ROMSOC

Numerical Solutions of the Fluid-Structure Interaction Problem in Membrane-Based Blood Pumps

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Reduced Order Modelling, Simulation and Optimization of Coupled Systems (ROMSOC)

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Blood Pumps: An Industrial Application

Left Ventricular Assist Devices (LVADs) support the activity of failed hearts by pumping blood into the ascending aorta.

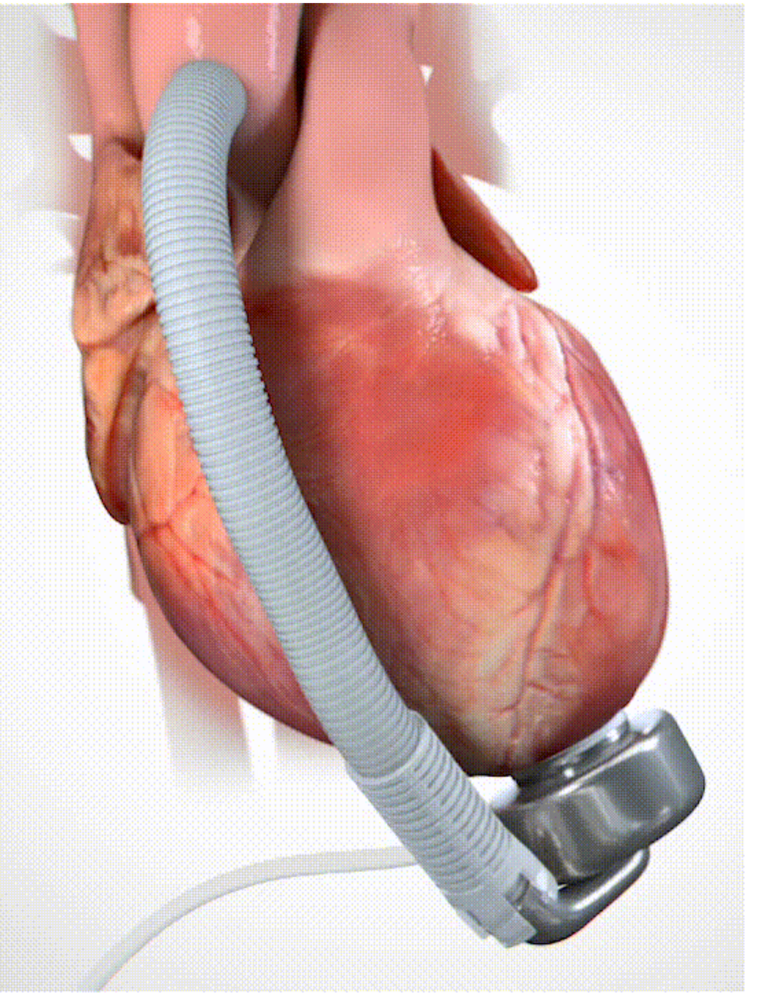
Medical applications:

- Bridge to recovery
- Bridge to transplantation
- Destination therapy

***CorWave Inc.* –** producer of new **membrane-based blood pumps**

→ **Wave propagation** technology

→ **Physiologic pulsatile** pump action



Video edited by CorWave Inc.

The Membrane-based Blood Pump

Video edited by CorWave Inc.

Advantages of In-Silico Models

In-silico simulations of the complex dynamics inside the pump allow to:

- Predict the **pump performance** under **different operating conditions,** varying:
  - **Frequence** of oscillation
  - **Amplitude** of oscillation
  - **Pressure gradient** between inlet and outlet

- Study the **3D vibrational modes** of the immersed elastic membrane

- Optimize the pump design to reduce the risk of **blood trauma** (e.g. hemolysis or thrombosis)

- Reduce the need of animal experimentations and make safer clinical trials

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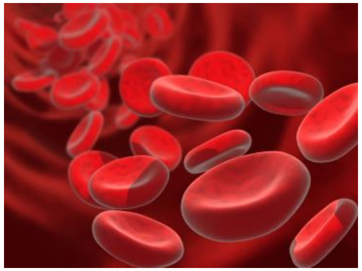
Fluid-Structure Interaction Model

Interface Coupling

Fluid Model

- The **blood** consists of a suspension of many cells (RBC, WBC, PT) in plasma



[C. Paddock, MedicalNewsToday]

- In most cases, it can be modeled as a **viscous incompressible Newtonian fluid** using **Navier-Stokes Equations**

Given the fluid domain Ω_t^f at time t, find fluid velocity $\mathbf{u}$ and pressure p such that:

$$\rho_f \left(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}\right) - \nabla \cdot \mathbf{T}^f(\mathbf{u}, p) = \mathbf{0} \quad \text{in } \Omega_t^f$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_t^f$$

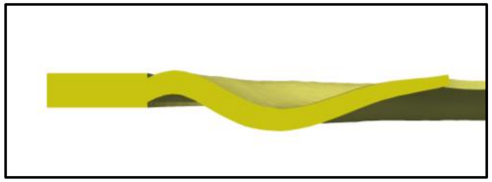
Structure Model

- The **elastic membrane** is made of an **homogeneous** and **isotropic** material

- **Small** displacements regime

→ **Linear elasticity** assumption

→ **Hooke's Law**



Given the solid domain $\widehat{\Omega}^s$ in the reference configuration, find displacement **d** such that:

$$\rho_s \partial_{tt} \widehat{\mathbf{d}} - \nabla \cdot \widehat{\mathbf{T}}^s(\widehat{\mathbf{d}}) = \mathbf{0} \quad \text{in } \widehat{\Omega}^s$$

$$\text{where } \widehat{\mathbf{T}}^s(\widehat{\mathbf{d}}) = \lambda_s (\nabla \cdot \widehat{\mathbf{d}})\, \mathbf{I} + \mu_s (\nabla \widehat{\mathbf{d}} + \nabla \widehat{\mathbf{d}}^T)$$

Fluid-Structure Interaction Model

The **coupling conditions** imposed at the fluid-structure interface should guarantee:

1) Geometric adherence of the fluid and solid domains (**geometric condition**)

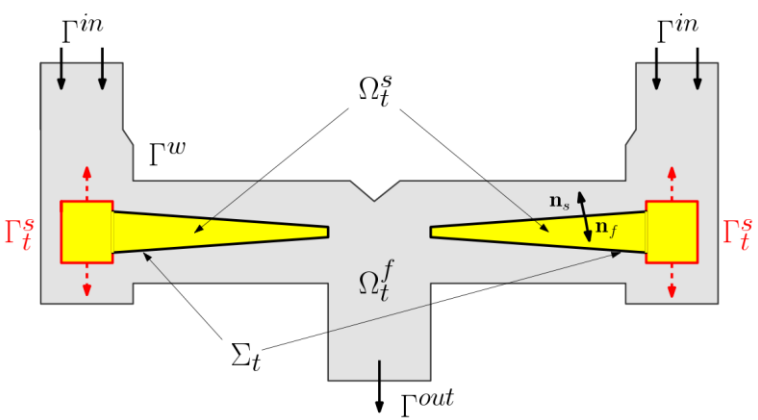
$$\mathbf{d}_f = \mathbf{d} \qquad \text{on } \partial\Omega_t^s$$

2) Continuity of the velocities at the interface (**kinematic condition**)

$$\mathbf{u} = \partial_t \mathbf{d} \qquad \text{on } \partial\Omega_t^s$$

3) Continuity of the forces at the interface (**dynamic conditions**)

$$\mathbf{T}^f(\mathbf{u}, p)\,\mathbf{n}_f = -\mathbf{T}^s(\mathbf{d})\,\mathbf{n}_s \qquad \text{on } \partial\Omega_t^s$$



$$\Omega = \Omega_t^f \cup \Omega_t^s$$

$\mathbf{d}_f$ - Fluid displacement

$$\Omega_t^f = \Omega^f(\mathbf{d}_f), \quad \Sigma_t = \Sigma(\mathbf{d}_f), \quad \Gamma_t^s = \Gamma^s(\mathbf{d}_f)$$

$$\partial\Omega_t^s = \Sigma_t \cup \Gamma_t^s, \quad \Sigma_t \cap \Gamma_t^s = \emptyset, \quad \Gamma_t^s, \Sigma_t \neq \emptyset$$

$$\mathcal{L}(t) : \widehat{\Omega}^s \longrightarrow \Omega^s(t) \qquad \mathbf{n} = \mathbf{n}_f = -\mathbf{n}_s$$

Boundary and Initial Conditions

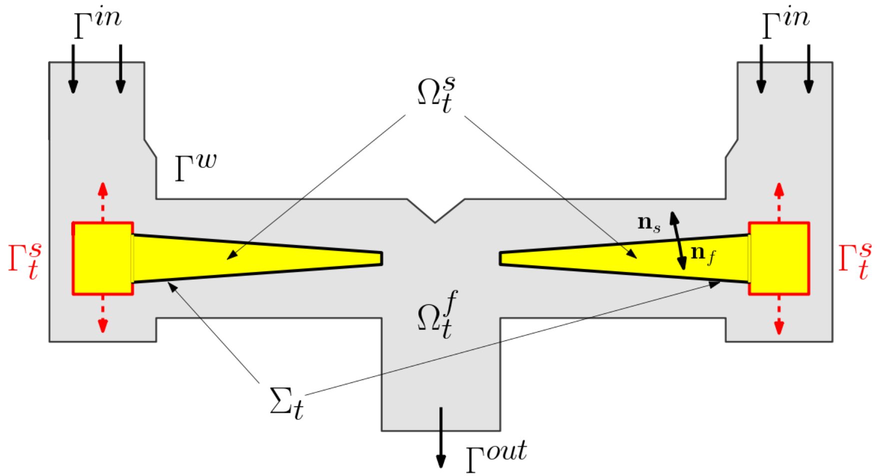
Boundary conditions:

$$\begin{cases} \mathbf{u}(r) = 2U\left(1-\left(\frac{r-R_m}{\Delta R}\right)^2\right)\mathbf{e}_z & \text{on } \Gamma^{in} \\ \mathbf{T}^f(\mathbf{u},p)\,\mathbf{n}^f = \mathbf{0} & \text{on } \Gamma^{out} \\ \mathbf{u} = \mathbf{0} & \text{on } \Gamma^{w} \\ \mathbf{d}(t) = \Phi \sin(2\pi f\, t)\,\mathbf{e}_z & \text{on } \Gamma^s_t \end{cases}$$

Initial conditions:

$$\begin{cases} \mathbf{u}(0) = \mathbf{0} & \text{in } \Omega^f_0 \\ \mathbf{d}(0) = \mathbf{0} & \text{in } \Omega^s_0 \\ \dot{\mathbf{d}}(0) = \mathbf{0} & \text{in } \Omega^s_0 \end{cases}$$

Sinusoidal oscillation imposed by the electromagnetic actuator



$$\partial\Omega^s_t = \Sigma_t \cup \Gamma^s_t, \qquad \Sigma_t \cap \Gamma^s_t = \emptyset, \qquad \Gamma^s_t, \Sigma_t \neq \emptyset$$

$$\Gamma^w = \partial\Omega^f_t \setminus (\partial\Omega^s_t \cup \Gamma^{in} \cup \Gamma^{out}), \qquad \Gamma^w \neq \emptyset$$

$$\mathbf{n} = \mathbf{n}_f = -\mathbf{n}_s$$

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Fitted vs. Unfitted Methods

Numerical issues:

- **Three-dimensional immersed** structure with **small thickness**
- **Large structure displacement** compared with the limited fluid free space
- Possible **contact** with the pump walls
- **High frequency** of oscillation (about 60-120 Hz)

Fitted vs. Unfitted Methods

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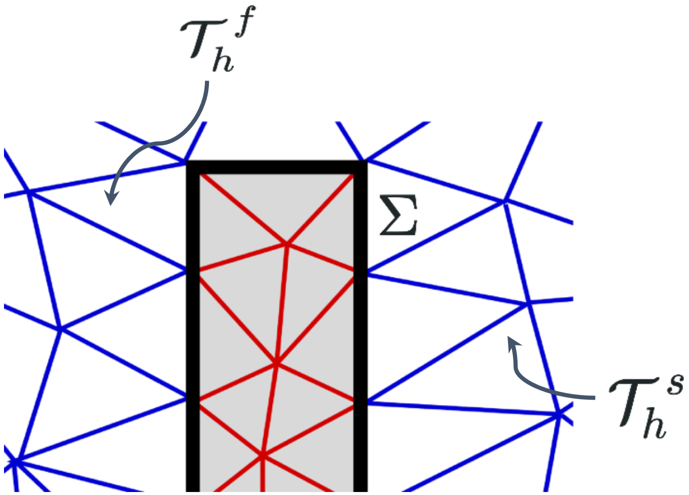
Fitted Methods: fluid and solid mesh are fitted at the inteface and move together.

✓ Simple and accurate

- Not suited to handle large deformations or the contact problem



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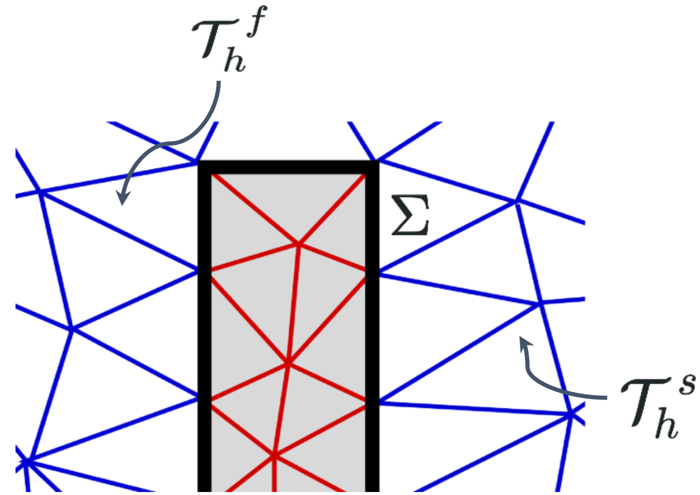
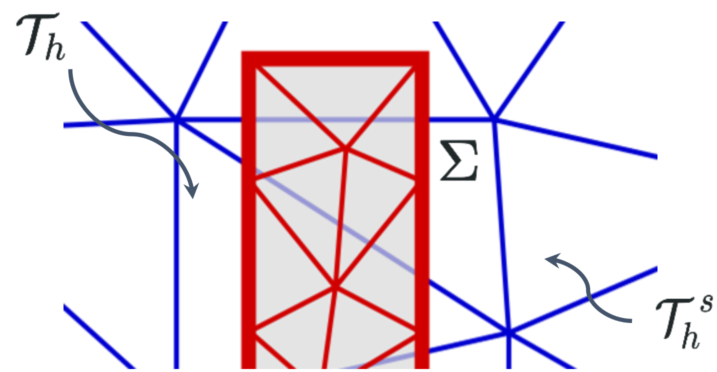


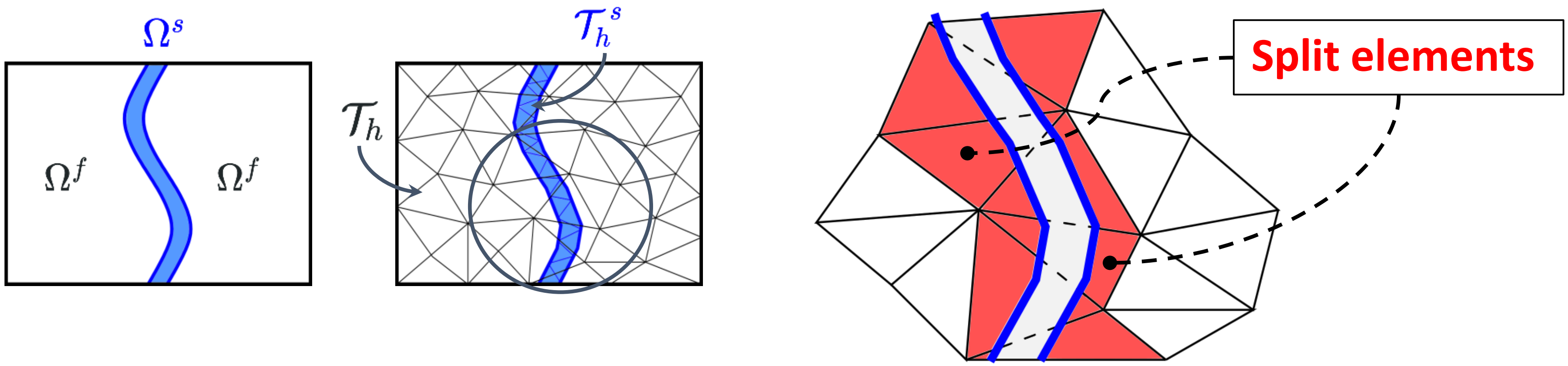
Unfitted Methods: the background fluid mesh is **not fitted** with the structure one at the interface, and it is **fixed** in time

✓ Suited for large deformations and contact
- Computationally more complex



XFEM-DG Technique

The **Extended-Finite Element Method** (XFEM) is an **unfitted technique** based on the enrichment of the functional space of the so-called **split elements.**



[Moës, Dolbow, Belytschko, IJNME (1999)], [Hansbo, Hansbo, CMAME (2002)]
[Burman, Fernández, CMAME (2014)], [Schott, Wall, CMAME (2014)], [Massing et al., CAMCoS (2015)],
[Alauzet, Fernández et al., CMAME (2016)], [Burman, Fernández, Gerbeau, C&F (2018)]

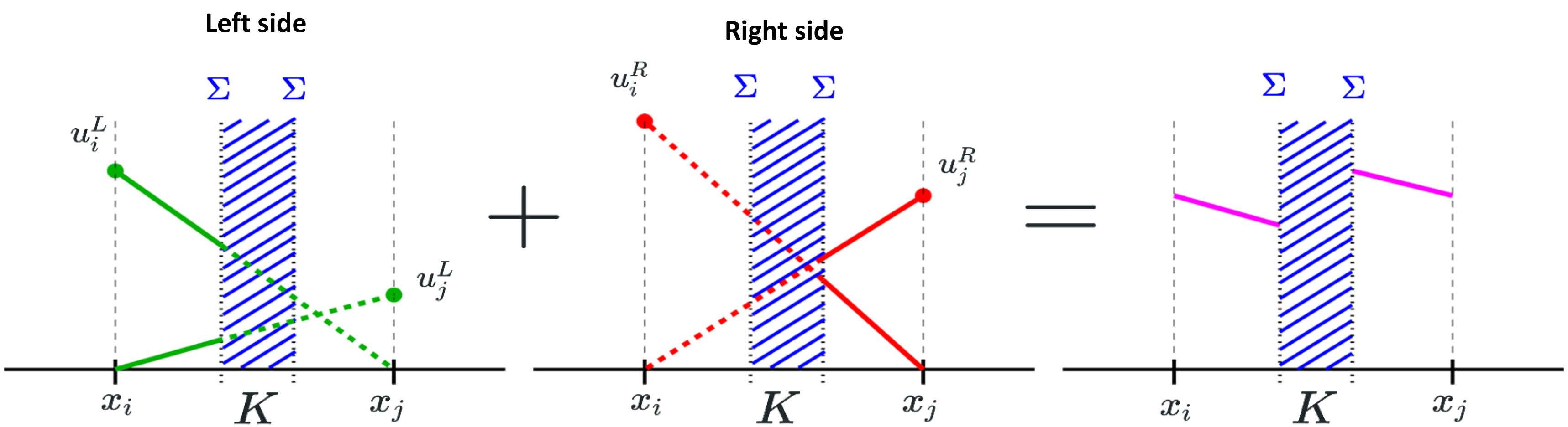
The **Discontinuous Galerkin mortaring** (DG) at the interface is employed to couple the fluid and the structure problems at the interface.

[Arnold et al., SIAM J Numer Anal (2001)]

[S. Zonca, C. Vergara, L. Formaggia. An unfitted formulation for the interaction of an incompressible fluid with a thick structure via an XFEM/DG approach. SIAM J. Sci. Comput. 40 (1) (2018), pp. B59-B84]

The Extension of the Finite Elements

The degrees of freedom (dofs) of each split element are **duplicated** to represent the fluid solution on both sides of the structure independently.



Representation of XFEM method in **1D** case scenario.

This approach allows to represent a discontinuity within the element, but using the same Lagrangian basis function.

Numerical Discretization

For any $t \in (0, T]$, find $\left(\mathbf{u}_h(t), p_h(t), \widehat{\mathbf{d}}_h(t)\right) \in \mathbf{V}_h \times \mathbf{Q}_h \times \mathbf{W}_h$ such that:

Fluid terms

$$\rho_f \int_{\Omega_t^f} \partial_t \mathbf{u}_h \cdot \mathbf{v}_h + \rho_f \int_{\Omega_t^f} \mathbf{u}_h \cdot \nabla \mathbf{u}_h \, \mathbf{v}_h + \int_{\Omega_t^f} 2\mu_f D(\mathbf{u}_h) : \nabla \mathbf{v}_h - \int_{\Omega_t^f} p_h \nabla \cdot \mathbf{v}_h + \int_{\Omega_t^f} q_h \nabla \cdot \mathbf{u}_h + s_h(\mathbf{u}_h, p_h, \mathbf{v}_h, q_h) +$$

Structure terms

$$+ \rho_s \int_{\widehat{\Omega}^s} \partial_{tt} \widehat{\mathbf{d}}_h \cdot \widehat{\mathbf{w}}_h + \int_{\widehat{\Omega}^s} \widehat{\mathbf{T}}^s(\widehat{\mathbf{d}}_h) : \nabla \widehat{\mathbf{w}}_h -$$

[Arnold et al., SIAM J Numer Anal (2001)]
[Burman, Fernández, CMAME (2009)]

$$- \left(\epsilon \mathbf{T}^f(\mathbf{u}_h, p_h)\mathbf{n} + (1-\epsilon)\mathbf{T}^s(\mathbf{d}_h)\mathbf{n}, \mathbf{v}_h - \mathbf{w}_h\right)_{\Sigma_t \cup \Gamma_t^s} - \left(\mathbf{u}_h - \partial_t \mathbf{d}_h, \epsilon \mathbf{T}^f(\mathbf{v}_h, -q_h)\mathbf{n} + (1-\epsilon)\mathbf{T}^s(\mathbf{w}_h)\mathbf{n}\right)_{\Sigma_t \cup \Gamma_t^s} +$$

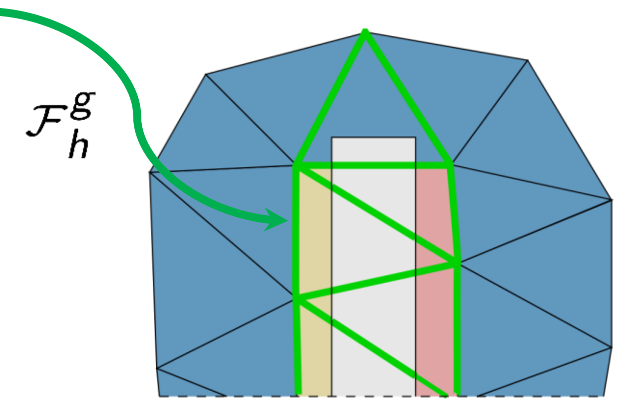
$$+ \frac{\gamma_\Sigma \mu_f}{h} (\mathbf{u}_h - \partial_t \mathbf{d}_h, \mathbf{v}_h - \mathbf{w}_h)_{\Sigma_t \cup \Gamma_t^s} +$$

DG Coupling terms

Ghost-penalty terms

[Burman, C R Math Acad Sci Paris (2010)]

$$+ \gamma_g \int_{\mathcal{F}_h^g} \mu_f h [[\nabla \mathbf{u}_h]]\mathbf{n} \cdot [[\nabla \mathbf{v}_h]]\mathbf{n}$$



for all $(\mathbf{v}_h, q_h, \widehat{\mathbf{w}}_h) \in \mathbf{V}_h \times \mathbf{Q}_h \times \mathbf{W}_h$

$$\mathbf{V}_h = \left\{ \mathbf{v} \in [X_h^f]^3 : \mathbf{v}|_{\Gamma^w} = \mathbf{0} \right\}, \quad \mathbf{Q}_h = \left\{ q \in X_h^f \right\},$$

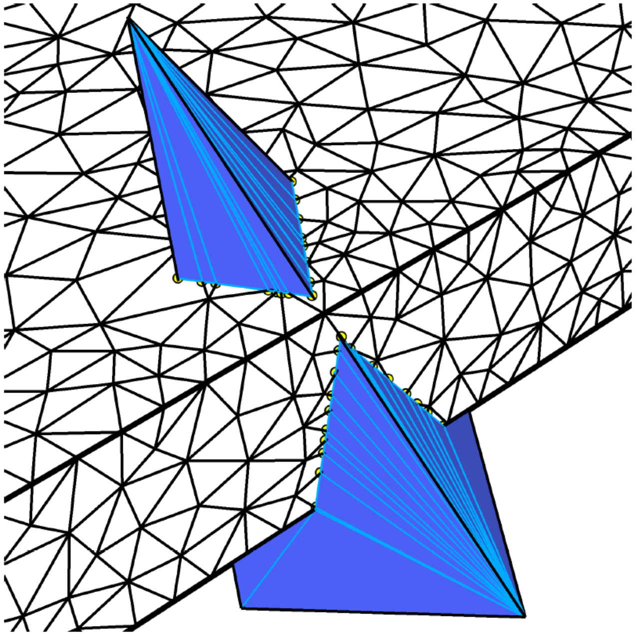
$$\mathbf{W}_h = \left\{ \mathbf{w} \in [X_h^s]^3 : \mathbf{w}|_{\widehat{\Gamma}^s} = \mathbf{0} \right\}.$$

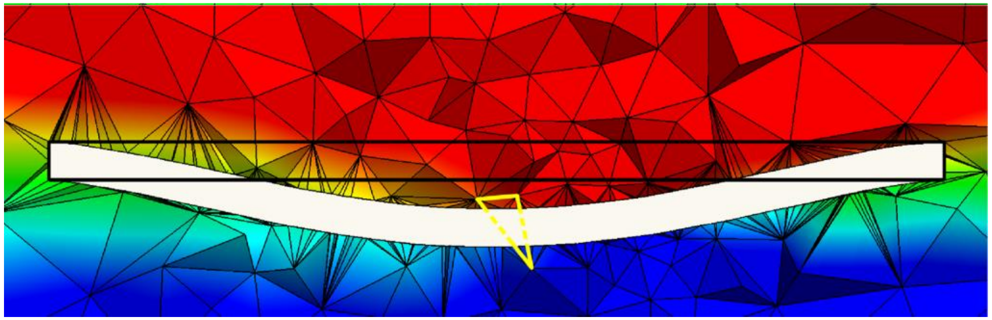
[S. Zonca, C. Vergara, L. Formaggia. An unfitted formulation for the interaction of an incompressible fluid with a thick structure via an XFEM/DG approach. SIAM J. Sci. Comput. 40 (1) (2018), pp. B59-B84]

The Geometric Complexity of XFEM Approach

At each time instant $t^n > 0$:

1. **Move** the solid mesh by $\mathbf{d}_{n-1}$
2. **Compute** the new intersections between the fluid and the structure meshes
3. **Double** the dofs of the split elements
4. **Sub-tetrahedralize** each polyhedron to integrate on tetrahedra (Gaussian rule)
5. **Solve** the problem to get displacement $\mathbf{d}_n$





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2) Mathematical formulation of the FSI problem
3) Numerical method: X-FEM/DG
4) **Results**
5) Conclusions

Settings of 3D FSI Simulations

Physiologic range

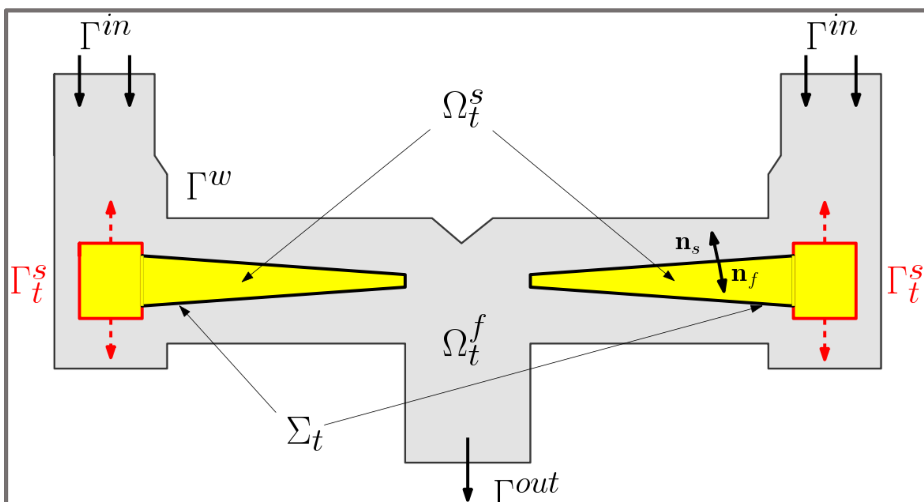
Physical settings:

- Membrane oscillation:
  - ϕ **= 0.9 cm**
  - **f = {10, 30, 60, 100 } Hz**
- Mean inlet velocity:
  - **U = {1.0, 3.6, 5.3, 7.1 } cm/s**

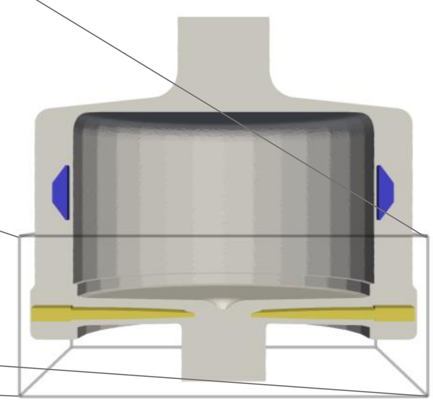
Discretization parameters:

- Space
  - Fluid FE space: $\mathbb{P}^1 - \mathbb{P}^1$
  - Solid FE space: $\mathbb{P}^1$
  - h_f = 0.03 cm, h_s = 0.02 cm
- Time
  - Δt = 5 e-4 s, T = 0.1 s
  - BDF order 1

Sketch of the cross section of the domain



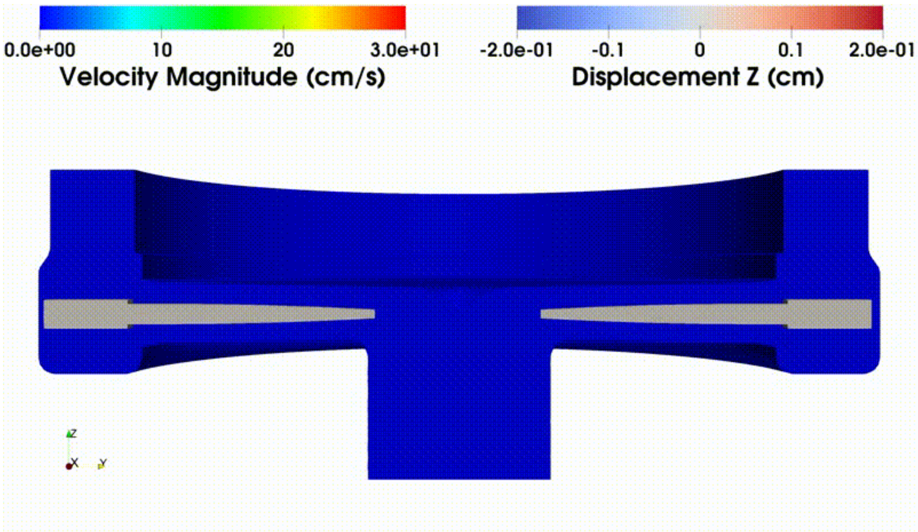
The region of interest is limited to the lower **pump head** region



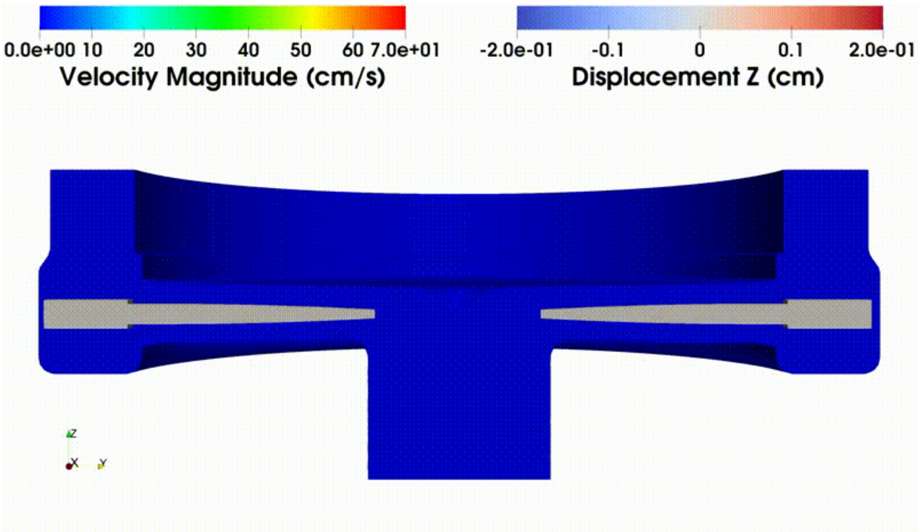
Velocity and Pressure Field

- **Outflow velocity** is modulated by the wave membrane propagation
- Reynolds number **increase** with oscillation frequency

<u>Low frequencies: f =10 Hz</u>



<u>High frequencies: f=100Hz</u>

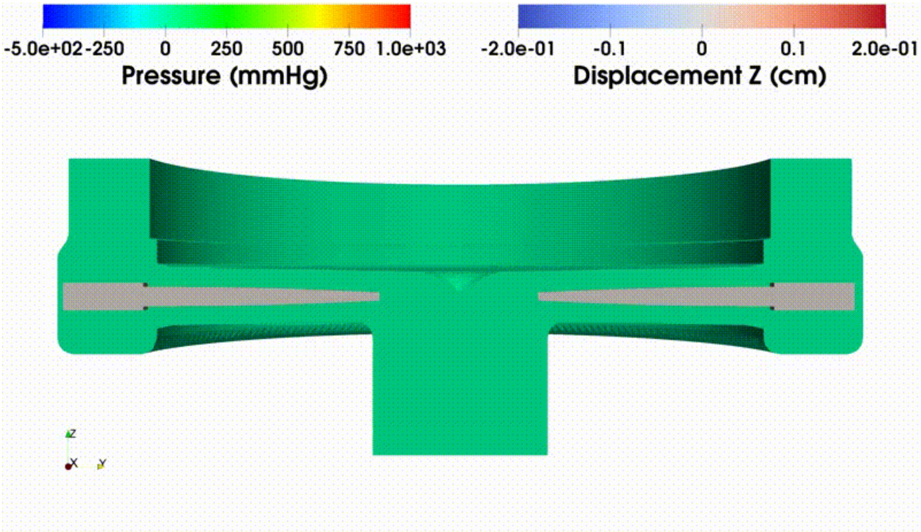


Due to small displacements, the fluid is **not** updated to reduce the computational cost.

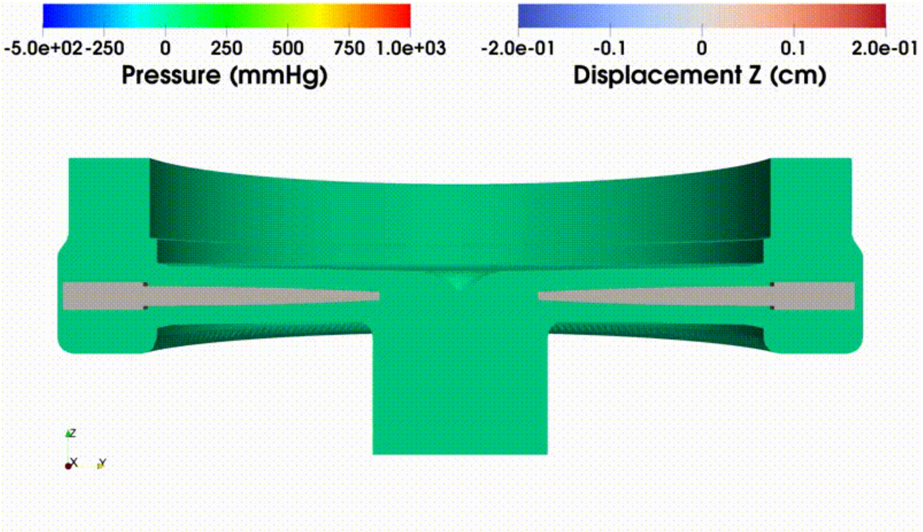
Velocity and Pressure Field

- **Outflow velocity** is modulated by the wave membrane propagation
- Reynolds number **increase** with oscillation frequency
- **Pressure gradient** between upside and downside the membrane alternates during oscillation period propelling the blood outwards

<u>Low frequencies: f =10 Hz</u>



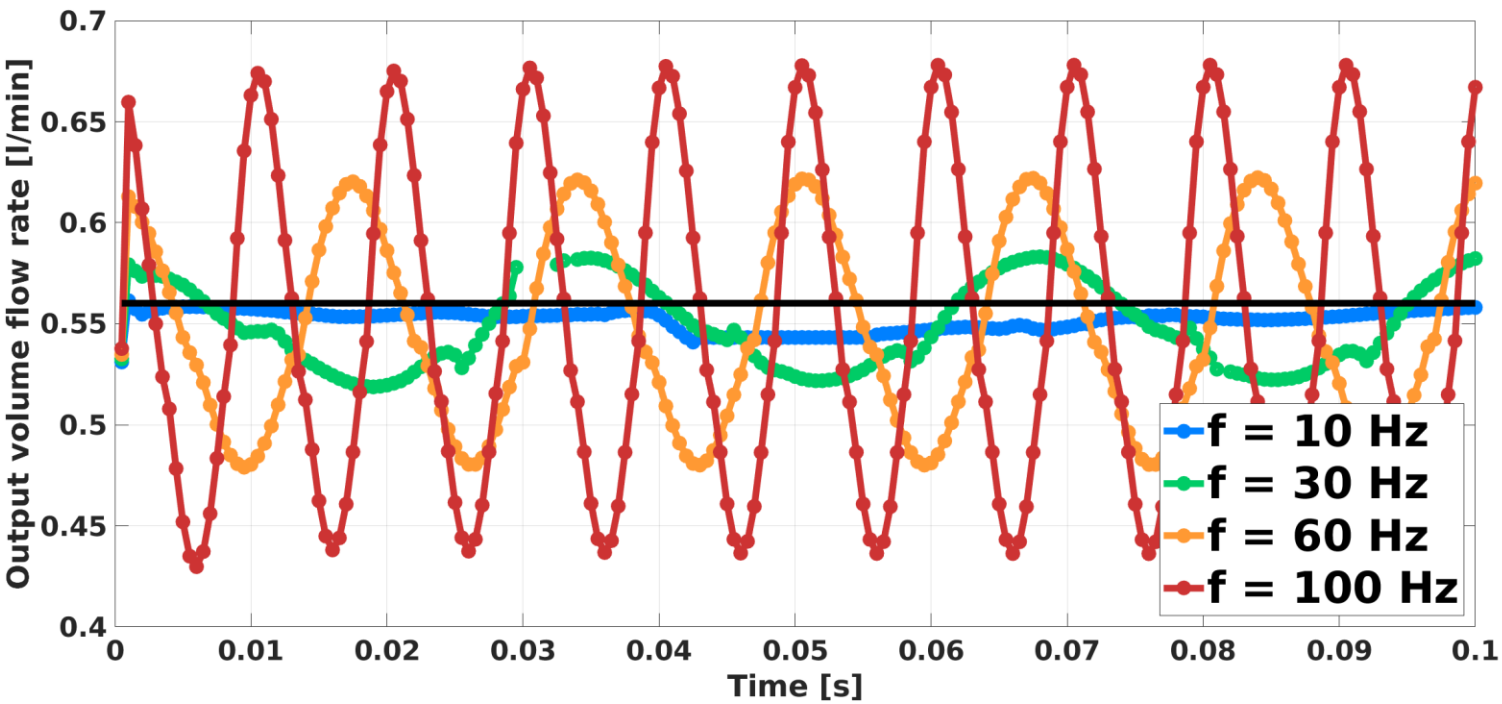
<u>High frequencies: f=100Hz</u>



Due to small displacements, the fluid is **not** updated to reduce the computational cost.

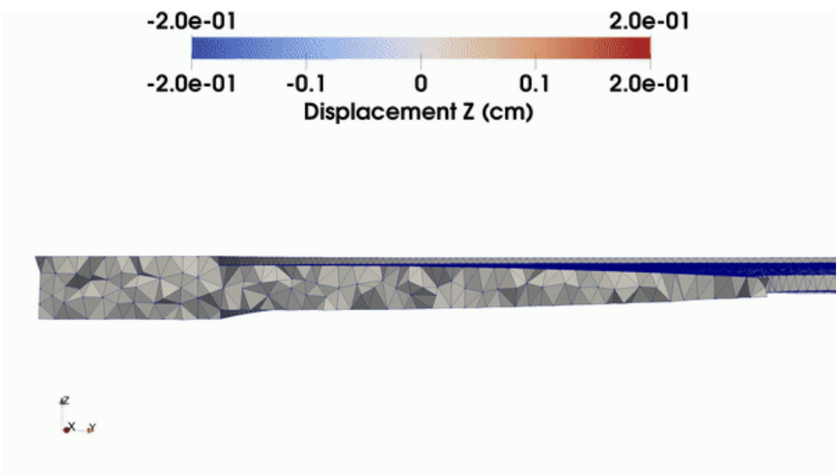
Pulsatility of the Outflow Volume Rate

- The oscillation imposed on the membrane motion reflects on a **pulsatile volume flow rate** at the outlet with the **same frequency**
- The amplitude of the volume outflow rate **increases** while increasing the vibration frequency f



Membrane Displacement

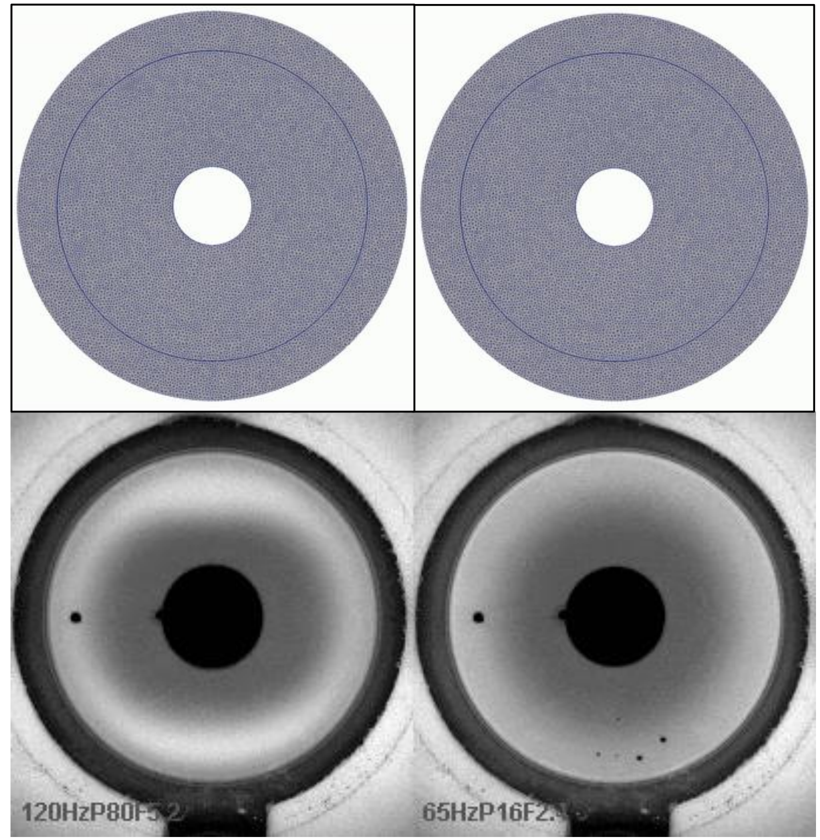
The simulations allow to study the membrane **wave propagations** and compare the displacement with **experimental data.**



Cross-section of membrane oscillating with frequency f = 30 Hz

F = 100 Hz

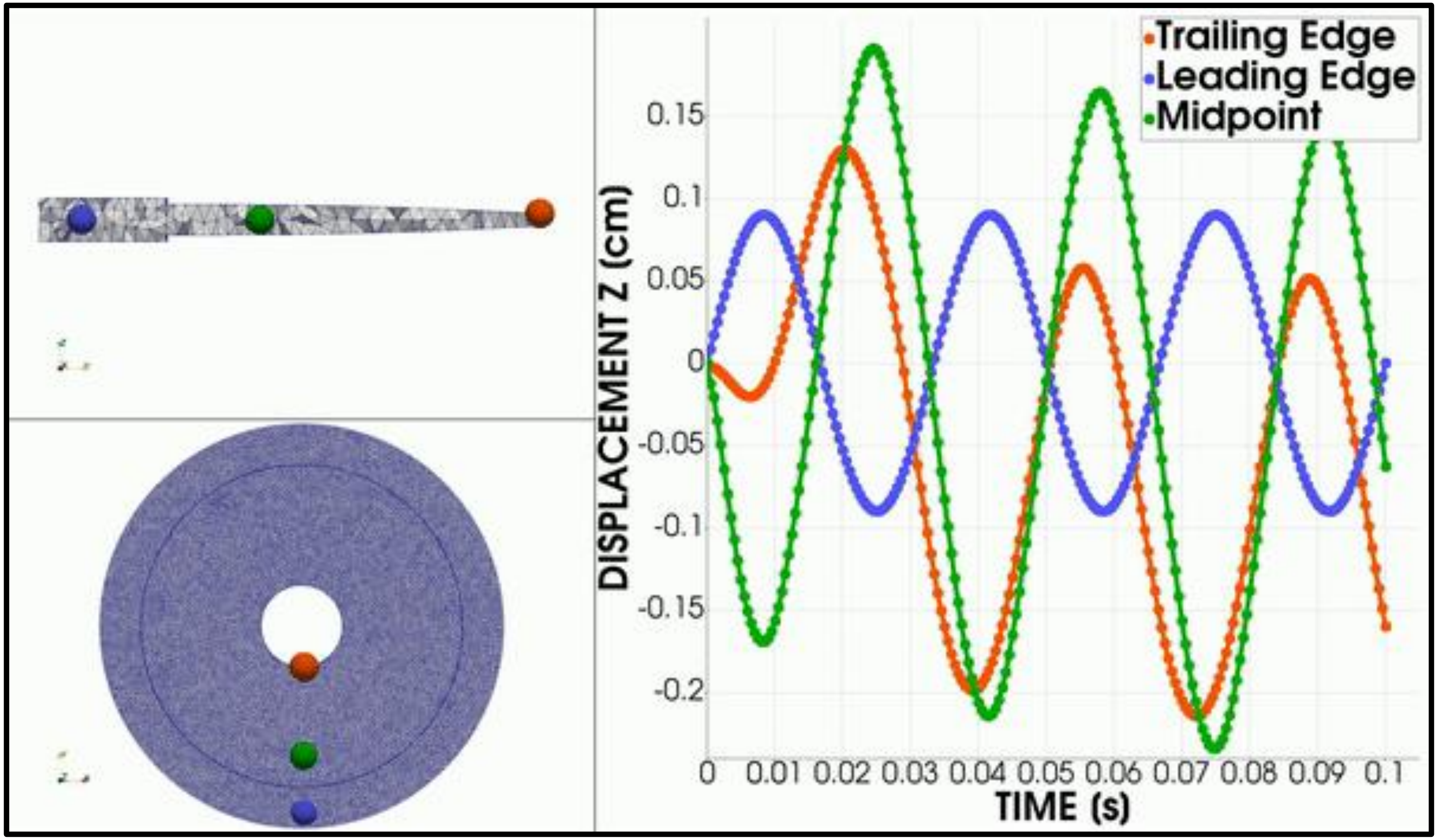
F = 60 Hz



Recording of membrane motion using high speed camera

Membrane Point Displacement Analysis

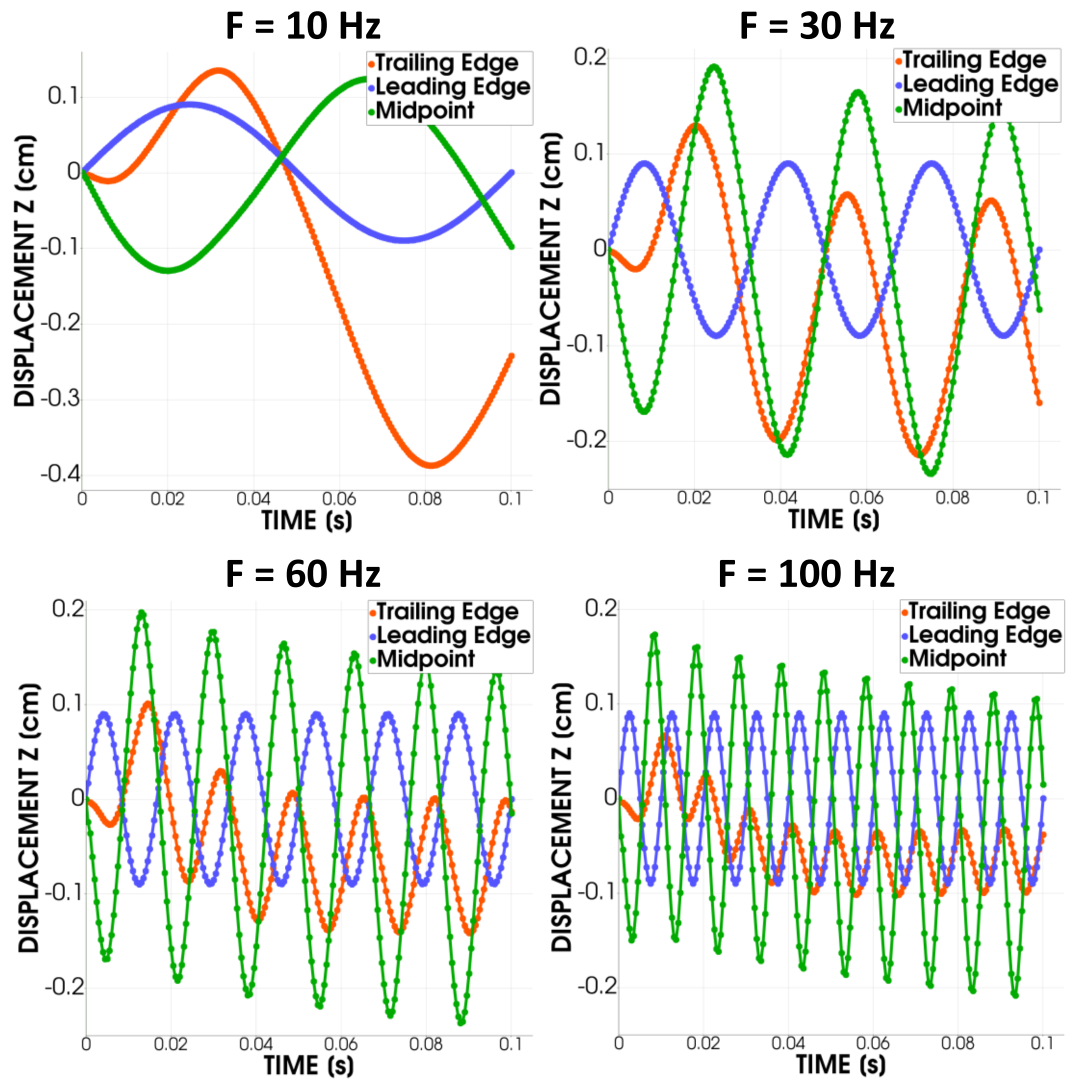
Registration of the displacements of **three key points** of the membrane



Membrane Point Displacement Analysis

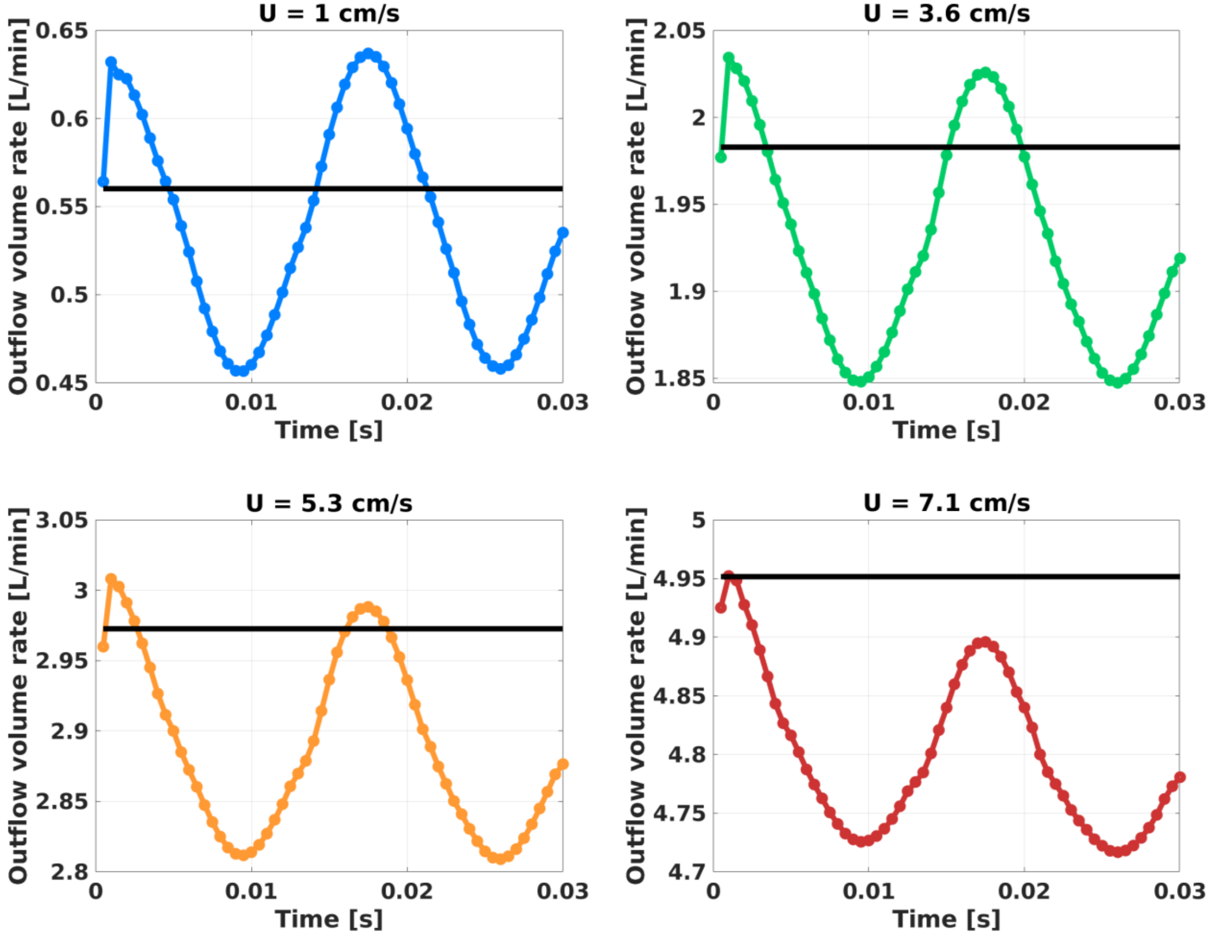
- **Leading edge** reflecting the given oscillation condition
- Stabilization of the **trailing edge** around a negative point
- Damping effect in the **trailing edge**, that **increases** with the frequency
- **Maximum** displacement achieved by the **midpoint** line around -2.2 mm

→ **contact** is more plausible to occur on the **lower pump wall**



(On)Going towards Higher Velocity Scales

Final goal: **U = 7.1 cm/s**, corresponding to **full cardiac support**



Increase in the Reynolds number results in a slight **decrease of mass flow through time**

→ High model sensitivity to stability parameters

Work In Progress: Need to further investigate the causes of such behavior.

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Conclusions & Next Steps

Conclusions:

- ❑ Application of **XFEM-DG** technique on an industrial **3D** problem
- ❑ Outflow and membrane motion studied under different **operating points**
- ❑ **Ouflow pulsatility** has the same frequency of the prescribed membrane propagation waves and its **amplitude** increases with higher frequencies

Next steps:

- ❑ **Update** the fluid mesh at each time instant
- ❑ Optimize model **robustness** over different physical parameters
- ❑ Include **contact conditions** to model posssible contact of membrane with pump walls

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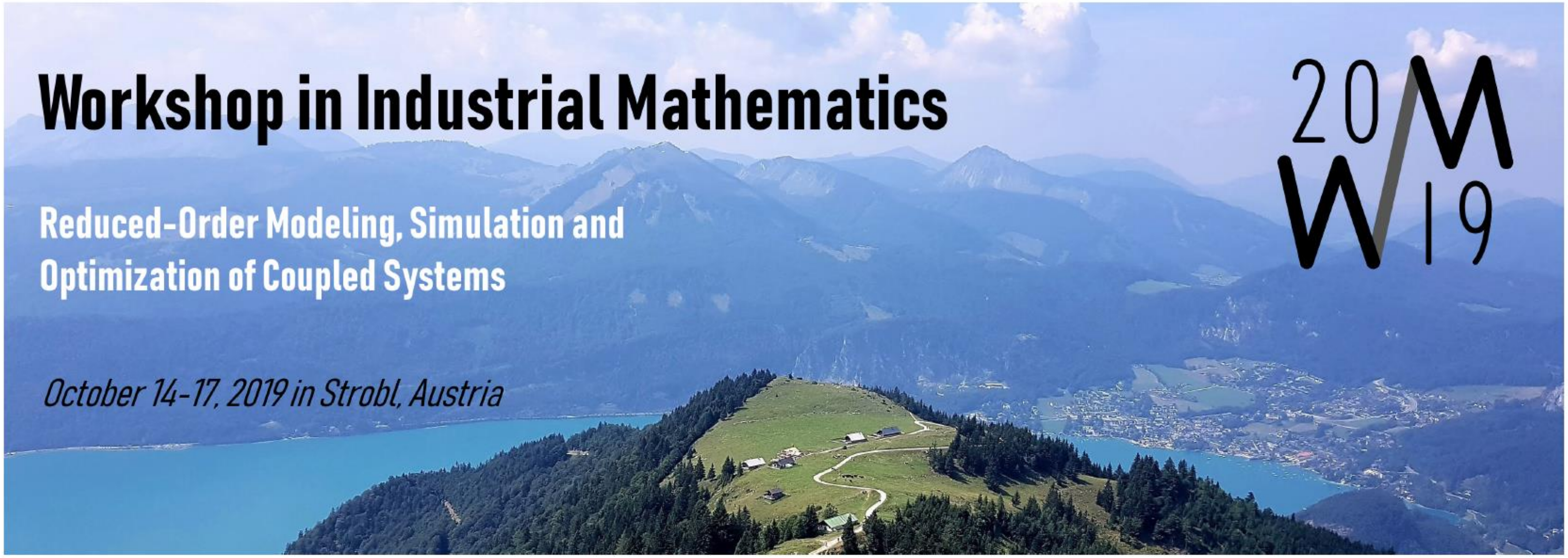

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