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Full Length Research Paper

Pairs Determination for Sine and Cosine Function in Modeling Nigerian Gross Domestic Product

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Gross Domestic Product (GDP) is one of the primary indicators that the government, economists and other stake holders use to gauge the stability of a country's economy. It measures the size of a country's economy as it represents the monetary value of all goods and services produced over a time period. GDP estimates are commonly used to measure the economic performance of a country and also measure the relative contribution of the industrial sector. In formulating the model, we considered a four pair model, six pair model, and eight pair model. The result shows that at best Nigeria GDP contains four pair of the sine and cosine functions.

Keywords: Fourier series, Gross Domestic Product, predicted, Dickey-Fuller

INTRODUCTION

Gross Domestic Product (GDP) is one of the primary indicators that the Government, Economists and other stake holders use to gauge the stability of a country's economy. It is the monetary value of finished goods and services produced within a country's borders in a specific time period. The GDP per capita of Nigeria expanded by 132% in the Sixties, and reached a peak growth of 283% in the Seventies. But this proved unsustainable and it consequently shrank by 66% in the Eighties. In the Nineties, diversification initiatives finally took effect and decadal growth was restored to 10% due to inflation. Per capita GDP today remains lower than in 1960 when Nigeria declared independence (World Bank Indicator). According to Abdulrahem, (2011) the importance of GDP growth of a country makes many researchers and economist to apply various time series, econometric, regression model etc., to forecast the GDP of a country. Thus, a country is said to have good economy if its GDP is relatively high. The GDP of any country helps in depicting the economic strength and/or development level of that country (Nigeria inclusive). Over the years Nigeria GDP has been evolving. Thus, there is need to study the changing pattern of the GDP in Nigeria.

The Fourier series model is useful in handling this kind of problems. Thus, it will form the core of this work. This paper focuses on building a Fourier series model that can be used to forecast the Gross Domestic Product (GDP) of Nigeria, using data that was collected from Central Bank of Nigeria from 1961 to 2015, and seeks to examine the trend pattern of the GDP, modeled Nigeria GDP using Fourier series. This research models Nigerian GDP using Fourier series analysis and in particular, suggests the pairs of sine and cosine function that best describe the data in frequency domain. Ensuing is extant literature on this subject matter. Section 3 is the method adopted; Sections 4 contains the results and discussion. We conclude with section 5 and make some recommendations.

Spectral analysis methods have a broad range of applications in the real world (Thomson and van Vuuren, 2016). The Fourier integral formula (due to Joseph Fourier) which enjoys widespread applications in Physics, Engineering and allied fields is regarded as one of the most fundamental results of modern Mathematical analysis (Debnath, 2012). The applications ranges economics to science including Spectroscopy, Crystallography, Imaging, Signal processing and Communications. Others are in the area weather and climate. Fourier series have gained more attraction as a tool in economics and econometrics, starting with the early works of Cunnyngham, (1963), Nerlove, (1964) and Granger, (1966). Other application ranges from simple economic time series in the 1960s to modern day applications in derivative pricing and wavelet analysis. Liu *et al.* (2012) argued that Fourier analysis could be used to detect cyclical behaviour in any type of time series data, although no cyclical behaviour was detected in the majority of the time series data tested and proposed that a natural extension to Fourier analysis was the use of wavelets. According to Tjostheim and Paulsen, (1983), Fourier analysis of the New York home prices data confirmed the glaring presence of strong seasonality with a cycle of 12 months. Thomson and van Vuuren (2016) measured the duration of the South African business cycle using log changes in nominal gross domestic product (GDP). Their findings suggest the effective of Fourier analysis in estimating the length of the business cycle, as well as in determining the current position (phase) of the economy in the business cycle.

Omekara *et al.* (2013) used Fourier series analysis to identify cycles in the Nigerian all-items monthly inflation rates from 2003 to 2011. A square root transformation was used to increase stability and normality of the inflation rate data. Periodogram analysis showed a short term and a long term cycle of 20 months and 51 months respectively with the long cycle corresponding to the length of two different government administrations that existed during the sample period. A general Fourier series model was then fitted to the data to make accurate short term monthly inflation rate forecasts from an out-of-sample period from September 2011 to September 2012.

Liu *et al.* (2012) investigated business and growth cycles in the frequency domain by performing Fourier analyses on several data sources including electricity demand, foreign currency data, monthly retail sales, quarterly GDP, labour market and productivity statistics from Statistics New Zealand. In their analysis of the GDP data, the data were transformed using natural logarithms and detrended using the Hodrick-Prescott filter before conducting Fourier analysis of the detrended transformed data. Using a Periodogram, definitive cycles corresponding to eight years and four-and-a-half years were found. Because of the distance between energy spikes in the periodogram, it was proposed that the cycle length varied between 4.5 to 8 years.

Konarasinghe and Abeynayake, (2015) focused on the application of Fourier transformation on the model fitting for Stock Returns of Sri Lanka share market and compared the forecasting ability of the above model with the Autoregressive Integrated Moving Average (ARIMA) model. They made use of the Fourier transformation to transform monthly returns to series of trigonometric and multiple regression analysis for the purpose of estimating amplitudes. The amplitudes were considered to be a set of orthogonal functions as proved by Wei, (2006). The result showed that Fourier transformation along with regression analysis was better and suitable for forecasting individual company returns of Sri Lanka share market than the ARIMA model. This paper focuses on building a Fourier series model that can be used to predict the Gross Domestic Product (GDP) of Nigeria, using data from Central Bank of Nigeria from 1960 to 2017.

METHODOLOGY

Annual data on the gross domestic product (GDP) of Nigeria that ranged from 1960 to 2017 collected from the Central Bank of Nigeria (CBN, 2014) website (www.cbn.gov.ng) was used for this research. Consider the general form of the infinite Fourier series:

$$f(t) = \alpha_0 + \sum_{k=1}^{\infty}\{\alpha_k \cos \pi kt + \beta_k \sin \pi kt\} \quad (1)$$

Where α_0 is the mean value of the Gross Domestic Product, k is the total number of period and α_k and β_k determines the shape of the output, and are known as the spectral coefficients of the signal $f(t)$. Assuming we sample a discrete dataset (y) from a continuous signal. Discrete Fourier Transform transforms the dataset $\{y\}$ into another dataset (y) containing the Fourier coefficients. The discrete Fourier transform of a sequence $y = \{y_1,..., y_n\}$ is defined as:

$$Y_k = \frac{1}{N}\sum_{n=1}^{N} y_n exp\{-i2\pi k(n-1)\} \quad (2)$$

To obtain the coefficients of the Fourier series, one may expand equation (2) using Euler's formula:

$$Y_k = \frac{1}{N}\sum_{n=1}^{N} y_k[\cos(2\pi k(n-1)) - i\sin(2\pi k(n-1))] \quad (3)$$

This is in the form of a complex number $a + ib$ where a and b are sequences of real and imaginary number respectively generated by Fast Fourier Transform. Equation (3) can be decomposed into even and odd functions respectively.

Table 1. Descriptive statistics of the percentage change in GDP.

	Mean	Std. Dv.	Minimum	Maximum	First - Case	Last - Case	N
% Change (GDP)	4.151905	8.335023	-15.7436	33.73578	1.000	55.000	55.000

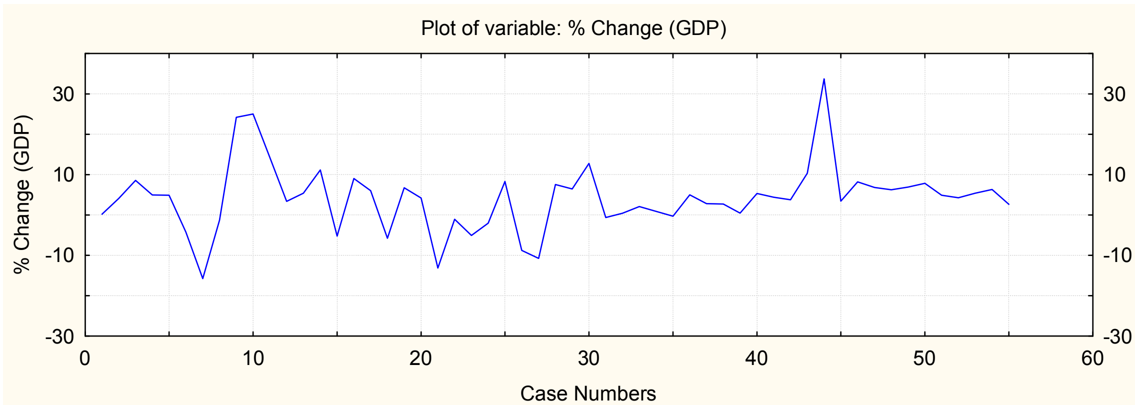

Figure 1. Time plot for data on Nigeria's GDP from 1961-2015.

$$\alpha_k = \frac{2}{N}\sum_{n=1}^{N} y_k \cos(2\pi k(n-1)) \qquad (4)$$

$$\beta_k = \frac{2}{N}\sum_{n=1}^{N} y_k \sin(2\pi k(n-1)) \qquad (5)$$

In determining the coefficients, we add stochastic term ε to the model

$$Y_k = \frac{1}{N}\sum_{n=1}^{N}\{\alpha_k \cos(2\pi k(n-1)) + \beta_k \sin(2\pi k(n-1))\} + \varepsilon \qquad (6)$$

Where ε is independently and identically distributed normal variable with mean zero and variance σ^2. The estimate $(\hat{Y}_t)$ can be obtained as a function of the frequency ω_i

$$\hat{Y}_t = \sum_{j=1}^{l}(\alpha_j \cos\omega_i t + \beta_j \sin\omega_i t) \qquad (7)$$

This can easily yield

$$SSE = (Y_t - \hat{Y}_t)^2 \qquad (8)$$

Since we are dealing with the annual Gross Domestic Product (GDP) which is discrete in nature, we employ discrete decomposition of Fourier series. The Fourier series for continuous function is then decomposed to give us a discrete representation which is given by:

$$IR_s^{d,s} = \alpha_0 + \sum_{n=1}^{N}\left(\alpha_k \cos\frac{2\pi k(n-1)}{N} + \beta_k \sin\frac{2\pi k(n-1)}{N}\right) \qquad (9)$$

Where

$$\alpha_k = \frac{2}{N}\sum_{n=1}^{N}\left(IR_{(n-1)}^{d,obs}\cos 2\pi k(n-1)\right) \qquad (10)$$

$$\beta_k = \frac{2}{N}\sum_{n=1}^{N}\left(IR_{(n-1)}^{d,obs}\sin 2\pi k(n-1)\right) \qquad (11)$$

$IR_m^{d,obs}$ is the discrete values observed, n is the individual data point, k the harmonic being considered. As each new harmonic (period) was added to the Fourier series, the sum of square errors were determined. The sum of square errors will be used in identifying the optimal period (k) associated with the Fourier series function. This is achieved by measuring the deviation between the estimate of the Fourier series function at each period (k) and the observed value as contained in equation (8). For Y being the observed value and $\hat{Y}$ is the estimated value. This leads to the formulation in Equation (12) below.

$$SSE = \sum_{n=1}^{N}\left(IR_s^{d,obs} - IR_s^{f,s}\right) \qquad (12)$$

Where $IR_s^{d,obs}$ and $IR_s^{f,s}$ are observed and estimated GDP.

The smaller the SSE, it is expected that, the better will be the predictive ability of the Fourier series function. An R program was written to execute this and generate the

Table 2. The model selection information for the GDP of Nigeria Data with Fourier Series of various complexity.

Model	AIC	BIC
Four Pair	390.4861[a]	406.5448[a]
Six Pair	393.5414	417.6294
Eight Pair	396.5245	428.6419

[a]Best Model with SSE= 3.614778e-30

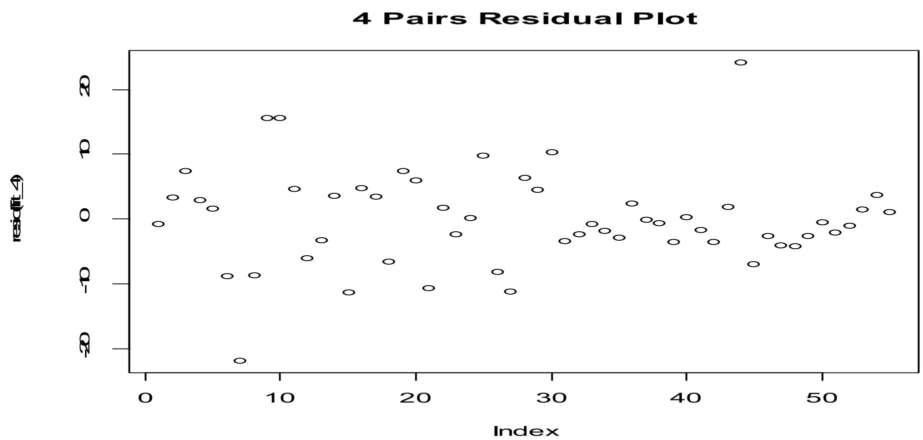

Figure 2. Residual plot for 4 pair.

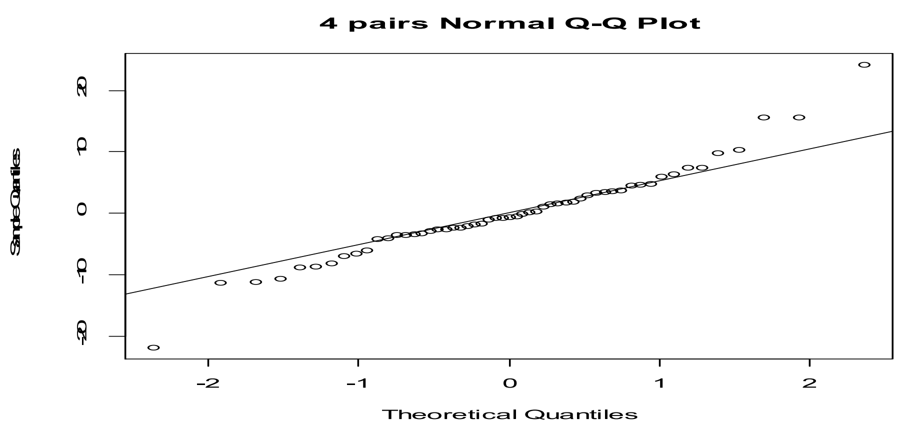

Figure 3. Normal Q-Q plot for four pair.

following results.

RESULTS AND DISCUSSION

Table 1 shown the average change in Nigerian GDP between 1961 and 2015 is about 4.2% with minimum of -15.74362821% in 1967 and maximum of 33.73577503% in 2004. Figure 1 shows the time series plot of the data at level. At level, the data was found to be stationary as indicated in figure 1 (the time series plot) and formal unit root (Augmented Dickey Fuller) test.

We considered a Fourier series model with 4 pair, 6 pair and 8 pair to see which one fit the data best (Table 2). In formulating the model we considered a four pair model, six pair model, and eight pair model. The BIC values for this different pair considered by the researcher were, 406.5448 for four pair, 417.6294 for six pair, 428.6419 for eight pair (Table 2). The four pair model was selected, since it has the least BIC value of 406.5448. Similarly, the four pair model has the lowest AIC of 390.4861 compared to six and eight pair with values 393.5414 and 396.5245 respectively. Figures 2 and 3 show the residual plot and the normal Q-Q plot of the selected model. The plot shows evidences of good fit.

Conclusion

In fitting the Fourier series, it is important to consider the number of components in the sinusoidal function. The GDP of Nigeria was found to at best contain four pair of the sine and cosine functions as suggested by the information criteria (AIC and BIC). The extent to which the selected model best fit the data is shown in the residual plot and the Q-Q plot. The value of sum of squares error (SSE= 3.614778e-30) indicates that four pair of sine and cosine function will give a reasonable forecast for Nigerian GDP. It is recommended that appropriate pair is used for sine and cosine function when modeling Fourier series in order to estimate the parameters and obtain a better forecast.

Authors' declaration

We declare that this study is an original research by our research team and we agree to publish it in the journal.

REFERENCES

Abdulrahem A (2011). Sectorial contributions to gross domestic product in Nigeria1971-2005, Nigeria Forum, 1-12.

Central Bank of Nigeria (2014). Statistical Bulletin Volume 25, December 2014. Garki-Abuja.

Debnath L (2012). A short biography of Joseph Fourier and historical development ofFourier series and Fourier transforms. *International Journal of Mathematical Education in Science and Technology*, 43(5): 1–24.

Granger CWJ (1966). "The Typical Spectra Shape of an Economic Variable" *Econometrica Vol. 34(1):150-161.*

Liu LE, Stonehouse J, You J (2012). Cycle Identification: An old approach to (relatively) new statistics. NZAE Annual Conference, Statistics New Zealand.

Konarasinghe WGS, Abeynayale NRL, (2015)." Spectral Analysis on Forecasting Sri Lankan Share Market Returns" *International Journal of Management and Applied Sciences (IJMAS) Vol. 2 (1): 22-27.*

Nerlove ML (1964). "On the optimality of Adaptive Forecasting," Journal of *Management Science, Vol.* 10: 207-29.

Omekara CO, Ekpenyong EJ, Ekerete MP (2013). Modeling the Nigerian Inflation Rates using Periodogram and Fourier Series Analysis, CBN Journal of Applied Statistics, ISSN 2476-8472, The Central Bank of Nigeria, Abuja, Vol. 4(2):51-68.

Thomson D, van Vuuren G (2016). Forecasting the South African Business Cycle Using Fourier Analysis, *International Business & Economics Research Journal* Vol. 15(4): 175-192.

Tjostheim D, Paulsen J (1983). "Bias of Some Commonly-Used Time Series Estimates"*Biometrika*, 70(2): 389-399.

Wei WS (2006). "Time Series Analysis: Univariate and Multivariate Methods" Second Edition, Pearson Addison Wesley, Boston USA.

World Bank National Accounts Data. Accessed on November 19th 2018 from http://data.worldbank.org/indicator