Spectral Efficiency Maximization for Spatial Modulation Aided Layered Division Multiplexing: An Injection Level Optimization Perspective

Yue Sun[1], Jintao Wang[1,2,3], Changyong Pan[1,2], and Longzhuang He[1]
[1]Electronic Engineering Department, Tsinghua University,
Beijing National Research Center for Information Science and Technology (BNRist), Beijing 100084, P. R. China
[2]National Engineering Lab. for DTV(Beijing), Beijing 100191, P. R. China
[3]Shenzhen City Key Laboratory of Digital TV System
Corresponding Author: Jintao Wang Email: wangjintao@tsinghua.edu.cn

***Abstract*—The layered division multiplexing (LDM) is recently combined with spatial modulation (SM) systems to provide a more efficiency way for broadcasting transmission. In SM aided LDM (SM-LDM) systems, the service of each layer, which is allocated with different power, is transmitted via SM scheme. In this paper, a gradient descent based iterative method is proposed to optimize the injection level, which can enhance the spectral efficiency (SE) in the two-layer SM-LDM systems with maximum ratio combining (MRC). In addition, the concavity analysis of this optimization problem is also conducted. Monte Carlo simulations are also provided to verify the effectiveness of our proposed injection level optimization method.**

***Keywords*—Spatial modulation (SM); layered division multiplexing (LDM); spectral efficiency (SE); gradient descent.**

I. Introduction

Recently, the concept of spatial modulation (SM) is proposed to provide a superior energy efficiency (EE) comparing with traditional multiple-input multiple-output (MIMO) architectures [1], such as the Vertical Bell Laboratories layered space-time (V-BLAST) scheme [2] and the space time block code (STBC) scheme [3]. In SM systems, only one transmit antenna (TA) is active in each time slot, so only one radio frequency (RF) chain is equipped in the transmitter, and the inter-channel interference (ICI) can be overcome [1] [4]. Based on the concept of SM, generalised spatial modulation (GenSM) is proposed to increase the SE by activating multiple TAs [5] [6]. In addition, SM is also combined with single carrier (SC) transmission and massive MIMO system to further increase the SE of the system [7]–[9],

In broadcasting transmission scenarios, recently the technology of layered division multiplexing (LDM) is proposed for SE enhancement, which has been adopted in the standard of the Advanced Television Systems Committee (ATSC) 3.0 [10] [11]. In LDM systems, different layers conveys different services, which are also allocated with different power levels [11]. In most typical cases, the LDM systems are assigned as two-layer LDM systems, and the upper layer (UL) and lower layer (LL) are utilized for providing services for mobile terminals and fixed terminals, respectively [12]. Therefore, the UL and LL are also known as mobile layer (ML) and fixed layer (FL), respectively. LDM systems can also be combined with traditional MIMO schemes, such as spatial multiplexing (SMX) systems [13].

The SM aided LDM (SM-LDM) scheme is recently proposed to provide a better SE performance than traditional single-TA LDM systems, in which both ML and FL utilize SM for information transmission [14]. In [14], a two-layer SM-LDM system framework is proposed, and the closed-form SE lower bound of SM-LDM systems with maximum ratio combining (MRC) is also formulated. However, the injection level optimization has not been considered yet, which can further increase the SE in SM-LDM systems. Therefore, in this paper, based on the theoretical SE lower bound derived in [14], we propose an injection level optimization algorithm utilizing the gradient descent method, which is intended for maximizing the sum rate of SM-LDM systems. Corresponding to the injection level, the function of sum rate in SM-LDM systems is quasi-concave [15], so our proposed algorithm can search out the optimized injection level.

The organization of this paper can be summarized as follows. Section II presents the system model of two-layer SM-LDM systems, and the SE lower bound of both ML and FL in SM-LDM systems with MRC. In Section III, the injection level optimization algorithm is proposed based on the gradient descent method, and the concavity of this problem is also analyzed. In Section IV, Monte Carlo simulations are presented to demonstrate the efficacy of our proposed SE maximization algorithm. In Section V, this paper is finally concluded.

Notations: In this paper, the operators $(\cdot)^T$ and $(\cdot)^H$ represent the transposition and the conjugate transposition, respectively. The abbreviations $\mathbf{A}(i,j)$ and $\det(\mathbf{A})$ indicate the element in the j-th column and i-th row and the determinant of matrix $\mathbf{A}$, respectively. $\mathcal{CN}(\mu,\sigma)$ represents a Gaussian random variable whose mean is μ and variance is σ. The abbreviation diag$(\mathbf{x})$ represents a diagonal matrix whose diagonal elements are $\mathbf{x}$, and $\mathbf{I}_n$ represents an identical matrix whose dimension is n-by-n.

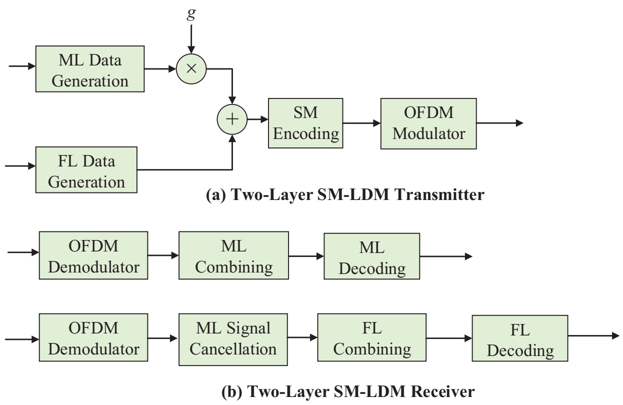

Fig. 1. System model of the two-layer SM-LDM system.

II. System Model

As shown in Fig. 1, the system model of the two-layer SM-LDM system is introduced in this section. We denote N_{T} as the number of TAs, N_{RM} as the number of receive antennas (RAs) in ML, and N_{RF} as the number of RAs in FL. At the transmitter, the ML information and FL information are generated separately, and the constellation symbols of both ML and FL are also independent. Besides, the active antennas of ML and FL are generated by the ML information and FL information, respectively. For ML, the injection level g is introduced to multiply the ML symbol. Then the ML symbol and FL symbol are added and transmitted by the orthogonal frequency division multiplexing (OFDM) modulator. Therefore, the transmitted symbol in SM-LDM systems can be denoted as follows:

$$\mathbf{x} = \sqrt{P_{\mathrm{ML}}}\mathbf{x}_{\mathrm{ML}} + \sqrt{P_{\mathrm{FL}}}\mathbf{x}_{\mathrm{FL}}, \tag{1}$$

where $\mathbf{x}_{\mathrm{ML}} = s_{\mathrm{ML}}\mathbf{e}_{\mathrm{ML}} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ and $\mathbf{x}_{\mathrm{FL}} = s_{\mathrm{FL}}\mathbf{e}_{\mathrm{FL}} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ are the transmit symbols of ML and FL, respectively. $s_{\mathrm{ML}} \sim \mathcal{CN}(0,1)$ and $s_{\mathrm{FL}} \sim \mathcal{CN}(0,1)$ denote the Gaussian constellation symbols of ML and FL, respectively, and $\mathbf{e}_{\mathrm{ML}} = [0,\ldots,0,1,0,\ldots,0]^T$ and $\mathbf{e}_{\mathrm{FL}} = [0,\ldots,0,1,0,\ldots,0]^T$ represent the spatial symbols of ML and FL, respectively. P_{ML} and P_{FL} are transmit power of ML and FL, respectively, and we have:

$$P_{\mathrm{ML}}/P_{\mathrm{FL}} = g, \quad g \geq 0, \quad P_{\mathrm{ML}} + P_{\mathrm{FL}} = P_{\mathrm{U}}, \tag{2}$$

where P_{U} is the total transmit power.

Since the transmitted symbol in (1) is in frequency domain, after OFDM modulation and OFDM demodulation, the received frequency domain symbol in ML can be denoted as follows:

$$\mathbf{y}_{\mathrm{ML}} = \sqrt{P_{\mathrm{ML}}}\mathbf{H}_{\mathrm{ML}}\mathbf{x}_{\mathrm{ML}} + \sqrt{P_{\mathrm{FL}}}\mathbf{H}_{\mathrm{ML}}\mathbf{x}_{\mathrm{FL}} + \mathbf{n}_{\mathrm{ML}}, \tag{3}$$

where $\mathbf{H}_{\mathrm{ML}} \in \mathbb{C}^{N_{\mathrm{RM}}\times N_{\mathrm{T}}}$ is the Wide Sense Stationary (WSS) Rayleigh fading channel matrix [12], and $\mathbf{H}_{\mathrm{ML}}(i,j) \sim \mathcal{CN}(0,1)$ is an independent and identically distributed (i.i.d.) Gaussian random variable. $\mathbf{n}_{\mathrm{ML}} \in \mathbb{C}^{N_{\mathrm{RM}}\times 1}$ is the additive white Gaussian noise (AWGN) in ML, and $\mathbf{n}_{\mathrm{ML}}(i) \sim \mathcal{CN}(0,\sigma^2_{\mathrm{ML}})$ is also i.i.d. For ML combining, the FL signal is treated as additive noise, since ML service is always allocated with a much higher transmit power than FL service.

Similarly, the received frequency domain symbol in FL can be denoted as follows:

$$\mathbf{y}_{\mathrm{FL}} = \sqrt{P_{\mathrm{ML}}}\mathbf{H}_{\mathrm{FL}}\mathbf{x}_{\mathrm{ML}} + \sqrt{P_{\mathrm{FL}}}\mathbf{H}_{\mathrm{FL}}\mathbf{x}_{\mathrm{FL}} + \mathbf{n}_{\mathrm{FL}}, \tag{4}$$

where $\mathbf{H}_{\mathrm{FL}} \in \mathbb{C}^{N_{\mathrm{RF}}\times N_{\mathrm{T}}}$ is also the WSS Rayleigh fading matrix with $\mathbf{H}_{\mathrm{FL}}(i,j) \sim \mathcal{CN}(0,1)$ [12]. $\mathbf{n}_{\mathrm{FL}} \in \mathbb{C}^{N_{\mathrm{RF}}\times 1}$ is the AWGN in FL with $\mathbf{n}_{\mathrm{FL}}(i) \sim \mathcal{CN}(0,\sigma^2_{\mathrm{FL}})$. In addition, from Fig. 1 (b), it can be seen that the ML signal is firstly cancelled before FL combining, and the ML signal cancellation can be assumed perfectly because in ML the signal-to-noise-ratio (SNR) is much higher than that in FL [12]. Moreover, the channel estimation (CE) can be assumed ideally, so the cross-layer interference (CLI) can be eliminated [12]–[14]. Therefore, after the ML signal cancellation, the received FL symbol can be denoted as follows:

$$\mathbf{y}_{\mathrm{FL}} = \sqrt{P_{\mathrm{FL}}}\mathbf{H}_{\mathrm{FL}}\mathbf{x}_{\mathrm{FL}} + \mathbf{n}_{\mathrm{FL}}, \tag{5}$$

The SE lower bound of ML and FL in SM-LDM systems with MRC is derived separately. According to [14], the closed-form SE lower bound of ML can be denoted as (6), where $\rho_{\mathrm{ML},n}$ denotes the reciprocal of the n-th signal-to-interference-plus-noise-ratio (SINR) in ML as follows:

$$\rho_{\mathrm{ML},n} = \frac{1}{\mathrm{SINR}_{\mathrm{ML},n}} = \frac{P_{\mathrm{U}}N_{\mathrm{T}} + P_{\mathrm{FL}}N_{\mathrm{RM}} + N_{\mathrm{T}}\sigma^2_{\mathrm{ML}}}{P_{\mathrm{ML}}N_{\mathrm{RM}}}, \tag{8}$$

and $\mathbf{\Sigma}_{\mathrm{ML},n}$ can be denoted as follows:

$$\mathbf{\Sigma}_{\mathrm{ML},n} = \mathrm{diag}\left\{\rho_{\mathrm{ML},1},...,\rho_{\mathrm{ML},N_{\mathrm{T}}}\right\} + N_{\mathrm{T}}\mathrm{diag}\{\hat{\mathbf{e}}_n\}, \tag{9}$$

where $\hat{\mathbf{e}}_n$ represents the n-th column of $\mathbf{I}_{N_{\mathrm{T}}}$.

From [14], the theoretical SE lower bound of FL in SM-LDM systems with MRC can be denoted as (7), where $\rho_{\mathrm{FL},m}$ represents the reciprocal of the m-th FL SINR, and $\rho_{\mathrm{FL},m}$ can be denoted as follows:

$$\rho_{\mathrm{FL},m} = \frac{1}{\mathrm{SINR}_{\mathrm{FL},m}} = \frac{P_{\mathrm{FL}}N_{\mathrm{T}} + N_{\mathrm{T}}\sigma^2_{\mathrm{FL}}}{P_{\mathrm{FL}}N_{\mathrm{RF}}}. \tag{10}$$

Besides, $\mathbf{\Sigma}_{\mathrm{FL},m}$ can be denoted as follows:

$$\mathbf{\Sigma}_{\mathrm{FL},m} = \mathrm{diag}\left\{\rho_{\mathrm{FL},1},...,\rho_{\mathrm{FL},N_{\mathrm{T}}}\right\} + N_{\mathrm{T}}\mathrm{diag}\{\hat{\mathbf{e}}_m\}, \tag{11}$$

where $\hat{\mathbf{e}}_m$ represents the m-th column of $\mathbf{I}_{N_{\mathrm{T}}}$.

The tightness of the closed-form SE lower bound for SM-LDM systems with MRC has also been verified via simulations in [14], and the simulations and bound have the same trend. Therefore, in next sections we will utilize the theoretical SE lower bound of SM-LDM systems with MRC as the objective function.

III. Injection Level Optimization

A. Proposed Gradient Descent Based Iterative Algorithm

The goal of injection level optimization is to maximize the sum rate in the two-layer SM-LDM systems, so the optimization problem can be formulated as follows:

$$\begin{aligned} &\text{maximize} \quad S_{\mathrm{ML}}(g) + S_{\mathrm{FL}}(g) \\ &\text{subject to} \quad g \geq 0. \end{aligned} \tag{12}$$

$$S_{\mathrm{ML}} = \log_2(N_{\mathrm{T}}) - N_{\mathrm{T}} + \frac{1}{N_{\mathrm{T}}}\left\{\sum_{n=1}^{N_{\mathrm{T}}}\log_2\left(1+\frac{N_{\mathrm{T}}}{\rho_{\mathrm{ML},n}}\right) - \sum_{n=1}^{N_{\mathrm{T}}}\log_2\left[\sum_{n'=1}^{N_{\mathrm{T}}}\frac{\det\left(\mathbf{\Sigma}_{\mathrm{ML},n}\right)}{\det\left(\mathbf{\Sigma}_{\mathrm{ML},n}+\mathbf{\Sigma}_{\mathrm{ML},n'}\right)}\right]\right\}, \tag{6}$$

$$S_{\mathrm{FL}} = \log_2(N_{\mathrm{T}}) - N_{\mathrm{T}} + \frac{1}{N_{\mathrm{T}}}\left\{\sum_{m=1}^{N_{\mathrm{T}}}\log_2\left(1+\frac{N_{\mathrm{T}}}{\rho_{\mathrm{FL},m}}\right) - \sum_{m=1}^{N_{\mathrm{T}}}\log_2\left[\sum_{m'=1}^{N_{\mathrm{T}}}\frac{\det\left(\mathbf{\Sigma}_{\mathrm{FL},m}\right)}{\det\left(\mathbf{\Sigma}_{\mathrm{FL},m}+\mathbf{\Sigma}_{\mathrm{FL},m'}\right)}\right]\right\}, \tag{7}$$

Aided by (2), (8) and (10), in SM-LDM systems with MRC, the ML SINRs and FL SINRs can be represented as functions of g. Besides, from (8) and (10), it can be seen that the ML SINRs are same for different n, and FL SINRs are also same for different m. Therefore, we denote ρ_{ML} as the reciprocal of ML SINRs as follows:

$$\rho_{\mathrm{ML}} = \left(1+\frac{1}{g}\right)\left(1+\frac{\sigma_{\mathrm{ML}}^2}{P_{\mathrm{U}}}\right)\frac{N_{\mathrm{T}}}{N_{\mathrm{RM}}}+\frac{1}{g}, \tag{13}$$

and we denoted ρ_{FL} as the reciprocal of FL SINRs as follows:

$$\rho_{\mathrm{FL}} = (1+g)\frac{N_{\mathrm{T}}}{N_{\mathrm{RF}}}\frac{\sigma_{\mathrm{FL}}^2}{P_{\mathrm{U}}}+\frac{N_{\mathrm{T}}}{N_{\mathrm{RF}}}. \tag{14}$$

In addition, the optimization problem in (12) can be simplified as follows:

$$\begin{aligned}\text{maximize} \quad & S(g) = S_{\mathrm{ML,C}} + S_{\mathrm{FL,C}} + S_{\mathrm{ML,S}} + S_{\mathrm{FL,S}} \\ \text{subject to} \quad & g \geq 0,\end{aligned} \tag{15}$$

where $S_{\mathrm{ML,C}}$ and $S_{\mathrm{FL,C}}$ correspond to the constellation domain mutual information (MI) of ML and FL, respectively, which can be denoted as follows:

$$\begin{aligned}& S_{\mathrm{C}}(\rho) = \log_2\left(1+\frac{N_{\mathrm{T}}}{\rho}\right), \\ & S_{\mathrm{ML,C}} = S_{\mathrm{C}}(\rho_{\mathrm{ML}}), \quad S_{\mathrm{FL,C}} = S_{\mathrm{C}}(\rho_{\mathrm{FL}}).\end{aligned} \tag{16}$$

The $S_{\mathrm{ML,S}}$ and $S_{\mathrm{FL,S}}$ correspond to the spatial domain MI of ML and FL, respectively, and the constant term is ignored, which can be simplified as follows:

$$\begin{aligned}& S_{\mathrm{S}}(\rho) = -\log_2\left[\frac{\rho(N_{\mathrm{T}}-1)(\rho+N_{\mathrm{T}})}{2^{N_{\mathrm{T}}-2}(2\rho+N_{\mathrm{T}})^2}+\frac{1}{2^{N_{\mathrm{T}}}}\right] \\ & S_{\mathrm{ML,S}} = S_{\mathrm{S}}(\rho_{\mathrm{ML}}), \quad S_{\mathrm{FL,S}} = S_{\mathrm{S}}(\rho_{\mathrm{FL}}).\end{aligned} \tag{17}$$

In order to solve the optimization problem in (15), the gradient should be calculated as follows:

$$\begin{aligned}\frac{\partial S}{\partial g} = & \frac{\partial S_{\mathrm{C}}(\rho_{\mathrm{ML}})}{\partial \rho_{\mathrm{ML}}}\frac{\partial \rho_{\mathrm{ML}}}{\partial g} + \frac{\partial S_{\mathrm{C}}(\rho_{\mathrm{FL}})}{\partial \rho_{\mathrm{FL}}}\frac{\partial \rho_{\mathrm{FL}}}{\partial g} \\ & + \frac{\partial S_{\mathrm{S}}(\rho_{\mathrm{ML}})}{\partial \rho_{\mathrm{ML}}}\frac{\partial \rho_{\mathrm{ML}}}{\partial g} + \frac{\partial S_{\mathrm{S}}(\rho_{\mathrm{FL}})}{\partial \rho_{\mathrm{FL}}}\frac{\partial \rho_{\mathrm{FL}}}{\partial g}.\end{aligned} \tag{18}$$

From (13) and (14), the partial derivatives, $\frac{\partial \rho_{\mathrm{ML}}}{\partial g}$ and $\frac{\partial \rho_{\mathrm{FL}}}{\partial g}$ can be formulated as follows:

$$\frac{\partial \rho_{\mathrm{ML}}}{\partial g} = -\frac{1}{g^2}-\frac{1}{g^2}\left(1+\frac{\sigma_{\mathrm{ML}}^2}{P_{\mathrm{U}}}\right)\frac{N_{\mathrm{T}}}{N_{\mathrm{RM}}}, \quad \frac{\partial \rho_{\mathrm{FL}}}{\partial g} = \frac{N_{\mathrm{T}}}{N_{\mathrm{RF}}}\frac{\sigma_{\mathrm{FL}}^2}{P_{\mathrm{U}}}. \tag{19}$$

From (16) and (17), the partial derivatives can be calculated as follows:

$$\begin{aligned}& \frac{\partial S_{\mathrm{C}}}{\partial \rho} = -\frac{1}{\ln 2}\frac{N_{\mathrm{T}}}{\rho^2+\rho N_{\mathrm{T}}}, \quad \frac{\partial S_{\mathrm{S}}}{\partial \rho} = -\frac{4}{\ln 2}\times \dots \\ & \frac{N_{\mathrm{T}}^2(N_{\mathrm{T}}-1)}{4(N_{\mathrm{T}}-1)(\rho^2+\rho N_{\mathrm{T}})(2\rho+N_{\mathrm{T}})+(2\rho+N_{\mathrm{T}})^3}.\end{aligned} \tag{20}$$

Then substituting (19) and (20) into (18), the gradient of S with respect to g can be formulated, and the gradient descent algorithm can be implemented based on our derived gradient. However, the inequality constraint $g \geq 0$ restricts the implementation of gradient descent algorithm, so we introduce the barrier method [15] to solve this problem, which can transform this constrained optimization problem into an unconstrained optimization problem. Therefore, the optimization problem in (15) can be reformulated as follows:

$$\text{minimize} \quad f(g) = -S(g) - \frac{1}{t}\ln(g), \tag{21}$$

where $t > 0$ is the parameter of the logarithmic barrier function. With a larger t, we can get a more accurate solution, but a larger t can also reduce the convergence speed.

Based on the above analysis, in Algorithm 1, the gradient descent based iterative method is proposed for optimizing the injection level.

Algorithm 1 Maximizing the Sum Rate over the Injection Level g:

1: *Initialization*: We randomly set $g > 0$ as the initialized g. We set $N_{\max} > 0$ as the maximum iteration times, $\epsilon > 0$ as the minimum gain rate, $\eta > 0$ as the step size, and $N = 1$ as the initialized iteration time.
2: *Gradient calculation*: Compute $\Delta g = \frac{\partial f}{\partial g} = -\frac{\partial S}{\partial g} - \frac{1}{t}\frac{1}{g}$ based on (18), (19) and (20).
3: *Update*: $\Delta f \leftarrow \frac{f(g+\eta\Delta g)-f(g)}{f(g)}$, $g \leftarrow g + \eta\Delta g$, and $N \leftarrow N+1$.
4: *Iteration*: Go to Step 2 until $\Delta f < \epsilon$ or $N > N_{\max}$.

B. Concavity Analysis

In this subsection, the concavity of constellation domain MI of ML and FL is firstly analyzed. Aided by (20), the second-order derivative of S_{C} can be derived as follows:

$$\frac{\partial^2 S_{\mathrm{C}}}{\partial \rho^2} = \frac{N_{\mathrm{T}}}{\ln 2}\frac{2\rho+N_{\mathrm{T}}}{(\rho^2+\rho N_{\mathrm{T}})^2}. \tag{22}$$

Since $\frac{\partial S_{\mathrm{C}}}{\partial \rho} < 0$ and $\rho > 0$ for both ML and FL, $S_{\mathrm{C}}(\rho)$ is a non-increasing convex function. For the SINR reciprocals of both ML and FL, the second-order derivative can be formulated as follows:

$$\frac{\partial^2 \rho_{\mathrm{ML}}}{\partial g^2} = \frac{2}{g^3}+\frac{2}{g^3}\left(1+\frac{\sigma_{\mathrm{ML}}^2}{P_{\mathrm{U}}}\right)\frac{N_{\mathrm{T}}}{N_{\mathrm{RM}}} \geq 0, \quad \frac{\partial^2 \rho_{\mathrm{FL}}}{\partial g^2} = 0, \tag{23}$$

so $\rho_{\mathrm{ML}}(g)$ is convex and $\rho_{\mathrm{FL}}(g)$ is both convex and concave. Because of the concavity of function of functions [15],

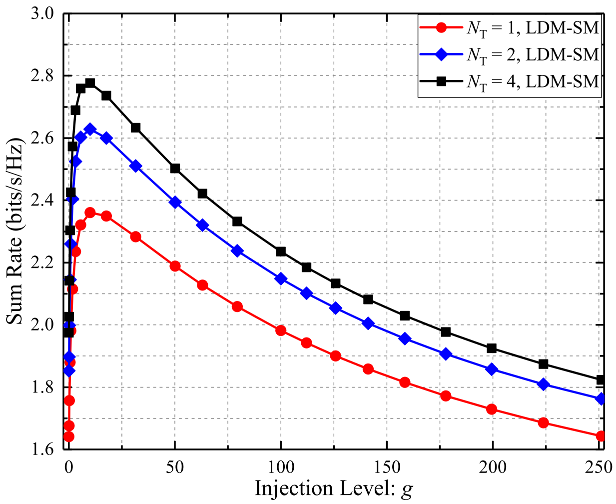

Fig. 2. Sum rate of the two-layer SM-LDM system with $N_{\mathrm{T}} \in \{1, 2, 4\}$, $N_{\mathrm{RM}} = 2$, $N_{\mathrm{RF}} = 2$, $\mathrm{SNR}_{\mathrm{ML}} = 0$ dB, and $\mathrm{SNR}_{\mathrm{FL}} = 20$ dB.

$S_{\mathrm{FL,C}}(g)$ is a convex function. Considering $S_{\mathrm{ML,C}}(g)$, it is indeed a concave function, and the second-order derivative can be derived as follows:

$$\frac{\partial^2 S_{\mathrm{ML,C}}}{\partial g^2} = \frac{\partial^2 S_{\mathrm{ML,C}}}{\partial \rho_{\mathrm{ML}}^2} \left(\frac{\partial \rho_{\mathrm{ML}}}{\partial g} \right)^2 + \frac{\partial S_{\mathrm{ML,C}}}{\partial \rho_{\mathrm{ML}}} \frac{\partial^2 \rho_{\mathrm{ML}}}{\partial g^2}. \tag{24}$$

We want to prove the inequality $\frac{\partial^2 S_{\mathrm{ML,C}}}{\partial g^2} \geq 0$. After substituting (20), (19), (22) and (23) into (24) and simplification, the proof is equivalent to the following inequation:

$$\left(\frac{1}{\rho_{\mathrm{ML}}} + \frac{1}{\rho_{\mathrm{ML}} + N_{\mathrm{T}}} \right) \frac{1}{g} \left[1 + \left(1 + \frac{\sigma_{\mathrm{ML}}^2}{P_{\mathrm{U}}} \right) \frac{N_{\mathrm{T}}}{N_{\mathrm{RM}}} \right] \leq 2. \tag{25}$$

From (13), we have $\rho_{\mathrm{ML}} > \frac{1}{g} \left[1 + \left(1 + \frac{\sigma_{\mathrm{ML}}^2}{P_{\mathrm{U}}} \right) \frac{N_{\mathrm{T}}}{N_{\mathrm{RM}}} \right]$, so (25) can be proved and $S_{\mathrm{ML,C}}(g)$ is concave.

Then we analyze the concavity of spatial domain MI, and aided by (20) the second-order derivative of S_{S} can be formulated as follows:

$$\frac{\partial^2 S_{\mathrm{S}}}{\partial \rho^2} = \frac{8 N_{\mathrm{T}}^3 (N_{\mathrm{T}} - 1)}{\ln 2} \times \ldots \\ \frac{12\rho^2 + 12\rho N_{\mathrm{T}} + 2N_{\mathrm{T}}^2 + N_{\mathrm{T}}}{[4(N_{\mathrm{T}} - 1)(\rho^2 + \rho N_{\mathrm{T}})(2\rho + N_{\mathrm{T}}) + (2\rho + N_{\mathrm{T}})^3]^2}. \tag{26}$$

Since $\frac{\partial S_{\mathrm{S}}}{\partial \rho} < 0$ and $\frac{\partial^2 S_{\mathrm{S}}}{\partial \rho^2} > 0$, $S_{\mathrm{S}}(\rho)$ is also a non-increasing convex function. Aided by (23), $S_{\mathrm{FL,S}}(g)$ is convex. However, for spatial domain MI of ML, after the derivation similar to that of $S_{\mathrm{ML,C}}(g)$, $S_{\mathrm{ML,S}}(g)$ is convex at the beginning and is concave later. Therefore, we utilize simulation rather than theoretical analysis for analyzing the concavity of the sum rate over g. The $\mathrm{SNR}_{\mathrm{ML}}$ and $\mathrm{SNR}_{\mathrm{FL}}$ denote the SNR of ML and FL, respectively. As shown in Fig. 2, the sum rate versus g is indeed a quasi-concave function [15]. Therefore, we can find the only extreme point by our proposed injection level optimization algorithm, and the following simulations also illustrate it.

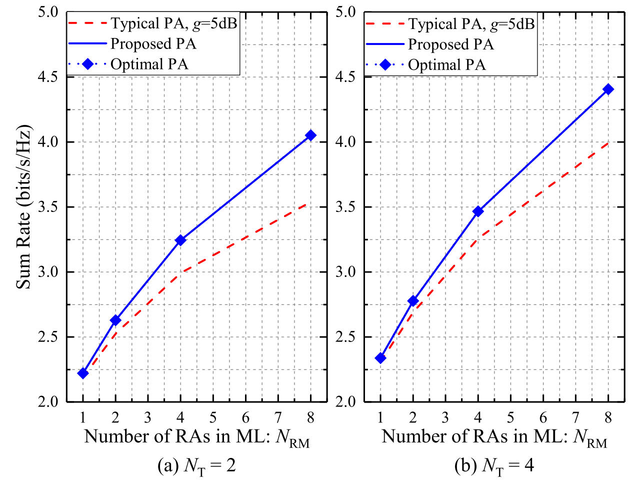

Fig. 3. The sum rate performance of typical PA, optimal PA and our proposed PA algorithm versus N_{RM} with $N_{\mathrm{RF}} = 2$, $\mathrm{SNR}_{\mathrm{ML}} = 0$ dB, $\mathrm{SNR}_{\mathrm{FL}} = 20$ dB. $N_{\mathrm{t}} = 2$ in (a) and $N_{\mathrm{t}} = 4$ in (b).

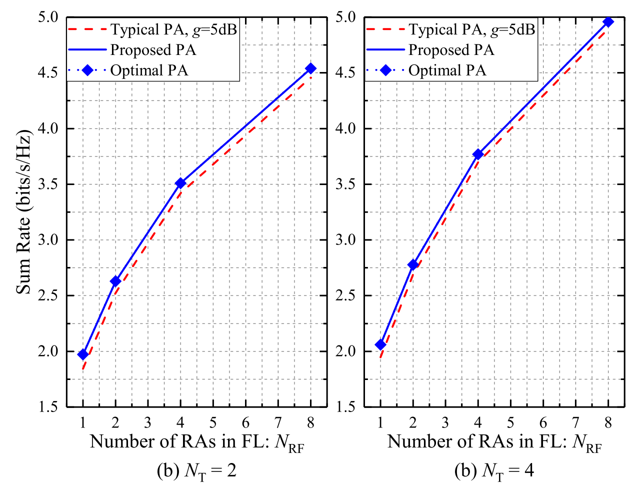

Fig. 4. The sum rate performance of typical PA, optimal PA and our proposed PA algorithm versus N_{RF} with $N_{\mathrm{RM}} = 2$, $\mathrm{SNR}_{\mathrm{ML}} = 0$ dB, $\mathrm{SNR}_{\mathrm{FL}} = 20$ dB. $N_{\mathrm{t}} = 2$ in (a) and $N_{\mathrm{t}} = 4$ in (b).

IV. Monte Carlo Simulation Results

In this section, simulations are provided to verify the efficiency of our proposed gradient descent based iterative algorithm, and the typical power allocation (PA) strategy and optimal PA strategy are shown as comparisons. For typical PA, the injection level is set as 5 dB [12]. Besides, the optimal strategy is achieved via exhaustive search. The perfect synchronization is assumed [16], and we assume the perfect channel estimation [17]–[19].

From Fig. 3, it can be seen that our proposed algorithm can achieve the optimal sum rate, which is much higher than the sum rate with typical PA. In addition, with the increasing of N_{RM}, the gap between our proposed PA and typical PA also becomes larger, which is because from (8) a larger N_{RM} brings a higher ML SINR, and in this case a larger g can increase the ML SE more efficiently. However, in Fig. 4, although

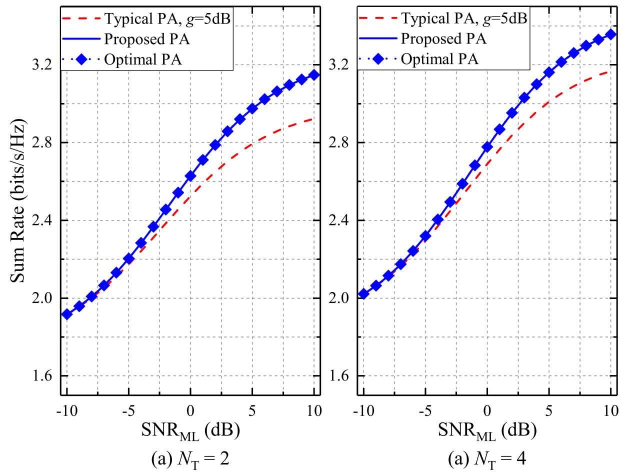

Fig. 5. The sum rate performance of typical PA, optimal PA and our proposed PA algorithm versus $\mathrm{SNR}_{\mathrm{ML}}$ with $N_{\mathrm{RM}} = 2$, $N_{\mathrm{RF}} = 2$ and $\mathrm{SNR}_{\mathrm{FL}} = 20$ dB. $N_{\mathrm{t}} = 2$ in (a) and $N_{\mathrm{t}} = 4$ in (b).

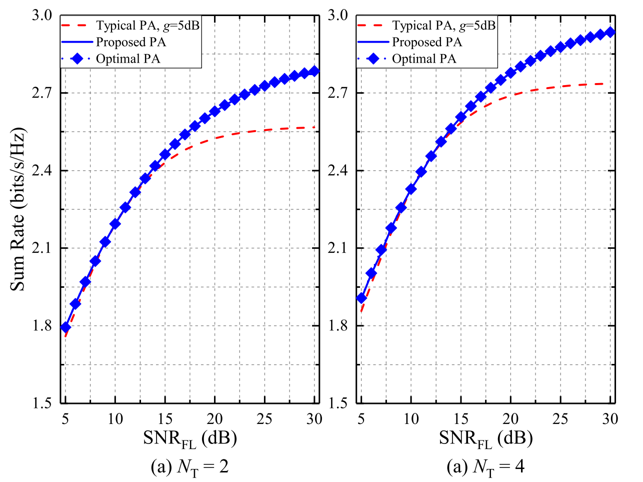

Fig. 6. The sum rate performance of typical PA, optimal PA and our proposed PA algorithm versus $\mathrm{SNR}_{\mathrm{FL}}$ with $N_{\mathrm{RM}} = 2$, $N_{\mathrm{RF}} = 2$ and $\mathrm{SNR}_{\mathrm{ML}} = 0$ dB. $N_{\mathrm{t}} = 2$ in (a) and $N_{\mathrm{t}} = 4$ in (b).

proposed algorithm can also achieve the optimal solution, the gap between the optimal PA and typical PA becomes much smaller. This is because $g = 5$ dB is almost the optimal injection level in this parameter configuration. Beyond that, from Fig. 3 and Fig. 4, it can be also illustrated that with more TAs or more RAs a larger sum rate can be achieved in the SM-LDM system.

As shown in Fig. 5 and Fig. 6, our proposed injection level optimization algorithm can also achieve the optimal sum rate with different $\mathrm{SNR}_{\mathrm{ML}}$ and $\mathrm{SNR}_{\mathrm{FL}}$. In addition, with the increasing of $\mathrm{SNR}_{\mathrm{ML}}$, our proposed PA can achieve a more and more higher sum rate than that with typical PA. With the increasing of $\mathrm{SNR}_{\mathrm{FL}}$, the gap between the sum rate of optimal PA and the sum rate of typical PA becomes firstly smaller and then larger. A larger $\mathrm{SNR}_{\mathrm{ML}}$ or a larger $\mathrm{SNR}_{\mathrm{FL}}$ can also increase the sum rate in SM-LDM systems, which is because a larger $\mathrm{SNR}_{\mathrm{ML}}$ and a larger $\mathrm{SNR}_{\mathrm{FL}}$ lead to a larger $\mathrm{SINR}_{\mathrm{ML}}$ and a larger $\mathrm{SINR}_{\mathrm{FL}}$, respectively.

V. Conclusions

In this paper, we propose the gradient descent based iterative algorithm in two-layer SM-LDM systems for injection level optimization. The concavity of this optimization problem is analyzed via both theoretical analysis and simulation. Since this optimization problem is quasi-concave, our proposed injection level optimization algorithm can achieve the optimal sum rate, which is also illustrated via simulation results. The proposed PA algorithm can always outperform the sum rate of typical PA strategy.

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