots of the OF
PMT/		Polar motion folder
m.dat	$\mathbf{m}(t_n)$	Cartesian components of polar motion at time t_n
m.dot	$\dot{\mathbf{m}}(t_n)$	Time derivative of cartesian components of polar motion at time t_n
RSL/		Relative sea-level folder
RSL.DAT	$RSL_{i,n}$	Synthetic RSL curves at all sites in the database
rslp-*code*.dat		RSL predictions at site identified by *code*
rsld-*code*.dat		RSL observations at site identified by *code*
rsl-SINGLE.gmt		GMT script to plot observed and predicted RSL curves at a single site
rsl-MULTI.gmt		GMT script to generate multiple plots of observed and predicted RSL
STK/		Stokes coefficients folder
stokes.dat	$(\dot{\overline{\delta c}}_{lm}, \dot{\overline{\delta s}}_{lm})$	Present-day rate of change of Stokes coefficients
TGS/		Tide-gauges folder
tg.dat	$\dot{\mathcal{S}}_{tg}$	Predictions of present-day $\dot{\mathcal{S}}$, $\dot{\mathcal{U}}$, $\dot{\mathcal{G}}$ and $\dot{\mathcal{N}}$ at tide-gauges
TOP/		Paleo-topography folder
topo.t_n^{BP}.dat	$T_{p,n}$	Paleo-topography at t_n^{BP} kyr BP
topo-REV2-nn52.gmt		GMT script to plot global paleo-topography maps

Table 4: Structure of the SELEN output depot.

A Configuration file for the example **I6G-R44-L128-I33** run

```
! --- Configuration file for SELEN4
!
! >>>> Basic parameters ----------------------------------------
!
 44 ! Tegmark resolution
  128 ! LMAX
   52 ! Number of time-steps in the ice history
    9 ! Number of v/e layers
     0.5 ! Ice history time step (kyr)
      2 ! 0 is no rot, 1 classical, 2 revised
       3 ! Number of external iterations
        3 ! Number of internal iterations
!
!
! >>>> Input files required for the execution ----------------
!
 ./DATA/ ! Directory for input files
!
 px-R44.dat ! pixel file
!
 sh-R44L128.bin ! SH file
!
 ebu-l1024-vm5a.dat ! U Green Function
 ebg-l1024-vm5a.dat ! G Green Function
!
 gamma-REV-vm5a.dat ! Rotational GF
!
 i6g-R44r-i.pix ! Pixelized ice model
!
 topo-R44.pix ! Pixelized present-day topo
!
 ofp-i6g-R44.pix ! Pixelized present-day OF (only for elastic models)
!
 geodetic-points-GS18.txt ! set of points for present-day velocity estimation
!
 sealevel-REV4.dat ! RSL database
!
 PMTF-REV-vm5a.dat ! PMTF
!
!
! >>>>>>> Output files --------------------------------------
!
top-R44-L128-I6G-VM5a-I33.bin
ofu-R44-L128-I6G-VM5a-I33.bin
shs-R44-L128-I6G-VM5a-I33.bin
shu-R44-L128-I6G-VM5a-I33.bin
shg-R44-L128-I6G-VM5a-I33.bin
shn-R44-L128-I6G-VM5a-I33.bin
psi-R44-L128-I6G-VM5a-I33.dat
sav-R44-L128-I6G-VM5a-I33.dat
!
! END OF FILE
!
```

B Partial transcript of the console output for the I6G-R44-L128-I33 run

```
$ sh make_sle.sh I6G-R44-L128-I33

Thu May 23 16:38:08 CEST 2019

::::::: Executing the script: make_sle.sh

+-------------------------------------------+
+------ Solving the Sea Level Equation -------+
+------------- This is SELEN 4 ---------------+
+--- Copyright G. Spada and D. Melini 2019 ---+
+-------------------------------------------+

======= The RUN label is: I6G-R44-L128-I33
Outputs will be stored in: RUN_I6G-R44-L128-I33

======= The config file is: config.sle.I6G-R44-L128-I33

======= The SLE solver is: sle.exe

======= The POST-PROCESSING program is: ppr.exe


- - - - Executing program: sle.exe

  ****
  **** This is the SELEN4 SLE solver
  ****


  >>>> Using configuration file: config.sle.I6G-R44-L128-I33


  ------------------
  >>>> 1. Data input
  ------------------

  >>>> Using these input data:
  ---- Pixelized grid: ./DATA/px-R44.dat
  ---- SHs at the pixels: ./DATA/sh-R44L128.bin
  ---- Geoid displac. GF: ./DATA/ebg-l1024-vm5a.dat
  ---- Vert. displac. GF: ./DATA/ebu-l1024-vm5a.dat
  ---- Rotational GFs ./DATA/gamma-REV-vm5a.dat
  ---- Ice sheets history: ./DATA/i6g-R44r-i.pix
  ---- Present topo: ./DATA/topo-R44.pix
  ---- Present OF: ./DATA/ofp-i6g-R44.pix

  >>>> Adopting the revised rotation theory

  >>>> Configuration of this run:
  ---- Ext/int iterations: 3 3
  ---- Degrees l_max, j_max: 128 8385
  ---- Number of v/e layers: 9
  ---- INPUT: Tegmark grid pixels data
  ---- Res, pixels and main pixels: 44 75692 13334
  ---- INPUT: Spherical harmonics Y_jp
  ---- INPUT: Present day topography T_pN
  ---- INPUT: Ice thickness history I_pn
  ---- INPUT: Coefficients beta^g_ln and beta^u_ln
  ---- INPUT: Coefficients beta^s_ln
```

```
---- INPUT: Coefficients gamma^g_n & gamma^u_n

>>>> INPUT: Done reading data from external files

-----------------------
>>>> 2. Initialisation
-----------------------

---- INIT: T_pn at 75692 pixels
==== OUT: T_pn
---- INIT: O_pn
==== OUT: O_pn
---- INIT: S^equ_n and S^ofu_n
---- INIT: S^ave_jn
---- INIT: S^(0)_jn

++++++++++++++++++++++++++++++++++++++++++++++++
Ext loop: 1 of 3
++++++++++++++++++++++++++++++++++++++++++++++++

---- EXT: Computing Q_p
---- EXT: Computing O_jn
    n= 0
    n= 52
---- EXT: Computing Z^av_n
---- - - - - - - - - - - - - - - -
---- EXT: Computing W_jn
    n= 0
    n= 52
---- EXT: Computing L^a_21n
---- EXT: Computing R^a_jn
---- EXT: Computing <R^a>_n
---- EXT: Computing R^pa_jn
---- EXT: Computing chi^a_pn
    p= 1
    p= 75692
---- EXT: Computing K^a_jn
    j= 1
    j= 8385
---- - - - - - - - - - - - - - - -
---- EXT: Computing X_jn
    n= 0
    n= 52
---- EXT: Computing L^c_21n
---- EXT: Computing R^c_jn
---- EXT: Computing <R^c>_n
---- EXT: Computing R^pc_jn
---- EXT: Computing chi^c_pn
    p= 1
    p= 75692
---- EXT: Computing K^c_jn
    j= 1
    j= 8385
---- - - - - - - - - - - - - - - -
---- Initializing the INTERNAL LOOP
---- EXT: Computing Z^0_jn

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Internal loop 1 of 3
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

```
 ---- INT: Computing L^b_21n
 ---- INT: Computing R^b_jn
 ---- INT: Computing <R^b>_n
 ---- INT: Computing R^pb_jn

 [....................................MORE OUTPUT....................................]
 [....................................MORE OUTPUT....................................]
 [....................................MORE OUTPUT....................................]
 [....................................MORE OUTPUT....................................]

 ---- - - - - - - - - - - - - - -
 ---- INT: Computing Psi^rig_n
 ---- INT: Computing c_n
 ---- INT: Computing Z^(IINT)_jn

 ====================
 END of INTERNAL loop
 ====================

 ---- EXT: Computing S^(IEXT)_jn
 ---- EXT: Computing S^(IEXT)_pn
     p= 1
     p= 75692
 ---- EXT: new topography T_pn
     p= 1
     p= 75692
 ==== OUT: T_pn
 ---- EXT: new Ocean Function O_pn
     p= 1
     p= 75692
 ==== OUT: O_pn
 ---- EXT: New S^ave_n

 o~o~o~o~o~o~o~o~o~o~o
 END of Ext loop
 o~o~o~o~o~o~o~o~o~o~+

 **** END of ALL ITERATIONS

 ==== OUT: S_jn
 >>>> Closing: U^a_jn
 >>>> Closing: U^b_jn
 >>>> Closing: U^c_jn
 ==== OUT: U_jn
 >>>> Closing: G^a_jn
 >>>> Closing: G^b_jn
 >>>> Closing: G^c_jn
 ==== OUT: G_jn
 >>>> Closing: N_jn
 ==== OUT: N_jn
 ==== OUT: psi^rig_n
 ==== OUT: s^ave_n

 -------------------------
    END of EXECUTION
 -------------------------

 good bye and good luck...

Thu May 23 16:54:38 CEST 2019
```

```
+---------------------------------+
+------ Done solving the SLE ------+
+---------------------------------+


+:::::::::::::::::::::::::::::+
+:::::: Post-processing ::::::+
+:::::::::::::::::::::::::::::+

Fri Jun 28 14:10:41 CEST 2019

---- Folder RUN_I6G-R44-L128-I33 is created with subfolders

- - - - Executing program: ppr.exe

 ****
 **** This is the SELEN4 post-processor
 ****


 >>>> Using configuration file: config.sle.I6G-R44-L128-I33


 **** WORKING on BASIC DATA:
 ---- Loading the Tegmark grid data...
 ---- Tegmark resolution: 44
 ---- number of pixels: 75692
 ---- number of main pixels: 13334
 ---- Time stamps
 ---- Degres and orders
 ---- Reading pixelized harmonics from file: ./DATA/sh-R44L128.bin
 ---- Loading the ice thickness data

 **** WORKING on TOPOGRAPHY:
 ---- Reading TOPO data from file: top-R44-L128-I6G-VM5a-I33.bin
 ---- Individual files for topography since the LGM...
         0 26.0 topo.26.0.dat
        52 00.0 topo.00.0.dat

 **** WORKING on the ICE MODEL:
 ---- Creating individual files for ice since the LGM...
         0 26.0 ice.26.0.dat
        52 00.0 ice.00.0.dat

 **** WORKING on the OCEAN FUNCTION:
 ---- Re-building the OF from the TOPO and ICE data
         0 26.0 ice.26.0.dat topo.26.0.dat
        52 00.0 ice.00.0.dat topo.00.0.dat

 **** WORKING on the fingerprints
 ---- Reading SHS data from file: shs-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHU data from file: shu-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHN data from file: shn-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHG data from file: shg-R44-L128-I6G-VM5a-I33.bin
 ---- Computing the fingeprints at the pixels...

 ---- Computing the present-day OCEAN averages
 ---- Average of S-DOT over the oceans (mm/yr): -8.1017911397982430E-003
 ---- Average of U-DOT over the oceans (mm/yr): -0.24356858716647581
 ---- Average of N-DOT over the oceans (mm/yr): -0.25182267294467803
 ---- Average of G-DOT over the oceans (mm/yr): -5.2008914209203401E-002
```

```
 ---- Computing the WHOLE EARTH averages
 ---- Average of S-DOT over the earth (mm/yr): -0.19981372069650266
 ---- Average of U-DOT over the earth (mm/yr): -8.5462482569912145E-008
 ---- Average of N-DOT over the earth (mm/yr): -0.19981380779493360
 ---- Average of G-DOT over the earth (mm/yr): -4.9059459805053571E-008


 **** WORKING on the Relative Sea level (RSL) sites
 ---- Reading data from file: ./DATA/sealevel-REV4.dat
 ---- Number of sites in the file: 451
 ---- Reading code, lat, lon, and names of sites...
 ---- Computing the SHs at the RSL sites coordinates
 ---- Reading SHS data from file: shs-R44-L128-I6G-VM5a-I33.bin
 ---- Computing sea level change at the RSL sites coordinates
 ---- Computing RSL at the RSL sites coordinates...
 ---- ... and dumping all curves in one file (rsl.dat)
 ---- Computing synthetic RSL curves at the sites
 ---- (one RSL curve in one file)
 ---- Files with RSL data at individual sites...

 **** WORKING on TIDE GAUGES:
 ---- Counting the number of TGs from file:
     ./DATA/geodetic-points-GS18.txt
 ---- Number of TGs: 2153
 ---- Computing the SHs at the TGs locations...
 ---- Computing Sdot, Udot, Ndot and Gdot at TGs locations
 ---- Reading SHS data from file: shs-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHU data from file: shu-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHN data from file: shn-R44-L128-I6G-VM5a-I33.bin
 ---- Reading SHG data from file: shg-R44-L128-I6G-VM5a-I33.bin
 ---- TG 1 of 2153
 ---- TG 2153 of 2153
 ---- For the predictions at TGs, see file: tg.dat

 **** WORKING on the STOKEs COEFFICIENTS:
 ---- Reading SHG data from file: shu-R44-L128-I6G-VM5a-I33.bin
 ---- For the STOKES COEFFICIENTS, see file: stokes.dat

 ---- Reading PSI data from file: psi-R44-L128-I6G-VM5a-I33.dat
 **** ANALYSIS OF POLAR MOTION (REVISED THEORY)
 ---- Number of rotational modes: 36
 ---- Reading PMTF data from file: ./DATA/PMTF-REV-vm5a.dat
 ---- Computing the polar motion
 ---- Computing the rate of polar motion
 **** Polar displacement and its rate
 ---- Output file for displacement: m.dat
 ---- Output file for the rate of displacement: m.dot

 **** DONE


---- Copying the CONFIG data into folder: RUN_I6G-R44-L128-I33/CON
---- Moving the OF data into folder: RUN_I6G-R44-L128-I33/OFU
---- Moving the TOPO data into folder: RUN_I6G-R44-L128-I33/TOP
---- Moving the ICE data into folder: RUN_I6G-R44-L128-I33/ICE
---- Moving the FINGERPRINTS data into folder: RUN_I6G-R44-L128-I33/FPR
---- Moving the RSL data into folder: RUN_I6G-R44-L128-I33/RSL
---- Moving the TG data into folder: RUN_I6G-R44-L128-I33/TGS
---- Moving the STOKES COEFF into folder: RUN_I6G-R44-L128-I33/STK
---- Moving the POLAR MOTION data into folder: RUN_I6G-R44-L128-I33/PMT
```

```
+::::::::::::::::::::::::::::::::::::+
+:::::: End of post-processing ::::::+
+::::::::::::::::::::::::::::::::::::+

Thu May 23 16:56:05 CEST 2019
```

C Copyright statement

References

Amante, C. and Eakins, B. (2009). *ETOPO1 Arc-Minute Global Relief Model: Procedures, Data Source and Analysis*, Tech. rep.

Farrell, W.E. and Clark, J.A. (1976). *On Postglacial Sea Level*, Geophys. J. R. Astron. Soc. **46** (3), 647-667, doi:10.1111/j.1365-246X.1976.tb01252.x

Lambeck, K. (2004). *Sea-level change through the last glacial cycle: geophysica, glaciological and palaeogeographic consequences*, Comptes Rendus Geoscience, 336, 677-689.

Milne, G. A. and Mitrovica, J. X. (1998). *Postglacial sea-level change on a rotating Earth*, Geophys. J. Int. , 133, 1–19.

Mitrovica, J. X. and Milne, G. (2003). *On post-glacial sea level: I. General theory*, Geophys. J. Int. 154, 253–267.

Mitrovica, J. X., Wahr, J., Matsuyama, I., and Paulson, A. (2005). *The rotational stability of an ice-age earth*, Geophys. J. Int. 30(161), 491–506.

Mitrovica, J. X. and Wahr, J. (2011). *Ice Age Earth Rotation*, Annual Review of Earth and Planetary Sciences, 39, 577–616.

Peltier, W.R. (1994). *Ice age paleotopography*, Science, 265, 1994.

Peltier, W., Argus, D., and Drummond, R. (2015). *Space geodesy constrains ice age terminal deglaciation: The global ICE-6G_C (VM5a) model*, J. Geophys. Res. , 120, 450–487.

Spada, G., Barletta, V. R., Klemann, V., Riva, R., Martinec, Z., Gasperini, P., Lund, B.,Wolf, D., Vermeersen, L., and King, M. (2011). *A benchmark study for glacial isostatic adjustment codes*, Geophys. J. Int. , 185, 106–132.

Spada, G. and Melini, D. (2019). *SELEN^4: a Fortran program for solving the gravitationally and topographically self-consistent sea-level equation in Glacial Isostatic Adjustment modeling*, manuscript submitted to Geoscientific Model Development.

Tegmark, M. (1996). *An icosahedron-based method for pixelizing the celestial sphere*, The Astrophysical Journal, 170, L81.

Tushingham, A. and Peltier, W. (1993). *Relative Sea Level Database*, IGCP PAGES/World Data Center-A Paleoclimatology Data Contribution Series, Tech. Rep. 93-106.

Wessel, P. and Smith, W. (1998). *New, improved version of generic mapping tools released*, Eos. Trans. AGU 79:579.

Wieczorek, M.A. and Meschede, M. (2018). *SHTools – Tools for working with spherical harmonics*, Geochemistry, Geophysics, Geosystems, 19, 2574-2592, doi:10.1029/2018GC007529.