Model Order Reduction for Boundary Condition Estimation in Casting Machinery

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Reduced Order Modelling, Simulation and Optimization of Coupled Systems (ROMSOC)

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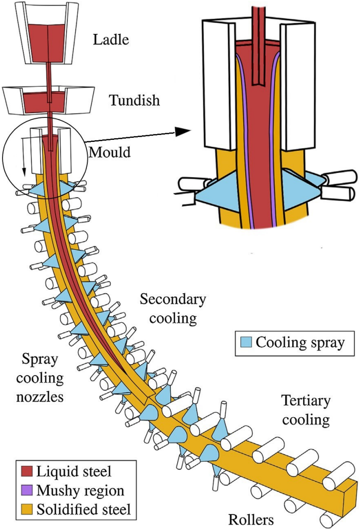
ROMSOC

Outline

Motivation



Credits: [4]

- Control mould behaviour
- Improve production quality
- Predict and avoid catastrofic events
- Increase machinery operative life

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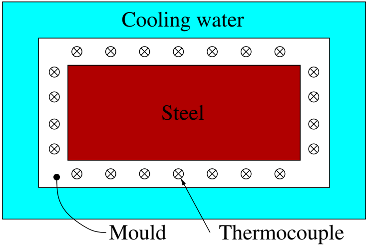
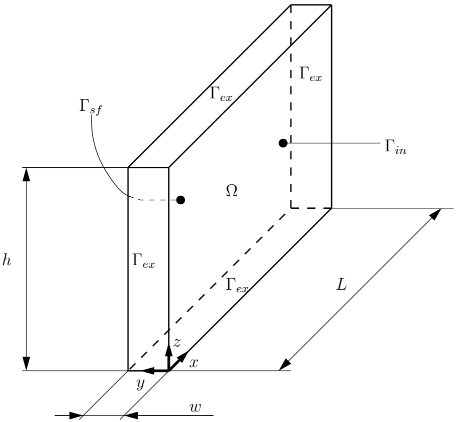
Real time boundary heat flux computation



Outline

Mathematical Model

- Steady-state thermal model
- Homogeneous and isotropic material
- Solid domain, Ω



Direct problem

Given $k \in \mathbb{R}^+$, $h \in \mathbb{R}^+$, $g \in L^2(\Gamma_{in})$ *and* $T_f \in L^2(\Gamma_{in})$. *Find* T *such that*

$$-k\Delta T(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = g(\mathbf{x}) & \forall \mathbf{x} \in \Gamma_{in}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{ex}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = h(T(\mathbf{x}) - T_f(\mathbf{x})) & \forall \mathbf{x} \in \Gamma_{sf}, \end{cases}$$

Discrete Model

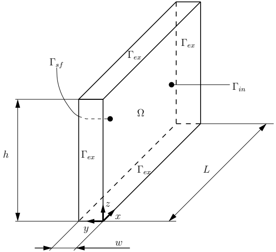
Discretization by **Finite Volumes Method**
For any cell, C, of the mesh, $\mathcal{T}$

$$-k \sum_{\sigma \in \mathcal{E}_C \setminus \Gamma} F_{C,\sigma} + \sum_{\sigma \in \mathcal{E}_C \cap \Gamma_{sf}} R_\sigma T_C = \sum_{\sigma \in \mathcal{E}_C \cap \Gamma_{sf}} R_\sigma T_{f_\sigma} - \sum_{\sigma \in \mathcal{E}_C \cap \Gamma_{in}} m(\sigma) g_\sigma,$$

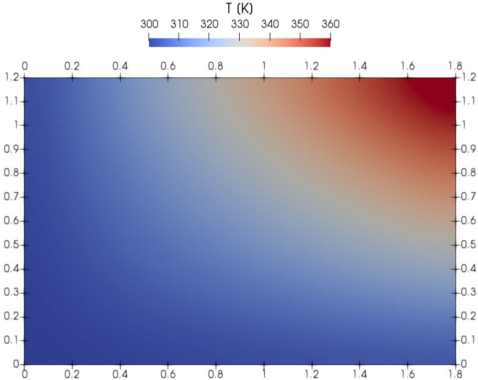
where

- $\mathcal{E}_C$ is the set of faces of the cell C, σ is the single face
- $F_{C,\sigma} := m(\sigma)\frac{T_L - T_C}{d_{C,L}}$
- $R_\sigma := \frac{hk}{hd_{C,\sigma}+k}$

Results



Parameter	Value
k	$300 \frac{\text{W}}{\text{m K}}$
g	$-x \cdot y \cdot 10^5 \frac{\text{W}}{\text{m}^2}$
h	$6 \cdot 10^3 \frac{\text{W}}{\text{m}^2\text{K}}$
T_f	300K
Grid type	Hexahedral, orthogonal
Grid size	$3 \cdot 10^5$



Numerical solution at $y = 0.5$ m.

Outline

Mathematical Formulation

Inverse problem

Given the temperature measurements $\tilde{T}(\mathbf{x}_i) \in \mathbb{R}^+$, $i = 1, 2, \ldots, M$, *find* $g(\mathbf{x}) \in L^2(\Gamma_{in})$ *which minimizes the functional*

$$J[g] = \frac{1}{2} \sum_{i=1}^{M} [T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2,$$

where $T[g](\mathbf{x})$ *is solution of the direct problem.*

- Thermocouples measurements within the domain
- Deterministic least-square approach
- Ill-posed problem
- Alifanov's regularization

Alifanov's Regularization

Set $g^0(\mathbf{x})$;

while $n \leq nMax$ **do**

  Solve direct problem;

  Compute J;

  if $J \leq Jtol$ **then**

    Stop;

  Solve adjoint problem;

  $\gamma^n = \dfrac{\int_{\Gamma_{s_{in}}} [J'_{g^n}(\mathbf{x})]^2 dx}{\int_{\Gamma_{s_{in}}} [J'_{g^{n-1}}(\mathbf{x})]^2 dx}$;

  Search direction, $P^n(\mathbf{x}) = J'_{g^n}(\mathbf{x}) + \gamma^n P^{n-1}(\mathbf{x})$;

  Solve sensitivity problem;

  $\beta^n = \arg\min_\beta J[g^n - \beta P^n] = \dfrac{\sum_{i=1}^M \{T[g^n](\mathbf{x}_i) - \hat{T}(\mathbf{x}_i)\}\delta T[P^n](\mathbf{x}_i)}{\sum_{i=1}^M (\delta T[P^n](\mathbf{x}_i))^2}$;

  $g^{n+1} = g^n - \beta^n P^n$;

  $n = n + 1$;

Alifanov's Regularization

It is a conjugate gradient method applied to the adjoint problem.

The solution of the ajoint problem is the gradient of J evaluated at g, so it gives us the search direction

Adjoint problem

$$\frac{1}{k}\Delta\lambda(\mathbf{x}) + \sum_{i=1}^{M}(T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i))\delta(\mathbf{x} - \mathbf{x}_i) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} \frac{1}{k}\nabla\lambda(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{in} \cup \Gamma_{ex}, \\ \frac{1}{k}\nabla\lambda(\mathbf{x}) \cdot \mathbf{n} + \frac{1}{k^2}h\lambda(\mathbf{x}) = 0 & \forall \mathbf{x} \in \Gamma_{sf}. \end{cases}$$

Alifanov's Regularization

It is a conjugate gradient method applied to the adjoint problem.

The step size along the search direction is given by the solution of the sensitivity problem

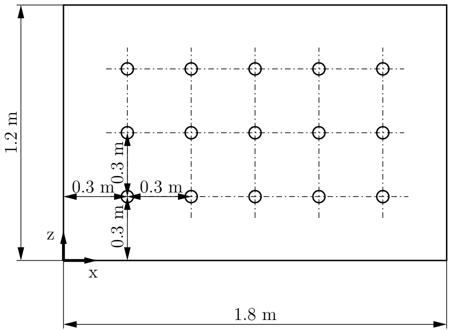
Sensitivity problem

$$-k\Delta\delta T(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = P^n(\mathbf{x}) & \forall \mathbf{x} \in \Gamma_{in}, \\ -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{ex}, \\ -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = h(\delta T(\mathbf{x})) & \forall \mathbf{x} \in \Gamma_{sf}. \end{cases}$$

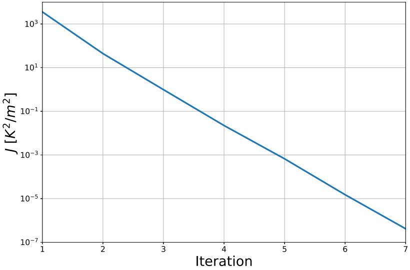
Numerical Symulation

- We want to estimate g at Γ_{in}
- Finite Volume discretization for all problems
- Virtual thermocouples
- Measurements $\tilde{T}$ taken from solution of the direct problem

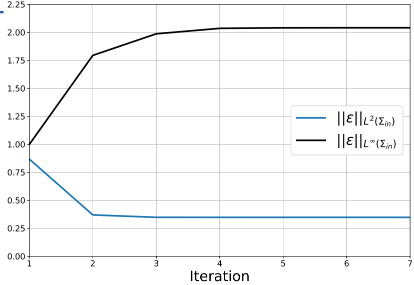


Thermocouples locations at the middleplane of the plate

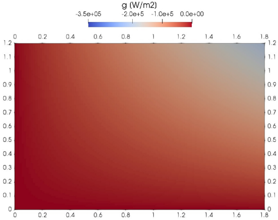
Results



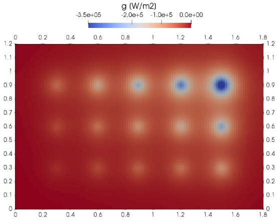
Convergence of the cost function J



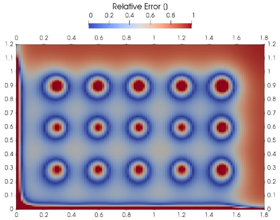
L^2 and L^∞ norm of the relative error, ϵ



$g(\mathbf{x}) = -x \cdot y \cdot 10^5$



Reconstructed BC

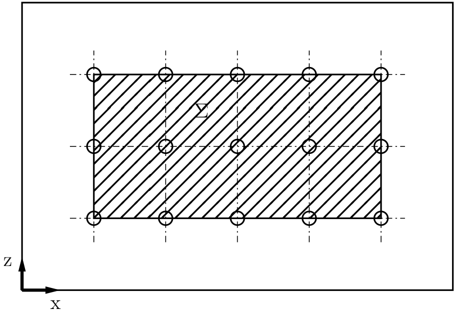


Relative error, ϵ

Measurements Interpolation

– Radial Basis Function interpolation

$$\tilde{T}(\mathbf{x}_i), i = 1, 2, \ldots, M$$
$$\Downarrow$$
$$\tilde{T}(\mathbf{x}) \forall \mathbf{x} \in \Sigma$$

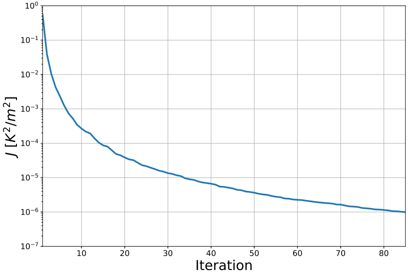


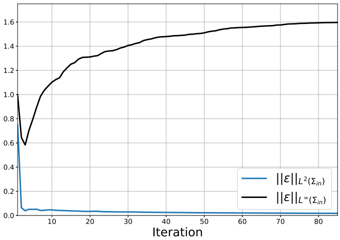
Measurements interpolation plane

Measurements Interpolation

Measurements	Cost functional
$\tilde{T}(\mathbf{x}_i), i = 1, 2, \ldots, M$	$J[g] = \frac{1}{2}\sum_{i=1}^{M}[T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2$
$\Downarrow$	$\Downarrow$
$\tilde{T}_{int}(\mathbf{x}) \forall \mathbf{x} \in \Sigma$	$J[g] = \frac{1}{2}\int_{\Sigma}(T[g](\mathbf{x}) - \tilde{T}_{int}(\mathbf{x}))^2 d\gamma$

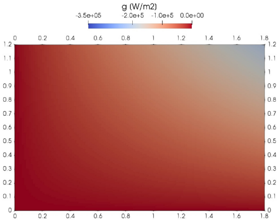
Results



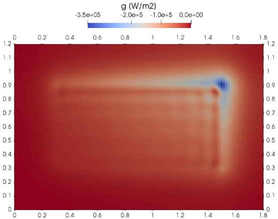


Convergence of the cost function J

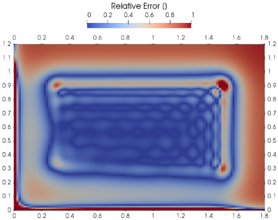
L^2 and L^∞ norm of the relative error, ϵ



$g(\mathbf{x}) = -x \cdot y \cdot 10^5$



Reconstructed BC



Relative error, ϵ

Outline

Methodology

- Parameters are the temperature measurements $\tilde{\boldsymbol{T}} \in \mathbb{R}^M$
- Solve the inverse problem for different values of the parameters
- At each iteration obtain a solution (snapshot) for $\mathbf{T}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\delta T}$

$$\mathbb{V}_T = \text{span}(\boldsymbol{T}_1, \boldsymbol{T}_2, \dots, \boldsymbol{T}_S), \quad \mathbb{V}_\lambda = \text{span}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \dots, \boldsymbol{\lambda}_S),$$

$$\mathbb{V}_{\delta T} = \text{span}(\boldsymbol{\delta T}_1, \boldsymbol{\delta T}_2, \dots, \boldsymbol{\delta T}_S)$$

- POD on solution spaces to obtain orthonormal basis $\boldsymbol{\phi}$, $\boldsymbol{\varphi}$,$\boldsymbol{\psi}$

$$\mathbb{V}_T = \text{span}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_S), \quad \mathbb{V}_\lambda = \text{span}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \dots, \boldsymbol{\varphi}_S),$$

$$\mathbb{V}_{\delta T} = \text{span}(\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \dots, \boldsymbol{\psi}_S)$$

Methodology

- Select the first few modes for each problem to have a reduced basis spaces $\mathbb{V}_{T_{RB}}$, $\mathbb{V}_{\lambda_{RB}}$, $\mathbb{V}_{\delta T_{RB}}$

$$\mathbb{V}_{T_{RB}} = \text{span}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_{N_T}) \quad \mathbb{V}_{\lambda_{RB}} = \text{span}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \dots, \boldsymbol{\varphi}_{N_\lambda})$$

$$\mathbb{V}_{\delta T_{RB}} = \text{span}(\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \dots, \boldsymbol{\psi}_{N_{\delta T}})$$

- Approximation of full order fields by linear combinations of the modes

$$T \approx \sum_{i=1}^{N_T} d_i \phi_i \quad \lambda \approx \sum_{j=1}^{N_\lambda} a_j \varphi_j \quad \delta T \approx \sum_{k=1}^{N_{\delta T}} s_k \psi_k$$

Methodology

– Galerkin projection of the full order models on these basis to obtain reduced models

$$L_T := \begin{bmatrix} | & & | \\ \boldsymbol{\phi}_1 & \dots & \boldsymbol{\phi}_{N_T} \\ | & & | \end{bmatrix} \in \mathbb{R}^{N_h \times N_T}, L_\lambda := \begin{bmatrix} | & & | \\ \boldsymbol{\varphi}_1 & \dots & \boldsymbol{\varphi}_{N_\lambda} \\ | & & | \end{bmatrix} \in \mathbb{R}^{N_h \times N_\lambda}$$

$$L_\delta T := \begin{bmatrix} | & & | \\ \boldsymbol{\psi}_1 & \dots & \boldsymbol{\psi}_{N_{\delta T}} \\ | & & | \end{bmatrix} \in \mathbb{R}^{N_h \times N_{\delta T}}$$

Full order		Reduced order
$A_T \boldsymbol{T} = \boldsymbol{b}_T$	$L_T^T A_T L_T \boldsymbol{d} = L_T^T \boldsymbol{b}_T$	$A_{T_r} \boldsymbol{d} = \boldsymbol{b}_{T_r}$
$A_\lambda \boldsymbol{\lambda} = \mathbf{b}_\lambda$	$L_\lambda^T A_\lambda L_\lambda \boldsymbol{a} = L_\lambda^T \mathbf{b}_\lambda$	$A_{\lambda_r} \boldsymbol{a} = \boldsymbol{b}_{\lambda_r}$
$A_{\delta T} \delta \mathbf{T}_r = \mathbf{b}_{\delta T}$	$L_{\delta T}^T A_{\delta T} L_{\delta T} \delta \mathbf{T}_r = L_{\delta T}^T \mathbf{b}_{\delta T_g}$	$A_{\delta T_r} \boldsymbol{s} = \boldsymbol{b}_{\delta T_r}$

Methodology

- Offline
  - Solve full problem, obtain snapshots
  - Perform POD on snapshots
  - Assemble linear systems
- Online
  - Solve reduced conjugate gradient

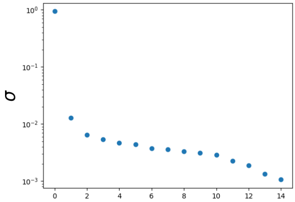
Parameter Space Reduction

10 samples per 15 parameters $\rightarrow 10^{15}$ full order inverse problem solutions

Parameter Space Reduction

Experimental temperature measurements available $\tilde{T}(\mathbf{x}_{TC}, t_i)$

$$Q := \begin{bmatrix} — & \tilde{\boldsymbol{T}}_1 & — \\ & \vdots & \\ — & \tilde{\boldsymbol{T}}_M & — \end{bmatrix}, \quad Q = U^T \Sigma V,$$

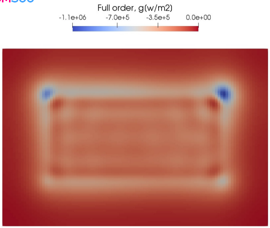


Singular values

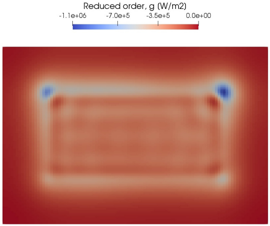
$$\tilde{\boldsymbol{T}} = \sum_{i=1}^{N_{\tilde{T}}} c_i \boldsymbol{\theta}_i,$$

where $\boldsymbol{\theta}_i$ is the i-th column of U

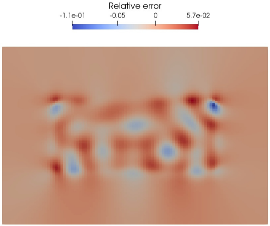
Results



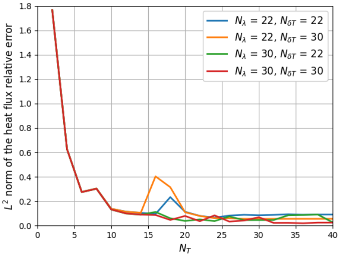
Full order



Reduced order



Relative error



Full	240s
Reduced	2s
Speedup	120

$$N_T = 60,\ N_\lambda = 40,\ N_\delta T = 40$$

Conclusions

Conclusions

- Implemented full order methodology
- Developed reduced method for inverse problem

Future Work

- Error estimate
- Study noise on input
- Bayesian approach

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Thank you