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1. "At-A-Station" Hydraulic Geometry Relationships (AHG)

1.1. Introduction

Simplified hydraulic geometry relationships representing the average conditions over longer reaches could reduce the need for detailed field surveys and minimize the computational burden while studying or carrying out numerical analyses and of the flow dynamics.

AHG relations are power-function as equations $1W$, $1\bar{Y}$, and $1U$ which are unanimously agreed among researchers [prior and after Richard (1973), e.g., Leopold and Maddock (1953), Knighton (1975), Ferguson (1986), Stewardson (2005), Dingman (2009), etc.] to be well representing hydraulics of the channel at a cross-section with respect to discharge variations:

$$W = a.Q^b \qquad equation\ 1W$$

$$\bar{Y} = c.Q^f \qquad equation\ 1\bar{Y}$$

$$U = k.Q^m \qquad equation\ 1U$$

that relate top-width (W), mean depth ($\bar{Y}$), and mean velocity (U) to the discharge (Q) at a stream cross section, which according to law of "Continuity", it is inferable that,

$$W.\bar{Y}.U = Q \qquad equation\ 2$$

therefore,

$$b + f + m = 1 \qquad equation\ 3,\ \text{and}$$

$$a.c.k = 1 \qquad equation\ 4$$

1.2. Combination of empirical and theoretical "at-a-station" hydraulic geometry relationships

the work made by Dingman (2007) which suggested an alternative approach for estimating the parameters of power-law AHG relations. He recognized the regression-based power-law relations (like equations 1W, 1Y, and 1U) as the "empirical" AHG relations. On the other hand, given the known values for bankfull top-width (W_{BF}) and bankfull maximum depth (Y_{BF}) at a channel cross-section, he proposed the "theoretical" AHG relations which can be analytically driven through the followings,

$$\bar{Y} = \left(\frac{r}{r+1}\right).Y \qquad equation\ 5$$

and

$$Y = Y_{BF}.\left(\frac{W}{W_{BF}}\right)^r \qquad equation\ 6$$

Which are providing approximated, key relationship between mean depth ($\bar{Y}$) versus maximum depth (Y) by equation 5 and maximum depth (Y) versus top-width (W) by equation 6, where in those relations r denotes the shape exponent of the power-law shaped idealized bed-geometry. Combining equations 5 and 6 will result in,

$$\bar{Y} = Y_{BF}.\left(\frac{1}{W_{BF}}\right)^r.\left(\frac{r}{r+1}\right).W^r \qquad equation\ 7$$

where $Y_{BF}.\left(\frac{1}{W_{BF}}\right)^r.\left(\frac{r}{r+1}\right)$ or $\bar{Y}_{BF}.\left(\frac{1}{W_{BF}}\right)^r$ can be represented by ω and thus equation 7 will become,

$$\bar{Y} = \omega.W^r \qquad equation\ 8$$

Equation 8 is a power-function relating top-width (W) to mean depth ($\bar{Y}$).

Suggested by Dingman (2007), the general forms of Chézy or Manning's uniform-flow equations, both considered as “theoretical” formulae, are widely used for estimating flow velocity (or discharge) based on hydraulic and geometrical properties of an open channel.

$$U = \frac{u_m}{n}.R^p.\left(\frac{dH}{dL}\right)^q \cong \frac{u_m}{n}.\left(\frac{dH}{dL}\right)^q.\bar{Y}^p \qquad equation\ 9$$

or,

$$Q = A.U = \frac{u_m}{n}.A.R^p.\left(\frac{dH}{dL}\right)^q \cong \frac{u_m}{n}.\left(\frac{dH}{dL}\right)^q.A.\bar{Y}^p \qquad equation\ 10$$

where, n is Manning’s roughness coefficient, p is the exponent of the hydraulic radius R (which can be approximated by the mean depth $\bar{Y}$ for wide channels, i.e., $W > 10\bar{Y}$), A is flow area, q is the exponent of the energy slope $\frac{dH}{dL}$ assumed to be equal to 0.5, and u_m is Unit conversion factor $3.281^{(1-p)}$ for English unit while being equal to unity concerning SI unit.

Afshari et al. (2017) followed the background of Dingman’s (2007) “theoretical” AHG relations (check out equations 17-30 of Dingman [2007]) and given the equation 8 here, they came across to the following relations between discharge (Q) versus channel hydraulics as mean depth ($\bar{Y}$), top-width (W), and mean velocity (U),

$$\bar{Y} = \omega^{\frac{1}{1+r+r.p}}.\left(\frac{n}{u_m}\right)^{\frac{r}{1+r+r.p}}.\left(\frac{dH}{dl}\right)^{\frac{-r.q}{1+r+r.p}}.Q^{\frac{r}{1+r+r.p}} \qquad equation\ 11$$

$$W = \omega^{\frac{-1-p}{1+r+r.p}}.\left(\frac{n}{u_m}\right)^{\frac{1}{1+r+r.p}}.\left(\frac{dH}{dl}\right)^{\frac{-q}{1+r+r.p}}.Q^{\frac{1}{1+r+r.p}} \qquad equation\ 12$$

$$U = \omega^{\frac{p}{1+r+r.p}}.\left(\frac{n}{u_m}\right)^{\frac{-r-1}{1+r+r.p}}.\left(\frac{dH}{dl}\right)^{\frac{r.q+q}{1+r+r.p}}.Q^{\frac{r.p}{1+r+r.p}} \qquad equation\ 13$$

Additionally, a relationship associated with flow area (A), can be driven by multiplying $\bar{Y}$ by W that gives:

$$A = \omega^{\frac{-p}{1+r+r.p+}}.\left(\frac{n}{u_m}\right)^{\frac{r+1}{1+r+r.p}}.\left(\frac{dH}{dl}\right)^{\frac{-r.q-q}{1+r+r.p}} Q^{\frac{r+1}{1+r+r.p}} \qquad equation\ 14$$

where in all equations 11, 12, and 13, r and p are being estimated respectively as

$\hat{r} = f/b \qquad equation\ 15$, given $f = \frac{r}{1+r+r.p}$ and $b = \frac{1}{1+r+r.p}$,

$$\hat{p} = m/f \qquad equation\ 16\text{ , given } m = \frac{r.p}{1+r+r.p} \text{ and } f = \frac{r}{1+r+r.p}$$

The data filtering approaches suggested by Afshari et al. (2017) was based upon the "empirical" or regression-based AHG relations which have been generated for over 4000 U.S. Geological Survey (USGS) stations in Conterminous United States (CONUS). Thus, by equating the parameters of "theoretical" AHG relations to the corresponding one in "empirical" relations we have,

$$\begin{cases} \omega^{\frac{1}{r+r.p+1}}.\left(\frac{n}{u_m}\right)^{\frac{r}{r+r.p+1}}.\left(\frac{dH}{dl}\right)^{\frac{-r.q}{r+r.p+1}} = c \\ \omega^{\frac{-1-p}{r+r.p+1}}.\left(\frac{n}{u_m}\right)^{\frac{1}{r+r.p+1}}.\left(\frac{dH}{dl}\right)^{\frac{-q}{r+r.p+1}} = a \end{cases} \qquad equation\ 17$$

This system of linear equations (equation 17) can be solved for ω and $\left(\frac{n}{u_m}\right).\left(\frac{dH}{dL}\right)^{-q}$, based on AHG parameters and equations 15 and 16 as,

$$\begin{bmatrix} \omega \\ \left(\frac{n}{u_m}\right).\left(\frac{dH}{dL}\right)^{-q} \end{bmatrix} = \begin{bmatrix} \frac{c}{a^{\frac{f}{b}}} \\ c^{\frac{m}{f}+1}.a \end{bmatrix} \qquad equation\ 18$$

Hence, equation 18, which derived from the combination of "experimental" and "theoretical" AHG relations, demonstrates the fact that given the reliable estimates of AHG coefficients and exponents at a site, one can predict: 1. the idealized shape of the river bed as power-law geometry (i.e., equation 10) given the estimates of the ω and r, 2. the geophysical features at/near by the monitoring station, i.e., the product of bed roughness (here as Manning's n) and the inversed form of the energy slope to the power of q, or $(dH/dl)^{-q}$.

All the above-mentioned information were computed for ~4000 USGS stations across the CONUS area and are included in the early columns of "USGS Table AHG Parameters And Supplementary Data" spreadsheet. To better comprehend the AHG parameters and their relations to geospatial relations to morphological and geophysical information, as an example, USGS site no. 05064500 at Red River of the North at Halstad, MN is randomly chosen. Concerning the AHG parameters, figure 1 is illustrating graphics as power-law relations of key hydraulics versus discharge (Q) for that station. In figure 1 blue-colored points are 'Reliable Flow Measures' which passed data-filtering procedure (Afshari et al. 2017) while gray-colored points are the original data. The blue-colored dashed line is the fitted power-function which is derived based on the 'Reliable Flow Measures'.

2-How basic, auxiliary morphological and geophysical information are driven?

NHDPlus[1] database are providing the basic geospatial information in current document, the steps and procedures for achieving required, primitive information from NHDPlus database will be discussed.

[1] http://www.horizon-systems.com/NHDPlus/NHDPlusV2_home.php

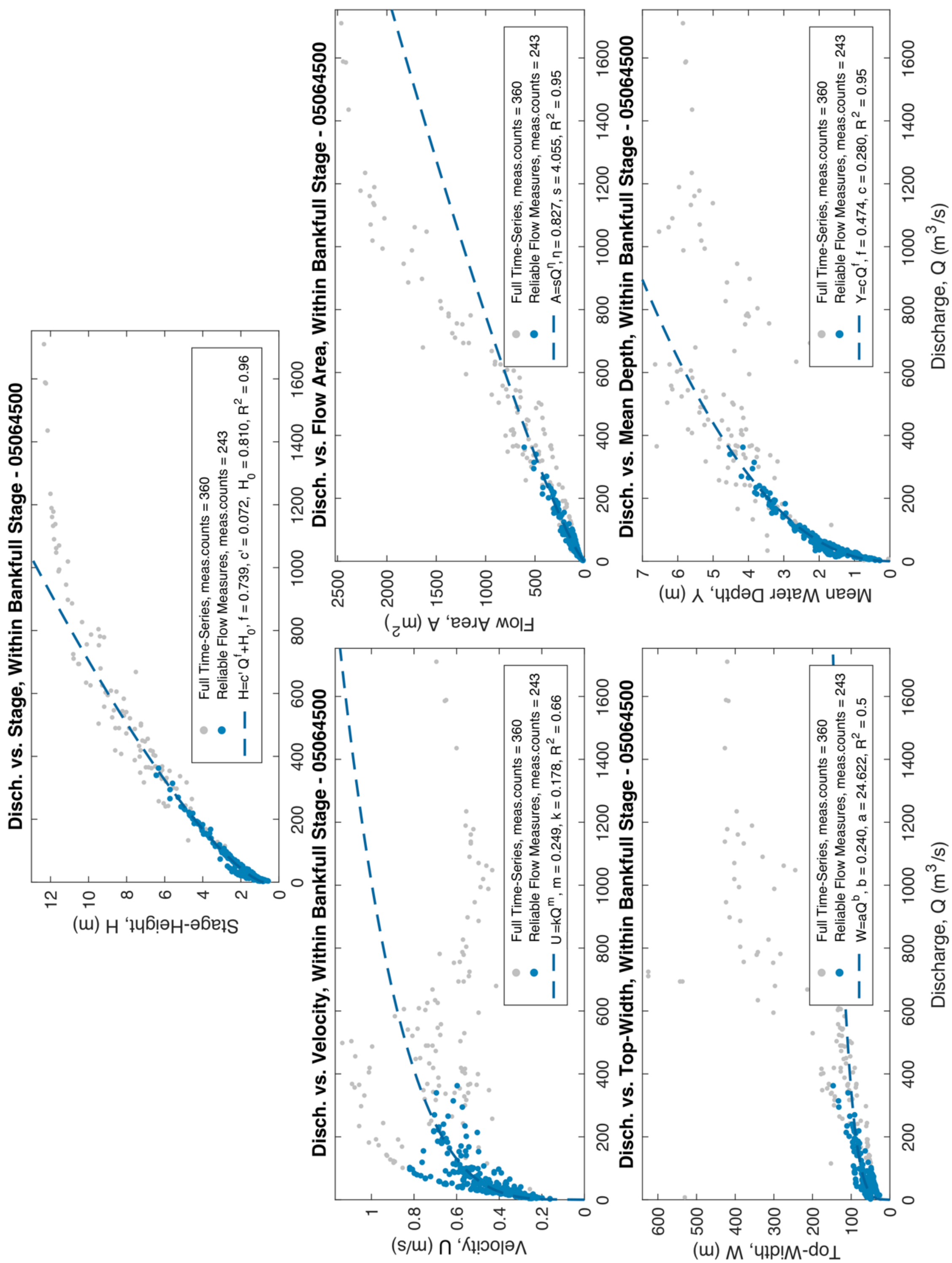

Figure 1. Visualizing AHG relations (i.e., discharge versus hydraulics) at USGS station (site no. 05064500). The grey-colored points are highlighting full time-series data while blue-colored points are representing reliable flow measures captured by data filtering procedure. The blue-colored dash-line is showing the power-law curve fitted to reliable flow measures.

Here, we are discussing an example to clarify steps proceeded for estimating stream order (or Strahler number), channel sinuosity, channel bed-slope and channel mean lateral (or overbank) slope. This is relying on connecting NHDPlus data (i.e., streamlines) to a USGS station (here, USGS site no. 05064500 at Red River of the North at Halstad, MN; figure 2).

Figure 2. General schematic view of steps made in establishing river streamlines produced through cutting main channel by circular buffers having 1000-5000m radii being centered at a USGS station (here, USGS 05064500). (a) overview of multiple NHDPlus lines and where the USGS stations is located, (b) highlighting main sub-channel labeled according to location with respect to USGS site, (c) showing main channel which is developed by merging NHDPlus main sub-channels, (d) highlighting how multiple circular buffers centered by USGS station.

Using either a GIS tool (e.g., ArcGIS) or GIS programming (e.g., for the purpose of this work, using "arcpy" library in Python programming tool) one can implicitly navigate through NHDPlus river network reaches and find the streamlines that are being touched by or passed through the USGS stations (figure 2a). For the ease of assessment, new columns will be added to attribute table of the NHDPlus river network reaches entitled as: "mid_1_NHDLine", "up_1_NHDLine", "down_1_NHDLine", and so forth up to the 9th category which sounded sufficient (i.e., "up_9_NHDLine" and "down_9_NHDLine"). The "up" and "down" prefixes are the short names for upstream and downstream indicating the direction of our movement through the main channel. Figure 2b is demonstrating such a progression, where, concerning selecting the next 'main' upstream and downstream streamlines (and not tributaries) it should be programmed in way to step through the right lane upward and downward. For instance, in progressing from "mid_1_NHDLine" to either "up_1_NHDLine" or "down_1_NHDLine" at a confluence point, it must decide between two (or in rare cases three) pathways. The modified NHDPlus table established and managed for this work, contains "Stream Order" and "Mean Annual Discharge and Velocity" (respectively denoted by Q0001E and V0001E) for individual NHDPlus streamline at each HUC2 region. Hence, it is coded in a way that it will choose the reach which has the higher stream order while progressing through upstream or downstream direction. However, in cases where upstream channels at the confluence point are possessing similar stream order value, Q0001E and V0001E can be utilized as indicators, that is, the upstream channel which has the larger Q0001E and V0001E will be chosen and so forth. As each streamline is being chosen, it will be attributed by the direction it is progressing (i.e., depending on being in upstream or downstream direction). If it is the first upstream NHDPlus line with respect to the mid-stream, it will be named as "up{USGS Site no.}" (or here as "up05064500"). Or, if it is the second downstream NHDPlus line with respect to the mid-stream, then the cell of the attribute table corresponding to the column titled as "down_2_NHDLine" and the row associated with the chosen NHDPlus line will be filled by "down{USGS Site no.}" (or here as "down05064500"). It should be noted that it is likely that two or more USGS monitoring stations be spatially close to each other. Concerning such cases, a NHDPlus line is programmed to provide information for more than one station, or being duplicated or even triplicated.

After assigning NHDPlus streamlines with "mid", "up", and "down" prefix labels, the lines possessing the so-called information in their attribute table must be chosen. In doing so, a new field (column) will be added to the end of the attribute table of the NHDPlus streamlines shapefile called as "River_Groupe_Number" (or "Ri_Grp_Nm" in the Python code) that actually is USGS site number. With regard to the NHDPlus streamlines having at least one record in either "mid-1-NHDLine" field or "{up/down}-{i}-NHDLine" (given i =1, 2,…, 9), the corresponding cell in the row of "River_Groupe_Number" field will be assigned by the USGS site no. Those rows which contain no information in "River_Groupe_Number" field will be assigned as "Null". For instance, in figure 2b, all the lines having USGS site no. of "05064500" will be identified and the corresponding cell in the row of "River_Groupe_Number" field will be filled by "05064500" (in string format) while the non-selected tributaries will remain as "Null". This will aid in finding and recognizing those NHDPlus streamlines having the same "River_Groupe_Number" values.

Figure 2c is highlighting (in red) the interconnected main channel lines passing through USGS 05064500. By merging these lines, a solid channel line will be achieved. Considering a USGS site as a center, multiple circles having radius 1000,2000, 3000, 4000, and 5000 meters can be drawn (figure 2d). These are acting exactly like cutting circles utilized for making different size cookies while here they are clipping a streamline by small to large circles. For instance, this is illustrated in figure 2d where the portion being cut by 1000m-radius buffer is highlighted in light-yellow while the blue-colored streamline is demonstrating the one being within 2000m-radius buffer. Those streamlines which do not have "River_Groupe_Number" records will be excluded for the upcoming computations. However, chances are that one or multiple meandering parts of a channel stick out from a clipping boundary while few remnants being left inside buffer zone (i.e., shown in figure 2d as blue-colored dashed line). Concerning such cases, the GIS programming is designed to recognize these anomalies and disregard the cutting leftovers by just including the streamline which passes through the USGS stations.

1.1-Estimating Channel Sinuosity

As an example, concerning figure 2d and employing GIS tools, one can estimate the "Actual Channel Length" by measuring the length of the meandering pattern that was clipped by circular buffer centered by a USGS station (here, USGS 05064500). Dividing "Actual Channel Length" by "Valley Length" will give an estimate of channel sinuosity (equation 1). As it was explained, this procedure was programmed in a way that main channel being clipped by 1000-5000m radii circular buffers (while figure 2d only highlights 1000 and 2000m radii). This approach also will provide us with direct distance between starting and ending points of a curved line which can be regarded as "Valley Length". The channel sinuosity, in turn, can be estimated as following which is graphically explained by figure 3,

$$\text{Sinuosity} = \frac{\text{Actual Channel Length}}{\text{Valley Length}} > 1 \qquad equation\ 19$$

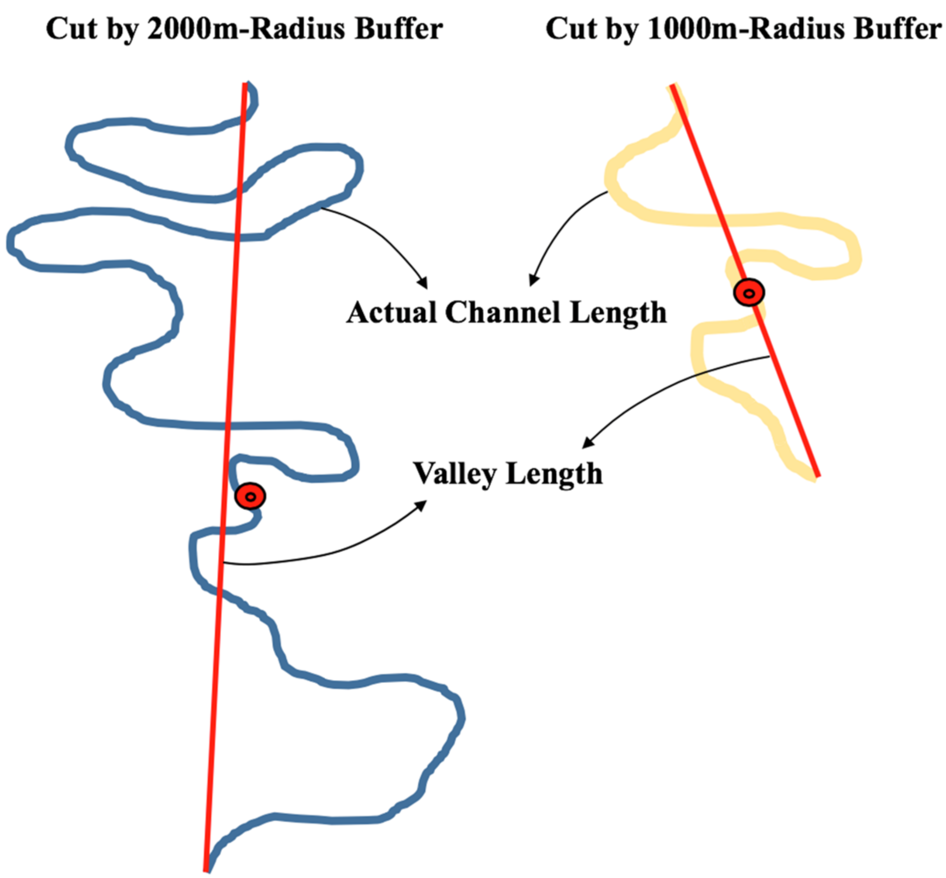

Figure 3. Schematic view of difference between "Actual Channel Length" and "Valley Length" both being used in current thesis for estimating channel sinuosity. These graphics (i.e., blue- and yellow-colored sinusoidal patterns) are the actual extents of the river channel.

All the procedures explained so far, can be executed over USGS stations. Given computed values for channel sinuosity, channel slope, etc., through circular buffers (i.e., 1000-5000 m radii), basic statistics of a morphological or a geophysical attribute, i.e., mean, maximum, minimum, and standard deviation of sinuosity, bed-slope, lateral slope, and reach length can be calculated for each USGS station. For instance, the analysis over USGS 05064500 will result in the following estimates for the abovementioned morphological or a geophysical attribute presented in table 1,

Table 1. NHDPlus information at USGS 05064500 monitoring station (Sources: https://waterdata.usgs.gov/nwis/sw and http://www.horizon-systems.com/NHDPlus/NHDPlusV2_home.php)

Information Derived from USGS 05064500

Mean Reach Slope	0.00025
Max. Reach Slope	0.0005
Min. Reach Slope	0.00012
Standard Devi. Reach Slope	0.00015
Mean Reach Len. (m)	13430
Max. Reach Len. (m)	23205.1
Min. Reach Len. (m)	3543.2
Standard Devi. Reach Length (m)	7417.4

Mean Reach Sinuosity	2.26
Max. Reach Sinuosity	2.62
Min. Reach Sinuosity	1.74
Standard Devi. Reach Sinuosity	0.35
Mean Channel Lateral (or Overbank) Slope	0.0097
Coeff. Vari. Channel Lateral (or Overbank) Slope (%)	204.3747
Geographic Names Information System (GNIS) ID	1035890
Geographic Names Information System (GNIS) Name	Red River N.
Stream Order (Stahler number)	7
Hydrologic Region	SR09

1.2-Estimating Channel slope

Figure 4 illustrates outline of two similar set of consecutively ordered reaches. Before being merged, circular buffers centered by the USGS station are created. Here these lines are coming from USGS 05064500 introduced earlier; figure 4b demonstrates extent of river within 1000m-radius circular buffer highlighted as a grey zone while figure 4a does the same expression but within 2000m-raduis circular buffer. Concerning figure 4b, the entire extent of mid-channel is within the 1000m buffer, while, only small portions of upstream (i.e., emphasized by the double-edge-arrow) and downstream streamlines are also included. Besides, figure 4a is showing the entire extent of mid-stream and first up- and downstream NHDPlus streamlines being inside the 2000m-raduis buffer area while second up- and dowmstream streamlines being partially inside the same area. Therefore, weighted slopes for each buffer areas must be computed as following,

$$\text{Weighted Slope within a } j-meter \text{ radius circular buffer}, \bar{S}_{j-\mathrm{m}} = \frac{\sum(S_i.L_i)}{\sum L_i}$$

$$equation\ 20$$

where, concerning equation 20, i indicates index of the NHDPlus streamlines (being either full length or a partial segment of a streamline), S_i and L_i are in turn denoting channel slope and length suggested by a NHDPlus flow line for streamline i within a buffer area made by a $j-meter$ radius circle (j = 1000, 2000, 3000, 4000, and 5000). This is applied for USGS 05064500 to estimate weighted slopes within 1000-m and 2000-m radii buffers as 0.000154 and 0.000133 respectively. Accordingly, given multiple weighted slopes derived for multiple buffers, statistics (i.e., mean, minimum, maximum, and standard deviation) of channel slope can be computed for USGS stations.

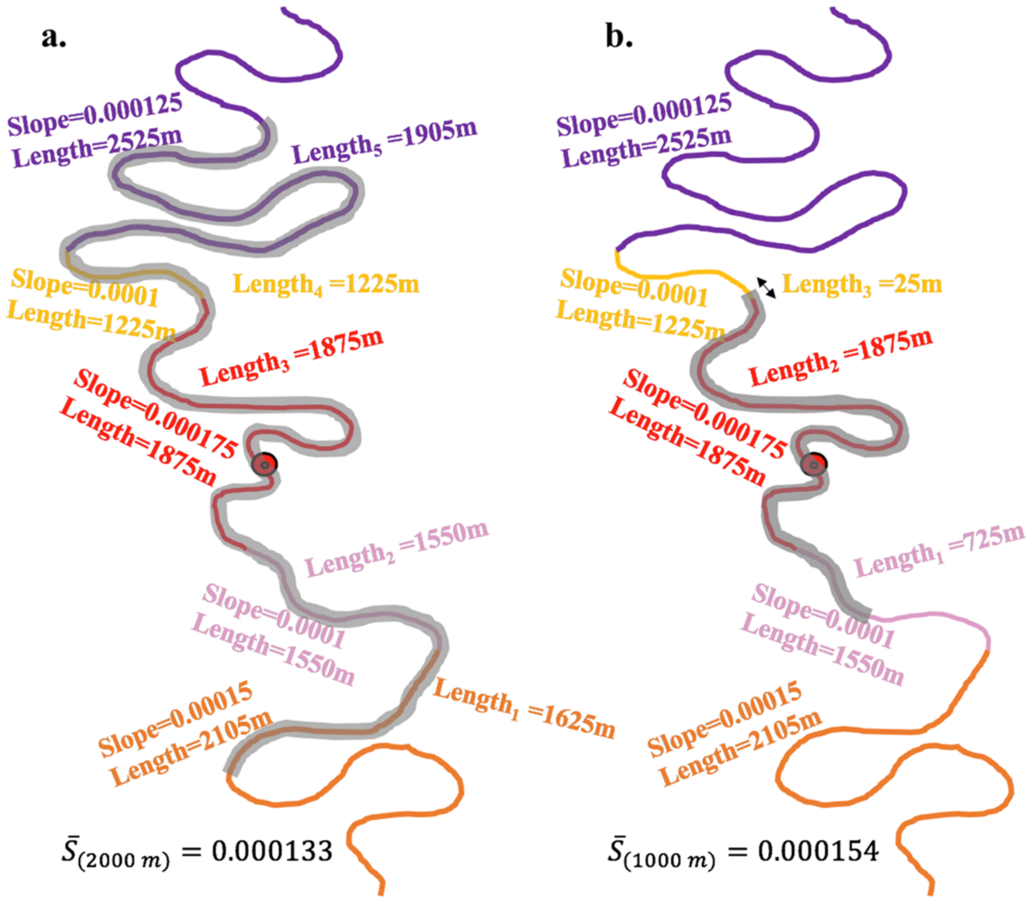

Figure 4. Graphical explanation of how channel slope is being estimated. These colorful sinusoidal lines each assigned by a length and slope within their spatial rang are the actual extents of the river channel highlighted in figure 2.4b. (b), (a): the gray-colored zone highlight the zones concerning 1000 and 2000m circular buffers centered at USGS 05064500 that clipped NHDPlus streamlines either fully or partially.

1.3-Estimating Channel mean lateral (or overbank) slope

The aim of the current subsection is to delineate the way we computed the lateral channel slope statistics (i.e., mean and coefficient of variation [%]) through depicting an example followed by highlighting the distribution of the frequency histogram of the mean lateral (or overbank) slope of channel at USGS stations.

Computation of lateral slope requires geospatial knowledge of bed-elevation along and across the main channel based on a wide extent. Here, the main source for elevation is Digital Elevation Models provided by USGS National Elevation Dataset (NED-DEM) based upon multiple spatial resolution. Concerning the purposes of this thesis, we applied the 1-arcsecond (~30m) resolution grids[2]. Due to numerous number of NHDPlus flow lines (centered by USGS stations) this procedure was coded (using Python which is compatible with ArcGIS) to estimate mean and

[2] rockyftp.cr.usgs.gov/vdelivery/Datasets/Staged/Elevation/1/ArcGrid/

coefficient of variation (%) of lateral slope at all the USGS stations, while, with no doubt, this would have been a tedious job if being made manually just via GIS tools. This programming process is explained visually for the USGS 05064500 illustrated in figure 5. Multiring buffers around a streamline (derived from clipping main channel by 1000-m radius circular buffer) provides basic statistics within each ring. This will be estimated for different radius of circular buffers, i.e., 50-, 100-, 200-, …, 900-, and 1000-m. The average of cell values resided in a ring will be a representative of that zone. The progression and variation of mean elevation through multiple buffers rings is represented by the coefficient of variation of all rings. This will statistically describe the variation of ground (or floodplain) elevation with respect to the lowest level which is the main channel. Table 1 also highlights the estimated values for channel mean lateral (or overbank) slope and coefficient of variation of that measure (in percent) for USGS 05064500.

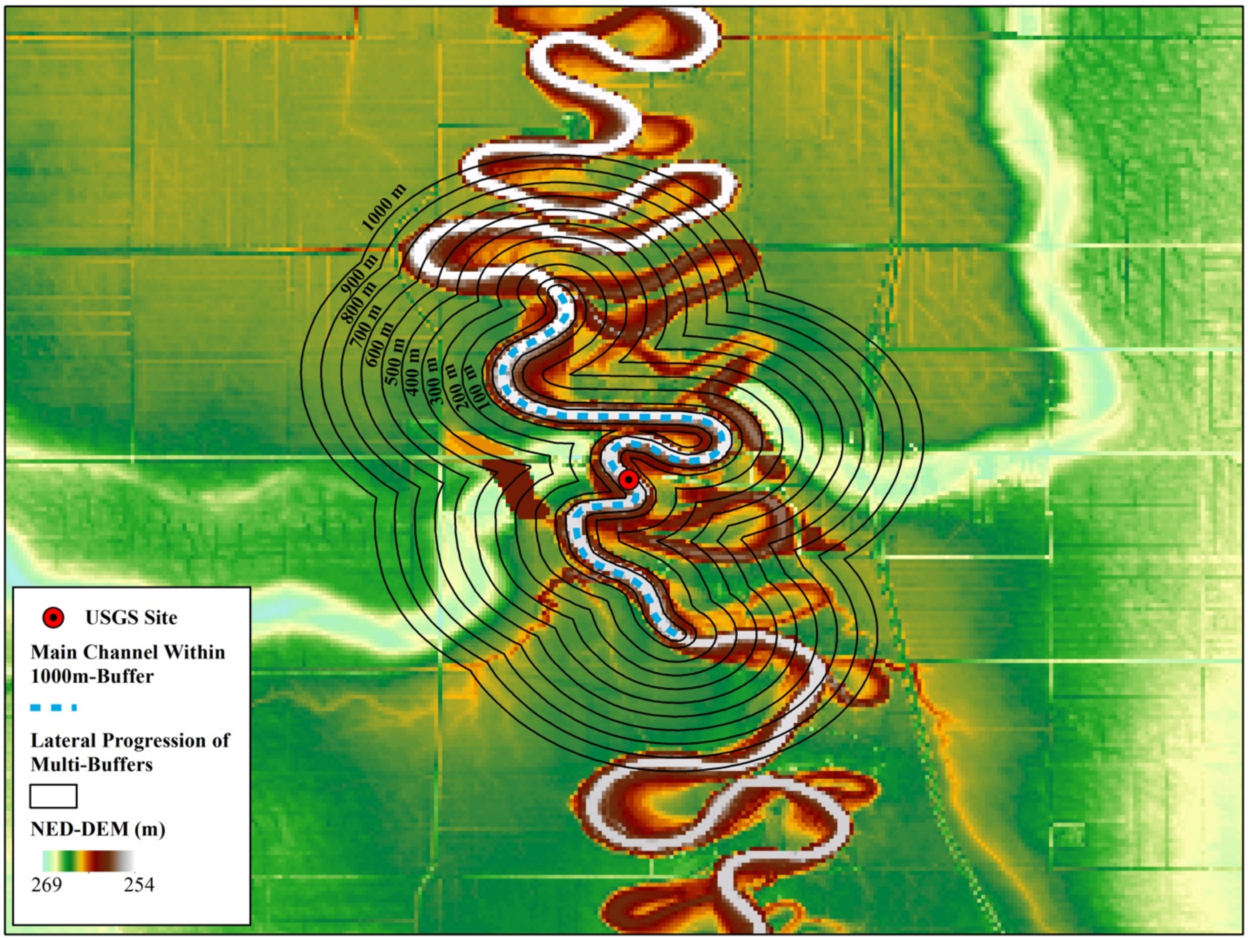

Figure 5. General schematic view of the procedures being made for computing mean lateral (or overbank) slope of a channel by computing arithmetic average of mean lateral slopes which is being computed as the mean of NED-DEM cells locating at multiple buffer lateral rings having 100-m thickness up to 1000-m buffer away from the main channel. Here, this is highlighting multi-ring buffers produced laterally with respect to a channel line centered by USGS 05064500 which clipped by 1000-m radius circular buffer.

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