This electronic companion is part of the manuscript entitled *"A Local Market Mechanism for Physical Storage Rights"* which has been exclusively submitted to **IEEE Transactions on Power Systems** for publication. It comprises the collection of Karush-Kuhn-Tucker (KKT) conditions for all models and the proofs for the equivalence of *Equilibrium Model* and *Optimization Model* (Appendix A). We show this way that market is efficient; the system is dispatched at the minimum social cost and no one is motivated to deviate from the market-clearing outcomes. We also prove market's revenue adequacy, which implies that the market operator never incurs a budget deficit (Appendix B). Moreover, in Appendix C we provide some technical parameters and data used in the case studies. We also give some information for the scenario reduction technique we implemented to obtain the 40 scenarios for the in-sample analysis in stochastic programming. Finally, we illustrate the 365 daily profiles of wind, PV production and price which have been used for the out-of-sample analysis.

Appendix A: KKTs for All Models

A. KKTs of Equilibrium Model

The KKT optimality conditions associated with the *Equilibrium Model* are given by (3) below. Note that $\mathcal{L}$ is the Lagrangian function with respect to each player's problem. Firstly, we provide the optimality conditions for the arbitrageur followed by the rest of market participants.

$$\Bigg\{ \text{(1ae), (1af), (1aj), (1ak)} \tag{3aa}$$

$$\frac{\partial \mathcal{L}}{\partial p^{c,\mathrm{DA}}_{i,s,t}} = \lambda^{\mathrm{loc,DA}}_{t} - \underline{\theta}^{c,\mathrm{DA}}_{i,s,t} + \overline{\theta}^{c,\mathrm{DA}}_{i,s,t} + \gamma^{e,\mathrm{DA}}_{i,s,t}\ \eta^{c}_{s} + \sum_{\omega}\left(\overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} - \underline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega}\right) = 0\ \forall t,s \tag{3ab}$$

$$\frac{\partial \mathcal{L}}{\partial p^{d,\mathrm{DA}}_{i,s,t}} = -\lambda^{\mathrm{loc,DA}}_{t} - \underline{\theta}^{d,\mathrm{DA}}_{i,s,t} + \overline{\theta}^{d,\mathrm{DA}}_{i,s,t} - \frac{\gamma^{e,\mathrm{DA}}_{i,s,t}}{\eta^{d}_{s}} + \sum_{\omega}\left(\overline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} - \underline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega}\right) = 0\ \forall t,s \tag{3ac}$$

$$\frac{\partial \mathcal{L}}{\partial e^{\mathrm{DA}}_{i,s,t}} = -\underline{\theta}^{e,\mathrm{DA}}_{i,s,t} + \overline{\theta}^{e,\mathrm{DA}}_{i,s,t} - \gamma^{e,\mathrm{DA}}_{i,s,t} + \gamma^{e,\mathrm{DA}}_{i,s,t+1} + \sum_{\omega}\left(\overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} - \underline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega}\right) = 0\ \forall s, t \leq 23 \tag{3ad}$$

$$\frac{\partial \mathcal{L}}{\partial e^{\mathrm{DA}}_{i,s,t}} = -\underline{\theta}^{e,\mathrm{DA}}_{i,s,t} + \overline{\theta}^{e,\mathrm{DA}}_{i,s,t} - \gamma^{e,\mathrm{DA}}_{i,s,t} - \tilde{\lambda} + \sum_{\omega}\left(\overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} - \underline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega}\right) = 0\ \forall s, t = 24 \tag{3ae}$$

$$\frac{\partial \mathcal{L}}{\partial p^{c,\max,\mathrm{DA}}_{i,s,t}} = \mu^{c,\mathrm{DA}}_{s,t} - \overline{\theta}^{c,\mathrm{DA}}_{i,s,t} - \sum_{\omega}\overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} = 0\ \forall t,s \tag{3af}$$

$$\frac{\partial \mathcal{L}}{\partial p^{d,\max,\mathrm{DA}}_{i,s,t}} = \mu^{d,\mathrm{DA}}_{s,t} - \overline{\theta}^{d,\mathrm{DA}}_{i,s,t} - \sum_{\omega}\overline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} = 0\ \forall t,s \tag{3ag}$$

$$\frac{\partial \mathcal{L}}{\partial e^{\max,\mathrm{DA}}_{i,s,t}} = \mu^{e,\mathrm{DA}}_{s,t} - \overline{\theta}^{e,\mathrm{DA}}_{i,s,t} - \sum_{\omega}\overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} = 0\ \forall t,s \tag{3ah}$$

$$\frac{\partial \mathcal{L}}{\partial p^{c,\mathrm{RT}}_{i,s,t,\omega}} = \lambda^{\mathrm{loc,RT}}_{t,\omega} - \underline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} + \gamma^{e,\mathrm{RT}}_{i,s,t,\omega}\ \eta^{c}_{s} = 0\ \forall t,s,\omega \tag{3ai}$$

$$\frac{\partial \mathcal{L}}{\partial p^{d,\mathrm{RT}}_{i,s,t,\omega}} = -\lambda^{\mathrm{loc,RT}}_{t,\omega} - \underline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} - \frac{\gamma^{e,\mathrm{RT}}_{i,s,t,\omega}}{\eta^{d}_{s}} = 0\ \forall t,s,\omega \tag{3aj}$$

$$\frac{\partial \mathcal{L}}{\partial e^{\mathrm{RT}}_{i,s,t,\omega}} = -\underline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} - \gamma^{e,\mathrm{RT}}_{i,s,t,\omega} + \gamma^{e,\mathrm{RT}}_{i,s,t+1,\omega} = 0\ \forall s,\omega, t \leq 23 \tag{3ak}$$

$$\frac{\partial \mathcal{L}}{\partial e^{\mathrm{RT}}_{i,s,t,\omega}} = -\underline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} - \gamma^{e,\mathrm{RT}}_{i,s,t,\omega} - \pi_{\omega}\tilde{\lambda} = 0\ \forall s,\omega, t = 24 \tag{3al}$$

$$0 \leq p^{c,\mathrm{DA}}_{i,s,t} \perp \underline{\theta}^{c,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3am}$$

$$0 \leq p^{d,\mathrm{DA}}_{i,s,t} \perp \underline{\theta}^{d,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3an}$$

$$0 \leq e^{\mathrm{DA}}_{i,s,t} \perp \underline{\theta}^{e,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3ao}$$

$$0 \leq \left(p^{c,\max,\mathrm{DA}}_{i,s,t} - p^{c,\mathrm{DA}}_{i,s,t}\right) \perp \overline{\theta}^{c,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3ap}$$

$$0 \leq \left(p^{d,\max,\mathrm{DA}}_{i,s,t} - p^{d,\mathrm{DA}}_{i,s,t}\right) \perp \overline{\theta}^{d,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3aq}$$

$$0 \leq \left(e^{\max,\mathrm{DA}}_{i,s,t} - e^{\mathrm{DA}}_{i,s,t}\right) \perp \overline{\theta}^{e,\mathrm{DA}}_{i,s,t} \geq 0\ \forall t,s \tag{3ar}$$

$$0 \leq \left(p^{c,\mathrm{DA}}_{i,s,t} + p^{c,\mathrm{RT}}_{i,s,t,\omega}\right) \perp \underline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3as}$$

$$0 \leq \left(p^{d,\mathrm{DA}}_{i,s,t} + p^{d,\mathrm{RT}}_{i,s,t,\omega}\right) \perp \underline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3at}$$

$$0 \leq \left(e^{\mathrm{DA}}_{i,s,t} + e^{\mathrm{RT}}_{i,s,t,\omega}\right) \perp \underline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3au}$$

$$0 \leq \left(p^{c,\max,\mathrm{DA}}_{i,s,t} - p^{c,\mathrm{DA}}_{i,s,t} - p^{c,\mathrm{RT}}_{i,s,t,\omega}\right) \perp \overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3av}$$

$$0 \leq \left(p^{d,\max,\mathrm{DA}}_{i,s,t} - p^{d,\mathrm{DA}}_{i,s,t} - p^{d,\mathrm{RT}}_{i,s,t,\omega}\right) \perp \overline{\theta}^{d,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3aw}$$

$$0 \leq \left(e^{\max,\mathrm{DA}}_{i,s,t} - e^{\mathrm{DA}}_{i,s,t} - e^{\mathrm{RT}}_{i,s,t,\omega}\right) \perp \overline{\theta}^{e,\mathrm{RT}}_{i,s,t,\omega} \geq 0\ \forall t,s,\omega \tag{3ax}$$

$$\Bigg\}\ \forall i \in \mathcal{A}.$$

$$\Bigg\{ \text{(1ae), (1af), (1aj), (1ak)} \tag{3ba}$$

$$\frac{\partial \mathcal{L}}{\partial p^{c,\mathrm{DA}}_{i,s,t}} = -\underline{\theta}^{c,\mathrm{DA}}_{i,s,t} + \overline{\theta}^{c,\mathrm{DA}}_{i,s,t} + \underline{\theta}^{g,\mathrm{DA}}_{i,t} + \gamma^{e,\mathrm{DA}}_{i,s,t}\ \eta^{c}_{s} + \sum_{\omega}\left(\overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} - \underline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{g,\mathrm{RT}}_{i,t,\omega}\right) = 0\ \forall t,s \tag{3bb}$$

$$\text{(3ac)} - \text{(3ah)}, \text{(3aj)} - \text{(3ax)} \tag{3bc}$$

$$\frac{\partial \mathcal{L}}{\partial p^{g,\mathrm{DA}}_{i,t}} = -\lambda^{\mathrm{loc,DA}}_{t} - \underline{\theta}^{g,\mathrm{DA}}_{i,t} + \overline{\theta}^{g,\mathrm{DA}}_{i,t} + \sum_{\omega}\left(\overline{\theta}^{g,\mathrm{RT}}_{i,t,\omega} - \underline{\theta}^{g,\mathrm{RT}}_{i,t,\omega}\right) = 0\ \forall t \tag{3bd}$$

$$\frac{\partial \mathcal{L}}{\partial p^{c,\mathrm{RT}}_{i,s,t,\omega}} = -\underline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{c,\mathrm{RT}}_{i,s,t,\omega} + \overline{\theta}^{g,\mathrm{RT}}_{i,t,\omega} + \gamma^{e,\mathrm{RT}}_{i,s,t,\omega}\ \eta^{c}_{s} = 0\ \forall t,s,\omega \tag{3be}$$

$$\frac{\partial \mathcal{L}}{\partial p^{g,\mathrm{RT}}_{i,t,\omega}} = -\lambda^{\mathrm{loc,RT}}_{t,\omega} - \underline{\theta}^{g,\mathrm{RT}}_{i,t,\omega} + \overline{\theta}^{g,\mathrm{RT}}_{i,t,\omega} = 0\ \forall t,\omega \tag{3bf}$$

$$0 \leq p_{i,t}^{g,\mathrm{DA}} \perp \underline{\theta}_{i,t}^{g,\mathrm{DA}} \; \forall t \quad (3bg)$$

$$0 \leq \left(G_{i,t}^{\mathrm{DA}} - \sum_{s} p_{i,s,t}^{c,\mathrm{DA}} - p_{i,t}^{g,\mathrm{DA}} \right) \perp \overline{\theta}_{i,t}^{g,\mathrm{DA}} \geq 0 \; \forall t \quad (3bh)$$

$$0 \leq \left(p_{i,t}^{g,\mathrm{DA}} + p_{i,t,\omega}^{g,\mathrm{RT}} \right) \perp \underline{\theta}_{i,t,\omega}^{g,\mathrm{RT}} \geq 0 \; \forall t, \omega \quad (3bi)$$

$$0 \leq \left[G_{i,t,\omega}^{\mathrm{RT}} - \sum_{s} \left(p_{i,s,t}^{c,\mathrm{DA}} + p_{i,s,t,\omega}^{c,\mathrm{RT}} \right) - p_{i,t}^{g,\mathrm{DA}} - p_{i,t,\omega}^{g,\mathrm{RT}} \right] \perp \overline{\theta}_{i,t,\omega}^{g,\mathrm{RT}} \geq 0 \; \forall t, \omega \quad (3bj)$$

$$\Big\} \; \forall i \in \mathcal{P}.$$

$$\Big\{ (1ae), (1af), (1aj), (1ak), (1cd), (1ce) \quad (3ca)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,s,t}^{c,\mathrm{DA}}} = -\underline{\theta}_{i,s,t}^{c,\mathrm{DA}} + \overline{\theta}_{i,s,t}^{c,\mathrm{DA}} + \gamma_{i,t}^{q,\mathrm{DA}} + \gamma_{i,s,t}^{e,\mathrm{DA}} \; \eta_s^c + \sum_{\omega} \left(\overline{\theta}_{i,s,t,\omega}^{c,\mathrm{RT}} - \underline{\theta}_{i,s,t,\omega}^{c,\mathrm{RT}} + \gamma_{i,t,\omega}^{q,\mathrm{RT}} \right) = 0 \; \forall t, s \quad (3cb)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,s,t}^{d,\mathrm{DA}}} = -\underline{\theta}_{i,s,t}^{d,\mathrm{DA}} + \overline{\theta}_{i,s,t}^{d,\mathrm{DA}} - \gamma_{i,t}^{q,\mathrm{DA}} - \frac{\gamma_{i,s,t}^{e,\mathrm{DA}}}{\eta_s^d} + \sum_{\omega} \left(\overline{\theta}_{i,s,t,\omega}^{d,\mathrm{RT}} - \underline{\theta}_{i,s,t,\omega}^{d,\mathrm{RT}} - \gamma_{i,t,\omega}^{q,\mathrm{RT}} \right) = 0 \; \forall t, s \quad (3cc)$$

$$(3ad) - (3ah) \quad (3cd)$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,t}^{\mathrm{DA}}} = -\lambda_t^{\mathrm{loc,DA}} + \gamma_{i,t}^{q,\mathrm{DA}} + \sum_{\omega} \gamma_{i,t,\omega}^{q,\mathrm{RT}} = 0 \; \forall t \quad (3ce)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,s,t,\omega}^{c,\mathrm{RT}}} = -\underline{\theta}_{i,s,t,\omega}^{c,\mathrm{RT}} + \overline{\theta}_{i,s,t,\omega}^{c,\mathrm{RT}} + \gamma_{i,t,\omega}^{q,\mathrm{RT}} + \gamma_{i,s,t,\omega}^{e,\mathrm{RT}} \; \eta_s^c = 0 \; \forall t, s, \omega \quad (3cf)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,s,t,\omega}^{d,\mathrm{RT}}} = -\underline{\theta}_{i,s,t,\omega}^{d,\mathrm{RT}} + \overline{\theta}_{i,s,t,\omega}^{d,\mathrm{RT}} - \gamma_{i,t,\omega}^{q,\mathrm{RT}} - \frac{\gamma_{i,s,t,\omega}^{e,\mathrm{RT}}}{\eta_s^d} = 0 \; \forall t, s, \omega \quad (3cg)$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,t,\omega}^{\mathrm{RT}}} = -\lambda_{t,\omega}^{\mathrm{loc,RT}} + \gamma_{i,t,\omega}^{q,\mathrm{RT}} = 0 \; \forall t, \omega \quad (3ch)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t,\omega}^{\mathrm{shed}}} = \pi_\omega V_i - \underline{\delta}_{i,t,\omega} + \overline{\delta}_{i,t,\omega} - \gamma_{i,t,\omega}^{q,\mathrm{RT}} = 0 \; \forall t, \omega \quad (3ci)$$

$$(3ak) - (3ax) \quad (3cj)$$

$$0 \leq p_{i,t,\omega}^{\mathrm{shed}} \perp \underline{\delta}_{i,t,\omega} \geq 0 \; \forall t, \omega \quad (3ck)$$

$$0 \leq \left(D_{i,t} - p_{i,t,\omega}^{\mathrm{shed}} \right) \perp \overline{\delta}_{i,t,\omega} \geq 0 \; \forall t, \omega \quad (3cl)$$

$$\Big\} \; \forall i \in \mathcal{C}.$$

$$\Big\{ (1ae), (1af), (1aj), (1ak), (1de), (1df) \quad (3da)$$

$$(3cb) - (3cl) \quad (3db)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t}^{\mathrm{PV,DA}}} = -\underline{\rho}_{i,t}^{\mathrm{DA}} + \overline{\rho}_{i,t}^{\mathrm{DA}} - \gamma_{i,t}^{q,\mathrm{DA}} + \sum_{\omega} \left(-\underline{\rho}_{i,t,\omega}^{\mathrm{RT}} + \overline{\rho}_{i,t,\omega}^{\mathrm{RT}} - \gamma_{i,t,\omega}^{q,\mathrm{RT}} \right) = 0 \; \forall t \quad (3dc)$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t,\omega}^{\mathrm{PV,RT}}} = -\underline{\rho}_{i,t,\omega}^{\mathrm{RT}} + \overline{\rho}_{i,t,\omega}^{\mathrm{RT}} - \gamma_{i,t,\omega}^{q,\mathrm{RT}} = 0 \; \forall t, \omega \quad (3dd)$$

$$0 \leq p_{i,t}^{\mathrm{PV,DA}} \perp \underline{\rho}_{i,t}^{\mathrm{DA}} \geq 0 \; \forall t \quad (3de)$$

$$0 \leq \left(G_{i,t}^{\mathrm{DA}} - p_{i,t}^{\mathrm{PV,DA}} \right) \perp \overline{\rho}_{i,t}^{\mathrm{DA}} \geq 0 \; \forall t \quad (3df)$$

$$0 \leq \left(p_{i,t}^{\mathrm{PV,DA}} + p_{i,t,\omega}^{\mathrm{PV,RT}} \right) \perp \underline{\rho}_{i,t,\omega}^{\mathrm{RT}} \geq 0 \; \forall t, \omega \quad (3dg)$$

$$0 \leq \left(G_{i,t,\omega}^{\mathrm{RT}} - p_{i,t}^{\mathrm{PV,DA}} - p_{i,t,\omega}^{\mathrm{PV,RT}} \right) \perp \overline{\rho}_{i,t,\omega}^{\mathrm{RT}} \geq 0 \; \forall t, \omega \quad (3dh)$$

$$\Big\} \; \forall i \in \mathcal{R}.$$

The optimality conditions for the storage owner when acting as a PSR provider are given by (3e):

$$\frac{\partial \mathcal{L}}{\partial p_{s,t}^{c,\mathrm{ri,DA}}} = -\mu_{s,t}^{c,\mathrm{DA}} - \underline{\phi}_{s,t}^{c,\mathrm{DA}} + \overline{\phi}_{s,t}^{c,\mathrm{DA}} = 0 \; \forall t, s \quad (3ea)$$

$$\frac{\partial \mathcal{L}}{\partial p_{s,t}^{d,\mathrm{ri,DA}}} = -\mu_{s,t}^{d,\mathrm{DA}} - \underline{\phi}_{s,t}^{d,\mathrm{DA}} + \overline{\phi}_{s,t}^{d,\mathrm{DA}} = 0 \; \forall t, s \quad (3eb)$$

$$\frac{\partial \mathcal{L}}{\partial e_{s,t}^{\mathrm{ri,DA}}} = -\mu_{s,t}^{e,\mathrm{DA}} - \underline{\phi}_{s,t}^{e,\mathrm{DA}} + \overline{\phi}_{s,t}^{e,\mathrm{DA}} = 0 \; \forall t, s \quad (3ec)$$

$$0 \leq p_{s,t}^{c,\mathrm{ri,DA}} \perp \underline{\phi}_{s,t}^{c,\mathrm{DA}} \geq 0 \; \forall t, s \quad (3ed)$$

$$0 \leq p_{s,t}^{d,\mathrm{ri,DA}} \perp \underline{\phi}_{s,t}^{d,\mathrm{DA}} \geq 0 \; \forall t, s \quad (3ee)$$

$$0 \leq e_{s,t}^{\mathrm{ri,DA}} \perp \underline{\phi}_{s,t}^{e,\mathrm{DA}} \geq 0 \; \forall t, s \quad (3ef)$$

$$0 \leq \left(P_s^{c,\max} - p_{s,t}^{c,\mathrm{ri,DA}} \right) \perp \overline{\phi}_{s,t}^{c,\mathrm{DA}} \geq 0 \; \forall t, s \quad (3eg)$$

$$0 \leq \left(P_s^{d,\max} - p_{s,t}^{d,\mathrm{ri,DA}} \right) \perp \overline{\phi}_{s,t}^{d,\mathrm{DA}} \geq 0 \; \forall t, s \quad (3eh)$$

$$0 \leq \left(E_s^{\max} - e_{s,t}^{\mathrm{ri,DA}} \right) \perp \overline{\phi}_{s,t}^{e,\mathrm{DA}} \geq 0 \; \forall t, s. \quad (3ei)$$

The optimality conditions for the grid owner are given by (3f) below:

$$\frac{\partial \mathcal{L}}{\partial p_t^{\mathrm{flow,DA}}} = -\lambda_t^{\mathrm{DA}} + \lambda_t^{\mathrm{loc,DA}} - \underline{\beta}_t^{\mathrm{DA}} + \overline{\beta}_t^{\mathrm{DA}} + \sum_{\omega} \left(\overline{\beta}_{t,\omega}^{\mathrm{RT}} - \underline{\beta}_{t,\omega}^{\mathrm{RT}} \right) = 0 \; \forall t \quad (3fa)$$

$$\frac{\partial \mathcal{L}}{\partial p_{t,\omega}^{\mathrm{flow,RT}}} = -\pi_\omega \lambda_{t,\omega}^{\mathrm{RT}} + \lambda_{t,\omega}^{\mathrm{loc,RT}} - \underline{\beta}_{t,\omega}^{\mathrm{RT}} + \overline{\beta}_{t,\omega}^{\mathrm{RT}} = 0 \; \forall t, \omega \quad (3fb)$$

$$0 \leq \left(L + p_t^{\mathrm{flow,DA}} \right) \perp \underline{\beta}_t^{\mathrm{DA}} \geq 0 \; \forall t \quad (3fc)$$

$$0 \leq \left(L - p_t^{\mathrm{flow,DA}} \right) \perp \overline{\beta}_t^{\mathrm{DA}} \geq 0 \; \forall t \quad (3fd)$$

$$0 \leq \left(L + p_t^{\mathrm{flow,DA}} + p_{t,\omega}^{\mathrm{flow,RT}} \right) \perp \underline{\beta}_{t,\omega}^{\mathrm{RT}} \geq 0 \; \forall t, \omega \quad (3fe)$$

$$0 \leq \left(L - p_t^{\mathrm{flow,DA}} - p_{t,\omega}^{\mathrm{flow,RT}} \right) \perp \overline{\beta}_{t,\omega}^{\mathrm{RT}} \geq 0 \; \forall t, \omega. \quad (3ff)$$

The shared constraints which express the provision of the physical storage rights as well as the power balance are expressed by (1ga) - (1ge).

B. KKTs of Optimization Model

It is straightforward to verify that the KKT optimality conditions of the *Optimization Model* are identical to those of *Equilibrium Model*. To avoid replicating the equations, we will only provide an example selecting a random player and derive its KKTs.

The Lagrangian function of the arbitrageur $i \in \mathcal{A}$ is given by (4):

$$\begin{aligned}
\mathcal{L} = {} & p_{i,s,t}^{c,\max,\text{DA}}\ \mu_{s,t}^{c,\text{DA}} + p_{i,s,t}^{d,\max,\text{DA}}\ \mu_{s,t}^{d,\text{DA}} + e_{i,s,t}^{\max,\text{DA}}\ \mu_{s,t}^{e,\text{DA}} \\
& - \lambda_t^{\text{loc,DA}} p_{i,s,t}^{d,\text{DA}} + \lambda_t^{\text{loc,DA}} p_{i,s,t}^{c,\text{DA}} - \tilde{\lambda}\ e_{i,s,T}^{\text{DA}} - \lambda_{t,\omega}^{\text{loc,RT}} p_{i,s,t,\omega}^{d,\text{RT}} \\
& + \lambda_{t,\omega}^{\text{loc,RT}} p_{i,s,t,\omega}^{c,\text{RT}} - \pi_\omega \tilde{\lambda}\ e_{i,s,T,\omega}^{\text{RT}} - \underline{\theta}_{i,s,t}^{c,\text{DA}} p_{i,s,t}^{c,\text{DA}} - \underline{\theta}_{i,s,t}^{d,\text{DA}} p_{i,s,t}^{d,\text{DA}} \\
& - \underline{\theta}_{i,s,t}^{e,\text{DA}} e_{i,s,t}^{\text{DA}} + \overline{\theta}_{i,s,t}^{c,\text{DA}} \left(p_{i,s,t}^{c,\text{DA}} - p_{i,s,t}^{c,\max,\text{DA}}\right) \\
& + \overline{\theta}_{i,s,t}^{d,\text{DA}} \left(p_{i,s,t}^{d,\text{DA}} - p_{i,s,t}^{d,\max,\text{DA}}\right) + \overline{\theta}_{i,s,t}^{e,\text{DA}} \left(e_{i,s,t}^{\text{DA}} - e_{i,s,t}^{\max,\text{DA}}\right) \\
& + \gamma_{i,s,t}^{e,\text{DA}} \left(-e_{i,s,t}^{\text{DA}} + e_{i,s,t-1}^{\text{DA}} + p_{i,s,t}^{c,\text{DA}} \eta_s^c - \frac{p_{i,s,t}^{d,\text{DA}}}{\eta_s^d}\right)\Bigg|_{t>1} \\
& + \gamma_{i,s,t}^{e,\text{DA}} \left(-e_{i,s,t}^{\text{DA}} + E_{i,s}^{\text{ini}} + p_{i,s,t}^{c,\text{DA}} \eta_s^c - \frac{p_{i,s,t}^{d,\text{DA}}}{\eta_s^d}\right)\Bigg|_{t=1} \\
& - \underline{\theta}_{i,s,t,\omega}^{c,\text{RT}} \left(p_{i,s,t}^{c,\text{DA}} + p_{i,s,t,\omega}^{c,\text{RT}}\right) - \underline{\theta}_{i,s,t,\omega}^{d,\text{RT}} \left(p_{i,s,t}^{d,\text{DA}} + p_{i,s,t,\omega}^{d,\text{RT}}\right) \\
& - \underline{\theta}_{i,s,t,\omega}^{e,\text{RT}} \left(e_{i,s,t}^{\text{DA}} + e_{i,s,t,\omega}^{\text{RT}}\right) \\
& + \overline{\theta}_{i,s,t,\omega}^{c,\text{RT}} \left(p_{i,s,t}^{c,\text{DA}} + p_{i,s,t,\omega}^{c,\text{RT}} - p_{i,s,t}^{c,\max,\text{DA}}\right) \\
& + \overline{\theta}_{i,s,t,\omega}^{d,\text{RT}} \left(p_{i,s,t}^{d,\text{DA}} + p_{i,s,t,\omega}^{d,\text{RT}} - p_{i,s,t}^{d,\max,\text{DA}}\right) \\
& + \overline{\theta}_{i,s,t,\omega}^{e,\text{RT}} \left(e_{i,s,t}^{\text{DA}} + e_{i,s,t,\omega}^{\text{RT}} - e_{i,s,t}^{\max,\text{DA}}\right) \\
& + \gamma_{i,s,t,\omega}^{e,\text{RT}} \left(-e_{i,s,t,\omega}^{\text{RT}} + e_{i,s,t-1,\omega}^{\text{RT}} + p_{i,s,t,\omega}^{c,\text{RT}} \eta_s^c - \frac{p_{i,s,t,\omega}^{d,\text{RT}}}{\eta_s^d}\right)\Bigg|_{t>1} \\
& + \gamma_{i,s,t,\omega}^{e,\text{RT}} \left(-e_{i,s,t,\omega}^{\text{RT}} + p_{i,s,t,\omega}^{c,\text{RT}} \eta_s^c - \frac{p_{i,s,t,\omega}^{d,\text{RT}}}{\eta_s^d}\right)\Bigg|_{t=1}.
\end{aligned} \tag{4}$$

The terms $p_{i,s,t}^{c,\max,\text{DA}}\ \mu_{s,t}^{c,\text{DA}}$, $p_{i,s,t}^{d,\max,\text{DA}}\ \mu_{s,t}^{d,\text{DA}}$, $e_{i,s,t}^{\max,\text{DA}}\ \mu_{s,t}^{e,\text{DA}}$ are obtained from the shared constraints for the physical storage rights (1ga)-(1gc), while the terms $-\lambda_t^{\text{loc,DA}} p_{i,s,t}^{d,\text{DA}}$, $\lambda_t^{\text{loc,DA}} p_{i,s,t}^{c,\text{DA}}$, $-\lambda_{t,\omega}^{\text{loc,RT}} p_{i,s,t,\omega}^{d,\text{RT}}$, $\lambda_{t,\omega}^{\text{loc,RT}} p_{i,s,t,\omega}^{c,\text{RT}}$ come from the power balance equations for DA (1gd) and RT (1ge), respectively. The terms $-\tilde{\lambda}\ e_{i,s,T}^{\text{DA}}$, $-\pi_\omega \tilde{\lambda}\ e_{i,s,T,\omega}^{\text{RT}}$ are obained from the objective function (2a) of the *Optimization Model*. The rest terms of the Lagrangian function are derived from the individual constraints of the player which, as already mentioned, remain the same for all players in both models. One may notice that the KKT optimality conditions to be derived with respect to (4) are identical to (3a). The process can be repeated for all players proving that the *Equilibrium Model* and the *Optimization Model* are equivalent.

Appendix B: Revenue Adequacy

This appendix proves that the *Equilibrium Model* is revenue-adequate. To this purpose, at the optimal solution, we multiply each expression within the physical storage rights equalities (1ga)-(1gc) by $\mu_{s,t}^{c,\text{DA}}$, $\mu_{s,t}^{d,\text{DA}}$, and $\mu_{s,t}^{e,\text{DA}}$, respectively. Similarly, all expressions within the power flow equalities (1gd) and (1ge) are multiplied by $\lambda_t^{\text{loc,DA}}$ and $\lambda_{t,\omega}^{\text{loc,RT}}$ at the optimal solution, respectively. Afterwards, we sum all the obtained equalities, i.e.:

$$\begin{aligned}
& \sum_s \left(p_{s,t}^{c,\text{ri,DA}*} \mu_{s,t}^{c,\text{DA}*} + p_{s,t}^{d,\text{ri,DA}*} \mu_{s,t}^{d,\text{DA}*} + e_{s,t}^{\text{ri,DA}*} \mu_{s,t}^{e,\text{DA}*}\right) \\
& + p_t^{\text{flow,DA}*} \lambda_t^{\text{loc,DA}*} + \sum_\omega p_{t,\omega}^{\text{flow,RT}*} \lambda_{t,\omega}^{\text{loc,RT}*} = \\
& \sum_{i \in \mathcal{I}, s} \Big(p_{i,s,t}^{c,\max,\text{DA}*}\ \mu_{s,t}^{c,\text{DA}*} + p_{i,s,t}^{d,\max,\text{DA}*}\ \mu_{s,t}^{d,\text{DA}*} + \\
& e_{i,s,t}^{\max,\text{DA}*}\ \mu_{s,t}^{e,\text{DA}*}\Big) + \sum_{i \in \mathcal{A}, s} \left[\left(p_{i,s,t}^{d,\text{DA}*} - p_{i,s,t}^{c,\text{DA}*}\right) \lambda_t^{\text{loc,DA}*}\right] + \\
& \sum_{i \in \mathcal{P}} \left[\left(p_{i,t}^{g,\text{DA}*} + \sum_s p_{i,s,t}^{d,\text{DA}*}\right) \lambda_t^{\text{loc,DA}*}\right] + \sum_{i \in (\mathcal{C} \cup \mathcal{R})} q_{i,t}^{\text{DA}*}\ \lambda_t^{\text{loc,DA}*} \\
& + \sum_{i \in \mathcal{A}, s, \omega} \left[\left(p_{i,s,t,\omega}^{d,\text{RT}*} - p_{i,s,t,\omega}^{c,\text{RT}*}\right) \lambda_{t,\omega}^{\text{loc,RT}*}\right] + \sum_{i \in (\mathcal{C} \cup \mathcal{R}), \omega} q_{i,t,\omega}^{\text{DA}*}\ \lambda_{t,\omega}^{\text{loc,RT}*} \\
& + \sum_{i \in \mathcal{P}, \omega} \left[\left(p_{i,t,\omega}^{g,\text{RT}*} + \sum_s p_{i,s,t,\omega}^{d,\text{RT}*}\right) \lambda_{t,\omega}^{\text{loc,RT}*}\right] \quad \forall t.
\end{aligned} \tag{5}$$

where superscript * stands for the optimal values.

According to (5), the total payment of the players for their obtained physical storage rights and the their purchased energy to the market operator for DA and RT, i.e., the right-hand side, equals to the total payment of the market operator to the physical storage rights supplier and to the grid owner. Therefore, the market operator never incurs a financial deficit, i.e., the market is revenue-adequate. Similarly, we can prove the revenue-adequacy for *Optimization Model*.

Appendix C: Parameters and Technical Data for the Case Studies

We provide here the technical data and the parameter values used in the Case studies in Section IV of the main paper. Table IV provides the storage (dis)charge efficiencies, the nominal (dis)charge and capacity rates, the value of lost load, as well as the rest parameters used in the case studies.

TABLE IV
Parameter values

Parameter	Value
Charge efficiency stor. type 1 η_s^c	0.81
Discharge efficiency stor. type 1 η_s^d	0.85
Charge efficiency stor. type 2 η_s^c	0.91
Discharge efficiency stor. type 2 η_s^d	0.95
Nominal dis(charge) rate (same for stor, type 1 and 2) $P_s^{c,\max}, P_s^{d,\max}$ [kW]	10
Nominal capacity (same for stor, type 1 and 2) $E_s^{\max}$ [kWh]	20
Grid line capacity L [kW]	59
Value of lost load V_i [euro/kWh]	4
Value of storage residual energy $\tilde{\lambda}$ [euro/kWh]	0.28

Fig. 5 illustrates the 40 scenarios for production (wind and PV) and energy price (RT), which were generated from historical time series. Specifically, the PV production is a source of uncertainty related with the prosumers. One should notice that the scenarios are not equiprobable but probability-weighted. The scenarios and their probabilities form a discrete approximation of the probability distribution of the data process.

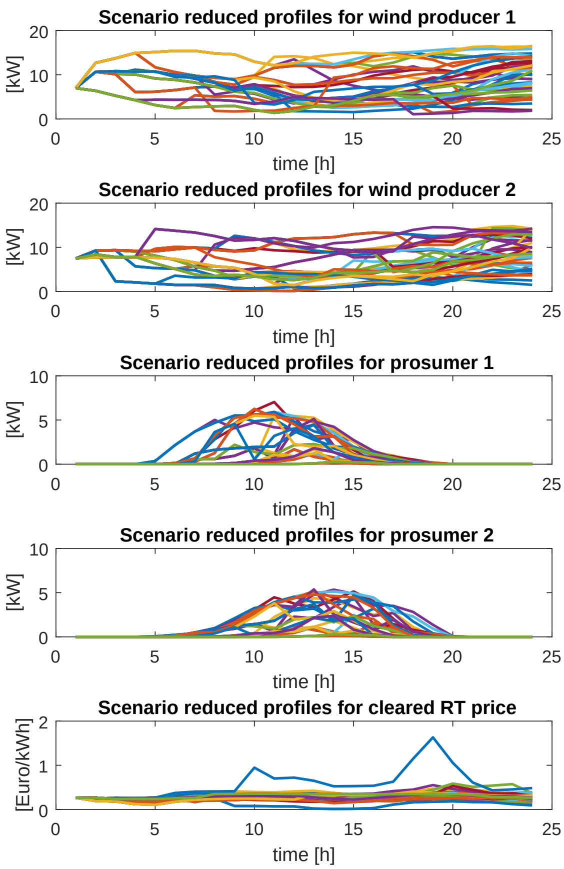

Fig. 5. The 40 scenarios for wind-PV power production and energy price used in the in-sample analysis

In this work, we used a scenario reduction technique based on the study of Growe-Kuska *et al.* ”Scenario reduction and scenario tree construction for power management problems” published in 2003 Bologna Power Tech. IEEE Conference. Specifically, we used this technique to aggregate all five sources of uncertainty into one. That is, a discrete probability has been assigned to each one of the 40 scenarios. Every scenario comprises five 24-hour vectors where each vector corresponds to a specific profile (two vector profiles for wind, two for PV production, and one for the price).

The data used for the out-of-sample simulations are presented in Fig. 6. Specifically, the RT market is cleared for each one of the 365 data vectors (including renewable generation values and RT prices) keeping fixed the DA values obtained from the in-sample analysis in stochastic optimization.

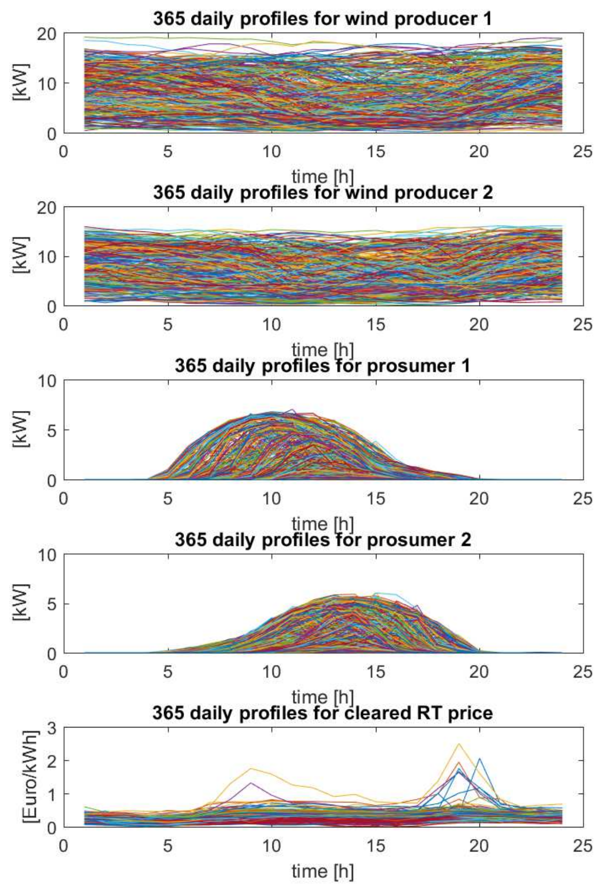

Fig. 6. The 365 daily profiles used in the out-of-sample simulations