## The Roman Towns and the geometry - Examples of Varatio

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**Abstract** This work is proposing some examples of a possible use of the geometry based on rectangular triangles, the varatio, in the planning of Roman towns. It is known that the planning of them is based on the centuriation, a grid of parallel and perpendicular streets, where the two main axes are the Decumanus and the Cardo. The ratio of the catheti of the rectangular triangles gives the angle the Decumanus is forming with the east-west direction.

Keywords: Centuriation, Orientation of Roman colonies, Varare, Varatio

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The planning of the Roman colonies was realized by the ancient land surveyors by means of a method which is known as *limitation* or *centuriation*. It is characterized by a layout in the form of a square or rectangular grids, created by parallel and perpendicular roads and canals, which subdivided the settlement into *insulae*, which were the house-blocks in the towns or the agricultural plots in the countryside. In some cases these plots were allocated to Roman army veterans [1]. The main axes of the grids were called *decumanus maximus* and *cardo maximus*. The parallel roads to them were the *decumani* and the *cardines*. Of course, the choice of the place of a town or colony was dictated by the local environment and the presence of rivers and roads.

To subdivide and label the plots of the land, the surveyor (*gromaticus*) identified a central viewpoint, known as the *umbilicus agri* or *umbilicus soli*. Taking position there and assuming the same orientation of the decumanus, the gromaticus defined the territory with the following names: *ultra*, the land he saw in front of him; *citra*, the land behind him; *dextera*, the land to his right; *sinistra*, the land to his left [2,3]. According to the Latin writer Frontinus [3], who was referring Varro, the Romans inherited the manner to subdivide the land from the Etruscan Doctrine.

We read in [3] that the *decumanus* runs from *oriens* to *occasus*, that is from the sunrise to the sunset. Of course, this is a natural manner to identify the plots of the land by a simple orientation from east to west and from north to south, in order to have a conventional method to label them. However, after Frontinus' words, some scholars, such as Barthel [2], argued for an orientation of the decumanus towards the point of the horizon were the sun was rising, *secundum coelum*, but

other scholars maintained that the orientation was just dictated by the local environment or by the main roads of the area, as stressed by Le Gall and Castagnoli [4,5].

In any case, to have the regular grids of the limitation matching the local environments, *secundum naturam*, the Roman surveyors had to rotate the grids. The surveyors had some methods that we can include under the term of *varatio*. We can find the *varatio* in the literature of the Gromatici, given in [6,7].

A remarkable work on the Roman land surveying is [8]. The author, Anne Roth-Congès, explains the methods used by the Romans and the Euclidean geometry involved in them. Let us note that Roth-Congès uses the verb *varare*, from which *varatio* is coming. This verb can be translated in the action of measuring by means of triangulation [9]. The method is based on the *vara*, a sort of *pertica*. The noun indicates in general "any of various branched structures or implements". It is a term derived from feminine of *vārus*, "bent outwards with converging extremities, bow-legged," of uncertain origin [9]. According to [8,10], the term *varatio*, also written *uaratio*, refers in general to the process of oblique surveying. It has two approaches: the *varatio fluminis*, by means of which the distance of an inaccessible point could be calculated by the construction of right angled triangles, and the *varatio in agris divisis*, that the surveyors could made by means of lines "along the hypotenuse of right angled triangles defined in a rectangular coordinate system" [10]. This second *varatio* is the one which is statistically analyzed in [10]. As stressed by John Peterson, when introducing his analysis, the variety of observed oblique relationships, that is (1:1, 1:2, 2:3, 3:4 ...), is limited.

Besides the study of *varatio* concerning the Hadrian's Wall and the Wetterau Limes [10], we can find the *varatio* considered for the orientation of the Roman towns in Spain [11,12]. In [13], we have analyzed two Roman towns in Italy, Como and Verona.

Here we show some example of *varatio* applied to the layout of Roman towns. The axis of reference for the *varatio* was the east-west line. It is quite easy to determine this direction by a gnomon, a pole fixed on a plane surface, its shadow and a circle. In the following discussion, we search for the hypotenuse of rectangular triangles as a segment of the east-west axis. We will try to find the catheti as multiples of the sizes of the town's *insulae*.



Figure 1: Thanks to Acme Mapper, <a href="https://mapper.acme.com">https://mapper.acme.com</a>, we can appreciate very well the terrain of Como (left panel) and Verona (right panel). The core of the Roman Verona is in the bend of the Adige river.

**Como and Verona** The cases of Como and Verona had been discussed in [13]. These two towns are showing the same geometry used for a quite different natural environment. For what concerns Como, is was founded by Julius Caesar in 59 BC, with the name of *Novum Comum*. In the case of Verona, its territory became Roman about 300 BC. Verona became a Roman colony in 89 BC, and it was classified as a *municipium* in 49 BC.

As we can see from the Figure 1, the terrain of the two towns is quite different. But, as shown in [13], the layout is the same. So let us consider the geometry of this layout. We assume that the *insulae* of the centuriation were squares. Let us note that, during the centuries, the *insulae* had been deformed and therefore it is difficult to see today the original Roman layout perfectly maintained in the town.

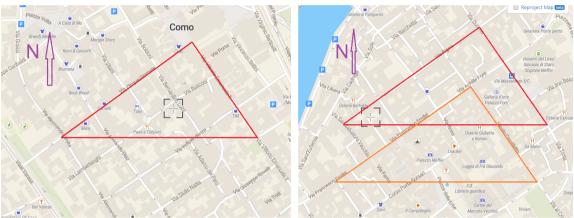
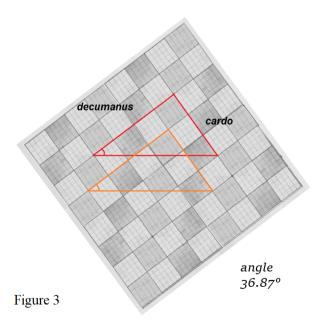


Figure 2: Como (on the left) and its rectangular triangle 3,4 and 5 shown on a epsg.io map. The same rectangular triangle on the right for Verona (© MapTiler © OpenStreetMap contributors).



In the Figures 2 we can see the rectangular triangles which could have been determined by the *varatio*. The hypotenuses are coincident to the east-west direction. The catheti seem to correspond to 3 and 4 *insulae* respectively [13]. So we have that the rectangular triangle is that of the fundamental Pythagorean triple 3,4 and 5. In the Figure 3, a corresponding ideal scheme is given. The two catheti give the directions of the *decumanus* and the *cardo*. The angle that the decumanus and the hypotenuse form is given by  $\alpha = \arctan(3/4) = 36.87^{\circ}$ .

Therefore the azimuth from true North of the *decumanus* is 53,13°.

These two towns have been proposed as oriented to solstices [14-16]. As shown in [13], we can see that, within an error of one degree, the direction of the sunrise on solstices is close to the direction of the decumanus given in the Figure 3 (ideal case). The same happens for the decumani

of Como and Verona. The fact that the towns were planned according to the varatio 3:4 seems well posed. But that this varatio was chosen to have the towns aligned to sunrise on solstices could be simply a coincidence.

**Torino (Julia Augusta Taurinorum).** About the foundation and the orientation of Torino, we discussed in [17-19]. Torino was probably found in 28/27 a.C. as the *colonia Julia Augusta Taurinorum*. The Roman town was placed close to the confluence of Dora and Po rivers, at the foot of the hills (see Figure 4). The geometry of Torino is based on the rectangular triangles 1:2. This geometry had been suggested by Gaetano Barbella in 2008 [20]. Triangles are shown in the Figure 5. In the image, the *Decumanus* and the *Cardo* are also shown such as the position of the four gates.

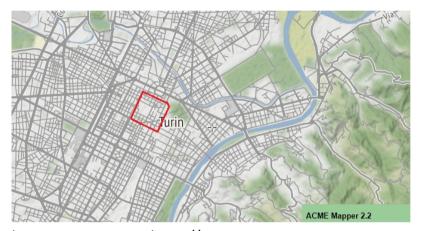


Figure 4: Thanks to Acme Mapper, https://mapper.acme.com, we can appreciate the position of Julia Augusta Taurinorum, the red square, and the terrain surrounding it.

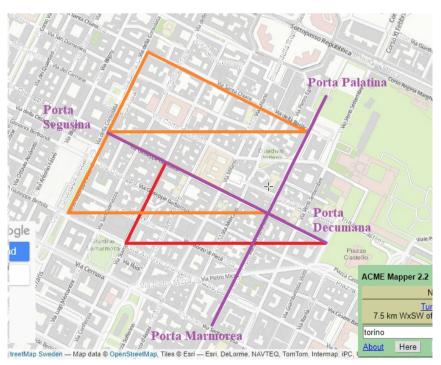


Figure 5. By means of Acme Mapper and © OpenStreetMap contributors, we can see the layout of Torino. OpenStreetMap (CC BY-SA). In the image, the *Decumanus* and the *Cardo* are also given.

Concordia Sagittaria The town was founded, probably in 42 BC, as Iulia Concordia by the Romans, under the Triumvirate. The town was placed where the Via Annia and the Via Postumia crossed each other. It was taken and destroyed by Attila in 452 AD. After the fall of the Western Roman Empire, it was ruled by the Lombard duchy of Cividale. In [21], we can see a map of the Roman town, as given from the report of an archaeological survey of 1880 [22].

In the case of Julia Concordia, the *insulae* are not squares. However, the decumanus has the same azimuth (within a degree) of Torino. Therefore the triangle is 1:2. Here in the Figure 6 we have the map and the triangle. The case of this town is very interesting, because of the rectangular *insulae*.

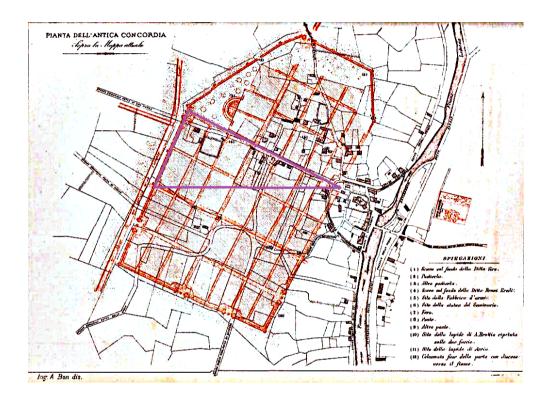


Figure 6. Map of Julia Concordia by Bertolini D., Notizie degli scavi di antichità, 1880.

**Pavia** In the Roman time, the town was known as Ticinum. It was an important municipality and military site. The Roman city most likely had its origin in a military camp, built by the consul Publius Cornelius Scipio in 218 BC to guard a wooden bridge that he ordered to build over the river Ticinum. The consul had received news concerning Hannibal, who was leading an army that was crossing the Alps. Rome and Carthage collided soon thereafter, and the Romans suffered the first of many defeats during the Second Punic War. The bridge was destroyed.

The fortified camp, which at the time was the westernmost military fort of Rome in the Po Valley, survived the war and gradually evolved into a town. The importance of Pavia grew with the extension of the Via Aemilia from Ariminum (Rimini) to the Po River (187 BC), which it crossed at Placentia (Piacenza).

In the Figure 7 we can see the terrain of Pavia. The rectangular triangle is 2:7, as shown in the Figure 8.



Figure 7: Pavia in the Acme Mapper, https://mapper.acme.com.

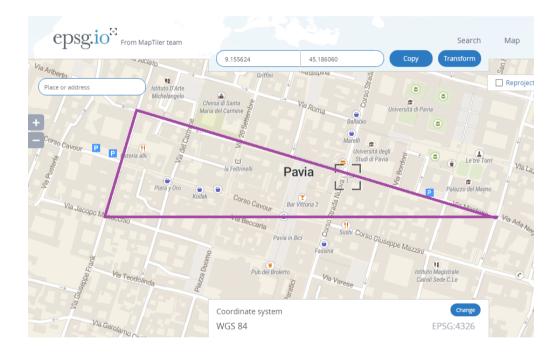


Figure 8: Pavia. The rectangular triangle is 2:7. Courtesy © MapTiler © OpenStreetMap contributors.

**Piacenza**. Piacenza and Cremona were founded as Roman military colonies in May 218 BC. The towns were planned in order to be constructed after the successful war with the Gauls, that ended in 219 BC. In 218 BC, after declaring war on Carthage, the Senate decided to accelerate the colonization. The layout that we see today is probably based on a second deduction of 190 BC [23]. In the Figure 9 we can see its triangle.

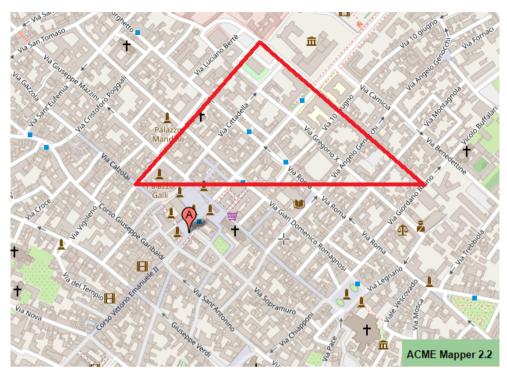


Figure 9: Piacenza The rectangular triangle is 5:6. Courtesy © OpenStreetMap contributors.

Other cases Let us mention the case of **Bologna**. The geometry of the Roman Bologna is given in [24] by Manuela Incerti. The rectangular triangle is 2:9. It is evident therefore, from [20] and [24], that the problem of the geometry on which the layout of the Roman towns had been founded, is a known problem, addressed in literature. The only way to continue the study is that of collecting, besides the maps of the towns, also the literature on their planning.

To conclude, let me mention book [25]; this can be the origin of further analyses on street networks. As told in [26], [25] is telling that the "Roman Empire used standardized street grids to efficiently lay out new towns and colonies during rapid imperial expansion". Therefore, the study of the geometry could be interesting to determine the standardization of the urban layout.

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oriente ad occasum, quod eo sol et luna spectaret, sicut quidam architecti delubra in occidentem recte spectare scripserunt. Aruspices altera linea ad septentrionem a meridiano diuiserunt terram, et a media ultra antica, citra postica nominauerunt.

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