Limits Problem Solving in Engineering Careers: Competences and Errors

Veronica Diaz Quezada

Abstract—In this article, the performance and errors are featured and analysed in the limit problems solving of a real-valued function, in correspondence to competency-based education in engineering careers, in the south of Chile. The methodological component is contextualised in a qualitative research, with a descriptive and explorative design, with elaboration, content validation and application of quantitative instruments, consisting of two parallel forms of open answer tests, based on limit application problems. The mathematical competences and errors made by students from five engineering careers from a public University are identified and characterized. Results show better performance only to solve routine-context problem-solving competence, thus they are oriented towards a rational solution or they use a suitable problem-solving method, achieving the correct solution. Regarding errors, most of them are related to techniques and the incorrect use of theorems and definitions of real-valued function limits of real variable.

Keywords—Engineering education, errors, limits, mathematics competences, problem solving.

I. INTRODUCTION

THE training of engineers capable of facing the challenges of the 21st century is a challenge for universities. In recent years, there has been an intensive review of the process of university training in engineering worldwide [1]. For instance, in the framework of the education of the future engineer, the contents and curricular activities which promote competency-based education have taken center stage, which is a trend observed in universities from both, Europe and America, which is a novel challenge for educators and new ways to approach learning, not as a new idiom, but as a necessary paradigm to face both, current and the future society. This competency-based training implies a new curricular design, in which the professional must demonstrate what he or she is capable of doing throughout performance indicators. References [2]-[4] reinforce the thesis that the development of competences itself represents a process of increasing complexity associated to people's performance.

On the other hand, the final interests of undergraduate educational processes are focused on academic results, understood in the framework of quality and approached towards development of capacities and competences. Nonetheless, the lack of understanding of key concepts of Calculus, in the learning of Mathematics in undergraduate education, such as: the notion of limits, and their variations, shows how difficult the conceptualization has, according to [5], come from mathematics rather than didactic.

The limit of a real-valued function of a real variable is an important aspect of the engineering curriculum. Research has shown that the concept of limit is a difficult notion for students [6]-[8]. One of the greatest difficulties comes from the complexity of understanding that all cognitive aspects of limits cannot be learnt starting from its mathematical definition. One of these cognitive aspects is the notion of approximation. After the first contact of students with this concept, they have it through the dynamic conception of limits [9].

According to [10], understanding the concept of the limit of a function is relevant in undergraduate. Researchers [11]-[17] describe a large quantity of students' misconceptions of limits. These researchers have found that students have three principal difficulties to understanding limits and the value of limits. Additionally, students use an incorrect methaphorical reasoning to understand limits [13]-[16], [18], which ends in both, errors in the resolution of exercises and application problems.

The findings of [19] suggest that at the end of the mathematical courses, many students of engineering do not consider the use of a formal definition of limits to solve mathematical problems based on limits. Most definitions of personal limit concepts are inoperable to solve boundary problems and inconsistent with the formal definition of boundary. In spite of its importance, students find too difficult the understanding of the concept of limits [20]. In this context, we conducted a research in engineering careers, whose curriculum is competency-based which includes problem solving; even though, an effective mathematical competence has not been researched yet. The general objective of this study was to identify and characterize the mathematical competences and errors of engineering students in the resolution of problems of limits of real-valued functions. The specific objectives were to elaborate, validate and apply assessment instruments of types of mathematical competences of limits applications in order to determine and analyze the performance of students in types of mathematical skills and to analyze the errors in relation to the types of mathematical skills. The study was guided by the following questions: Do engineering students have the necessary mathematical competences to solve problems of application of limits of a function? Which are the biggest errors made when solving problems of limits of a real-valued function of a real variable? From this research problem, and to provide an answer to these questions, we designed a study for five engineering careers.

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II. THEORETICAL FRAMEWORK

In the framework of formal mathematics education, students will face problems. These problems may come from mathematics itself, as well as real life, [21] that involve facts and contexts that can be modeled in mathematics. When the student is ready to provide a strategy to get a solution to a mathematical problem, the enquiry is not a problem but an exercise [22]. According to [23], mathematical thinking is developed throughout formation and development of habits which are necessary in problem solving [24].

A. Types of Mathematical Competences

For this work, we took the types of mathematical competences for problem solving, from the authors [25], [26]. Hereafter, it is presented the classification of types of mathematical competences by [27] which is part of the theoretical framework. Three types of competences will be considered, as described in Fig. 1.
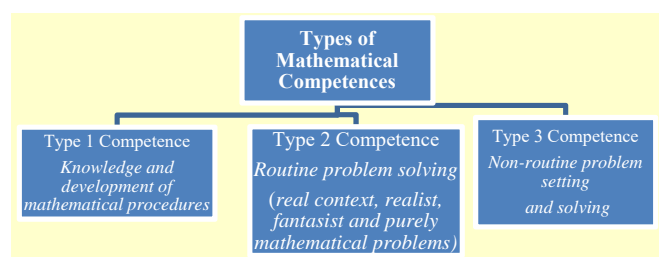

Fig. 1 Types of mathematical competences

1) *Type 1 Competence*: Knowledge and development of mathematical procedures, which include comprehending and managing the extension of mathematical concepts and mathematical argumentation. Basically, it consists of problems with calculation and definitions of most common type that appear in conventional evaluation of mathematics.
2) *Type 2 Competence*: Routine problem solving includes to propose, formulate and resolve types of routine problems in real, realistic, fantasist and purely mathematical context. The routine problems are similar to those resolved during instruction courses; the student follows a sequence that implies understanding and algorithms to outcome valid solutions.
   Context problems:
   a) *Real context problem*: A context is real if it is produced in reality and compromises the actions of the student in it.
   b) *Realistic context problem*: A context is realistic if it is susceptible to be produced. It is about a simulation of reality or a part of it.
   c) *Fantasist context problem*: A context is fantasy if it is fruit of imagination and is unfounded in reality.
   d) *Purely mathematical context*: A context is purely mathematical if makes exclusive reference to mathematical objects: numbers, relations and arithmetic operations, geometric figures, etc.
3) *Type 3 Competence:* Non-routine problems setting and solving, includes the decoding of diverse forms of

presenting mathematical situations, translating from natural language into symbolic/formal, that is to say, it consists of mathematical thinking which includes capacity of generalization. A problem is non-routine if a student does not know a pre-established answer, procedure or routine to find the answer.

It should be noted that non-routine problems may also be classified according to context.

B. Errors in Mathematics

It is a reality reckoned by studies in mathematics education, that students make mistakes in their performance when working in any mathematical domain, specially, when solving application problems. Several researchers have found that errors and wrong ideas that students present in their attempts to solve mathematical problems contribute to perpetuate their low performance when learning mathematics [28]-[33].

In general, the wrong concepts are presented through errors. An error might be an error, a calculation error or a wrong judgement, and such category underlines non-systematic errors [34]. The challenge related to wrong concepts is that many people present difficulties to quit wrong concepts, because false concepts may be deeply anchored in an individual's mind map. Due to importance of errors associated to learning, diverse categorizations have been proposed.

The authors [35] consider the research of errors as necessary, even to find out if a certain teaching style is associated to certain errors, in particular. For the purposes of the theoretical framework of this research, it has been considered the category proposed by these authors which is proposed hereafter:

1) *Errors due to misused data*: It includes errors that may be related to a disagreement between data from the problem and how the student processed it.
2) *Errors due to language misinterpretation*: It includes errors that arise from mistranslation of mathematical facts to common language, and vice versa.
3) *Errors due to non-valid inferences on logic*: It includes errors made by incorrect reasoning. This novel invalid information is then used to solve the problem set, causing a wrong answer.
4) *Errors due to use of theorems or deformed definitions*: It includes the errors that appear due to a distortion of a principle, a rule, theorem or definition. In this category, there are errors by theorem applications without the necessary conditions due to application of wrong properties, due to the application of an incorrect validation of a definition, theorem or formula.
5) *Errors due to lack of solution verification:* It includes errors made during the final result yet not in the process, in other words, every step taken by the examinee is right itself yet the final result, as it is presented, is not the solution for the given problem. In this category there are included the errors that, if there were any verification, by the student, they would had been discarded.
6) *Technical errors*: It includes calculation errors, the errors in data taken from the tables, the errors in use of

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elemental algebraic symbols, among others.

III. METHODOLOGY

A qualitative methodology has been used, which was framed in the descriptive and exploratory [36], due to lack of similar researches registered to support the appropriation of mathematical competences for problem solving, in competency-based engineering careers.

The study was conducted at Los Lagos University, campus Osorno and Puerto Montt and considered a 51 students non-probabilistic intended sample, corresponding to four semesters of environmental engineering (4), IT civil engineering (6), civil industrial engineering (10) all from campus Puerto Montt, and IT civil engineering (9), business engineering (22) from campus Osorno. This intended selection is based on the fact that all of them were taking Calculus I (Differential and Integral Calculus in one variable) during second semester in 2018, whose curriculum approach, in this institution, is competency-based and problem solving is part of the assessment protocol. The subject students correspond to groups already formed and all working with different Calculus I teachers, ad port as the end of semester, which means, that all of them had already been assessed in Limits. This program includes numbers and the real line, limits, continuity, derivatives and integrals, with the use of Geogebra or MATLAB software in most units.

A. Instruments

With the objective of assessing the students' performance in types of mathematical competences, a quantitative instrument has been used: a mathematical knowledge test of the limit of a real function of a real variable based on open answer problems and to follow two parallel forms (Form A and Form B). It has been validated previously by content, through judgment from ten experts and piloted in such way that the final test included only problems with the 75% of positive results, or upper, whose results were used to elaborate final versions about application problems of real-valued function limits.

For the application of the test in November 2018, in each form, students had 2 hours and 30 minutes to take it. Tests were applied in regular classrooms and schedules for Calculus I and with a difference of one week for each test form. Hereafter, Table I shows the distribution of problems from test in forms A and B, according to the classification of types of mathematical competences [27] with seven problems and with equal type of competence in each form of the test.

TABLE I
DISTRIBUTION ACCORDING TO TYPES OF MATHEMATICS COMPETENCES,
FORM A AND FORM B

Problem	Types of mathematics competences
Problem 1	Type 2 competence: realist context
Problem 2	Type 2 competence: fantasist context
Problem 3	Type 3 competence
Problem 4	Type 2 competence: realist context
Problem 5	Type 1 competence
Problem 6	Type 3 competence
Problem 7	Type 2 competence: fantasist context

The assessment of student's performance, was considered in relation to the student's advance degree in mathematical problem solving, and it has been estimated according to the grading scale [27]. A five-point scale has been associated with this scale, which indicates the students' progress levels towards the correct solution of the problems. This score scale registers every detail in the students' attempt to find the solution, and it is presented hereafter in Table II.

TABLE II
PROBLEM-SOLVING GRADING SCALE

Score	Solution stages
0	*No Start*: The student is unable to start the problem or delivers work which is meaningless
1	*Focus*: The student focuses the problem with a meaningful work, indicating comprehension of the problem, yet faces difficulties easily.
2	*Substance*: Sufficient details show that the student has been oriented to a rational solution, yet relevant error s or wrong interpretations prevent the process of the correct resolution.
3	*Result:* The problem is about to be resolved, yet few mistakes lead to a wrong final solution.
4	*Completion:* The proper method has been used and it has led to the correct solution.

IV. RESULTS

A. Problem Solving Tests

In *Type 1 Competence*: Knowledge and development of mathematical procedures, it has been included a problem for each form of the test.
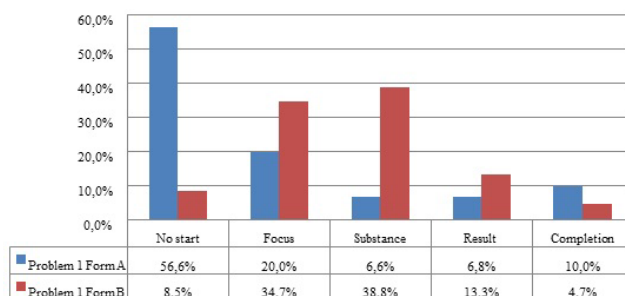

Fig. 2 Percentage of performance in type 1 competence

Fig. 2 shows the students' development of five engineering in problem N°5 in both forms of the test. Most students present a low performance, with a 65.1% of them who never started the solution of the problem. The 45.4% of the students achieve to approach towards the rational solution and proceed properly, nevertheless, in both cases, only 14.7% achieved the correct solution of the problem.

In *Type 2 Competence:* Resolution of routine problems, there were included four routine context problems in each form of the test: problem 1 and 4 realistic routine context, and problems 2 and 7 fantasist routine context.

In relation to type 2 competence, form A, according to Fig. 3, the performance in students was high in problems N°2 and N°7, both routine-fantasist problems, achieving a 66.6% and 49.3% of achievement respectively. In general, in these problems, the students are oriented towards a rational solution, and show the problem nearly to be finished, or use a proper work method, achieving a correct solution.

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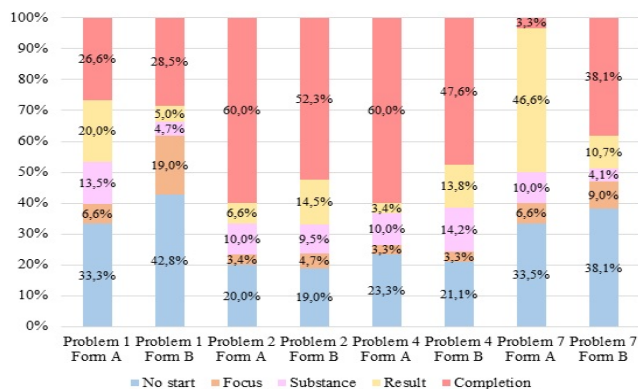International Journal of Educational and Pedagogical Sciences
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Fig. 3 Percentage of performance in type 2 competence form A and form B

Concerning the type 2 competence, of form B, according to Fig. 3, students' performance was again high in problems N°. 2 and N°. 7, both in routine fantasist context, achieving a 66.8 % and 48.8% respectively, when considering the result and completion stages, since in these problems they only had either small solution errors, or the correct solution. Below, there is one of the fantasist context routine problems from form A, and an example of a student's response.

"The bank offers the credit card "Master Plop". Through data obtained in the past, they have determined that the percentage of collection of the ones given in one month is in function of time past after granting them. This function is:

$$P = f(t) = 0,9\left(1 - 3^{-0,08t}\right)$$

P is percentage of accounts receivable "t" months after granting the card. Which percentage is expected to be collected after 2 and 5 months?. If the number of months past from the "Master Plop Card" granting time growths indefinitely, determine the percentage expected to be collected".
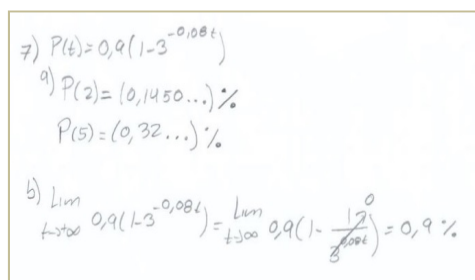

Fig. 4 Answer by a student to problem 7

In what concerns to the students' answer, although he/she understands the problem, he/she leaves the decimals expressed as percentages, he/she does not present full development of the problem. It should be noted that the fantasist context routine problems are fruit of imagination and are designed without any foundation of reality, never the less they were better approached and answered by the students.

In type 3 competence approach and resolution of non-routine problems, two problems were included in each form of the test: problems 3 and 6. Next, Fig. 5 shows the results

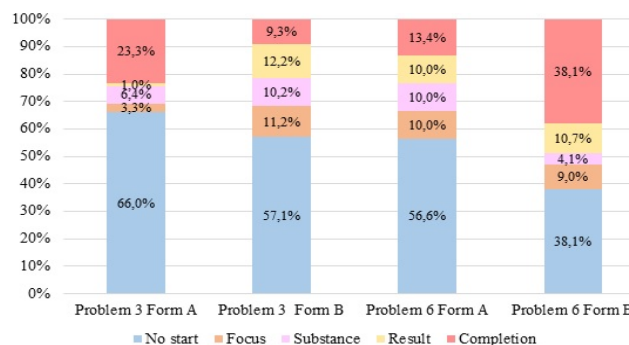obtained by the study-subject students of the five engineering careers.

Fig. 5 Percentage of performance in type 3 competence form A and form B

In general, the engineering students showed a very low level of performance in the limit problem solving of real-valued function test form A. In problem 3 and problem 6, the 66% and the 56.6% respectively, did not achieve any type of solution or delivered an incorrect development, staying in non-developed stage. Between both stages, the 13.3% achieved to approach, significantly indicating comprehension of the problem, yet only 36.7 of them could achieve the correct solution, achieving the completion stage.

According to Fig. 5, in test form B about limit problem solving of a real-valued function, the students of the 5 engineering careers showed a low level of performance, yet there were small differences which favored those in form A. In problem 3 and problem 6, the 57.1% and the 38.1% respectively, did not achieved any type of solution, staying in non-developed stage. Between both problems, the 14.3% was oriented towards a rational solution, yet crucial errors prevented the achievement of a correct solution, and finally the 47.4% of them achieved the solution, staying in completion stage.

B. Errors and Types of Mathematical Competences

The students' errors have been identified, and the analyzed data has been grouped according to the [35] classification of errors, and the problems according to the type of mathematical competence by [27]. The following results have been obtained.

Considering results from Table III, the knowledge and development of mathematical procedures type 1 competence presents the highest connection to errors due to the use of theorems and deformed definitions. In this competence, the students from the five engineering careers have to face problems which are not necessarily related to daily life context; hence to solve them, they needed to manage all concepts and theorems about real-valued functions limits. The career that presented the highest percentage of errors was IT civil engineering campus Osorno.

It shall be noticed that the assessment in this type of competences, the 100% of students from the five engineering careers do not present errors due to misused data, nor errors

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due to misinterpretation of language.

TABLE III
RELATION BETWEEN ERRORS AND TYPE 1 COMPETENCE

				Misused data	Incorrect interpretation of the language	Inferences logically invalid	Use of theorems and deformed definitions	Lack of verification of the solution	Technical errors
Type 1 Competence	Environmental Engineering Puerto Montt	Problem	N°5				25%		
	Industrial Civil Engineering Puerto Montt	Problem	N°5				36.4%		9.1%
	IT Civil Engineering Osorno	Problem	N°5			22.2%	44.4%	22.2%	22.2%
	IT Civil Engineering Puerto Montt	Problem	N°5				60%		40%
	Business Engineering Osorno	Problem	N°5				22.7%		

TABLE IV
RELATION BETWEEN ERRORS AND TYPE 2 COMPETENCE

				Misused data	Incorrect interpretation of the language	Inferences logically invalid	Use of theorems and deformed definitions	Lack of verification of the solution	Technical errors
Type 2 Competence	Environmental Engineering Puerto Montt	Problem	N°1			25%	25%		50%
			N°2	25%	25%			50%	25%
			N°4						25%
			N°7						25%
	Industrial Civil Engineering Puerto Montt	Problem	N°1			18.2%	18.2%	18.2%	27.3%
			N°2	9.1%	9.1%			18.2%	9.1%
			N°4				9.1%	9.1%	27.3%
			N°7	9.1%			9.1%		
	IT Civil Engineering Osorno	Problem	N°1	11.1%	11.1%	44.4%	33.3%	33.3%	44.4%
			N°2		11.1%		11.1%		11.1%
			N°4		11.1%		11.1%	11.1%	11.1%
			N°7	11.1%			77.8%		11.1%
	IT Civil Engineering Puerto Montt	Problem	N°1				20%	60%	60%
			N°2						
			N°4				20%		
			N°7		20%		20%		
	Business Engineering Osorno	Problem	N°1			13.6%	40.9%	4.5%	18.2%
			N°2	4.5%	4.5%	4.5%	4.5%		13.6%
			N°4			4.5%		9.1%	13.6%
			N°7	13.6%	13.6%		18.2%		4.5%

According to Table IV, type 2 competence about routine problem solving presents a strong relation in higher frequency and percentage only with technical mistakes, which also agrees with the students' performance in the test, since in this competence, the students face real life and fantasist context problems, which due to the connection to daily life context allow major comprehension and favors the resolution of them, except for small mistakes which lead to wrong answer. Likewise, errors are detected due to lack of verification of solution.

The career that presented more errors was business engineering campus Osorno, and the career that presented less was IT civil engineering campus Puerto Montt.

In what refers to type 3 competence about approach and non-routine problem solving, according to Table V, errors can be seen in major degree due to the use of theorems or deformed definitions and technical errors. This agrees with the students' performance in both forms of the test, as a non-routine problem, the method to be solved cannot be determined directly, hence it requires an experienced, mature reasoning to understand the real purpose of the problem given. It must be pointed out that the careers which presented more errors in this type of competence were industrial civil engineering campus Puerto Montt, and business engineering campus Osorno. A small number of students made mistakes due to lack of verification of the solution.

V. DISCUSSION AND CONCLUSIONS

The objective of this study was to identify and feature the mathematical competences and errors made by engineering students in limits problem solving of a real-valued function. Nevertheless, this study evidences that the teaching of mathematics in engineering careers is so far being taught in a traditional method and oriented to an objective approach. This study reveals how necessary is to install a competency-based curriculum, methodology and approach, in order to revert these low results and to improve the standards of future

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engineers.

TABLE V
RELATION BETWEEN ERRORS AND TYPE 3 COMPETENCE

			Misused data	Incorrect interpretation of the language	Inferences logically invalid	Use of theorems and deformed definitions	Lack of verification of the solution	Technical errors
Type 3 Competence	Environmental Engineering Puerto Montt	Problem N°3				11.1%		
		N°6						25%
	Industrial Civil Engineering Puerto Montt	Problem N°3				27.3%		
		N°6	18.2%	18.2%	27.3%	27.3%		36.4%
	IT Civil Engineering Osorno	Problem N°3			11.1%	55.6%	11.1%	11.1%
		N°6				44.4%		
	IT Civil Engineering Puerto Montt	Problem N°3				40%		40%
		N°6				20%		20%
	Environmental Engineering Puerto Montt	Problem N°3				18.2%		4.5%
		N°6	4.5%	9.1%	4.5%	18.2%		9.1%

The students from five engineering careers presented similar results. They all showed a better performance in type 2 competences about realistic and fantasist routine context problems. The lowest performances were presented in type 1 competence knowledge and development of mathematical procedures and in type 3 competence about non-routine problem solving. In type competence 1, 65.1% stays in non-development stage, and only 34.8% grasps the right result. Basically, they correspond to problems that recurrently form part of the conventional evaluations in Calculus. They only involve the understanding and application of limit definitions of a real variable function and theorems associated with them.

With respect to type 3 competition, more than 50% of the students could not begin to solve these problems. Our results coincide with those of [37], when it indicates that students do not have the skills to solve non-routine problems or with higher levels of difficulty. Unlike routine problems that require only regular calculation applications and have already practiced them. The non-routine problems do not have a direct way to approach the question, yet they require the use of creative thinking and strategies, so the problem can be understood and as a consequence to find the right way to solve it [38]. Thus, non-routine problems tend to be more complex and difficult than routine problems. The authors [39] declared that the problem solving is taught independently from the basic tools and basic thinking. In time, students build a repertoire of techniques for problem solving. In the end, the difference between someone who is good and one who is not good at solving non-routine problems is the capacity to solve novel problems. Even more, the more experienced student may find common a non-routine problem.

In relation to errors and type of mathematics competences, the students in general present the use of deformed theorems and definitions, and technical errors. In this category, there are errors due to the application of theorems without the necessary conditions, due to the application of incorrect properties and due to a disvalued use of a definition, theorem or formula. It was also registered important frequency of technical errors that include calculation, data extraction from the tables, in the use of elemental algebraic symbols, among others.

We are conscious that obtained results cannot be generalized by the type of qualitative study, yet match with researches which point out that when mathematics are isolated from the use in engineering, students lose a chance to engage with the perception of the real value of its applicability, in a wide range. Even more, the students' perceptions would improve significantly if the undergraduate engineering programs included proper examples for the application of mathematics in engineering [40].

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