

BABYLONIAN Tonal System

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Abstract

Proposal for the BABYLONIAN Tonal System, based on consecutive heptachords.¹

Keywords. Ancient MESOPOTAMIAN music, Old BABYLONIAN music, MESOPOTAMIA, BABYLON, UR, SUMER, AKKADIAN, 1800 BCE to 500 BCE, Sexagesimal arithmetic and numbers, Musical cuneiform tablets, Seven diatonic Heptachords, Tuning system.

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‡Revised version.

¹Original research, based on [1] and an idea by composer CARL NIELSEN (1865-1931).

Contents

| | | |
|----------|-------------------------------|-----------|
| I | Heptachords and Scales | 3 |
| 1 | Introduction | 3 |
| 1.1 | Heptachords | 3 |
| 1.2 | Scale Range | 6 |
| 1.3 | Scale Cycle | 7 |
| 1.4 | Tuning | 8 |
| 2 | <i>išartum</i> | 10 |
| 2.1 | Scale 1 | 10 |
| 2.2 | Scale 2 | 12 |
| 3 | <i>embūbum</i> | 14 |
| 3.1 | Scale 1 | 14 |
| 3.2 | Scale 2 | 16 |
| 4 | <i>kitmum</i> | 18 |
| 4.1 | Scale 1 | 18 |
| 4.2 | Scale 2 | 20 |
| 5 | <i>pītum</i> | 22 |
| 5.1 | Scale 1 | 22 |
| 5.2 | Scale 2 | 24 |
| 6 | <i>qablītum</i> | 26 |
| 6.1 | Scale 1 | 26 |
| 6.2 | Scale 2 | 28 |
| 7 | <i>nīš tuḫrim</i> | 30 |
| 8 | <i>nād qablīm</i> | 34 |
| A | Appendix | 39 |
| A.1 | Octatonic Scale | 39 |
| A.1.1 | Scale 1 | 39 |
| A.1.2 | Scale 2 | 39 |
| A.2 | Plots | 40 |
| A.2.1 | Heptachords | 40 |
| A.2.2 | Octaves | 41 |

Part I

Heptachords and Scales

1 Introduction

1.1 Heptachords

Table 1: Heptachords

| | Heptachord | Pattern | Scales | Symmetric Center |
|---|-------------------|--|--------|------------------|
| 1 | <i>išartum</i> | $[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$ | 2 | G |
| 2 | <i>embūbum</i> | $[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$ | 2 | A |
| 3 | <i>kitmum</i> | $[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$ | 2 | D |
| 4 | <i>pītum</i> | $[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$ | 2 | E |
| 5 | <i>qablītum</i> | $[1, 1, \frac{1}{2}, 1, 1, \frac{1}{2}]$ | 2 | C |
| 6 | <i>nīš tuḥrim</i> | $[1, 1, \frac{1}{2}, 1, 1, 1]$ | 1 | F |
| 7 | <i>nīd qablim</i> | $[1, 1, 1, \frac{1}{2}, 1, 1]$ | 1 | B |

2

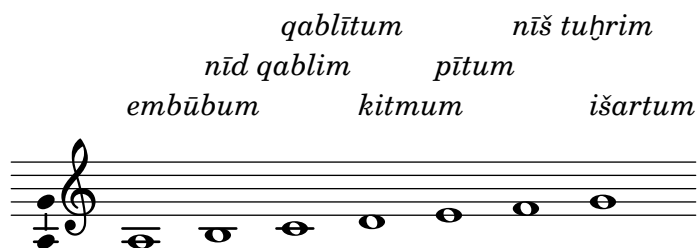


Figure 1: Symmetric Center

² Rising Scale.

Table 2: Heptachords II

| | Heptachord | Keys per Scale | Key Signature Δ | Sum Σ |
|---|-------------------|----------------|------------------------|--------------|
| 1 | <i>išartum</i> | 6 | 2 | 12 |
| 2 | <i>embūbum</i> | 6 | 2 | 12 |
| 3 | <i>kitmum</i> | 6 | 2 | 12 |
| 4 | <i>pītum</i> | 6 | 2 | 12 |
| 5 | <i>qablātum</i> | 6 | 2 | 12 |
| 6 | <i>nīš tuḫrim</i> | 12 | 5 | 60 |
| 7 | <i>nād qablim</i> | 12 | 5 | 60 |

1: 𐎶

2: 𐎶

5: 𐎶

6: 𐎶

12: 𐎶

60: 𐎶 ANU

Example. Consecutive Heptachords.^{3,4}

The image shows a musical staff with a treble clef. It contains three heptachords of the *kitmum* type, each consisting of seven notes. The first heptachord is D-*kitmum* (1 flat), with notes D, E, F, G, A, B, and C. The second is C-*kitmum* (3 flats), with notes C, D, E, F, G, A, and B. The third is B \flat -*kitmum* (5 flats), with notes B \flat , C, D, E, F, G, and A. Each heptachord is marked with a flat sign and its name. The notes are connected by semitone intervals, indicated by brackets labeled "semitone" below the staff. The notes are: D, E, F, G, A, B, C, C, D, E, F, G, A, B, B \flat , C, D, E, F, G, A.

Figure 2: Consecutive *kitmum* Heptachords

³ From the main theme of CARL NIELSEN *Symphony No. 5* CNW 29 (1922).

⁴ Flatward shift for higher pitches.

1.2 Scale Range

Table 3: Scale Range

| | Heptachord | Center | Semitones | Octaves | i , Cent | Cent per semitone |
|---|-------------------|--------|-----------|---------|------------|-------------------|
| 1 | <i>išartum</i> | G | 60 | 5 | 6000 | 100 |
| 2 | <i>embūbum</i> | A | 60 | 5 | 6000 | 100 |
| 3 | <i>kitmum</i> | D | 60 | 5 | 6000 | 100 |
| 4 | <i>pītum</i> | E | 60 | 5 | 6000 | 100 |
| 5 | <i>qablītum</i> | C | 60 | 5 | 6000 | 100 |
| 6 | <i>nāš tuhrim</i> | F | 120 | 10 | 12000 | 100 |
| 7 | <i>nād qablim</i> | B | 120 | 10 | 12000 | 100 |

Definition. Cent per octave: 1200.

Definition. Cent per semitone: 100, Equal temperament (12-TET⁵ or 12-EDO⁶).

Definition. Scale Range: $-3000 \dots 0$ (Center) $\dots +3000$ Cent.⁷

Theorem. *The 7 Heptachords of the Babylonian Tonal System are based on an Equal Temperament system.*

⁵ Twelve-tone equal temperament.

⁶ Equal division of the octave.

⁷ Semitone: $i \leftarrow i + 100$, $i \leftarrow i - 100$. Whole tone: $i \leftarrow i + 200$, $i \leftarrow i - 200$.

1.3 Scale Cycle

Table 4: Scale Cycle

| | Heptachord | Center | +3000 Cent | -3000 Cent |
|---|-----------------|--------|------------|------------|
| 1 | <i>išartum</i> | G | D \flat | C \sharp |
| 2 | <i>embūbum</i> | A | E \flat | D \sharp |
| 3 | <i>kitnum</i> | D | A \flat | G \sharp |
| 4 | <i>pītum</i> | E | B \flat | A \sharp |
| 5 | <i>qablītum</i> | C | G \flat | F \sharp |

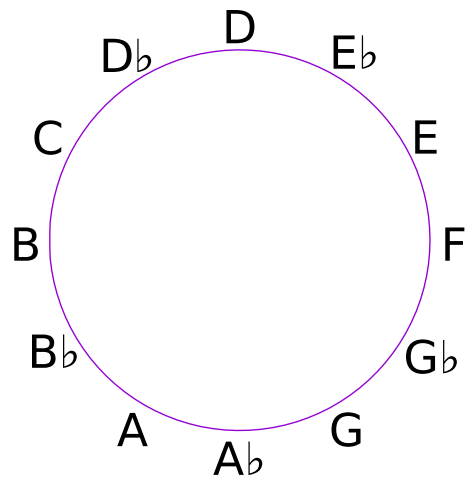


Figure 3: Tone Circle

1.4 Tuning

Conjecture. *Equal Temperament Tuning Procedure.*

Table 5: Tuning

| | Heptachord | Center | i , Cent | f , Hz |
|---|-------------------|--------|------------|----------|
| 1 | <i>išartum</i> | G4 | +500 | |
| 2 | <i>embūbum</i> | A3 | -500 | 220 |
| 3 | <i>kitmum</i> | D4 | 0 | |
| 4 | <i>pītum</i> | E4 | +200 | |
| 5 | <i>qablītum</i> | C4 | -200 | |
| 6 | <i>nāš tuḫrim</i> | F4 | +300 | |
| 7 | <i>nād qablim</i> | B3 | -300 | |

8

Definition. Tuning Range: -500 ... 0 ... +500 Cent.

⁸ Center in Scientific Pitch Notation.

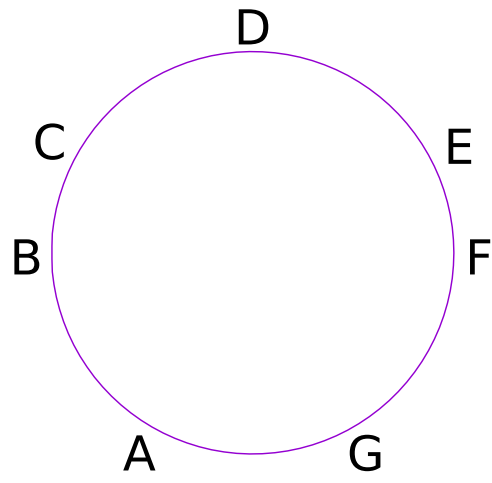


Figure 4: Tuning

2 *išartum*

$[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$

2.1 Scale 1

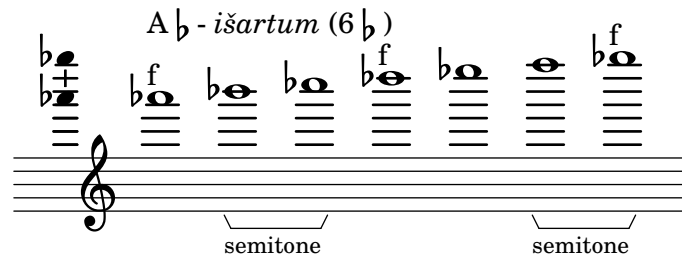


Figure 5: Scale I

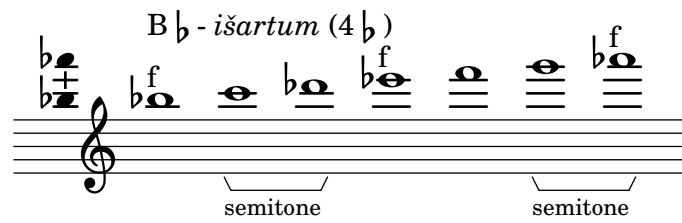


Figure 6: Scale II

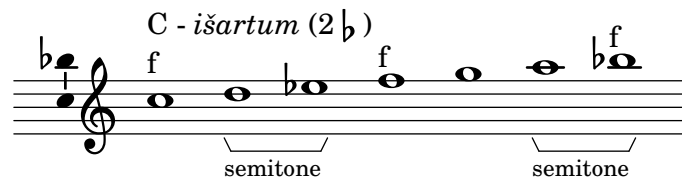


Figure 7: Scale III

D - *išartum* (0 ♭)

Figure 8: Scale IV

E - *išartum* (2 ♯)

Figure 9: Scale V

F♯ - *išartum* (4 ♯)

Figure 10: Scale VI

G♯ - *išartum* (6 ♯)

Figure 11: Scale VII

2.2 Scale 2

$D \flat$ - *išartum* (7 \flat)

semitone semitone

Figure 12: Scale I

$E \flat$ - *išartum* (5 \flat)

semitone semitone

Figure 13: Scale II

F - *išartum* (3 \flat)

semitone semitone

Figure 14: Scale III

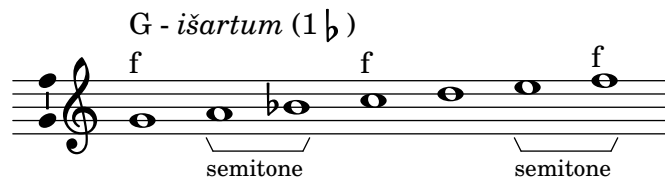


Figure 15: Scale IV

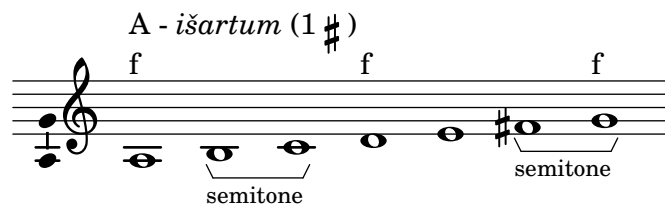


Figure 16: Scale V

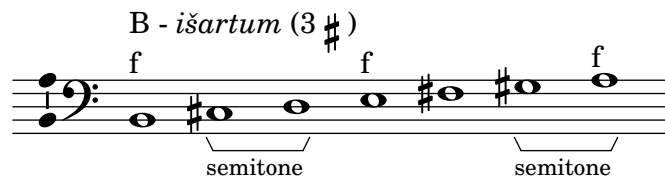


Figure 17: Scale VI

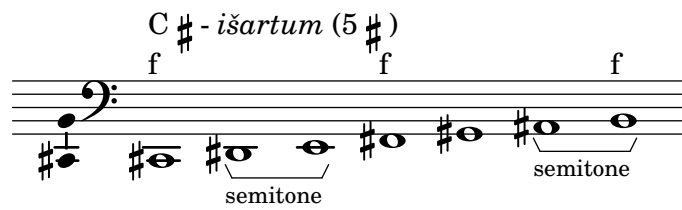


Figure 18: Scale VII

3 *embūbum*

$[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$

3.1 Scale 1

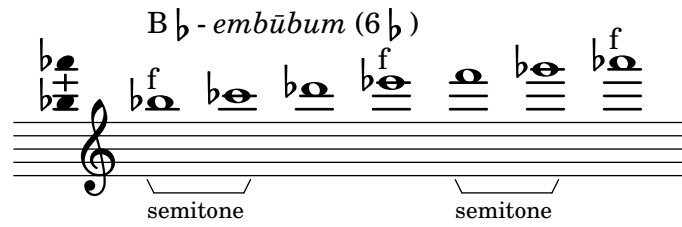


Figure 19: Scale I

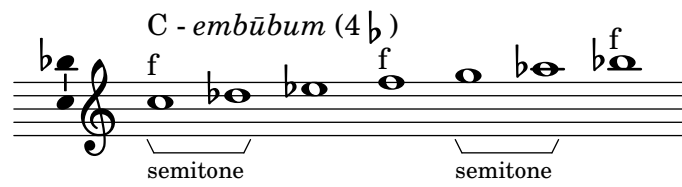


Figure 20: Scale II

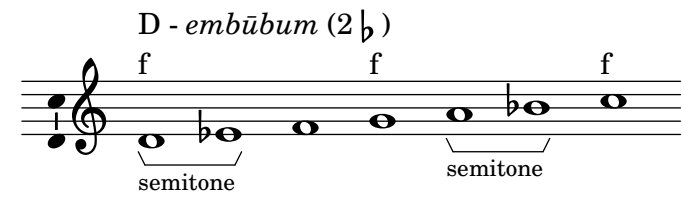


Figure 21: Scale III

E - *embūbum* (0 ♭)

f f f

semitone

Figure 22: Scale IV

F # - *embūbum* (2 #)

f f f

semitone

Figure 23: Scale V

G # - *embūbum* (4 #)

f f f

semitone

Figure 24: Scale VI

A # - *embūbum* (6 #)

f f f

semitone

Figure 25: Scale VII

3.2 Scale 2

E \flat - *embūbum* (7 \flat)

semitone semitone

Figure 26: Scale I

F - *embūbum* (5 \flat)

semitone semitone

Figure 27: Scale II

G - *embūbum* (3 \flat)

semitone semitone

Figure 28: Scale III

A - *embūbum* (1 ♭)

Figure 29: Scale IV

B - *embūbum* (1 ♯)

Figure 30: Scale V

C - *embūbum* (3 ♯)

Figure 31: Scale VI

D ♯ - *embūbum* (5 ♯)

Figure 32: Scale VII

4 *kitmum*

$[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$

4.1 Scale 1

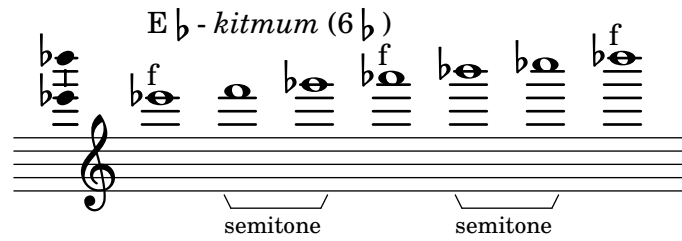


Figure 33: Scale I

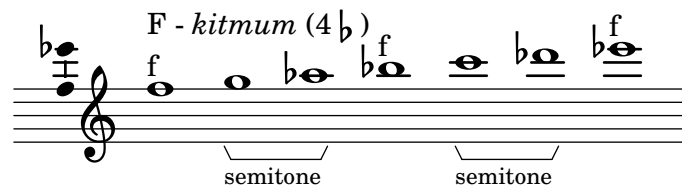


Figure 34: Scale II

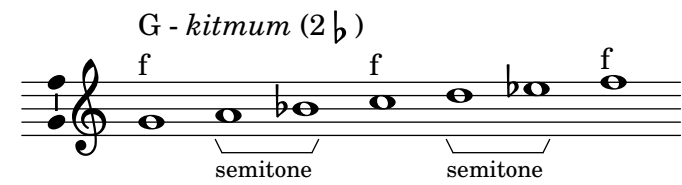


Figure 35: Scale III

A - *kitmum* (0 ♭)

Figure 36 shows Scale IV in treble clef. The scale starts on A4 (f) and ends on A5 (f). The notes are A4, B4, C5, D5, E5, and A5. Brackets indicate semitone intervals between B4 and C5, and between E5 and A5.

Figure 36: Scale IV

B - *kitmum* (2 #)

Figure 37 shows Scale V in bass clef. The scale starts on B3 (f) and ends on B4 (f). The notes are B3, C#4, D4, E4, F#4, and B4. Brackets indicate semitone intervals between C#4 and D4, and between F#4 and B4.

Figure 37: Scale V

C # - *kitmum* (4 #)

Figure 38 shows Scale VI in bass clef. The scale starts on C#4 (f) and ends on C#5 (f). The notes are C#4, D#4, E4, F#4, G#4, and C#5. Brackets indicate semitone intervals between D#4 and E4, and between G#4 and C#5.

Figure 38: Scale VI

D # - *kitmum* (6 #)

Figure 39 shows Scale VII in bass clef. The scale starts on D#4 (f) and ends on D#5 (f). The notes are D#4, E#4, F#4, G#4, A#4, and D#5. Brackets indicate semitone intervals between E#4 and F#4, and between A#4 and D#5.

Figure 39: Scale VII

4.2 Scale 2

A \flat - *kitmum* (7 \flat)

semitone semitone

Figure 40: Scale I

B \flat - *kitmum* (5 \flat)

semitone semitone

Figure 41: Scale II

C - *kitmum* (3 \flat)

semitone semitone

Figure 42: Scale III

D - *kitmum* (1 ♭)

Figure 43: Scale IV

E - *kitmum* (1 #)

Figure 44: Scale V

F # - *kitmum* (3 #)

Figure 45: Scale VI

G # - *kitmum* (5 #)

Figure 46: Scale VII

5 *pītum*

$[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$

5.1 Scale 1

F - *pītum* (6 ♭)

f b b b b b f

semitone semitone

Figure 47: Scale I

G - *pītum* (4 ♭)

f b b b b f

semitone semitone

Figure 48: Scale II

A - *pītum* (2 ♭)

f b b f b f

semitone semitone

Figure 49: Scale III

B - *pītum* (0 ♭)

Figure 50: Scale IV

C # - *pītum* (2 #)

Figure 51: Scale V

D # - *pītum* (4 #)

Figure 52: Scale VI

E # - *pītum* (6 #)

Figure 53: Scale VII

5.2 Scale 2

Musical notation for the B-flat major scale (7 flats) in treble clef. The scale is written as a sequence of notes: B-flat, C, D, E-flat, F, G, A-flat, B-flat. The notes are grouped into two pairs, each labeled "semitone" with a bracket underneath. The first pair is B-flat and C, and the second pair is G and A-flat. The notes are marked with "f" (forte) and "b" (flat) symbols.

Figure 54: Scale II

Musical notation for the C major scale (5 flats) in treble clef. The scale is written as a sequence of notes: C, D, E-flat, F, G, A-flat, B-flat, C. The notes are grouped into two pairs, each labeled "semitone" with a bracket underneath. The first pair is E-flat and F, and the second pair is G and A-flat. The notes are marked with "f" (forte) and "b" (flat) symbols.

Figure 55: Scale II

Musical notation for the D major scale (3 flats) in treble clef. The scale is written as a sequence of notes: D, E-flat, F, G, A-flat, B-flat, C, D. The notes are grouped into two pairs, each labeled "semitone" with a bracket underneath. The first pair is E-flat and F, and the second pair is G and A-flat. The notes are marked with "f" (forte) and "b" (flat) symbols.

Figure 56: Scale III

E - *pītum* (1 ♭)

f f f

semitone semitone

Figure 57: Scale IV

F # - *pītum* (1 #)

f f f

semitone semitone

Figure 58: Scale V

G # - *pītum* (3 #)

f f f

semitone semitone

Figure 59: Scale VI

A # - *pītum* (5 #)

f f f

semitone semitone

Figure 60: Scale VII

6 *qablītum*

[1, 1, $\frac{1}{2}$, 1, 1, $\frac{1}{2}$]

6.1 Scale 1

D \flat - *qablītum* (6 \flat)

semitone semitone

Figure 61: Scale I

E \flat - *qablītum* (4 \flat)

semitone semitone

Figure 62: Scale II

F - *qablītum* (2 \flat)

semitone semitone

Figure 63: Scale III

G - *qablītum* (0 ♭)

f f f

semitone semitone

Figure 64: Scale IV

A - *qablītum* (2 ♯)

f f f

semitone semitone

Figure 65: Scale V

B - *qablītum* (4 ♯)

f f f

semitone semitone

Figure 66: Scale VI

C ♯ - *qablītum* (6 ♯)

f f f

semitone semitone

Figure 67: Scale VII

6.2 Scale 2

G \flat - *qablītum* (7 \flat)

semitone semitone

Figure 68: Scale I

A \flat - *qablītum* (5 \flat)

semitone semitone

Figure 69: Scale II

B \flat - *qablītum* (3 \flat)

semitone semitone

Figure 70: Scale III

C - *qablītum* (1 ♭)

Figure 71: Scale IV

D - *qablītum* (1 #)

Figure 72: Scale V

E - *qablītum* (3 #)

Figure 73: Scale VI

F # - *qablītum* (5 #)

Figure 74: Scale VII

7 *nīš tuḥrim*

[1, 1, $\frac{1}{2}$, 1, 1, 1]

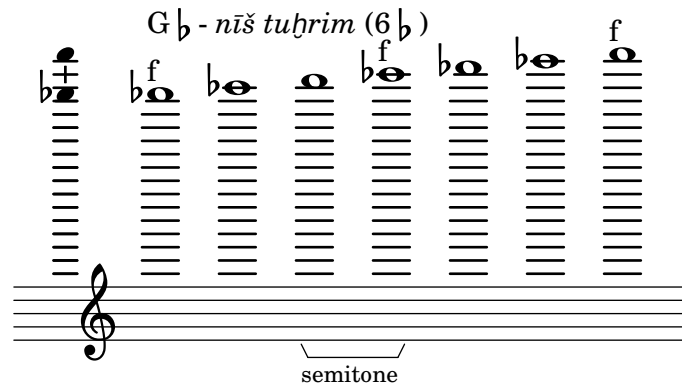


Figure 75: Scale I

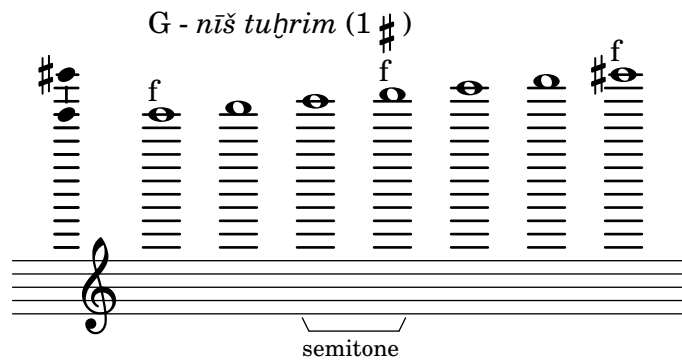


Figure 76: Scale II

A \flat - *nīš tuḥrim* (4 \flat)

semitone

Figure 77: Scale III

A - *nīš tuḥrim* (3 \sharp)

semitone

Figure 78: Scale IV

B \flat - *nīš tuḥrim* (2 \flat)

semitone

Figure 79: Scale V

B - *nīš tuḥrim* (5 \sharp)

semitone

Figure 80: Scale VI

C - *nīš tuḥrim* (0 ♭)

f f f

semitone

Figure 81: Scale VII

D ♭ - *nīš tuḥrim* (5 ♭)

f f f

semitone

Figure 82: Scale VIII

D - *nīš tuḥrim* (2 ♯)

f f f

semitone

Figure 83: Scale IX

E ♭ - *nīš tuḥrim* (3 ♭)

f f f

semitone

Figure 84: Scale X

E - *nīš tuḥrim* (4 #)

f f f

Figure 85: Scale XI

F - *nīš tuḥrim* (1 ♭)

f f f

Figure 86: Scale XII

F # - *nīš tuḥrim* (6 #)

f # f # f

Figure 87: Scale XIII

8 *nīd qablim*

[1, 1, 1, $\frac{1}{2}$, 1, 1]

B - *nīd qablim* (6 \flat)

f

semitone

Figure 88: Scale I

C - *nīd qablim* (1 \sharp)

f

semitone

Figure 89: Scale II

D \flat - *nīd qablim* (4 \flat)

f

semitone

Figure 90: Scale III

D - *nīd qablim* (3 #)

Figure 91: Scale IV

E \flat - *nīd qablim* (2 \flat)

Figure 92: Scale V

E - *nīd qablim* (5 #)

Figure 93: Scale VI

F - *nīd qablim* (0 \flat)

Figure 94: Scale VII

G \flat - *nīd qablim* (5 \flat)

f f f

semitone

Figure 95: Scale VIII

G - *nīd qablim* (2 \sharp)

f f f

semitone

Figure 96: Scale IX

A \flat - *nīd qablim* (3 \flat)

f f f

semitone

Figure 97: Scale X

A - *nīd qablim* (4 \sharp)

f f f

semitone

Figure 98: Scale XI

B \flat - *nīd qablim* (1 \flat)

f f f

semitone

Figure 99: Scale XII

B - *nīd qablim* (6 \sharp)

f f f

semitone

Figure 100: Scale XIII

References

- [1] Leon Crickmore. New Light on the Babylonian Tonal System. In Richard Dumbrill and Irving Finkel, editors, *ICONEA 2008: Proceedings of the International Conference of Near Eastern Archaeomusicology*, volume 24, pages 11–22. London: Iconea Publications, 2010. Held at the British Museum, December 4-6, 2008. 209 pp. 1

A Appendix

A.1 Octatonic Scale

9

A.1.1 Scale 1

$[1, \frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}]$ ¹⁰

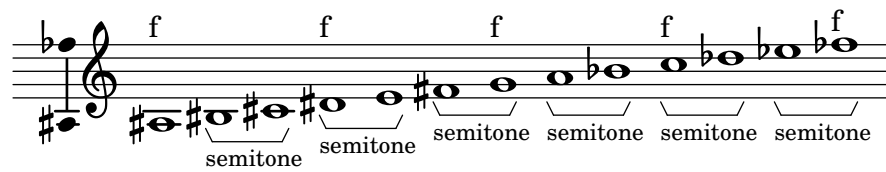


Figure 101: Scale 1, Center G

A.1.2 Scale 2

$[\frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}, 1]$

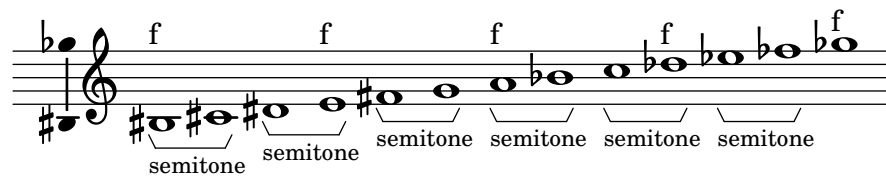


Figure 102: Scale 2, Center A

⁹ Derived from CARL NIELSEN *Symphony No. 5* CNW 29.

¹⁰ OLIVIER MESSIAEN's *second mode* of limited transposition.

A.2 Plots

A.2.1 Heptachords

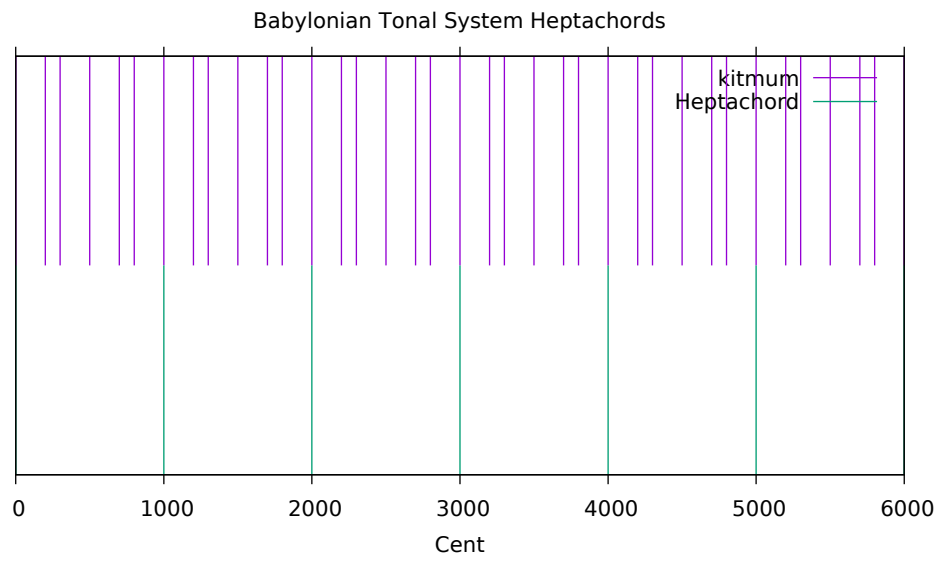


Figure 103: *kitnum* vs. Heptachord

A.2.2 Octaves

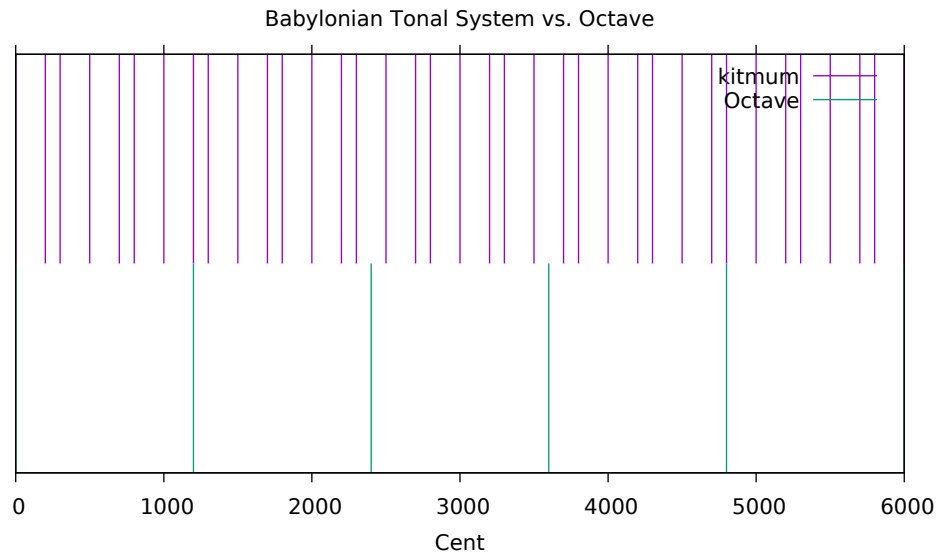


Figure 104: *kitnum* vs. Octave