THE TRUE PROBABILITY OF A CONFIDENCE INTERVAL

Giacomo Lorenzoni
qualifications/experience: http://orcid.org/0000-0002-2329-2881
email: info@giacomo.lorenzoni.name; info@pec.giacomo.lorenzoni.name

ABSTRACT

The calculation of a confidence interval, which together with the hypothesis testing is the best known procedure of inferential statistics, has as result the probability that a certain statistical parameter is contained in a certain part of the real line. However, this result does not enjoy of unanimity because it is widely believed the not be strictly a probability and that must be called only confidence. To this is added the perplexity of being able to replace, as is highlighted in the article, the said probability with many other equally reliable.

These uncertainties are tackled by distinguishing, among all those of the same event, only one probability true and therefore not merely conventional, and then choosing, as result of the determination of a confidence interval, the true inherent probability which, although it is not exactly calculable, however is unlimitedly approximable.

For this purpose, it is preliminarily dedicated much care in defining the symbology and the concepts of logic and set theory needed for the subsequent deductions, substantially taking again notions of [1] such as the original algorithmic definitions of relations and operations between sets, the unusual formulation concerning the equality between the intersection of products and the product of intersections, and an expanded form of the important tautology that includes the known "law of contraposition".

The treatment of events and probabilities exposed in [1] is summarized, simplified and integrated by new decisive positions. It is thoroughly analyzed the event constituted by the happen an unknown constant into a certain part of the real line and its probability, because fundamental for the treatment of the confidence interval which is then deduced and specified in detail for the two cases, of great importance in the experimental sciences, that are had when the statistical parameter is the mean or the variance of a normal random variable.

1. INTRODUCTION

The calculation of a confidence interval is, together with the hypothesis testing, the more known procedure of inferential statistics. This procedure determines, by means of a sample, a confidence (i.e. probability) that a parameter of the inherent population is contained in an arbitrary part (e.g. interval) of the real line.

However the perplexity of this probability is immediately revealed by the fact that the replacement of the said sample with an its subset determines a probability generally different and equally credible of the same event. Moreover many authors believe that the confidence in question is not a real probability (but, in truth, with arguments that do not seem decisive in the face of logical coherence of the following deductions).

In this work a remedy to this situation is achieved by defining a probability that, among all those of the same event, stands out as true and hence not is merely conventional, and that in the case of the confidence interval, also if not is exactly calculable, is however unlimitedly approximable.

1 PRELIMINARIES OF LOGIC AND SET THEORY

In relation to the following logical concepts, reference is made to [1], [2], [3], [4].

A proposition is a set of graphic symbols. A name is a proposition that relates and represents a certain object, which alone expresses a meaning (e.g. "home") or not (e.g. "A"), and which attributes to such object the properties indicated by its eventual meaning. An object is identified by the set of all its properties.

An $\mathrm{A} \equiv \mathrm{B}$ affirms that A and B are two names of a same object and thereby reciprocally replaceable. Consequently an $\mathrm{A} \equiv \mathrm{B}$ implies that A has also the possible meaning of B (and *vice versa*).

A pairing of two names A and B is a third name (e.g. $\mathrm{A_B}$) that has both meanings of the other two, therefore if A has a meaning then this is also of $\mathrm{A_B}$ (and analogously for B).

In identifying the members of an expression, each "≡" is considered, coherently with the parentheses, at last (and analogously "≢", "=", "≠"). Is intended $\text{ş}\langle\text{ş}\rangle \equiv \text{Ş}_{\text{ş}}$, $\wedge \equiv \mathrm{AND} \equiv$ "conjunction", $\vee \equiv \mathrm{OR} \equiv$ "inclusive disjunction", $\dot\vee \equiv \mathrm{XOR} \equiv$ "exclusive disjunction".

Being $\mathcal{P}$, $\mathcal{P}_\mathrm{A}$ e $\mathcal{P}_\mathrm{B}$ three propositions, is meant, coherently with the use of parentheses "$\{\}$" or "$\lbrace\!\lbrace\ \rbrace\!\rbrace$" to delimit respectively a generic proposition or that defines an event, $\lbrace\!\lbrace \mathcal{P} \rbrace\!\rbrace \equiv$ "$\mathcal{P}$ is true", $\neg\lbrace\!\lbrace \mathcal{P} \rbrace\!\rbrace \equiv$ "$\mathcal{P}$ is false", $\neg\mathcal{P}$ the proposition true if $\neg\lbrace\!\lbrace \mathcal{P} \rbrace\!\rbrace$ and false if $\lbrace\!\lbrace \mathcal{P} \rbrace\!\rbrace$, $\{\mathcal{P}_\mathrm{A} \equiv \mathcal{P}_\mathrm{B}\} \equiv \{\neg\mathcal{P}_\mathrm{A} \equiv \neg\mathcal{P}_\mathrm{B}\}$,

$\{\mathcal{P}_A \mid \mathcal{P}_B\} \equiv$ "$\mathcal{P}_A$ subjected to the condition $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ of which $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ where $\mathcal{P}_B$"

"Is implicit $\mathcal{P}_B$" $\equiv \{\mathcal{P}_A \equiv \{\mathcal{P}_A \mid \mathcal{P}_B\};\ \forall \mathcal{P}_A\}$ (1)

and $\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$ a set of propositions from which is logically deducible $\mathcal{P}_B$ being such propositions all true except $\mathcal{P}_A$ that can be true or false.

Indicating $\Rightarrow$ and $\underline{\rightarrow}$ the two logical connectives called respectively *entailment* or *logical implication* or *logical consequence* and *material conditional* or *material implication* or *material consequence*, is placed

$\{\mathcal{P}_A \Rightarrow \mathcal{P}_B\} \equiv \{\mathcal{P}_B \Leftarrow \mathcal{P}_A\} \equiv \exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle \equiv$ "$\mathcal{P}_B$ is logically deducible from $\mathcal{P}_A$" $\equiv$ "An argumentation leads from $\mathcal{P}_A$ to $\mathcal{P}_B$" $\equiv$

"$\mathcal{P}_B$ is logically demonstrable starting from $\mathcal{P}_A$" $\equiv$ "$\mathcal{P}_B$ is a logical consequence of $\mathcal{P}_A$"

$\{\mathcal{P}_A \Leftrightarrow \mathcal{P}_B\} \equiv \{\mathcal{P}_A \Rightarrow \mathcal{P}_B\} \wedge \{\mathcal{P}_A \Leftarrow \mathcal{P}_B\}$ $\{\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B\} \equiv \{\mathcal{P}_B \underline{\leftarrow} \mathcal{P}_A\} \equiv \{$"$\{\mathcal{P}_A\} \Rightarrow \{\mathcal{P}_B\}$"$\} \underline{\leftarrow} \{\mathcal{P}_A \Rightarrow \mathcal{P}_B\}$

$\{\mathcal{P}_A \underline{\leftrightarrow} \mathcal{P}_B\} \equiv \{\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B\} \wedge \{\mathcal{P}_A \underline{\leftarrow} \mathcal{P}_B\} \equiv \{\{\mathcal{P}_A\} \Leftrightarrow \{\mathcal{P}_B\}\} \equiv \{\mathcal{P}_A \equiv \mathcal{P}_B\}$ (2)

$\{$from: $A_1;\ A_2;\ldots;\ A_{ɨ};$ follows $B_0{\circ}\square_1{\circ}B_1{\circ}\square_2{\circ}B_2\ldots{\circ}\square_{ɨ}{\circ}B_{ɨ}{\circ}\square_{ɨ+1}{\circ}B_{ɨ+1}\ldots{\circ}\square_{ɨ+j}{\circ}B_{ɨ+j}\} \equiv \{A_1 \underline{\rightarrow} \{B_0{\circ}\square_1{\circ}B_1\};\ A_2 \underline{\rightarrow} \{B_1{\circ}\square_2{\circ}B_2\};\ldots;\ A_{ɨ} \underline{\rightarrow} \{B_{ɨ-1}{\circ}\square_{ɨ}{\circ}B_{ɨ}\}\}$

where: each of $\{{\circ}\square_1,{\circ}\square_2{\circ},\ldots,{\circ}\square_{ɨ+j}\}$ is a generally different relational symbol, as for example one of $\{\equiv,\not\equiv,=,\neq\}$; $\{{\circ}\square_{ɨ+1}{\circ}B_{ɨ+1}\ldots{\circ}\square_{ɨ+j}{\circ}B_{ɨ+j}\}$ may be absent and if is present the validity of its presence is considered evident; each of $\{A_1,A_2,\ldots,A_{ɨ}\}$ is replaced by symbol "ƀ" when is considered evident (or is highlighted after) the validity of the corresponding element of $\{\{B_0{\circ}\square_1{\circ}B_1\},\{B_1{\circ}\square_2{\circ}B_2\},\ldots,\{B_{ɨ-1}{\circ}\square_{ɨ}{\circ}B_{ɨ}\}\}$.

A $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ is a $\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$ of which is considered conventionally only $\mathcal{P}_A$, in the sense that all its propositions certainly true (i.e. all except $\mathcal{P}_A$) are implicitly treated as such and are then contextually ignored as obvious. This highlights immediately $\{\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B\} \underline{\rightarrow} \exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$. Moreover this identity of $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ and the always possible faculty of considering as said conventionally the only $\mathcal{P}_A$ of a $\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$ show also $\exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle \underline{\rightarrow} \{\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B\}$. Therefore is had $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv \exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$. This and $\mathcal{P}_A \Rightarrow \mathcal{P}_B \equiv \exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle$ entail $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv \mathcal{P}_A \Rightarrow \mathcal{P}_B$.

In conformity with the (2.1.1.1) of [1] is had

$\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv \neg\mathcal{P}_B \underline{\rightarrow} \neg\mathcal{P}_A \equiv$ "$\{\mathcal{P}_A\}$ is sufficient for $\{\mathcal{P}_B\}$" $\equiv$ "$\{\mathcal{P}_B\}$ is necessary for $\{\mathcal{P}_A\}$" $\equiv$ "$\{\mathcal{P}_B\}$ if $\{\mathcal{P}_A\}$" $\equiv$ "$\{\mathcal{P}_A\}$ only if $\{\mathcal{P}_B\}$" $\equiv$

$\{\mathcal{P}_A \equiv \{\mathcal{P}_A \mid \mathcal{P}_B\}\} \equiv \{\mathcal{P}_B;\ \forall\mathcal{P}_A\} \equiv \exists\underline{\boldsymbol{\mathcal{P}}}\langle \mathcal{P}_B \,\vert\, \mathcal{P}_A\rangle \equiv$ "from $\mathcal{P}_A$ follows $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ entails $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ show $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ gives rise to $\mathcal{P}_B$" $\equiv$

"$\mathcal{P}_A$ highlights $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ implies $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_B$ is due to $\mathcal{P}_A$" $\equiv$ "$\mathcal{P}_B$ is obtainable from $\mathcal{P}_A$" $\equiv$ "$\mathcal{P}_B$ is a direct consequence of $\mathcal{P}_A$" (3)

whose $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv$ "$\{\mathcal{P}_A\}$ is sufficient for $\{\mathcal{P}_B\}$" $\equiv$ "$\{\mathcal{P}_B\}$ is necessary for $\{\mathcal{P}_A\}$" is in [5], whose parentheses "$\{$" and "$\}$" can evidently be removed without risk of misunderstandings, and that, on the basis of $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv \mathcal{P}_A \Rightarrow \mathcal{P}_B$, includes the tautology $\mathcal{P}_A \Rightarrow \mathcal{P}_B \equiv \neg\mathcal{P}_B \Rightarrow \neg\mathcal{P}_A$ known as *law of contraposition* (a tautology is a proposition always true anyway are changed its variable arguments).

The (3) and (2) give rise to

"$\mathcal{P}_A$ is necessary and sufficient for $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ if and only if $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ is equivalent to $\mathcal{P}_B$" $\equiv$ "$\mathcal{P}_A$ means $\mathcal{P}_B$" $\equiv \{\mathcal{P}_A \equiv \mathcal{P}_B\}$ (4)

whose subscripts are exchangeable in each of the four members.

The (3) entails that $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ give rise to $\neg\{\mathcal{P}_A\}$, and hence entails also the kind of argumentation known as *demonstratio per absurdum* and consisting in the deduce $\{\mathcal{P}_A\}$ from $\neg\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ or $\neg\{\mathcal{P}_A\}$ from $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ (and consistent thus ultimately in the establish false a $\mathcal{P}_A$ which implies a $\mathcal{P}_B$ false).

Is implicit

$Æ\langle A / B / C\rangle \equiv \{$the being A a specification of B of which C$\}$

where "$/C$" may be absent causing so the absence of "of which C".

It is said that B is a specification of A for understand that B has all the properties of A. So, on the base of first three paragraphs of this section, A_B is a specification of A if this name has a meaning. From: this; (2.1.1.3) of [1]; follows

$Æ\langle A / B\rangle \equiv \{A \equiv \{A \wedge B\}\} \equiv \{A \underline{\rightarrow} B\}$ (5)

where is intended that A is a name which has a meaning.

In relation to the following concepts of set theory reference is made to [1], [6], [7], [3], [8].

Intending $\{ş_i;\ i=1,ɨ\} \equiv \{ş_1,ş_2,\ldots ş_{ɨ}\} \equiv \wedge_{i=1,ɨ}(ş_i)$, a sequence and a set, both made up of ɨ elements, are respectively indicated $(ş_i;\ i=1,ɨ)$ and $\{ş_i;\ i=1,ɨ\}$, and they differ because in the second case it is irrelevant to the order defined by $\{a < b\} \equiv \{ş_a$ precedes $ş_b\}$ and called sequential such as the one typically own of every sequence. Therefore, a sequence is also a set but not *vice versa*. Is indicated $\{ş / \mathcal{P}\}$ a set consisting of all the different specifications of ş contextually possible when there is the condition $\mathcal{P}$. Is implicit

$\{i=1,\text{ɨ}\}\equiv\{i;\ i=1,\text{ɨ}\}$.

Is meant $\mathfrak{N}\langle\underline{A}\rangle$ the numerosity of the set $\underline{A}$ i.e. the number of elements that constitute $\underline{A}$, $\mathfrak{E}\langle\underline{A}\rangle\equiv\{\text{ş}\,|\,\text{ş}\in\underline{A}\},\neg\underline{A}$ the set of elements that do not belong to $\underline{A}$, $\varnothing$ the empty set since $\mathfrak{N}\langle\varnothing\rangle=0$, $\neg\varnothing$ the set constituted by each element.

The equality of the sets $\underline{A}$ and $\underline{B}$ is indicated $\underline{A}=\underline{B}$ and affirms that every element of $\underline{A}$ is also a element of $\underline{B}$ and *vice versa*. The addition of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A}+\underline{B}$ and constituted by all the elements of $\underline{A}$ and all the elements of $\underline{B}$. The intersection of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A}\cap\underline{B}$ and constituted by each element that belongs both to $\underline{A}$ and $\underline{B}$. The difference between $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A}-\underline{B}$ and constituted by each element of $\underline{A}$ that do not also belongs to $\underline{B}$. The union of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A}\cup\underline{B}$ and constituted by each element that belongs to $\underline{A}$ but not to $\underline{A}\cap\underline{B}$, or to $\underline{B}$ but not to $\underline{A}\cap\underline{B}$, or to $\underline{A}\cap\underline{B}$. The Cartesian product of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A}\times\underline{B}$ and constituted by each different pair which can be made by choosing its elements respectively belonging to $\underline{A}$ and $\underline{B}$.

These definitions, intending $\underline{A}\equiv\{A_h;\ h=1,\text{ħ}\}$ and $\underline{B}\equiv\{B_k;\ k=1,\text{ꝁ}\}$, are specified by

$$\{\underline{A}=\underline{B}\}\equiv\{\mathbf{i}_{ABh}=1;\ h=1,\text{ħ}\}\wedge\{\mathbf{i}_{BAk}=1;\ k=1,\text{ꝁ}\}\qquad \underline{A}\cap\underline{B}\equiv\{\{A_h\,|\,\mathbf{i}_{ABh}=1\};\ h=1,\text{ħ}\}\qquad \underline{A}-\underline{B}\equiv\{\{A_h\,|\,\mathbf{i}_{ABh}=0\};\ h=1,\text{ħ}\}$$
$$\underline{A}\cup\underline{B}\equiv\{\underline{A}+\underline{B}\}-\{\underline{A}\cap\underline{B}\}\tag{6}$$

whose $\{\mathbf{i}_{ABh};\ h=1,\text{ħ}\}$ is determined by the following steps (and similarly $\{\mathbf{i}_{BAk};\ k=1,\text{ꝁ}\}$):

- it is placed $\{\mathbf{i}_{ABh}=0;\ h=1,\text{ħ}\}$;
- they are carried out the $\mathfrak{N}\langle\underline{B}\rangle$ iterations indicated by $\{k=1,\text{ꝁ}\}$;
- at the k-th iteration is searched for a $h\in\{h=1,\text{ħ}\}$ that verifies the $\{\mathbf{i}_{ABh}=0, A_h\equiv B_k\}$ and is placed $\mathbf{i}_{ABh}=1$ if there is a such $\{h\in\{h;\ h=1,\text{ħ}\}\,|\,\mathbf{i}_{ABh}=0, A_h\equiv B_k\}$.

A $\underline{A}\cap\underline{B}\neq\varnothing$ implies that at least one of the two sets $\{\underline{A},\underline{B}\}$ is the addition of a subset whose elements are also elements of the other set and of another subset that does not has this property. Being then such addition and $\underline{A}\cap\underline{B}\neq\varnothing$ respective specifications of $\mathcal{P}_B$ and $\mathcal{P}_A$ in (3), is had a *demonstratio per absurdum* of $\underline{A}\cap\underline{B}=\varnothing$ if the addition in question must be deemed to be false because it is unjustifiable the inherent distinction between elements of a same set.

Is had

$$\{\underline{A}\subseteq\underline{B}\}\equiv\{\underline{A}=\underline{A}\cap\underline{B}\}\equiv\{\underline{B}=\underline{A}\cup\underline{B}\}\tag{7}$$

A permutation of N elements is one of their different N! possible sequences. Intending

$$\square_{i=1,\text{ɨ}}(\text{ş}_i)\equiv\text{ş}_1\,\square\,\text{ş}_2\,\square\ldots\square\,\text{ş}_{\text{ɨ}}\qquad \{\square,\square\}\equiv\{\{\Sigma,+\}\,\dot\vee\,\{\Pi,\cdot\}\,\dot\vee\,\{\wedge,\wedge\}\,\dot\vee\,\{\vee,\vee\}\,\dot\vee\,\{\dot\vee,\dot\vee\}\,\dot\vee\,\{\cap,\cap\}\,\dot\vee\,\{\cup,\cup\}\,\dot\vee\,\{\dot\cup,\dot\cup\}\}$$

$\square_{i=1,\text{ɨ}}(\text{ş}_i)$ has the commutative property (i.e. $\square_{i=1,\text{ɨ}}(\text{ş}_i)\equiv\square_{i=1,\text{ɨ}}(\text{ş}_{P\langle i\rangle})$ with $(P_i;\ i=1,\text{ɨ})$ any one of the i! permutations of $(i=1,\text{ɨ})$) and associative, with the exception of the case $\{\square,\square\}\equiv\{\Pi,\cdot\}$ which has the only associativity if every ş_i is a set.

De Morgan's laws in propositional logic and set theory are

$$\neg\vee_{k=1,\text{ꝁ}}(\text{ş}_k)\equiv\wedge_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\qquad \neg\wedge_{k=1,\text{ꝁ}}(\text{ş}_k)\equiv\vee_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\qquad \neg\cup_{k=1,\text{ꝁ}}(\underline{A}_k)=\cap_{k=1,\text{ꝁ}}(\neg\underline{A}_k)\qquad \neg\cap_{k=1,\text{ꝁ}}(\underline{A}_k)=\cup_{k=1,\text{ꝁ}}(\neg\underline{A}_k)\tag{8}$$

The symbols "$\dot\vee$" and "$\dot\cup$" are specifications of the respective "$\vee$" e "$\cup$". The $Æ\langle\dot\vee\,/\,\vee\rangle$, (5) and the first two of (8) give rise to the first two of

$$\neg\dot\vee_{k=1,\text{ꝁ}}(\text{ş}_k)\,\underline{\rightarrow}\,\wedge_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\qquad \dot\vee_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\,\underline{\rightarrow}\,\neg\wedge_{k=1,\text{ꝁ}}(\text{ş}_k)\qquad \neg\dot\cup_{k=1,\text{ꝁ}}(\text{ş}_k)\,\underline{\rightarrow}\,\cap_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\qquad \dot\cup_{k=1,\text{ꝁ}}(\neg\text{ş}_k)\,\underline{\rightarrow}\,\neg\cap_{k=1,\text{ꝁ}}(\text{ş}_k)\tag{9}$$

whose second two are deduced in the way obviously analogous. The $\neg\text{ş}\equiv\mathit{ş}$ and $\text{ş}\equiv\neg\mathit{ş}$ they highlight how, in each of (8) and (9), $\{\text{ş}_k,\neg\text{ş}_k\}$ can be substituted by $\{\neg\text{ş}_k,\text{ş}_k\}$.

Being the $\{\underline{A}_k;\ k=1,\text{ꝁ}\}$ ꝁ sets, is had $\cap_{k=1,\text{ꝁ}}(\underline{A}_k)\subseteq\cup_{k=1,\text{ꝁ}}(\underline{A}_k)$, $⊡_{k=1,\text{ꝁ}}(\underline{A})\equiv\underline{A}$ of which $⊡\equiv\cap\,\dot\vee\,\cup$, and $\dot\cup_{k=1,\text{ꝁ}}(\underline{A}_k)$ a $\cup_{k=1,\text{ꝁ}}(\underline{A}_k)$ of which $\{\underline{A}_a\cap\underline{A}_b=\varnothing;\ \forall\{a,b\}\subseteq\{k=1,\text{ꝁ}\}\}$.

From: (7); $\{\underline{A}=\underline{B}\}\equiv\{\neg\underline{A}=\neg\underline{B}\}$; fourth of (8); (7); follows

$$\{\underline{A}\subseteq\underline{B}\}\equiv\{\underline{A}=\underline{A}\cap\underline{B}\}\equiv\{\neg\underline{A}=\neg\{\underline{A}\cap\underline{B}\}\}\equiv\{\neg\underline{A}=\{\neg\underline{A}\cup\neg\underline{B}\}\}\equiv\{\neg\underline{B}\subseteq\neg\underline{A}\}\tag{10}$$

In the following are treated (with reference to [9] and [10]) dispositions, permutations and combinations "simple" i.e. "without repetitions". A disposition of class K of N objects is a sequence of K elements of a set consisting of N elements, so two dispositions may also differ only for the respective sequential orders. Instead a combination of class K of N objects is a subset of numerosity K of a set of numerosity N, so the sequential order of the elements of a combination is irrelevant as in the case of the sets. A disposition of class N of N objects is called also permutation, and a disposition of class K of N objects is also called permutation of N objects taken K at a time. The respective number of all the possible different dispositions and combinations of class K of N objects is $N!/(N-K)!$ and $Б\langle N,K\rangle$ with the second which is the binomial coefficient of which $Б\langle N,K\rangle\equiv N!\,/\,((N-K)!\cdot K!)$).

Calling $\underline{k}\langle c,b,a\rangle$ the a-th element of the b-th different combination of class c of the $\{k=1,\text{ꝁ}\}$, is had

$$\{\{\underline{k}_{cba};\ a=1,c\};\ b=1,Б\langle\text{ꝁ},c\rangle\}\Leftrightarrow\{\{k=1,\text{ꝁ}\}-\{\underline{k}_{cba};\ a=1,c\};\ b=1,Б\langle\text{ꝁ},c\rangle\}\equiv\{\{\underline{\mathit{k}}_{cba};\ a=1,\text{ꝁ}-c\};\ b=1,Б\langle\text{ꝁ},\text{ꝁ}-c\rangle\}\tag{11}$$

The (2.2.36) and (2.2.37) of [1] affirm the respective

$$\mathfrak{N}\langle\cup_{k=1,k}(\underline{A}_k)\rangle = \Sigma_{c=1,k}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle k,c\rangle}(\mathfrak{N}\langle\cap_{a=1,c}(\underline{A}_{k\langle c,b,a\rangle})\rangle)) \qquad \mathfrak{N}\langle\dot\cup_{k=1,k}(\underline{A}_k)\rangle = \Sigma_{k=1,k}(\mathfrak{N}\langle\underline{A}_k\rangle) \tag{12}$$

The $\bowtie_{k=1,k}(\underline{A}) \equiv \underline{A}$ entails $\mathfrak{N}\langle\bowtie_{k=1,k}(\underline{A})\rangle = \mathfrak{N}_{\underline{A}}$ which is coherent with the first of (12) and the verify $\Sigma_{c=1,k}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle k,c\rangle}(1)) = \Sigma_{c=1,k}((-1)^{c+1}\cdot Б\langle k,c\rangle) = 1$.

Inherently the $h\cdot k$ sets $\{\underline{A}_{hk};\ h=1,h;\ k=1,k\}$, in section 2.2 of [1] is had $\cap_{h=1,h}(\Pi_{k=1,k}(\underline{A}_{hk})) \supseteq \Pi_{k=1,k}(\cap_{h=1,h}(\underline{A}_{hk}))$ and (2.2.30) i.e.

$$\neg\exists\{\{ş\ /ş \equiv ş;\ ş\in\underline{A}_{ac}\} \neq \{ş\ /ş\equiv ş;\ ş\in\underline{A}_{bc}\};\ ş\in\underline{A}_{ac};\ ş\in\underline{A}_{bc};\ \{a,b\}\subseteq\{h=1,h\};c\in\{k=1,k\}\} \underline{\rightarrow}$$

$$\{\cap_{h=1,h}(\Pi_{k=1,k}(\underline{A}_{hk})) = \Pi_{k=1,k}(\cap_{h=1,h}(\underline{A}_{hk}))\} \tag{13}$$

which both can result from computer verifications if each $\underline{A}_{hk}$ is a finite set, while (13) can result by representing the products as rectangular parallelepipeds k-dimensional if each $\underline{A}_{hk}$ is an interval of real numbers.

A univocal (i.e. non-injective and surjective) correspondence between $\underline{A}$ and $\underline{B}$ is a set of $\mathfrak{N}_{\underline{A}}$ pairs indicated $\underline{A}\Rightarrow\underline{B}$ and defined by a $\underline{A}\Rightarrow\underline{B} \equiv \{A_h,B_{k\langle h\rangle};\ h=1,h\}$ of which $\{k_h\in\{k=1,k\};\ h=1,h\}$, $\{k\in\{k_h;\ h=1,h\};\ k=1,k\}$. Therefore a $\underline{A}\Rightarrow\underline{B}$ makes to correspond to each $€\langle\underline{A}\rangle$ a only $€\langle\underline{B}\rangle$ and in such pairs appear all the elements of $\underline{A}$ and $\underline{B}$.

A bijection, i.e. a biunivocal correspondence, i.e. a one-to-one (injective) and onto (surjective) correspondence, between $\underline{A}$ and $\underline{B}$ of which $\mathfrak{N}_{\underline{A}} = \mathfrak{N}_{\underline{B}}$ is a set of $\mathfrak{N}_{\underline{A}}$ pairs indicated $\underline{A}\Leftrightarrow\underline{B}$ and defined by a $\underline{A}\Leftrightarrow\underline{B} \equiv \{A_h,B_{k\langle h\rangle};\ h=1,h\}$ of which $\{k_h;\ h=1,h\} = \{k=1,k\}$. Therefore a such $\underline{A}\Leftrightarrow\underline{B}$ makes to correspond to each $€\langle\underline{A}\rangle$ a only $€\langle\underline{B}\rangle$ and *vice versa*.

2 EVENTS AND PROBABILITY

For the following concepts of probability and statistics are referred [1], [11], [12], [6], [13], [14], [15], [7], [16], [17]. This section summarizes, simplifies and integrates the section 3 of [1] for present purposes.

A event E is biunivocally associated to its set of modalities $\underline{M}\langle E\rangle$ whose elements are all the different modalities with which E can occur namely all the different possibilities that E has to happen. Is underlined the name of an event, with exclusion of subscripts and prefix "$\neg$", to indicate its set of modalities, in the sense of $\underline{E} \equiv \underline{M}_E$ and $\neg\underline{E}_A \equiv \underline{M}\langle\neg E_A\rangle$. The elements of $\underline{E}$ are modalities mutually exclusive of a single happening: an E occur with (i.e. "as") a only $€\langle\underline{E}\rangle$ that is indicated $M\langle E\rangle$ and this property is called "uniqueness of M_E". A $€\langle\underline{E}\rangle$ can be considered as a set of modalities which has an only element and that is then own of the event constituted by the happening of such element.

The event $\neg E$ happens if E does not happen but could happen, $E_\varnothing$ is the event impossible because $\underline{E}_\varnothing = \varnothing$.

A name of an event also means its happen that in turn means its truth intended as alternative to the falsity established by its not happen. Therefore is intended $E \equiv$ "the happen of E" $\equiv \{E\}$.

For two events A and B, is had $\{\underline{A}=\underline{B}\} \equiv \{A\equiv B\}$. An $A\not\equiv B$ has like sufficient condition the happen of A and B in different places or times and, if is due only to this condition, A and B are two different happenings of the same event.

A $A \twoheadrightarrow B$ affirms that the happen of A implies the happen of B and is a univocal correspondence between $\underline{A}$ and a subset of $\underline{B}$, constituted by pairs such that the properties of first element are agree in asserting that its happen implies the happen of the second. So $A\twoheadrightarrow B$ means that the happen of each $€\langle\underline{A}\rangle$ entails the happen of a only $€\langle\underline{B}\rangle$ being such $€\langle\underline{A}\rangle$ and $€\langle\underline{B}\rangle$ the elements respectively first and second of one of said pairs, and is had

$$\{A\twoheadrightarrow B\} \equiv \{\{\underline{A}\Rightarrow\underline{b}\}_{\twoheadrightarrow}\ |\ \underline{b}\subseteq\underline{B}\} \tag{14}$$

where the subscript "$\twoheadrightarrow$" indicates a univocal correspondence of the particular type just said.

Generally a name of a proposition does not mean also the happening of the event consisting in the being true such proposition. Instead a name of an event always means also its happening and its "truth" in the sense of $E \equiv$ "the happen of E" $\equiv \{E\}$. Coherently with this and $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B \equiv \mathcal{P}_A \twoheadrightarrow \mathcal{P}_B$ (in section 1) is implicit that $\mathcal{P}_A \underline{\rightarrow} \mathcal{P}_B$ is specifiable as $A\twoheadrightarrow B$.

Therefore in particular (5) entails $\{A\twoheadrightarrow B\} \equiv Æ\langle A\ /\ B\rangle$.

The (14) and $A \leftrightarrow B$ imply $a_1 \twoheadrightarrow b$ and $b\twoheadrightarrow a_2$ of which $a_1\in\underline{A}$, $b\in\underline{B}$, $a_2\in\underline{A}$ (and $\mathfrak{N}\langle\underline{M}\langle a_1\rangle\rangle = \mathfrak{N}\langle\underline{b}\rangle = \mathfrak{N}\langle\underline{a}_2\rangle = 1$). The uniqueness of $M\langle A\rangle$ and this be a_2 implied by a_1 show that a_1 and a_2 are a same $€\langle\underline{A}\rangle$ i.e. $a_1 \equiv a_2 \equiv M_A$, following so a $\{A\leftrightarrow B\} \equiv \{\underline{A}\Leftrightarrow\underline{B}\}_{\leftrightarrow}$ whose second member is a bijection constituted by pairs such that the properties of both elements agree that the happen of one implies the happen of the other.

The (7) highlights how $\underline{A}\subseteq\underline{B}$ entails that A happen as a $€\langle\underline{A}\cap\underline{B}\rangle$, and so coherently with second of (6) highlights the first two members of

$$\{\underline{A}\subseteq\underline{B}\} \underline{\rightarrow} \{M\langle A\rangle \equiv \{M\langle A\rangle\ |\ M\langle A\rangle \equiv M\langle B\rangle\}\} \underline{\rightarrow} \{A\twoheadrightarrow B\} \tag{15}$$

From: this; uniqueness of M_E (for which M_E is in both cases a same modality); (15) and (2); follows

$$\{\underline{A}\subseteq\underline{E},\underline{B}\subseteq\underline{E}\} \underline{\rightarrow} \{\{M\langle A\rangle \equiv \{M\langle A\rangle\ |\ M\langle A\rangle \equiv M\langle E\rangle\},\{M\langle B\rangle \equiv \{M\langle B\rangle\ |\ M\langle B\rangle \equiv M\langle E\rangle\}\} \underline{\rightarrow} \{\{A\twoheadrightarrow B\} \underline{\rightarrow} \{\underline{A}\subseteq\underline{B}\}\} \equiv$$

$$\{\{A\twoheadrightarrow B\} \equiv \{\underline{A}\subseteq\underline{B}\}\} \tag{16}$$

From (2) follows $\{\mathsf{A} \Leftrightarrow \mathsf{B}\} \equiv \{\mathsf{A} \equiv \mathsf{B}\}$ which, for $\{\mathsf{A} \equiv \mathsf{B}\} \equiv \{\underline{\mathsf{A}} = \underline{\mathsf{B}}\}$ and $\{\mathsf{A} \Leftrightarrow \mathsf{B}\} \equiv \{\underline{\mathsf{A}} \Leftrightarrow \underline{\mathsf{B}}\}_{\leftrightarrow}$, gives rise to $\{\underline{\mathsf{A}} = \underline{\mathsf{B}}\} \equiv \{\underline{\mathsf{A}} \Leftrightarrow \underline{\mathsf{B}}\}_{\leftrightarrow}$. This is confirmed by the first of (6) and by the said properties of $\{\underline{\mathsf{A}} \Leftrightarrow \underline{\mathsf{B}}\}_{\leftrightarrow}$ which allow to consider each pair as two names of the same object.

Is placed

$$\mathcal{B}\langle\mathsf{E}\rangle \equiv \text{“E is a sure event”} \equiv \text{“E happens surely”} \equiv \text{“E is happened or will happen”} \underline{\rightarrow} \text{“is known at least a definition of E”} \tag{17}$$

and regarding its latest member is noted that ignore, voluntarily or involuntarily, an event is a mere limitation of knowledge and not a logical error that could make the results unreliable.

In relation to $\not{k}$ events $\{\mathsf{e}_k;\, k=1,\not{k}\}$, $\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle$ means that this events are independent namely that the set of modalities of each of them not is modified by the happen of any of the others.

On the basis of the first two members of (16) and uniqueness of $\mathbb{M}_{\mathsf{E}}$, a $\{\underline{\mathsf{A}} \subseteq \underline{\mathsf{E}}, \underline{\mathsf{B}} \subseteq \underline{\mathsf{E}}\}$ implies that, if happens A, B can only happen with a $\mathbb{M}\langle\mathsf{B}\rangle$ that verifies $\mathbb{M}\langle\mathsf{B}\rangle \equiv \{\mathbb{M}\langle\mathsf{B}\rangle \,|\, \mathbb{M}\langle\mathsf{B}\rangle \equiv \mathbb{M}\langle\mathsf{A}\rangle\}$ and hence is had $\{\underline{\mathsf{A}} \subseteq \underline{\mathsf{E}}, \underline{\mathsf{B}} \subseteq \underline{\mathsf{E}}\} \underline{\rightarrow} \neg\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle$. Is furthermore evident $\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle \underline{\rightarrow} \{\mathbb{I}\langle\mathsf{a},\mathsf{b}\rangle \,|\, \underline{\mathsf{a}} \subseteq \underline{\mathsf{A}}, \underline{\mathsf{b}} \subseteq \underline{\mathsf{B}}\}$. Therefore is had

$$\{\mathcal{B}\langle\mathsf{E}\rangle \wedge \exists\{\underline{\mathsf{A}} \subseteq \underline{\mathsf{E}}, \underline{\mathsf{B}} \subseteq \underline{\mathsf{E}} \,|\, \{\mathsf{A},\mathsf{B}\} \subseteq \{\mathsf{e}_k;\, k=1,\not{k}\}\}\} \underline{\rightarrow} \neg\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle \qquad \mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle \underline{\rightarrow} \{\mathbb{I}\langle e_k;\, k=1,\not{k}\rangle \,|\, \underline{e}_k \subseteq \underline{\mathsf{e}}_k;\, k=1,\not{k}\}$$

$$\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle \underline{\rightarrow} \neg\{\mathsf{A} \twoheadrightarrow \mathsf{B}\} \tag{18}$$

for which (based on (3)) a $\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle$ exists only if is ignored each E that makes true the first member of the first of (18).

A $\mathsf{A} \twoheadrightarrow \mathsf{B}$ affirms, as said on the occasion of (14), that the happen of A implies the happen of a only B. This highlights $\{\mathsf{A} \twoheadrightarrow \mathsf{B}\} \underline{\rightarrow} \neg\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle$ from which is deduced, for (3), $\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle \underline{\rightarrow} \neg\{\mathsf{A} \twoheadrightarrow \mathsf{B}\}$ e $\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle \underline{\rightarrow} \neg\{\mathsf{B} \twoheadrightarrow \mathsf{A}\}$, and hence that the first member of (19) implies the second. Being also immediately evident the reverse implication, is had

$$\mathbb{I}\langle\mathsf{A},\mathsf{B}\rangle \equiv \neg\{\mathsf{A} \twoheadrightarrow \mathsf{B}\} \wedge \neg\{\mathsf{B} \twoheadrightarrow \mathsf{A}\} \tag{19}$$

2.1 Composite events

Are composite events $\mathsf{E}_\cap$, $\mathsf{E}_\cup$, $\mathsf{E}_{\dot\cup}$, $\mathsf{E}_\bowtie$, $\mathsf{E}_\wedge$ and $\mathsf{E}_\vee$ of which $\mathsf{E}_\cap \equiv \cap_{k=1,\not{k}}(\mathsf{e}_k)$, $\mathsf{E}_\cup \equiv \cup_{k=1,\not{k}}(\mathsf{e}_k)$, $\mathsf{E}_{\dot\cup} \equiv \dot\cup_{k=1,\not{k}}(\mathsf{e}_k) \equiv \{\mathsf{E}_\cup \,|\, \underline{\mathsf{e}}_a \cap \underline{\mathsf{e}}_b = \varnothing;\, \forall a \neq b\}$, $\mathsf{E}_\bowtie \equiv \bowtie_{k=1,\not{k}}(\mathsf{e}_k)$, $\mathsf{E}_\wedge \equiv \wedge_{k=1,\not{k}}(\mathsf{e}_k)$ and $\mathsf{E}_\vee \equiv \vee_{k=1,\not{k}}(\mathsf{e}_k)$.

The $\mathsf{E}_\cap$ and $\mathsf{E}_\wedge$ both mean the happen of all the elements of $\{\mathsf{e}_k;\, k=1,\not{k}\}$. The $\mathsf{E}_\cup$ and $\mathsf{E}_\vee$ both mean the happen of at least one of the elements of $\{\mathsf{e}_k;\, k=1,\not{k}\}$. However these four events differ because their sets of modalities are

$$\underline{\mathsf{E}}_\bowtie \equiv \bowtie_{k=1,\not{k}}(\underline{\mathsf{e}}_k) \qquad \underline{\mathsf{E}}_\wedge \equiv \{(e_k;\, k=1,\not{k}) \,/\, e_k \in \underline{\mathsf{e}}_k;\, k=1,\not{k}\} \qquad \underline{\mathsf{E}}_\vee \equiv \{\vee_{k=1,\not{k}}(e_k) \,/\, e_k \in \underline{\mathsf{e}}_k;\, k=1,\not{k}\} \tag{20}$$

thus having, in particular and with reference to (17), $\mathcal{B}\langle\mathsf{E}_\wedge\rangle \equiv \wedge_{k=1,\not{k}}(\mathcal{B}\langle\mathsf{e}_k\rangle)$.

An $\mathsf{E}_\bowtie$, of which $\mathsf{E}_\bowtie \not\equiv \mathsf{E}_\varnothing$, is defined only if $\exists\{\mathsf{E} \,|\, \underline{\mathsf{E}} \subseteq \underline{\mathsf{e}}_k;\, k=1,\not{k}\}$, because *vice versa* the elements of $\underline{\mathsf{E}}_\bowtie$ would not be modalities mutually exclusive of a single happening and would be contradicted the uniqueness of $\mathbb{M}\langle\mathsf{E}_\bowtie\rangle$. An example of the absence of this necessary condition is obtainable with $\mathsf{e}_1 \equiv \wedge_{k=2,\not{k}}(\mathsf{e}_k)$, when it (i.e. $\exists\{\mathsf{E} \,|\, \underline{\mathsf{E}} \subseteq \underline{\mathsf{e}}_k;\, k=1,\not{k}$) would not be prevented by $\{\mathsf{e}_k \cap \mathsf{e}_1 \equiv \mathsf{E}_\varnothing;\, k=2,\not{k}\}$ but by the fact that would imply relations of type $\text{€}\langle\underline{\mathsf{E}}\rangle \equiv \text{€}\langle\underline{\mathsf{e}}_1\rangle \equiv \text{€}\langle\neg\underline{\mathsf{e}}_k\rangle$ with $k \neq 1$ and so with a such $\text{€}\langle\underline{\mathsf{E}}\rangle$ that it would be impossible since e_1 and $\neg\mathsf{e}_k$ they cannot happen together.

The first of (20) entails $\underline{\mathsf{e}}_k \subseteq \underline{\mathsf{E}}_\bowtie$ and so, based on first of (18), shows that only consider $\underline{\mathsf{E}}_\bowtie$ gives anyway rise to $\neg\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle$.

The $\underline{\mathsf{E}}_{\dot\cup} \equiv \dot\cup_{k=1,\not{k}}(\underline{\mathsf{e}}_k)$ (due to first of (20)) and uniqueness of each $\mathbb{M}_{\mathsf{E}}$ imply that $\{\mathsf{e}_k;\, k=1,\not{k}\}$ are mutually exclusive when is considered $\mathsf{E}_{\dot\cup}$ and imply $\mathsf{E}_\cup \equiv \mathsf{E}_{\dot\cup}$ of $\{\mathsf{e}_k;\, k=1,\not{k}\}$ mutually exclusive.

From a $\mathcal{I}_1$, of which $\mathcal{I}_1 \equiv \{\underline{\mathsf{e}}_k \subseteq \underline{\mathsf{E}}, \mathsf{e}_k \twoheadrightarrow \mathsf{A};\, k=1,\not{k}\}$, is not immediately deducible $\mathsf{E}_\cup \twoheadrightarrow \mathsf{A}$, because to the properties of a $\text{€}\langle\underline{\mathsf{e}}_a\rangle$ that contribute to determine a $\mathsf{e}_a \twoheadrightarrow \mathsf{A}$ when it is not considered a e_b, when instead is also considered e_b can be added other which contradict those said (this possibility will be highlighted by the example in section 3.1). So, having in each case $\mathsf{E}_\cap \twoheadrightarrow \mathsf{E}_\cup$ (due to $\underline{\mathsf{E}}_\cap \subseteq \underline{\mathsf{E}}_\cup$ and (15)) and intending $\mathcal{I}_2 \equiv \neg\exists\{\underline{\mathsf{e}} \cup \underline{\mathsf{E}}_\cup \neq \underline{\mathsf{E}}_\cup \,|\, \mathsf{e} \twoheadrightarrow \mathsf{A}\}$, is had first of

$$\{\mathcal{I}_1 \underline{\rightarrow} \{\mathsf{E}_\cap \twoheadrightarrow \mathsf{E}_\cup \twoheadrightarrow \mathsf{A}\}\} \underline{\rightarrow} \mathcal{I}_3 \qquad \{\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{B}\langle\mathsf{E}\rangle\} \underline{\rightarrow} \{\mathsf{E}_\cup \equiv \mathsf{A}\}\} \underline{\rightarrow} \mathcal{I}_3 \tag{21}$$

where $\mathcal{I}_3$ (that is necessary analogously to $\mathcal{P}_{\mathsf{B}}$ of (3)) consists in being the properties of each $\text{€}\langle\underline{\mathsf{E}}_\cup\rangle$, which are determined by the consider all the $\{\mathsf{e}_k;\, k=1,\not{k}\}$, all unanimous in implying a $\text{€}\langle\underline{\mathsf{A}}\rangle$, namely in being the names of each $\text{€}\langle\underline{\mathsf{E}}_\cup\rangle$ coherent in determining such implication; and it is also evident that the second follows from the first because in this $\mathsf{E}_\cup \twoheadrightarrow \mathsf{A}$ can be replaced by $\mathsf{E}_\cup \equiv \mathsf{A}$ if $\mathcal{I}_2$ prevents of increasing $\mathfrak{N}\langle\underline{\mathsf{E}}_\cup\rangle$ and $\mathcal{B}\langle\mathsf{E}\rangle$.

The only $\mathsf{E}_\wedge \equiv \wedge_{k=1,\not{k}}(\mathsf{e}_k)$ makes to deduce, coherently to $\mathfrak{N}\langle\Pi_{k=1,\not{k}}(\underline{\mathsf{A}}_k)\rangle = \Pi_{k=1,\not{k}}(\mathfrak{N}\langle\underline{\mathsf{A}}_k\rangle)$ in (2.2.29) of [1],

$$\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle \equiv \{\underline{\mathsf{E}}_\wedge = \Pi_{k=1,\not{k}}(\underline{\mathsf{e}}_k)\} \equiv \{\mathfrak{N}\langle\underline{\mathsf{E}}_\wedge\rangle = \Pi_{k=1,\not{k}}(\mathfrak{N}\langle\underline{\mathsf{e}}_k\rangle)\} \qquad \neg\mathbb{I}\langle\mathsf{e}_k;\, k=1,\not{k}\rangle \equiv \{\underline{\mathsf{E}}_\wedge \subset \Pi_{k=1,\not{k}}(\underline{\mathsf{e}}_k)\} \equiv \{\mathfrak{N}\langle\underline{\mathsf{E}}_\wedge\rangle < \Pi_{k=1,\not{k}}(\mathfrak{N}\langle\underline{\mathsf{e}}_k\rangle)\} \tag{22}$$

as well as $\underline{\mathsf{E}}_\wedge \subseteq \underline{\mathsf{C}}_\wedge$ of which $\mathsf{C}_\wedge \equiv \wedge_{k=1,\not{k}}(\mathsf{C}_k)$ e $\{\underline{\mathsf{e}}_k \subseteq \underline{\mathsf{C}}_k;\, k=1,\not{k}\}$.

This $\underline{\mathsf{E}}_\wedge \subseteq \underline{\mathsf{C}}_\wedge$ can be specified as $\underline{\mathbb{E}}_{\wedge k} \subseteq \underline{\mathsf{C}}_\wedge$ of which $\mathbb{E}_{\wedge k} \equiv \wedge_{k=1,\not{k}}(\mathbb{e}_{kk})$, $\{\mathbb{e}_{kk} \equiv \mathsf{C}_k;\, \forall k \neq k\}$, $\mathbb{e}_{kk} \equiv \mathsf{e}_k$, $\underline{\mathsf{E}}_\wedge \subseteq \underline{\mathbb{E}}_{\wedge k}$. The said meaning own of both the $E_\cap$ and $E_\wedge$ entails that $\cap_{k=1,\not{k}}(\mathbb{E}_{\wedge k})$ is the happening of all the $\{ⓔ_{kk};\ k=1,\not{k};\ k=1,\not{k}\}$. The definition of $ⓔ_{kk}$ entails that this happen is that of all the $\{e_k,C_k;\ k=1,\not{k}\}$ which, for $e_k \rightarrow C_k$ (due to (15) and $\underline{e}_k \subseteq \underline{C}_k$) and $\{e_k \rightarrow C_k\} \equiv \{e_k \equiv \{e_k,C_k\}\}$ (affirmed by (2.1.1.3) of [1]), is necessary and sufficient for the happen of $E_\wedge$. Therefore is had $\cap_{k=1,\not{k}}(\mathbb{E}_{\wedge k}) \equiv E_\wedge$.

Substituting in this $\{e_k;\ k=1,\not{k}\}$ with $\{\neg e_k;\ k=1,\not{k}\}$ is had $\wedge_{k=1,\not{k}}(\neg e_k) \equiv \cap_{k=1,\not{k}}(\wedge_{k=1,\not{k}}(ⓔ_{kk}{}^{\neg}))$ of which $\{ⓔ_{kk}{}^{\neg} \equiv C_k;\ \forall k \neq k\}$, $ⓔ_{kk}{}^{\neg} \equiv \neg e_k$, and so $\neg\wedge_{k=1,\not{k}}(\neg e_k) \equiv \neg\cap_{k=1,\not{k}}(\wedge_{k=1,\not{k}}(ⓔ_{kk}{}^{\neg}))$ which, for (8), becomes $E_\vee \equiv \cup_{k=1,\not{k}}(\neg\wedge_{k=1,\not{k}}(ⓔ_{kk}{}^{\neg}))$ whose $\neg\wedge_{k=1,\not{k}}(ⓔ_{kk}{}^{\neg})$, if $\mathcal{B}\langle C_\wedge\rangle$ i.e. if $C_\wedge$ is sure, may be replaced by $\mathbb{E}_{\wedge k}$.

Therefore if $\mathcal{B}\langle C_\wedge\rangle$ is had

$$E_\wedge \equiv \cap_{k=1,\not{k}}(\mathbb{E}_{\wedge k}) \qquad E_\vee \equiv \cup_{k=1,\not{k}}(\mathbb{E}_{\wedge k}) \tag{23}$$

of which $\underline{E}_\wedge \subseteq \underline{E}_\vee \subseteq \underline{C}_\wedge$ (compliant to $\underline{E}_\cap \subseteq \underline{E}_\cup$) and which allows to place each $\mathbb{E}_{\wedge k}$ in a space evidently analogous to a Cartesian space $\not{k}$-dimensional. By specifying $\{e_k;\ k=1,\not{k}\}$ as $\{C_k;\ k=1,\not{k}\}$ in (23), these become $C_\wedge \equiv \cap_{k=1,\not{k}}(C_\wedge)$ and $\vee_{k=1,\not{k}}(C_k) \equiv \cup_{k=1,\not{k}}(C_\wedge)$ of which $\cap_{k=1,\not{k}}(C_\wedge) \equiv \cup_{k=1,\not{k}}(C_\wedge) \equiv C_\wedge$ (due to $\bowtie_{k=1,\not{k}}(\underline{A}) \equiv \underline{A}$) and which therefore show $\vee_{k=1,\not{k}}(C_k) \equiv C_\wedge$ that is coherent with $\mathcal{B}\langle E_\wedge\rangle \equiv \wedge_{k=1,\not{k}}(\mathcal{B}\langle e_k\rangle)$ in the sense that if $\mathcal{B}\langle E_\wedge\rangle$ each e_k is implicitly present although not mentioned.

2.2 Probability

Are placed, coherently with (7), the first two of

$$\rho\langle A \mid B\rangle \equiv \mathfrak{N}\langle \underline{A} \cap \underline{B}\rangle / \mathfrak{N}\langle \underline{B}\rangle \qquad \{\rho\langle A \mid B\rangle = \mathfrak{N}\langle \underline{A}\rangle / \mathfrak{N}\langle \underline{B}\rangle;\ \forall \underline{A} \subseteq \underline{B}\} \qquad \rho\langle A \mid B\rangle + \rho\langle \neg A \mid B\rangle = 1 \tag{24}$$

whose third is affirmed by the (3.2.1.2) of [1].

Is meant $\underline{\mathfrak{R}} \equiv (-\infty,\infty)$ with ∞ a number unlimitedly large. Coherently with (24) and (4.2.2) of [1] is had

$$\rho\langle a \le s \le b \mid s \in \underline{\mathfrak{R}}\rangle = \mathfrak{N}\langle \underline{M}\langle a \le s \le b\rangle\rangle / \mathfrak{N}\langle \underline{M}\langle s \in \underline{\mathfrak{R}}\rangle\rangle = \int_{a,b}(\mathfrak{p}\langle s\rangle(x)\cdot dx) = \int_{-\infty,b}(\mathfrak{p}_s(x)\cdot dx) - \int_{-\infty,a}(\mathfrak{p}_s(x)\cdot dx) \tag{25}$$

where s is a random variable and $\mathfrak{p}_s(x)$ its probability density function (PDF). In such $\mathfrak{p}_s(x)$ x has also the identity of value of s.

It is understood IPM $\equiv$ {the first member of}. From: (24), $\underline{E}_\wedge \subseteq \underline{C}_\wedge$; $\mathbb{I}\langle e_k;\ k=1,\not{k}\rangle$, (22); (24), $\underline{e}_k \subseteq \underline{C}_k$; follows IPM

$$\{\rho\langle E_\wedge \mid C_\wedge\rangle = \mathfrak{N}\langle \underline{E}_\wedge\rangle / \mathfrak{N}\langle \underline{C}_\wedge\rangle = \Pi_{k=1,\not{k}}(\mathfrak{N}\langle \underline{e}_k\rangle / \mathfrak{N}\langle \underline{C}_k\rangle) = \Pi_{k=1,\not{k}}(\rho\langle e_k \mid C_k\rangle)\} \underline{\leftarrow} \mathbb{I}\langle e_k;\ k=1,\not{k}\rangle \tag{26}$$

With reference to (17), is placed

$$\mathfrak{C}\langle E\rangle \equiv \mathcal{A}\langle E\rangle \wedge \mathcal{B}\langle E\rangle \wedge \mathcal{C}\langle E\rangle \qquad \mathcal{A}\langle E\rangle \equiv \text{“}E \text{ is identified univocally”}$$

$\mathcal{C}\langle E\rangle \equiv$ “all the elements of $\underline{E}$ have the same potentiality to be the modality with which happens E”

which makes evident $\mathfrak{C}\langle \cup_{k=1,\not{k}}(e_k)\rangle \equiv \dot{\vee}_{k=1,\not{k}}(\mathfrak{C}\langle e_k\rangle)$ and for $\{\mathcal{P}_A \wedge \mathcal{P}_B\} \underline{\rightarrow} \mathcal{P}_B$, tautology known as *conjunction elimination*, $\mathfrak{C}\langle E\rangle \underline{\rightarrow} \mathcal{B}\langle E\rangle$.

Calling $\mathbb{P}\langle A\rangle$ the probability of A, is had

$$\mathfrak{C}\langle B\rangle \underline{\rightarrow} \{\mathbb{P}\langle A\rangle \equiv \mathbb{P}\langle A \cap B\rangle = \rho\langle A \mid B\rangle\} \qquad \mathbb{P}\langle A\rangle + \mathbb{P}\langle \neg A\rangle = 1 \tag{27}$$

whose second follows from the definition of $\neg E$.

The $\underline{A} \subseteq \underline{B}$ implies $\mathfrak{N}\langle \underline{A} \cap \underline{C}\rangle \le \mathfrak{N}\langle \underline{B} \cap \underline{C}\rangle$. This and first of (27) give rise to the only $\{\mathfrak{C}\langle C\rangle, \underline{A} \subseteq \underline{B}\} \underline{\rightarrow} \{\mathbb{P}\langle A\rangle = \rho\langle A \mid C\rangle \le \rho\langle B \mid C\rangle = \mathbb{P}\langle B\rangle\}$. Nevertheless first of (27) and the only meaning of $A \rightarrow B$ also allow

$$\{\mathfrak{C}\langle C\rangle, A \rightarrow B\} \underline{\rightarrow} \{\rho\langle A \mid C\rangle = \mathbb{P}\langle A\rangle \le \mathbb{P}\langle B\rangle\} \tag{28}$$

The (27) indicates that $\mathbb{P}\langle A\rangle$ does not have nature absolute and universal, but relative and contingent as that of the inherent B. Indeed (27) makes possible all the generally different $\mathbb{P}\langle A\rangle$ that correspond to the different choices of B, by following that each of these probabilities has the eminently conventional nature of the being inherent only the particular context determined by choice of corresponding B.

However, being evident that $\mathbb{P}\langle A\rangle$ is more significant if B represents better the context in which are interesting information about A and in particular if subsists $\underline{A} \subseteq \underline{B}$ for which intervenes the entire $\underline{A}$ and not the only $\underline{A} \cap \underline{B}$ of the case $\underline{A} \neq \underline{A} \cap \underline{B}$, is also evident that a $\mathbb{P}\langle A\rangle$ can be considered as the only true and not merely conventional probability of A, and in this case is indicated $\mathfrak{P}\langle A\rangle$, if B is the event which has the lesser $\mathfrak{N}\langle \underline{B}\rangle$ compatible with $\underline{A} \subseteq \underline{B}$ and

$$\neg\exists \boldsymbol{\mathcal{P}}_{AB} \neq \{\{\neg\underline{B} \subseteq \neg\underline{A}\} \underline{\rightarrow} \{\mathfrak{E}\langle \neg\underline{B}\rangle \equiv \mathfrak{E}\langle \neg\underline{A}\rangle\}\} \tag{29}$$

where $\mathcal{P}_{AB}$ is a set of propositions from which is logically deducible $\mathfrak{E}\langle\neg\underline{B}\rangle \equiv \mathfrak{E}\langle\neg\underline{A}\rangle$, with $\neg\underline{B} \subseteq \neg\underline{A}$ that is deduced from $\underline{A} \subseteq \underline{B}$ and (10), and being such (29) equivalent to the absence of every relation that may be involved between A and $\neg B$ by their respective properties. Such evidence, i.e. the definition just said of $\mathfrak{P}\langle A\rangle$, is based on the exclude any modality that has no relation with A and *vice versa* in the include any modality that has relation with A. Coherently with this, in (28) is had $\mathbb{P}\langle B\rangle \equiv \mathfrak{P}\langle B\rangle$ if no relation between $\neg C$ and B may be involved from their properties.

2.3 An application of composite events

Is considered $\mathcal{B}\langle Q_\wedge\rangle$ of which $Q_\wedge \equiv \wedge_{d=1,\text{đ}}(q_d)$, $q_d \equiv p_d \dot\cup \neg p_d$, $\mathbb{I}\langle q_d;\, d=1,\text{đ}\rangle$, and the $\{q_a \not\equiv q_b;\, \forall\{a,b\} \subseteq \{k=1,\text{ƙ}\}\}$ due to the sole fact that $\{d=1,\text{đ}\}$ indicates đ several days i.e. with each $q_a \not\equiv q_b$ caused only by the happen q_a and q_b in the respective and different days a-th and b-th. This implies both $\{q_d;\, d=1,\text{đ}\}$ as đ happenings of same q and $\{p_d;\, d=1,\text{đ}\}$ as đ happenings of same p, thus having also $q \equiv p \dot\cup \neg p$.

Placing the

$$R_{\wedge kb} \equiv \wedge_{d=1,\text{đ}}(r_{u\langle k,b,d\rangle}) \qquad \{r_{u\langle k,b,d\rangle} \equiv p_{u\langle k,b,d\rangle};\, d=1,k\} \qquad \{r_{u\langle k,b,d\rangle} \equiv \neg p_{u\langle k,b,d\rangle};\, d=k+1,\text{đ}\} \qquad \{u_{kbd};\, d=1,\text{đ}\} = \{d=1,\text{đ}\} \quad (30)$$

is had

$$\{\underline{R}_{\wedge kb} \subseteq \underline{Q}_\wedge, R_{\wedge kb} \twoheadrightarrow P_k \equiv \{\text{in đ days happens } k \text{ times } p \text{ and } \text{đ} - k \text{ times } \neg p\};\, b=1,N_R\} \quad (31)$$

whose $\underline{R}_{\wedge kb} \subseteq \underline{Q}_\wedge$ is due to $\underline{r}_d \subseteq \underline{q}_d$, and of which is had $N_R = \text{đ}!$ as immediate consequence of the being $(u_{kbd};\, d=1,\text{đ})$ a b-th permutation of $\{d=1,\text{đ}\}$ affirmed by the last of (30).

Such $N_R = \text{đ}!$ is confirmed by the evident possibility of placing $N_R = N_{RA} \cdot N_{RB}$ with N_{RA} the number of dispositions of class k of đ objects (i.e. $N_{RA} = \text{đ}!/(\text{đ} - k)!$) and N_{RB} the number of permutations of $\text{đ} - k$ objects (i.e. $N_{RB} = (\text{đ} - k)!$), or vice versa with N_{RA} the number of dispositions of class $\text{đ} - k$ of đ objects (i.e. $N_{RA} = \text{đ}!/k!$) and N_{RB} the number of permutations of k objects (i.e. $N_{RB} = k!$).

From: $Æ\langle\mathcal{B}\langle Q_\wedge\rangle,(31) \,/\, \text{first member of (21)}\rangle$; the mere hypothesize the possibility of cases such as $\{R_{\wedge ka} \equiv R_{\wedge kb} \,|\, a \neq b\}$, commutativity and associativity of the union, $\{A \equiv B\} \xrightarrow{} \{A \cup B \equiv A\}$; follows

$$P_k \equiv \cup_{b=1,N\langle R\rangle}(R_{\wedge kb}) \equiv \cup_{b=1,N\langle S\rangle}(S_{\wedge kb}) \equiv \dot\cup_{b=1,N\langle S\rangle}(S_{\wedge kb}) \quad (32)$$

of which $\{S_{\wedge kb};\, b=1,N_S\} \subseteq \{R_{\wedge kb};\, b=1,N_R\}$ with N_S the maximum compatible with $\{S_{\wedge kr} \not\equiv S_{\wedge ks};\, \forall\{r,s\} \subseteq \{b=1,N_S\}\}$, and whose last member is due to mutual exclusivity of such $\{S_{\wedge kb};\, b=1,N_S\}$ which is substantially highlighted by the fact that each of these N_S events is a specification of $Q_\wedge$ whose happen excludes that of any other.

From: last of (30); commutativity of $\wedge_{i=1,\text{ɨ}}(\text{ş}_i)$, (4); follows

$$\{\{u_{kmd};\, d=1,k\} \neq \{u_{knd};\, d=1,k\}\} \equiv \{\{u_{kmd};\, d=k+1,\text{đ}\} \neq \{u_{knd};\, d=k+1,\text{đ}\}\} \equiv \{R_{\wedge km} \not\equiv R_{\wedge kn}\}$$

This and (11) give rise to

$$\{S_{\wedge kb};\, b=1,N_S\} \Leftrightarrow \{\{\underline{d}_{kba};\, a=1,k\};\, b=1,Б\langle\text{đ},k\rangle\} \Leftrightarrow \{\{\underline{d}_{kba};\, a=1,\text{đ} - k\};\, b=1,Б\langle\text{đ},\text{đ} - k\rangle\}$$

where $\underline{d}\langle k,b,a\rangle$ is the a-th element of the b-th combination of class k of the $\{d=1,\text{đ}\}$, $\{\underline{d}_{kba};\, a=1,\text{đ} - k\} = \{d=1,\text{đ}\} - \{\underline{d}_{kba};\, a=1,k\}$.

So every $S_{\wedge kb}$ corresponds to a different combination of class k (and/or $\text{đ} - k$) of the elements of $\{d=1,\text{đ}\}$ as is specified by

$$N_S = Б\langle\text{đ},k\rangle \qquad S_{\wedge kb} \equiv \wedge_{a=1,k}(p_{\underline{d}\langle k,b,a\rangle}) \wedge \wedge_{a=1,\text{đ}-k}(\neg p_{\underline{d}\langle k,b,a\rangle}) \quad (33)$$

From: $Q_\wedge \equiv \wedge_{d=1,\text{đ}}(q_d)$, $\mathbb{I}\langle q_d;\, d=1,\text{đ}\rangle$, (22); $\mathfrak{N}\langle\underline{q}_d\rangle = \mathfrak{N}\langle\underline{q}\rangle$ due to being the $\{q_d;\, d=1,\text{đ}\}$ đ happenings of a same q; follows $\mathfrak{N}\langle\underline{Q}_\wedge\rangle = \Pi_{d=1,\text{đ}}(\mathfrak{N}\langle\underline{q}_d\rangle) = (\mathfrak{N}\langle\underline{q}\rangle)^{\text{đ}}$. Besides $\mathbb{I}\langle q_d;\, d=1,\text{đ}\rangle$, $\{\underline{p}_d \subseteq \underline{q}_d, \neg\underline{p}_d \subseteq \underline{q}_d\}$ (due to $q_d \equiv p_d \dot\cup \neg p_d$), and second of (18) entail $\mathbb{I}\langle\{p_{\underline{d}\langle k,b,a\rangle};\, a=1,k\},\{\neg p_{\underline{d}\langle k,b,a\rangle};\, a=1,\text{đ} - k\}\rangle$. From: this, (22), second of (33); $\mathfrak{N}\langle\underline{p}_d\rangle = \mathfrak{N}\langle\underline{p}\rangle$ e $\mathfrak{N}\langle\neg\underline{p}_d\rangle = \mathfrak{N}\langle\neg\underline{p}\rangle$ due to being the $\{p_d;\, d=1,\text{đ}\}$ đ happenings of a same p; follows

$$\mathfrak{N}\langle\underline{S}_{\wedge kb}\rangle = \Pi_{a=1,k}(\mathfrak{N}\langle\underline{p}_{\underline{d}\langle k,b,a\rangle}\rangle) \cdot \Pi_{a=1,\text{đ}-k}(\mathfrak{N}\langle\neg\underline{p}_{\underline{d}\langle k,b,a\rangle}\rangle) = (\mathfrak{N}\langle\underline{p}\rangle)^k \cdot (\mathfrak{N}\langle\neg\underline{p}\rangle)^{\text{đ}-k}$$

From: $\underline{S}_{\wedge kb} \subseteq \underline{Q}_\wedge$ (due to $\{S_{\wedge kb};\, b=1,N_S\} \subseteq \{R_{\wedge kb};\, b=1,N_R\}$ and $\underline{R}_{\wedge kb} \subseteq \underline{Q}_\wedge$), second of (24); previous expressions of $\mathfrak{N}\langle\underline{Q}_\wedge\rangle$ and $\mathfrak{N}\langle\underline{S}_{\wedge kb}\rangle$; $\{\underline{p} \subseteq \underline{q}, \neg\underline{p} \subseteq \underline{q}\}$ (due to $q \equiv p \dot\cup \neg p$); third of (24); follows

$$\rho\langle S_{\wedge kb} \,¦\, Q_\wedge\rangle = \mathfrak{N}\langle\underline{S}_{\wedge kb}\rangle / \mathfrak{N}\langle\underline{Q}_\wedge\rangle = (\mathfrak{N}\langle\underline{p}\rangle / \mathfrak{N}\langle\underline{q}\rangle)^k \cdot (\mathfrak{N}\langle\neg\underline{p}\rangle / \mathfrak{N}\langle\underline{q}\rangle)^{\text{đ}-k} = (\rho\langle p \,¦\, q\rangle)^k \cdot (\rho\langle\neg p \,¦\, q\rangle)^{\text{đ}-k} = (\rho\langle p \,¦\, q\rangle)^k \cdot (1 - \rho\langle p \,¦\, q\rangle)^{\text{đ}-k} \quad (34)$$

From: (32); $\rho\langle E_{\dot\cup} \,¦\, B\rangle = \Sigma_{k=1,\text{ƙ}}(\rho\langle e_k \,¦\, B\rangle)$ affirmed by (3.2.1.11) of [1]; (34), first of (33); folows

$$\rho\langle P_k \,¦\, Q_\wedge\rangle = \rho\langle\dot\cup_{b=1,N\langle S\rangle}(S_{\wedge kb}) \,¦\, Q_\wedge\rangle = \Sigma_{b=1,N\langle S\rangle}(\rho\langle S_{\wedge kb} \,¦\, Q_\wedge\rangle) = Б\langle\text{đ},k\rangle \cdot (\rho\langle p \,¦\, q\rangle)^k \cdot (1 - \rho\langle p \,¦\, q\rangle)^{\text{đ}-k} \quad (35)$$

The (32) and $\underline{S}_{\wedge kb} \subseteq \underline{Q}_{\wedge}$ imply $\underline{P}_k \subseteq \underline{Q}_{\wedge}$, and moreover the definition of P_k (in (31)) highlights

$\{P_h \rightarrow \mathbb{P}_k \equiv$ {in d days p happens at least k times}; h=k,d}

From: this and $\mathcal{B}\langle Q_{\wedge}\rangle$, (21), $\{\underline{P}_a \cap \underline{P}_b = \varnothing; \forall \{a,b\} \subseteq \{h=k,d\}\}$; þ; (35); follows

$$\rho\langle \mathbb{P}_k \mid Q_{\wedge}\rangle = \rho\langle \cup_{h=k,d}(P_h) \mid Q_{\wedge}\rangle = \Sigma_{h=k,d}(\rho\langle P_h \mid Q_{\wedge}\rangle) = \Sigma_{h=k,d}(Б\langle d,h\rangle \cdot (\rho\langle p \mid q\rangle)^{h} \cdot (1 - \rho\langle p \mid q\rangle)^{d-h}) \quad (36)$$

The first of (27) gives rise to

$$\mathfrak{C}\langle Q_{\wedge}\rangle \rightarrow \{\mathbb{P}\langle P_k\rangle = \rho\langle P_k \mid Q_{\wedge}\rangle, \mathbb{P}\langle \mathbb{P}_k\rangle = \rho\langle \mathbb{P}_k \mid Q_{\wedge}\rangle\}$$

which together with (35) and (36) expresses two probabilities notable as a result of the application of properties of composite e-vents.

3 THE PROBABILITY OF AN UNKNOWN CONSTANT

A $G \in \underline{\mathfrak{R}}$, where G is a quantity, implies $\underline{\mathfrak{R}} \subseteq \underline{\mathbb{R}}$ and means that G has a value equal to that of one of the elements of the set $\underline{\mathfrak{R}}$. By calling $\underline{\mathbb{R}}\langle G\rangle$ the set of different values that can have G (and intending that a subscript can also represent a character string empty i.e. absent), is had $\{G \in \underline{\mathbb{R}}_G\}_A \equiv \{G \in \underline{\mathbb{R}}\}_A$ and $\{G \in \neg\underline{\mathbb{R}}_G\}_A \equiv \{G \in \neg\underline{\mathbb{R}}\}_A \equiv E_{\varnothing}$. Is considered implicit that a $\{G \in \underline{\mathfrak{R}}\}_A$ can happen only if $\mathcal{B}\langle\{G \in \underline{\mathbb{R}}\}_A\rangle$ and so is had $\neg\{G \in \underline{\mathfrak{R}}\}_A \equiv \{G \in \underline{\mathbb{R}} - \underline{\mathfrak{R}}\}_A$. Being evident the first of

$$\{\underline{\mathfrak{R}}_a \subseteq \underline{\mathfrak{R}}_b\} \equiv \{\underline{M}\langle\{G \in \underline{\mathfrak{R}}_a\}_A\rangle \subseteq \underline{M}\langle\{G \in \underline{\mathfrak{R}}_b\}_A\rangle\} \qquad \bowtie_{m=1,m}(\{G \in \underline{\mathfrak{R}}_m\}_A) \equiv \{G \in \bowtie_{m=1,m}(\underline{\mathfrak{R}}_m)\}_A \quad (37)$$

the second of them is shown by the first of (20), uniqueness of $\mathbb{M}_{E}$ (said in second paragraph of section 2) and by the fact that a $\mathfrak{E}\langle \underline{M}\langle G \in \underline{\mathfrak{R}}\rangle\rangle$ regards a only value of G.

From: $\underline{A} = \{\underline{A} \cap \underline{B}\} \cup \{\underline{A} \cap \neg\underline{B}\}$ (in (2.2.23) of [1]); $\underline{A} \cap \underline{B} = \underline{A} - \neg\underline{B}$ (in (2.2.7) of [1]); second of (37); $\{G \in \neg\underline{\mathbb{R}}\}_A \equiv E_{\varnothing}$; $\neg\{G \in \underline{\mathfrak{R}}\}_A \equiv \{G \in \underline{\mathbb{R}} - \underline{\mathfrak{R}}\}_A$; follows

$$\{G \in \neg\underline{\mathfrak{R}}\}_A \equiv \{G \in \{\underline{\mathbb{R}} \cap \neg\underline{\mathfrak{R}}\} \cup \{\neg\underline{\mathfrak{R}} \cap \neg\underline{\mathbb{R}}\}\}_A \equiv \{G \in \{\underline{\mathbb{R}} - \underline{\mathfrak{R}}\} \cup \{\neg\underline{\mathfrak{R}} \cap \neg\underline{\mathbb{R}}\}\}_A \equiv \{G \in \underline{\mathbb{R}} - \underline{\mathfrak{R}}\}_A \cup \{\{G \in \neg\underline{\mathfrak{R}}\}_A \cap \{G \in \neg\underline{\mathbb{R}}\}_A\} \equiv$$

$$\{G \in \underline{\mathbb{R}} - \underline{\mathfrak{R}}\}_A \equiv \neg\{G \in \underline{\mathfrak{R}}\}_A$$

The $\{G \in \neg\underline{\mathfrak{R}}\}_A \equiv \{G \in \underline{\mathbb{R}} - \underline{\mathfrak{R}}\}_A$ entails that in relation to a $\{G \in \neg\underline{\mathfrak{R}}\}_A$ is implied the conventional $\neg\underline{\mathfrak{R}} \equiv \underline{\mathbb{R}} - \underline{\mathfrak{R}}$. From: this; $\underline{\mathfrak{R}} \subseteq \underline{\mathbb{R}}$; follows $\underline{\mathfrak{R}} \dot\cup \neg\underline{\mathfrak{R}} = \underline{\mathfrak{R}} \dot\cup \{\underline{\mathbb{R}} - \underline{\mathfrak{R}}\} = \underline{\mathbb{R}}$.

How much a moment ago and second of (37) give rise to

$$\{\{G \in \underline{\mathbb{R}}\}_A \equiv \{G \in \underline{\mathfrak{R}}\}_A \dot\cup \neg\{G \in \underline{\mathfrak{R}}\}_A, \neg\{G \in \underline{\mathfrak{R}}\}_A \equiv \{G \in \neg\underline{\mathfrak{R}}\}_A, \neg\underline{\mathfrak{R}} \equiv \underline{\mathbb{R}} - \underline{\mathfrak{R}}\} \leftarrow \mathcal{B}\langle\{G \in \underline{\mathbb{R}}\}_A\rangle \quad (38)$$

Is intended that X is a an unknown constant and are placed its $e \equiv \{X \in \underline{\mathfrak{R}}\}$, $\bar{e} \equiv \{X \in \underline{\mathbb{R}}\}$. The being X a constant entails that e_A not is the happening of one of the values of X which are elements of $\underline{\mathfrak{R}}$ when $\underline{e}_A$ would be an addition of subsets each corresponding to a different $\mathfrak{E}\langle\underline{\mathfrak{R}}\rangle$, but it is instead the happening of a set $\underline{\mathfrak{R}}$ of which is element the only value that can have X and concerning therefore each $\mathfrak{E}\langle\underline{e}_A\rangle$ this same value. However this neither influences the information expressed by e_A on X nor prevents a $\mathbb{I}\langle\bar{e}_A,\bar{e}_B\rangle$.

The second of (37) entails $\{\cap_{m=1,m}(\{G \in \underline{\mathfrak{R}}_m\}_A) \mid \cap_{m=1,m}(\underline{\mathfrak{R}}_m) = \varnothing\} \equiv \{G \in \varnothing\}_A$ whose second member is impossible (i.e. $E_{\varnothing}$) for which are considered impossible also events such as its first member of which on the other hand is not definable any non-zero probability. Instead the events of type $\{\wedge_{m=1,m}(\{X \in \underline{\mathfrak{R}}_m\}_m) \mid \cap_{m=1,m}(\underline{\mathfrak{R}}_m) = \varnothing\}$, even being able to calculate their nonzero probabilities, are however neglected as rendered evidently impossible by the constancy of X and coherently with the ignore a $\mathbb{P}\langle E\rangle > 0$ of an E impossible i.e. the replace it with $\mathbb{P}\langle E\rangle = 0$ because erroneously resulting by approximate knowledge of the hypothetical happen of E.

Is placed

$$\{\underline{\bar{e}}_A \cap \underline{\bar{e}}_B \neq \varnothing\} \equiv \{\underline{\bar{e}}_A \cap \underline{\bar{e}}_B = \{\underline{\bar{e}}_A \vee \underline{\bar{e}}_B\}\} \quad (39)$$

because, being evident that the second member implies the first, if this does not imply the second is had an impossibility to justify such as that of the k-th paragraph of page x.

From: (39), (7); (8); follows

$$\{\underline{\bar{e}}_A \cap \underline{\bar{e}}_B = \varnothing\} \equiv \neg\{\{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_B\} \vee \{\underline{\bar{e}}_B \subseteq \underline{\bar{e}}_A\}\} \equiv \neg\{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_B\} \wedge \neg\{\underline{\bar{e}}_B \subseteq \underline{\bar{e}}_A\} \quad (40)$$

Is called $\underline{\mathcal{I}}$ a set of which $\underline{\mathcal{I}} \equiv \{\bar{e}_t; t=1,t\}$ and whose numerousness is maximum subordinately to the condition

$$\{\underline{\bar{e}}_A \cap \underline{\bar{e}}_B = \varnothing;\ \forall\{\bar{e}_A,\bar{e}_B\} \subseteq \underline{I}\} \tag{41}$$

which, together with (40), highlights that $\underline{I}$ can not have elements of type $\cup_{k=1,\text{ꝁ}}(\bar{e}_k)$ and $\cap_{k=1,\text{ꝁ}}(\bar{e}_k)$ because this would prevent that ŧ is a maximum.

Coherently with (41) and $\underline{e}_t \subseteq \underline{\bar{e}}_t$ (due to first of (37)), are introduced the events $\bar{e}_{\dot\cup}$ and $e_{\dot\cup}$ of which

$$\underline{\mathbb{M}}\langle e_{\dot\cup}\rangle \equiv \dot\cup_{t=1,\text{ŧ}}(\underline{e}_t) \subseteq \dot\cup_{t=1,\text{ŧ}}(\underline{\bar{e}}_t) \equiv \underline{\mathbb{M}}\langle \bar{e}_{\dot\cup}\rangle \tag{42}$$

In conformity with the first two paragraphs of section 1, $\bar{e}$ has the meaning implicated by $\bar{e} \equiv \{X\in\mathfrak{R}\}$. This, on the basis of (5) and the paragraph that introduces it, implies $\bar{e}_A \twoheadrightarrow \bar{e}$.

The $\underline{\bar{e}}_{\dot\cup} \equiv \dot\cup_{t=1,\text{ŧ}}(\underline{\bar{e}}_t)$ and $\bar{e}_A \twoheadrightarrow \bar{e}$ respectively give rise to the two members of each t-th element of $\{\underline{\bar{e}}_t \subseteq \underline{\bar{e}}_{\dot\cup}, \bar{e}_t \twoheadrightarrow \bar{e};\ t=1,\text{ŧ}\}$ which specifies the $\mathcal{E}_1$ of (21). It is understood $\mathcal{B}\langle\bar{e}_{\dot\cup}\rangle$ which excludes any $\neg\bar{e}_t \equiv \{X\in\neg\mathfrak{R}\}_t \equiv \mathsf{E}_\varnothing$ (*vice versa* implicated by the second of (38) and $\mathcal{B}\langle\bar{e}_t\rangle$), so is had the specification of the $\mathcal{E}_3$ of (21) consisting of being the properties of each $\text{€}\langle\bar{e}_{\dot\cup}\rangle$ coherent in the imply a $\text{€}\langle\bar{e}\rangle$. From: this and second of (21); $\neg\exists\mathcal{P}_A \equiv \{\neg\mathcal{P}_A;\ \forall A\}$; (7); follows

$$\{\bar{e} \equiv \bar{e}_{\dot\cup}\} \underline{\leftarrow} \{\neg\exists\underline{\bar{e}}_{\dot\cup} \cup \underline{\bar{e}}_A \neq \underline{\bar{e}}_{\dot\cup}\} \equiv \{\underline{\bar{e}}_{\dot\cup} \cup \underline{\bar{e}}_A = \underline{\bar{e}}_{\dot\cup};\ \forall\bar{e}_A\} \equiv \{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_{\dot\cup};\ \forall\bar{e}_A\} \tag{43}$$

of which

$$\neg\{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_{\dot\cup}\} \equiv \mathcal{P}_{A1}\ \dot\vee\ \mathcal{P}_{A2}\ \dot\vee\ \mathcal{P}_{A3} \qquad \mathcal{P}_{A1} \equiv \{\underline{\bar{e}}_{\dot\cup} \subset \underline{\bar{e}}_A\} \qquad \mathcal{P}_{A2} \equiv \{\underline{\bar{e}}_{\dot\cup} \cap \underline{\bar{e}}_A \subset \underline{\bar{e}}_{\dot\cup}\} \wedge \{\underline{\bar{e}}_{\dot\cup} \cap \underline{\bar{e}}_A \subset \underline{\bar{e}}_A\}$$
$$\mathcal{P}_{A3} \equiv \{\underline{\bar{e}}_{\dot\cup} \cap \underline{\bar{e}}_A = \varnothing\} \tag{44}$$

The $\mathcal{P}_{A1}$ and $\mathcal{P}_{A2}$ are both false since each of them implies for each $\{\bar{e}_{\dot\cup},\bar{e}_A\}$ an impossibility to justify analogous to that of x-th paragraph of page x. Also $\mathcal{P}_{A3}$ is false because it is incoherent with the definition of $\underline{I}$ i.e. with the be ŧ the maximum compatible with (41). These falsities, first of (44) and (43) give rise to $\bar{e} \equiv \bar{e}_{\dot\cup}$. This and $\underline{e} \subseteq \underline{\bar{e}}$ entail $\underline{e} \subseteq \underline{\bar{e}}_{\dot\cup}$ for which $\underline{e}$ is constituted by all the elements of $\underline{\bar{e}}_{\dot\cup}$ compatible with the meaning of e and hence $e \equiv e_{\dot\cup}$.

From: $\bar{e} \equiv \bar{e}_{\dot\cup}$; first of (27); follows

$$\mathbb{C}\langle\bar{e}_{\dot\cup}\rangle \equiv \mathbb{C}\langle\bar{e}\rangle \underline{\rightarrow} \{\mathbb{P}\langle e\rangle = \rho\langle e \mid \bar{e}\rangle\} \tag{45}$$

Is placed $\mathscr{N}\langle\underline{\bar{e}}_t\rangle = \infty$ on the basis of the unlimited greatness of $\mathscr{N}\langle\mathfrak{R}\rangle$ (and coherently with section 4.1 of [1]). From: second of (24), $\underline{e}_A \subseteq \underline{\bar{e}}_A$; $\bar{e} \equiv \bar{e}_{\dot\cup}$, $e \equiv e_{\dot\cup}$; second of (12); $\mathscr{N}\langle\underline{\bar{e}}_t\rangle = \infty$; second of (24), $\underline{e}_t \subseteq \underline{\bar{e}}_t$; follows

$$\rho\langle e \mid \bar{e}\rangle = \mathscr{N}\langle\underline{e}\rangle / \mathscr{N}\langle\underline{\bar{e}}\rangle = \mathscr{N}\langle\dot\cup_{t=1,\text{ŧ}}(\underline{e}_t)\rangle / \mathscr{N}\langle\dot\cup_{t=1,\text{ŧ}}(\underline{\bar{e}}_t)\rangle = \Sigma_{t=1,\text{ŧ}}(\mathscr{N}\langle\underline{e}_t\rangle) / \Sigma_{t=1,\text{ŧ}}(\mathscr{N}\langle\underline{\bar{e}}_t\rangle) = \text{ŧ}^{-1}\cdot\Sigma_{t=1,\text{ŧ}}(\mathscr{N}\langle\underline{e}_t\rangle / \mathscr{N}\langle\underline{\bar{e}}_t\rangle) = \text{ŧ}^{-1}\cdot\Sigma_{t=1,\text{ŧ}}(\rho\langle e_t \mid \bar{e}_t\rangle) \tag{46}$$

The $\{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_{\dot\cup} = \underline{\bar{e}} \supseteq \underline{e}_{\dot\cup} = \underline{e};\ \forall\bar{e}_A\}$ (above deducted) shows that in (45) $\underline{\bar{e}}_{\dot\cup}$ can not be replaced by a $\underline{\mathsf{B}}$ such that $\mathscr{N}\langle\underline{\mathsf{B}}\rangle < \mathscr{N}\langle\underline{\bar{e}}_{\dot\cup}\rangle$, $\underline{e} \subseteq \underline{\mathsf{B}}$ and of which can not be deduced a $\text{€}\langle\neg\underline{\mathsf{B}}\rangle \equiv \text{€}\langle\neg\underline{e}\rangle$ without using $\{\neg\underline{\mathsf{B}} \subseteq \neg\underline{e}\} \underline{\rightarrow} \{\text{€}\langle\neg\underline{\mathsf{B}}\rangle \equiv \text{€}\langle\neg\underline{e}\rangle\}$. Therefore the $\mathbb{P}\langle e\rangle$ of (45) is, according to the last paragraph of section 2.2, the only true probability of e i.e. the $\mathfrak{P}\langle e\rangle$. This, (45) and (46) give rise to

$$\mathbb{C}\langle\bar{e}_{\dot\cup}\rangle \equiv \mathbb{C}\langle\bar{e}\rangle \underline{\rightarrow} \{\mathfrak{P}\langle e\rangle = \text{ŧ}^{-1}\cdot\Sigma_{t=1,\text{ŧ}}(\rho\langle e_t \mid \bar{e}_t\rangle)\} \tag{47}$$

From: $\mathbb{C}\langle\dot\cup_{k=1,\text{ꝁ}}(e_k)\rangle \equiv \dot\vee_{k=1,\text{ꝁ}}(\mathbb{C}\langle e_k\rangle)$, $\bar{e}_{\dot\cup} \equiv \dot\cup_{t=1,\text{ŧ}}(\bar{e}_t)$; $e_t \twoheadrightarrow e$; (28); follows

$$\mathbb{C}\langle\bar{e}_{\dot\cup}\rangle \equiv \dot\vee_{t=1,\text{ŧ}}(\mathbb{C}\langle\bar{e}_t\rangle) \equiv \dot\vee_{t=1,\text{ŧ}}(e_t \twoheadrightarrow e, \mathbb{C}\langle\bar{e}_t\rangle) \underline{\rightarrow} \dot\vee_{t=1,\text{ŧ}}(\rho\langle e_t \mid \bar{e}_t\rangle = \mathbb{P}\langle e_t\rangle \leq \mathbb{P}\langle e\rangle) \tag{48}$$

which shows how in the absence of (47) would be impossible to have a practically useful information on the probability of e, since they would exist only the following two alternatives, replace $\mathbb{P}\langle e\rangle$ with a $\mathbb{P}\langle e_t\rangle$ or choose a $\mathbb{P}\langle e_t\rangle \leq \mathbb{P}\langle e\rangle$ and exclude all remaining, that would involve however both a decision unjustifiably arbitrary.

From $\underline{I} \equiv \{\bar{e}_t;\ t=1,\text{ŧ}\}$ is deducible (with any criterion) a $\underline{I} \equiv \{\bar{e}_{mn};\ n=1,\text{ꞥ}_m;\ m=1,\text{m̶}\}$. From this is had, coherently with $\bar{e} \equiv \bar{e}_{\dot\cup}$, $e \equiv e_{\dot\cup}$ and (42),

$$\{\underline{\bar{e}}_{mn} \subseteq \underline{\bar{e}}_m \subseteq \underline{\bar{e}},\ \underline{e}_{mn} \subseteq \underline{e}_m \subseteq \underline{e};\ n=1,\text{ꞥ}_m;\ m=1,\text{m̶}\}$$

defined by

$$\bar{e}_m \equiv \dot\cup_{n=1,\text{ꞥ}\langle m\rangle}(\bar{e}_{mn}) \qquad \bar{e} \equiv \dot\cup_{m=1,\text{m̶}}(\bar{e}_m) \qquad e_m \equiv \dot\cup_{n=1,\text{ꞥ}\langle m\rangle}(e_{mn}) \qquad e \equiv \dot\cup_{m=1,\text{m̶}}(e_m) \tag{49}$$

How $\mathscr{N}\langle\underline{\bar{e}}_t\rangle = \infty$ is had also $\mathscr{N}\langle\underline{\bar{e}}_{mn}\rangle = \infty$. From: first of (49); second of (12); $\mathscr{N}\langle\underline{\bar{e}}_{mn}\rangle = \infty$; follows $\mathscr{N}\langle\underline{\bar{e}}_m\rangle = \mathscr{N}\langle\dot\cup_{n=1,\text{ꞥ}\langle m\rangle}(\underline{\bar{e}}_{mn})\rangle = \Sigma_{n=1,\text{ꞥ}\langle m\rangle}(\mathscr{N}\langle\underline{\bar{e}}_{mn}\rangle) = \text{ꞥ}_m\cdot\infty$. From: (24), $\underline{e} \subseteq \underline{\bar{e}}$; (49); second of (12), $\mathscr{N}\langle\underline{\bar{e}}_{mn}\rangle = \infty$; (24), $\underline{e}_{mn} \subseteq \underline{\bar{e}}_{mn}$, $\text{ŧ} = \Sigma_{m=1,\text{m̶}}(\text{ꞥ}_m)$; follows

$$\rho\langle e \mid \underline{\bar{e}}\rangle = \mathfrak{N}\langle \underline{e}\rangle / \mathfrak{N}\langle \underline{\bar{e}}\rangle = \mathfrak{N}\langle \dot{\cup}_{m=1,\not{m}}(\dot{\cup}_{n=1,\not{n}\langle m\rangle}(\underline{e}_{mn}))\rangle / \mathfrak{N}\langle \dot{\cup}_{m=1,\not{m}}(\dot{\cup}_{n=1,\not{n}\langle m\rangle}(\underline{\bar{e}}_{mn}))\rangle = \Sigma_{m=1,\not{m}}(\Sigma_{n=1,\not{n}\langle m\rangle}(\mathfrak{N}\langle \underline{e}_{mn}\rangle / \mathfrak{N}\langle \underline{\bar{e}}_{mn}\rangle)) / \Sigma_{m=1,\not{m}}(\not{n}_m) =$$

$$\not{t}^{-1}\cdot\Sigma_{m=1,\not{m}}(\Sigma_{n=1,\not{n}\langle m\rangle}(\rho\langle e_{mn} \mid \bar{e}_{mn}\rangle)) = \not{t}^{-1}\cdot\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\rho\langle e_m \mid \bar{e}_m\rangle) \tag{50}$$

whose last member is due to

$$\rho\langle e_m \mid \bar{e}_m\rangle = \not{n}_m^{-1}\cdot\Sigma_{n=1,\not{n}\langle m\rangle}(\rho\langle e_{mn} \mid \bar{e}_{mn}\rangle)$$

that is deduced in the evidently analogous way.

The (47), (46) e (50) entail

$$\mathfrak{C}\langle \bar{e}_{\dot{\cup}}\rangle \equiv \mathfrak{C}\langle \bar{e}\rangle \underline{\rightarrow} \{\mathfrak{P}\langle e\rangle = \not{t}^{-1}\cdot\Sigma_{m=1,\not{m}}(\Sigma_{n=1,\not{n}\langle m\rangle}(\rho\langle e_{mn} \mid \bar{e}_{mn}\rangle)) = \not{t}^{-1}\cdot\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\rho\langle e_m \mid \bar{e}_m\rangle)\} \tag{51}$$

Is intended $\{\bar{e}_A,\bar{e}_B\} \subseteq \underline{I}$ and hence $\{\underline{\bar{e}}_A \subseteq \underline{\bar{e}}_{\dot{\cup}},\underline{\bar{e}}_B \subseteq \underline{\bar{e}}_{\dot{\cup}}\}$ which coherently with first of (18) gives rise to $\mathcal{B}\langle \bar{e}_{\dot{\cup}}\rangle \underline{\rightarrow} \neg\mathbb{I}\langle \bar{e}_A,\bar{e}_B\rangle$ which by (3) entails $\mathbb{I}\langle \bar{e}_A,\bar{e}_B\rangle \underline{\rightarrow} \neg\mathcal{B}\langle \bar{e}_{\dot{\cup}}\rangle$. Moreover the having deduced $\bar{e} \equiv \bar{e}_{\dot{\cup}}$ implying $\mathcal{B}\langle \bar{e}_{\dot{\cup}}\rangle$ is equivalent, by (1), to $\{\bar{e} \equiv \bar{e}_{\dot{\cup}}\} \equiv \{\bar{e} \equiv \bar{e}_{\dot{\cup}} \,\big|\, \mathcal{B}\langle \bar{e}_{\dot{\cup}}\rangle\}$ which is equivalent, by (3), to $\neg\mathcal{B}\langle \bar{e}_{\dot{\cup}}\rangle \underline{\rightarrow} \neg\{\bar{e} \equiv \bar{e}_{\dot{\cup}}\}$. This gives rise to $\mathbb{I}\langle \bar{e}_A,\bar{e}_B\rangle \underline{\rightarrow} \{\bar{e} \not\equiv \bar{e}_{\dot{\cup}}\}$ for which the elements of $\underline{I}$ are grouped on the basis of mutual independence placing the

$$\underline{I} \equiv \{\{\bar{e}_{mn};\ n=1,\not{n}_m;\ m=1,\not{m}\} \,\big|\, \neg\mathbb{I}\langle \bar{e}_{mh},\bar{e}_{mk}\rangle,\{\mathbb{I}\langle \bar{e}_{ah},\bar{e}_{bk}\rangle;\ \forall a \neq b\}\} \tag{52}$$

of which (52) $\underline{\rightarrow} \{\bar{e} \not\equiv \bar{e}_{\dot{\cup}}\}$ whose second member is true only if is ignored $\bar{e}_{\dot{\cup}}$.

Therefore (52) (as also everything that is deduced from it) is in force only if $\bar{e} \not\equiv \bar{e}_{\dot{\cup}}$ i.e. is ignored $\bar{e}_{\dot{\cup}}$ i.e. are ignored second and last of (49) as well as the $\{\underline{\bar{e}}_m \subseteq \underline{\bar{e}},\underline{e}_m \subseteq \underline{e};\ m=1,\not{m}\}$ by them implicated.

The $\{\mathbb{I}\langle \bar{e}_{ah},\bar{e}_{bk}\rangle;\ \forall a \neq b\}$ of (52) entails $\mathbb{I}\langle \bar{e}_m;\ m=1,\not{m}\rangle$ that by last of (18) and $\underline{e}_m \subseteq \underline{\bar{e}}_m$ entails $\mathbb{I}\langle e_m;\ m=1,\not{m}\rangle$.

3.1 A confirmation

The (23) has, coherently with (52), the specification

$$\ddot{e}_\wedge \equiv \wedge_{m=1,\not{m}}(\ddot{e}_m) \equiv \cap_{m=1,\not{m}}(\wedge_{m=1,\not{m}}(\ddot{\ddot{e}}_{mm})) \qquad \ddot{e}_\vee \equiv \vee_{m=1,\not{m}}(\ddot{e}_m) \equiv \cup_{m=1,\not{m}}(\wedge_{m=1,\not{m}}(\ddot{\ddot{e}}_{mm})) \tag{53}$$

of which $\ddot{e}_m \equiv \{X \in \underline{\mathcal{R}}_m\}_m$, $\{\ddot{\ddot{e}}_{mm} \equiv \bar{e}_m;\ \forall m \neq m\}$, $\ddot{\ddot{e}}_{mm} \equiv \ddot{e}_m$, $\underline{\ddot{e}}_\wedge \subseteq \underline{\ddot{e}}_\vee \subseteq \underline{\bar{E}}$, being in particular $\bar{E}$, of which $\bar{E} \equiv \wedge_{m=1,\not{m}}(\bar{e}_m)$, the specification of $C_\wedge$ and hence being worth $\bar{E} \equiv \vee_{m=1,\not{m}}(\bar{e}_m)$ if $\mathcal{B}\langle \bar{E}\rangle$ as it is understood in this section.

The evident $\mathbb{I}\langle e_m;\ m=1,\not{m}\rangle \underline{\rightarrow} \mathbb{I}\langle \ddot{e}_m;\ m=1,\not{m}\rangle$ and the said $\mathbb{I}\langle e_m;\ m=1,\not{m}\rangle$ entail $\mathbb{I}\langle \ddot{e}_m;\ m=1,\not{m}\rangle$. This and (26) imply

$$\rho\langle \ddot{e}_\wedge \mid \bar{E}\rangle = \mathfrak{N}\langle \underline{\ddot{e}}_\wedge\rangle / \mathfrak{N}\langle \underline{\bar{E}}\rangle = \Pi_{m=1,\not{m}}(\mathfrak{N}\langle \underline{\ddot{e}}_m\rangle / \mathfrak{N}\langle \underline{\bar{e}}_m\rangle) = \Pi_{m=1,\not{m}}(\rho\langle \ddot{e}_m \mid \bar{e}_m\rangle) \tag{54}$$

From: last of (24); first of (8); (54), $\neg\ddot{e}_m \equiv \{X \in \neg\underline{\mathcal{R}}_m\}_m$ of which $\neg\underline{\mathcal{R}}_m \equiv \underline{\mathfrak{R}} - \underline{\mathcal{R}}_m$ (as can be deduced from $\mathcal{B}\langle \bar{E}\rangle$ and (38)); follows

$$\rho\langle \ddot{e}_\vee \mid \bar{E}\rangle = 1 - \rho\langle \neg\ddot{e}_\vee \mid \bar{E}\rangle = 1 - \rho\langle \wedge_{m=1,\not{m}}(\neg\ddot{e}_m) \mid \bar{E}\rangle = 1 - \Pi_{m=1,\not{m}}(\rho\langle \neg\ddot{e}_m \mid \bar{e}_m\rangle) = 1 - \Pi_{m=1,\not{m}}(1 - \rho\langle \ddot{e}_m \mid \bar{e}_m\rangle) \tag{55}$$

From: second of (53); $\underline{\ddot{e}}_\wedge = \Pi_{m=1,\not{m}}(\underline{\ddot{e}}_m)$ (due to $\mathbb{I}\langle \ddot{e}_m;\ m=1,\not{m}\rangle$ and (22)); (12), the being true the first member of (13) when the $\{\underline{A}_{hk};\ h=1,\not{h};\ k=1,\not{k}\}$ are specified by $\{\underline{\ddot{\ddot{e}}}_{\underline{m}\langle c,b,a\rangle m};\ a=1,c;\ m=1,\not{m}\}$; $\mathfrak{N}\langle \Pi_{k=1,\not{k}}(\underline{A}_k)\rangle = \Pi_{k=1,\not{k}}(\mathfrak{N}\langle \underline{A}_k\rangle)$, $\{\ddot{\ddot{e}}_{mm} \equiv \bar{e}_m;\ \forall m \neq m\}$, $\ddot{\ddot{e}}_{mm} \equiv \ddot{e}_m$; follows

$$\mathfrak{N}\langle \underline{\ddot{e}}_\vee\rangle = \mathfrak{N}\langle \cup_{m=1,\not{m}}(\underline{M}\langle \wedge_{m=1,\not{m}}(\ddot{\ddot{e}}_{mm})\rangle)\rangle = \mathfrak{N}\langle \cup_{m=1,\not{m}}(\Pi_{m=1,\not{m}}(\underline{\ddot{\ddot{e}}}_{mm}))\rangle = \Sigma_{c=1,\not{m}}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle\not{m},c\rangle}(\mathfrak{N}\langle \Pi_{m=1,\not{m}}(\cap_{a=1,c}(\underline{\ddot{\ddot{e}}}_{\underline{m}\langle c,b,a\rangle m}))\rangle)) =$$

$$\Sigma_{c=1,\not{m}}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle\not{m},c\rangle}(\Pi_{a=1,c}(\mathfrak{N}\langle \underline{\ddot{e}}_{\underline{m}\langle c,b,a\rangle}\rangle)\cdot\Pi_{a=c+1,\not{m}}(\mathfrak{N}\langle \underline{\bar{e}}_{K\langle c,b,a\rangle}\rangle))) \tag{56}$$

of which $\{K_{cba};\ a=c+1,\not{m}\} = \{m=1,\not{m}\} - \{\underline{m}\langle c,b,a\rangle;\ a=1,c\}$.

From: $\underline{\ddot{e}}_\vee \subseteq \underline{\bar{E}}$, (24); (56), $\mathfrak{N}\langle \underline{\bar{E}}\rangle = \Pi_{m=1,\not{m}}(\mathfrak{N}\langle \underline{\bar{e}}_m\rangle)$ (due to $\mathfrak{N}\langle \underline{\ddot{e}}_\wedge\rangle = \Pi_{m=1,\not{m}}(\mathfrak{N}\langle \underline{\ddot{e}}_m\rangle$ that is implied by $\mathbb{I}\langle e_m;\ m=1,\not{m}\rangle$ e (22)); $\underline{\ddot{e}}_m \subseteq \underline{\bar{e}}_m$, (24); follows

$$\rho\langle \ddot{e}_\vee \mid \bar{E}\rangle = \mathfrak{N}\langle \underline{\ddot{e}}_\vee\rangle / \mathfrak{N}\langle \underline{\bar{E}}\rangle = \Sigma_{c=1,\not{m}}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle\not{m},c\rangle}(\Pi_{a=1,c}(\mathfrak{N}\langle \underline{\ddot{e}}_{\underline{m}\langle c,b,a\rangle}\rangle / \mathfrak{N}\langle \underline{\bar{e}}_{\underline{m}\langle c,b,a\rangle}\rangle))) =$$

$$\Sigma_{c=1,\not{m}}((-1)^{c+1}\cdot\Sigma_{b=1,Б\langle\not{m},c\rangle}(\Pi_{a=1,c}(\rho\langle \ddot{e}_{\underline{m}\langle c,b,a\rangle} \mid \bar{e}_{\underline{m}\langle c,b,a\rangle}\rangle))) \tag{57}$$

It is remarkable the difference between (55) and (57) in expressing the same $\rho\langle \ddot{e}_\vee \mid \bar{E}\rangle$, as well as the being the first numerically much more convenient because the second requires a computation time that as m increases soon becomes hardly available. Moreover, intending $\underline{\cup A} \equiv \cup_{k=1,\not{k}}(\underline{A}_k)$, the associative property of the union entails $\underline{\cup A} = \{\ldots\{\{\underline{A}_1 \cup \underline{A}_2\} \cup \underline{A}_3\} \cup \ldots \underline{A}_{\not{k}}\}$, for which $\underline{\cup A}$ is the result of a succession of $\not{k} - 1$ unions between two sets and hence each of type $\underline{A} \cup \underline{B}$ of which $\mathfrak{N}\langle \underline{A} \cup \underline{B}\rangle = \mathfrak{N}\langle \underline{A}\rangle + \mathfrak{N}\langle \underline{B}\rangle - \mathfrak{N}\langle \underline{A} \cap \underline{B}\rangle$ (due to first of (12)). Therefore $\mathfrak{N}\langle \underline{\cup A}\rangle$ can be defined iteratively, placing initially $\mathfrak{N}\langle \underline{\cup A}\rangle = \mathfrak{N}\langle \underline{A}_1\rangle$ and then executing the steps indicated by $\{k;\ k=2,\not{k}\}$ and constituted by the replace, at the k-th step, $\mathfrak{N}\langle \underline{\cup A}\rangle$ with $\mathfrak{N}\langle \underline{\cup A}\rangle + \mathfrak{N}\langle \underline{A}_k\rangle - \mathfrak{N}\langle \underline{\cup A} \cap \underline{A}_k\rangle$. The evident analogy, between first of (12) (to which are alternatives the iterations just said) and (57), makes to deduce, as alternative to this expression of $\rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle$, the following procedure: is placed $P=\rho\langle\ddot{e}_1 \,¦\, \bar{e}_1\rangle$, are carried out the steps indicated by $\{m=2,\not{m}\}$ and constituted by the replace P with $P+\rho\langle\ddot{e}_m \,¦\, \bar{e}_m\rangle - P\cdot\rho\langle\ddot{e}_m \,¦\, \bar{e}_m\rangle$ at step m-th, is placed $\rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle = P$ at the end of these steps. The computing time required by this procedure is very near to that of (55).

A $€\langle\underline{\ddot{e}}_\wedge\rangle$ (as well as a $€\langle\underline{\ddot{e}}_\vee\rangle$) is an $\not{m}$-tuple $\{€\langle\underline{\ddot{e}}_m\rangle; m=1,\not{m}\}$ element of $\underline{\bar{E}}$ and then, being X a constant (and neglecting as impossible any $\{\ddot{e}_\wedge \,|\, \cap_{m=1,\not{m}}(\mathscr{R}_m)=\varnothing\}$), implies a $€\langle\underline{\ddot{e}}_\cap\rangle$ of which $\ddot{e}_\cap \equiv \{X\in\cap_{m=1,\not{m}}(\mathscr{R}_m)\}$. Therefore is had $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$ that, for (28) and if $\mathbb{C}\langle\bar{E}\rangle$, gives rise to $\rho\langle\ddot{e}_\wedge \,¦\, \bar{E}\rangle \le \mathscr{P}\langle\ddot{e}_\cap\rangle$ i.e. $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[\rho\langle\ddot{e}_\wedge \,¦\, \bar{E}\rangle,1]$.

As these are also $\ddot{e}_{\wedge p} \twoheadrightarrow \ddot{e}_\cap$ and $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge p},1]$ of which $\ddot{e}_{\wedge p} \equiv \wedge_{m=1,\not{m}}(\ddot{e}_{mp})$, $P_{\wedge p} \equiv \rho\langle\ddot{e}_{\wedge p} \,¦\, \bar{E}\rangle$, $\ddot{e}_{mp} \equiv \{X\in\mathscr{R}_{\mu\langle p,m\rangle}\}_m$ with $\{\mu_{pm}; m=1,\not{m}\}$ the p-th of the $\not{m}!$ permutations of $\{m=1,\not{m}\}$.

If the $\{\ddot{e}_{\wedge p}; p=1,\not{m}!\}$ could be concomitant (i.e. joined) and independent, as indicated by $\wedge_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$ and $\mathbb{I}\langle\ddot{e}_{\wedge p}; p=1,\not{m}!\rangle$, then it could be considered $\wedge_{p=1,\not{m}!}(\ddot{e}_{\wedge p} \twoheadrightarrow \ddot{e}_\cap)$ which would allow to establish $\ddot{e}_{\wedge \not{p}} \twoheadrightarrow \ddot{e}_\cap$ of which $\not{p} \equiv \{p \,|\, P_{\wedge p} = \max\langle P_{\wedge p}; p=1,\not{m}!\rangle\}$ and that on the basis of (28) would allow to deduce $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[\rho\langle\ddot{e}_{\wedge \not{p}} \,¦\, \bar{E}\rangle,1]$ if $\mathbb{C}\langle\bar{E}\rangle$.

However $\mathscr{B}\langle\bar{E}\rangle$, $\{\underline{\ddot{e}}_{\wedge p}\subseteq\underline{\bar{E}}; p=1,\not{m}!\}$ and second of (20) imply that the said concomitance not is representable by $\wedge_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$, but must instead be represented by $\cap_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$. Moreover $\mathscr{B}\langle\bar{E}\rangle$, $\{\underline{\ddot{e}}_{\wedge p}\subseteq\underline{\bar{E}}; p=1,\not{m}!\}$ and first of (18) imply $\neg\mathbb{I}\langle\ddot{e}_{\wedge p}; p=1,\not{m}!\rangle$. Lastly, on the base of (13) and second of (37), is deducible the equivalence between $\cap_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$ and $\wedge_{m=1,\not{m}}(\ddot{e}_{\cap m})$, and thus the concomitance in question is discretionally eliminable by means of the reduce the complexity of the first to the only and mere second.

For these reasons is excluded $\wedge_{p=1,\not{m}!}(\ddot{e}_{\wedge p} \twoheadrightarrow \ddot{e}_\cap)$ and is instead admitted $\dot{\vee}_{p=1,\not{m}!}(\ddot{e}_{\wedge p} \twoheadrightarrow \ddot{e}_\cap)$. Hence, corresponding $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge p},1]$ to each $\ddot{e}_{\wedge p} \twoheadrightarrow \ddot{e}_\cap$, is had $\dot{\vee}_{p=1,\not{m}!}(\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge p},1])$. From: second of (9); $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[0,1]$, $P_{\wedge p}\in[0,1]$; $\wedge_{k=1,\not{k}}(B\in\underline{A}_k)\equiv B\in\cap_{k=1,\not{k}}(\underline{A}_k)$; follows

$$\dot{\vee}_{p=1,\not{m}!}(\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge p},1]) \underline{\twoheadrightarrow} \neg\wedge_{p=1,\not{m}!}(\neg\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge p},1]) \equiv \neg\wedge_{p=1,\not{m}!}(\mathscr{P}\langle\ddot{e}_\cap\rangle\in[0,P_{\wedge p})) \equiv \neg\mathscr{P}\langle\ddot{e}_\cap\rangle\in\cap_{p=1,\not{m}!}([0,P_{\wedge p})) \equiv \neg\mathscr{P}\langle\ddot{e}_\cap\rangle\in[0,P_{\wedge \not{P}}) \equiv \mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge \not{P}},1]$$

of which $\not{P} \equiv \{p \,|\, P_{\wedge p} = \min\langle P_{\wedge p}; p=1,\not{m}!\rangle\}$.

Therefore in what follows it is implied that $\ddot{e}_\wedge$, in order to obtain $\mathscr{P}\langle\ddot{e}_\cap\rangle\in[P_{\wedge \not{P}},1]$ i.e. $\rho\langle\ddot{e}_{\wedge \not{P}} \,¦\, \bar{E}\rangle \le \mathscr{P}\langle\ddot{e}_\cap\rangle$, is replaced by $\ddot{e}_{\wedge \not{P}}$. Hence $Æ\langle\wedge_{m=1,\not{m}}(\neg\ddot{e}_m) \,/\, \ddot{e}_\wedge\rangle$ (for which is applied to $\wedge_{m=1,\not{m}}(\neg\ddot{e}_m)$ the replacement analogous to that just said for $\ddot{e}_\wedge$) and first of (8) imply that $\neg\ddot{e}_\vee$ is replaced by $\neg\ddot{e}_{\vee \not{P}}$ of which $\neg\ddot{e}_{\vee p} \equiv \wedge_{m=1,\not{m}}(\neg\ddot{e}_{mp})$, $\not{P} \equiv \{p \,|\, P_{\vee p}{}^\neg = \min\langle P_{\vee p}{}^\neg; p=1,\not{m}!\rangle\}$, $P_{\vee p}{}^\neg \equiv \rho\langle\neg\ddot{e}_{\vee p} \,¦\, \bar{E}\rangle$, and therefore that $\ddot{e}_\vee$ is replaced by $\ddot{e}_{\vee \not{P}}$.

Coherently with last of (24) is had $P_{\vee \not{P}} = 1 - P_{\vee \not{P}}{}^\neg$ of which $P_{\vee p} \equiv \rho\langle\ddot{e}_{\vee p} \,¦\, \bar{E}\rangle$, $\ddot{e}_{\vee p} \equiv \vee_{m=1,\not{m}}(\ddot{e}_{mp})$. This and the being $P_{\vee \not{P}}{}^\neg$ a minimum imply that $P_{\vee \not{P}}$ is a maximum and hence give rise to $\not{P} \equiv \{p \,|\, P_{\vee p} = \max\langle P_{\vee p}; p=1,\not{m}!\rangle\}$. In the same way from (24) and second of (8) follows $P_{\wedge \not{P}} = 1 - P_{\wedge \not{P}}{}^\neg$ of which $P_{\wedge p}{}^\neg \equiv \rho\langle\neg\ddot{e}_{\wedge p} \,¦\, \bar{E}\rangle$, $\neg\ddot{e}_{\wedge p} \equiv \vee_{m=1,\not{m}}(\neg\ddot{e}_{mp})$. This and the being $P_{\wedge \not{P}}$ a minimum imply that $P_{\wedge \not{P}}{}^\neg$ is a maximum and hence give rise to $\not{P} \equiv \{p \,|\, P_{\wedge p}{}^\neg = \max\langle P_{\wedge p}{}^\neg; p=1,\not{m}!\rangle\}$.

The $\underline{\ddot{e}}_\wedge \subseteq \underline{\ddot{e}}_\vee$ (said in (53) occasion) entails $P_{\wedge \not{P}} \le P_{\vee \not{P}}$. This and the being $P_{\wedge \not{P}}$ a minimum entail $P_{\wedge \not{P}} \le P_{\vee \not{P}}$. This does not contradict the $\underline{\ddot{e}}_\wedge \subseteq \underline{\ddot{e}}_\vee$ that is had with the said replacements of $\ddot{e}_\wedge$ with $\ddot{e}_{\wedge \not{P}}$ and $\ddot{e}_\vee$ with $\ddot{e}_{\vee \not{P}}$.

Being therefore conserved the properties of $\{\ddot{e}_\wedge,\ddot{e}_\vee\}$ by its implicit substitution with $\{\ddot{e}_{\wedge \not{P}},\ddot{e}_{\vee \not{P}}\}$, (54) and (55) are replaced by

$$\rho\langle\ddot{e}_\wedge \,¦\, \bar{E}\rangle = \Pi_{m=1,\not{m}}(\rho\langle\{X\in\mathscr{R}_{\mu\langle \not{P},m\rangle}\}_m \,¦\, \bar{e}_m\rangle) \qquad \rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle = 1 - \Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\mathscr{R}_{\mu\langle \not{P},m\rangle}\}_m \,¦\, \bar{e}_m\rangle) \tag{58}$$

where $\not{P}$ and $\not{P}$ are respectively a p that minimizes $\Pi_{m=1,\not{m}}(\rho\langle\{X\in\mathscr{R}_{\mu\langle p,m\rangle}\}_m \,¦\, \bar{e}_m\rangle)$ and $\Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\mathscr{R}_{\mu\langle p,m\rangle}\}_m \,¦\, \bar{e}_m\rangle)$.

Is placed $\ddot{e}_\cup \equiv \{X\in\cup_{m=1,\not{m}}(\mathscr{R}_m)\}$. From: second of (38), (8); substitution of $\mathscr{R}_m$ whit $\neg\mathscr{R}_m$ in $\neg\ddot{e}_\cap \twoheadrightarrow \neg\ddot{e}_\wedge$ (due to $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$ and (3)); (8); follows $\ddot{e}_\cup \equiv \neg\{X\in\cap_{m=1,\not{m}}(\neg\mathscr{R}_m)\} \twoheadrightarrow \neg\wedge_{m=1,\not{m}}(\neg\ddot{e}_m) \equiv \ddot{e}_\vee$. The $\ddot{e}_\cup \twoheadrightarrow \ddot{e}_\vee$ is the (3.1.21) of [1] and is implied also by (3) and $\neg\ddot{e}_\vee \twoheadrightarrow \neg\ddot{e}_\cup$ which is deduced analogously to $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$.

The $\cap_{m=1,\not{m}}(\neg\mathscr{R}_m) \ne \varnothing$ of $\neg\wedge_{m=1,\not{m}}(\{X\in\neg\mathscr{R}_m\}_m) \equiv \ddot{e}_\vee$ (analogous to $\cap_{m=1,\not{m}}(\mathscr{R}_m) \ne \varnothing$ of $\ddot{e}_\wedge$) is equivalent, for (8), to the need to imply $\cup_{m=1,\not{m}}(\mathscr{R}_m) \ne \neg\varnothing$ inherent to $\ddot{e}_\vee$, but this condition is obviously always obtained because $\neg\varnothing$ is the set that contains each set.

The name (and hence properties) of a $€\langle\underline{\ddot{e}}_\vee\rangle$ is the addition of the names of the elements of such $\not{m}$-tuple. The (53) show $\underline{\ddot{e}}_\vee = \underline{\ddot{e}}_\wedge + \underline{E}$ such that an $\not{m}$-tuple $€\langle\underline{E}\rangle$ may be contradictory because it can happen that some names of its $\not{m}$ elements affirm and the remaining deny that X falls into a certain $\mathscr{R}_m$. This entails that such $€\langle\underline{E}\rangle$ can not imply a $€\langle\underline{\ddot{e}}_\cup\rangle$, and then highlights the erroneousness of $\ddot{e}_\vee \twoheadrightarrow \ddot{e}_\cup$ which, on the basis of (38), (8) and (3), would be tantamount to $\ddot{e}_\cap \twoheadrightarrow \ddot{e}_\wedge$.

The $\{\ddot{E}_{\wedge k}; k=1,\not{k}\}$, of which $\ddot{E}_{\wedge k} \equiv \wedge_{m=1,\not{m}}(\{X\in\mathscr{R}_{mk}\}_m)$ and $\cap_{m=1,\not{m}}(\mathscr{R}_{mk}) = \cap_{m=1,\not{m}}(\mathscr{R}_m)$, verify, analogously to $\ddot{e}_\wedge$ of which are specifications, $\{\underline{\ddot{E}}_{\wedge k}\subseteq\underline{\bar{E}}, \ddot{E}_{\wedge k} \twoheadrightarrow \ddot{e}_\cap; k=1,\not{k}\}$. However is not accepted on the basis of (21) $_\cup\ddot{E}_\wedge \twoheadrightarrow \ddot{e}_\cap$ (nor $_\cup\ddot{E}_\wedge \equiv \ddot{e}_\cap$) of which $_\cup\ddot{E}_\wedge \equiv \cup_{k=1,\not{k}}(\ddot{E}_{\wedge k})$, because, being also $€\langle_\cup\underline{\ddot{E}}_\wedge\rangle$ an $\not{m}$-tuple element of $\underline{\bar{E}}$, not subsists necessarily the inherent specification of $\mathcal{E}_3$ that is the condition for which the properties of each $€\langle_\cup\underline{\ddot{E}}_\wedge\rangle$, determined by considering all the $\{\ddot{E}_{\wedge k}; k=1,\not{k}\}$, they agree in implying a $€\langle\underline{\ddot{e}}_\cap\rangle$. These $_\cup\ddot{E}_\wedge \twoheadrightarrow \ddot{e}_\cap$ and $_\cup\ddot{E}_\wedge \equiv \ddot{e}_\cap$ are not accepted also because for the $\{\ddot{E}_{\wedge k}; k=1,\not{k}\}$ we have the evident considerations analogous to those which above have induced to neglect $\wedge_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$ and consider $\dot{\vee}_{p=1,\not{m}!}(\ddot{e}_{\wedge p})$. Furthermore the $\{\ddot{E}_{\wedge k} \twoheadrightarrow \ddot{e}_\cap; k=1,\not{k}\}$ highlight that $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$ and (21) are not sufficient for $\ddot{e}_\wedge \equiv \ddot{e}_\cap$ since it is not obtainable true the specification of $\mathcal{E}_2$.

The $\neg\ddot{e}_\vee \twoheadrightarrow \neg\ddot{e}_\cup$ and (21) are not sufficient for $\neg\ddot{e}_\vee \equiv \neg\ddot{e}_\cup$, since a single $\wedge_{m=1,\not{m}}(\ddot{\mathbb{e}}_{mm}{}^\neg) \twoheadrightarrow \neg\ddot{e}_\cup$, of which $\{\ddot{\mathbb{e}}_{mm}{}^\neg \equiv \bar{e}_m; \forall m \ne \mathrm{m}\}$ and $\ddot{\mathbb{e}}_{mm}{}^\neg \equiv \neg\ddot{e}_\cup$, is enough to prevent the specification of $\mathcal{E}_2$, noting in this regard also that a $\neg\ddot{e}_\vee \cup \wedge_{m=1,\not{m}}(\ddot{\mathbb{e}}_{mm}{}^\neg) \twoheadrightarrow \neg\ddot{e}_\cup$ is prevented by the absence of the specification of $\mathcal{E}_3$.

From $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$ is deduced (for (28) and if $\mathbb{C}\langle\bar{E}\rangle$) $\rho\langle\ddot{e}_\wedge \,¦\, \bar{E}\rangle \le \mathscr{P}\langle\ddot{e}_\cap\rangle$, but not is had an analogous of (28) for deducing from $\ddot{e}_\cup \twoheadrightarrow \ddot{e}_\vee$ an upper bound of $\mathscr{P}\langle\ddot{e}_\cup\rangle$. However even this limitation can be achieved as follows. The $\neg\ddot{e}_\vee \twoheadrightarrow \neg\ddot{e}_\cup$ entails, for (28) and if $\mathbb{C}\langle\bar{E}\rangle$, $\rho\langle\neg\ddot{e}_\vee \,¦\, \bar{E}\rangle \le \mathscr{P}\langle\neg\ddot{e}_\cup\rangle$. This, for last of (24), is equivalent to $1-\rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle \le 1-\mathscr{P}\langle\ddot{e}_\cup\rangle$ that shows $\mathscr{P}\langle\ddot{e}_\cup\rangle \le \rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle$. Therefore they are had both the $\rho\langle\ddot{e}_\wedge \,¦\, \bar{E}\rangle = 1-\rho\langle\neg\ddot{e}_\wedge \,¦\, \bar{E}\rangle \le \mathscr{P}\langle\ddot{e}_\cap\rangle$ and $\mathscr{P}\langle\ddot{e}_\cup\rangle \le \rho\langle\ddot{e}_\vee \,¦\, \bar{E}\rangle = 1-\rho\langle\neg\ddot{e}_\vee \,¦\, \bar{E}\rangle$.

Specifying in these $\ddot{e}_\wedge$ and $\ddot{e}_\vee$ as the respective $\ddot{e}_{\wedge\mathscr{R}}$ and $\ddot{e}_{\vee\mathscr{R}}$ of which $\ddot{e}_{\wedge\mathscr{R}} \equiv \wedge_{m=1,\not{m}}(\{X\in\underline{A}_m\}_m)$, $\cap_{m=1,\not{m}}(\underline{A}_m) = \mathscr{R}$,

$\ddot{e}_{\vee\underline{\mathfrak{R}}} \equiv \vee_{m=1,\not{m}}(\{X\in\underline{B}_m\}_m)$, $\cup_{m=1,\not{m}}(\underline{B}_m)=\underline{\mathfrak{R}}$, the $\ddot{e}_\cap$ and $\ddot{e}_\vee$ are specified, on the basis of $e\equiv\{X\in\underline{\mathfrak{R}}\}\equiv\{\ddot{e}_\cap \mid \cap_{m=1,\not{m}}(\underline{\mathfrak{R}}_m)=\underline{\mathfrak{R}}\}\equiv\{\ddot{e}_\cup \mid \cup_{m=1,\not{m}}(\underline{\mathfrak{R}}_m)=\underline{\mathfrak{R}}\}$, both by e, following

$$\rho\langle\ddot{e}_{\wedge\underline{\mathfrak{R}}} \mid \bar{E}\rangle = 1-\rho\langle\neg\ddot{e}_{\wedge\underline{\mathfrak{R}}} \mid \bar{E}\rangle \le \mathfrak{P}\langle e\rangle \le \rho\langle\ddot{e}_{\vee\underline{\mathfrak{R}}} \mid \bar{E}\rangle = 1-\rho\langle\neg\ddot{e}_{\vee\underline{\mathfrak{R}}} \mid \bar{E}\rangle \tag{59}$$

which has $\mathfrak{C}\langle\bar{E}\rangle$ as sufficient condition, of which by (8) is had $\neg\ddot{e}_{\wedge\underline{\mathfrak{R}}}\equiv\vee_{m=1,\not{m}}(\{X\in\neg\underline{A}_m\}_m)$ and $\neg\ddot{e}_{\vee\underline{\mathfrak{R}}}\equiv\wedge_{m=1,\not{m}}(\{X\in\neg\underline{B}_m\}_m)$, of which by (8) is had $\cup_{m=1,\not{m}}(\neg\underline{A}_m)=\cap_{m=1,\not{m}}(\neg\underline{B}_m)=\neg\underline{\mathfrak{R}}$, and which results therefore coherent with (8.4) and (8.5) of [1] if is considered that in how much moment ago the use of $\underline{A}_m$ is equivalent to using $\neg\underline{A}_m$.

The $\neg\bar{E}\equiv\vee_{m=1,\not{m}}(\neg\bar{e}_m)$ and the fact that $\neg\bar{e}_m$ does not affect by no means X imply that no relation between $\neg\bar{E}$ and e can be implicated from their properties. Therefore, on the base of the last paragraph of section 2.2 and meaning $\rho_A=\rho\langle\ddot{e}_{\wedge\underline{\mathfrak{R}}} \mid \bar{E}\rangle$ and $\rho_B=\rho\langle\ddot{e}_{\vee\underline{\mathfrak{R}}} \mid \bar{E}\rangle$, (59) can be written as $\rho_A\le\mathfrak{P}\langle e\rangle\le\rho_B$, of which $\underline{\rho}_A\le\rho_A\le\bar{\rho}_A$ and $\underline{\rho}_B\le\rho_B\le\bar{\rho}_B$ because ρ_A and ρ_B are variables dependent on choice of the respective $\{\underline{A}_m;\, m=1,\not{m}\}$ and $\{\underline{B}_m;\, m=1,\not{m}\}$ which moreover, as shown by $\cap_{m=1,\not{m}}(\underline{A}_m)=\cup_{m=1,\not{m}}(\underline{B}_m)=\underline{\mathfrak{R}}$, are different in the sense that always verify $\{\underline{A}_m;\, m=1,\not{m}\}\ne\{\underline{B}_m;\, m=1,\not{m}\}$ with the single exception of the case $\{\underline{A}_m=\underline{B}_m=\underline{\mathfrak{R}};\, m=1,\not{m}\}$.

The $\rho_A=\rho\langle\ddot{e}_{\wedge\underline{\mathfrak{R}}} \mid \bar{E}\rangle$ and first of (58) entail $\rho_A=\Pi_{m=1,\not{m}}(\rho\langle\{X\in\underline{A}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle)$ where Q is a p that minimizes $\Pi_{m=1,\not{m}}(\rho\langle\{X\in\underline{A}_{\mu\langle p,m\rangle}\}_m \mid \bar{e}_m\rangle)$. This, $\cap_{m=1,\not{m}}(\underline{A}_m)=\underline{\mathfrak{R}}$, $\rho\langle\ddot{e}_m \mid \bar{e}_m\rangle=\mathfrak{N}\langle\underline{\ddot{e}}_m\rangle/\mathfrak{N}\langle\underline{\bar{e}}_m\rangle$ (due to second of (24)) and the being $\mathfrak{N}\langle\underline{\ddot{e}}_m\rangle$ growing with the extension of $\underline{\mathfrak{R}}_m$ (due to first of (37)),)), $\{X\in\underline{\mathfrak{R}}\}\equiv e$ e $\{X\in\underline{\mathfrak{R}}\}\equiv\bar{e}$ entail

$$\underline{\rho}_A=\{\Pi_{m=1,\not{m}}(\rho\langle\{X\in\underline{A}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle) \mid \underline{A}_m=\underline{\mathfrak{R}};\, m=1,\not{m}\}=\Pi_{m=1,\not{m}}(\rho\langle e_m \mid \bar{e}_m\rangle)$$
$$\bar{\rho}_A=\{\Pi_{m=1,\not{m}}(\rho\langle\{X\in\underline{A}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle) \mid \{\underline{A}_m=\underline{\mathfrak{R}};\, \forall m\ne M\},\underline{A}_M=\underline{\mathfrak{R}}\}=\rho\langle e_M \mid \bar{e}_M\rangle \tag{60}$$

of which $M\equiv\{m \mid \rho\langle e_m \mid \bar{e}_m\rangle=\min\langle\rho\langle e_m \mid \bar{e}_m\rangle;\, m=1,\not{m}\rangle\}$.

The $\rho_B=\rho\langle\ddot{e}_{\vee\underline{\mathfrak{R}}} \mid \bar{E}\rangle$ and second of (58) entail $\rho_B=1-\Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\underline{B}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle)$ where Q is a p that minimizes $\Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\underline{B}_{\mu\langle p,m\rangle}\}_m \mid \bar{e}_m\rangle)$. This, $\cap_{m=1,\not{m}}(\neg\underline{B}_m)=\neg\underline{\mathfrak{R}}$ and last of (24) entail

$$\underline{\rho}_B=1-\{\Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\underline{B}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle) \mid \{\neg\underline{B}_m=\underline{\mathfrak{R}};\, \forall m\ne M\},\neg\underline{B}_M=\neg\underline{\mathfrak{R}}\}=1-\rho\langle\neg e_M \mid \bar{e}_M\rangle=\rho\langle e_M \mid \bar{e}_M\rangle \tag{61}$$
$$\bar{\rho}_B=1-\{\Pi_{m=1,\not{m}}(\rho\langle\{X\in\neg\underline{B}_{\mu\langle Q,m\rangle}\}_m \mid \bar{e}_m\rangle) \mid \neg\underline{B}_m=\neg\underline{\mathfrak{R}};\, m=1,\not{m}\}=1-\Pi_{m=1,\not{m}}(\rho\langle\neg e_m \mid \bar{e}_m\rangle)=1-\Pi_{m=1,\not{m}}(1-\rho\langle e_m \mid \bar{e}_m\rangle)$$

of which $M\equiv\{m \mid \rho\langle\neg e_m \mid \bar{e}_m\rangle=\min\langle\rho\langle\neg e_m \mid \bar{e}_m\rangle;\, m=1,\not{m}\rangle\}\equiv\{m \mid \rho\langle e_m \mid \bar{e}_m\rangle=\max\langle\rho\langle e_m \mid \bar{e}_m\rangle;\, m=1,\not{m}\rangle$.

The $\ddot{e}_\wedge \twoheadrightarrow \ddot{e}_\cap$ has the specification $\bar{E}\twoheadrightarrow\bar{e}$. The $\neg\bar{E}\equiv\vee_{m=1,\not{m}}(\neg\bar{e}_m)$ (that is had by (8)) and (14) show $\neg\exists\{\underline{e}\cup\underline{\bar{E}}\ne\underline{\bar{E}} \mid e\twoheadrightarrow\bar{e}\}$. This and (21) give rise to $\bar{e}\equiv\bar{E}$.

This, $\mathfrak{C}\langle\bar{E}\rangle \underline{\rightarrow} \{\rho_A\le\mathfrak{P}\langle e\rangle\le\rho_B\}$, $\underline{\rho}_A\le\rho_A\le\bar{\rho}_A$, $\underline{\rho}_B\le\rho_B\le\bar{\rho}_B$, (60) and (61) entail

$$\mathfrak{C}\langle\bar{E}\rangle\equiv\mathfrak{C}\langle\bar{e}\rangle \underline{\rightarrow} \{\rho\langle e_M \mid \bar{e}_M\rangle\le\mathfrak{P}\langle e\rangle\le\rho\langle e_M \mid \bar{e}_M\rangle\} \tag{61}$$

From: (52) $\underline{\rightarrow}$ $\{\bar{e}\ne\bar{e}_\cup\}$ (and (3)); follows

$$\{\bar{e}\equiv\bar{e}_\cup\} \underline{\rightarrow} \neg(52) \underline{\rightarrow} \neg\{\bar{e}\equiv\bar{E}\}$$

that, by (3), implies

$$\{\bar{e}\equiv\bar{e}_\cup\}\equiv\{\bar{e}\equiv\bar{e}_\cup \mid \bar{e}\ne\bar{E}\} \qquad \{\bar{e}\equiv\bar{E}\}\equiv\{\bar{e}\equiv\bar{E} \mid \bar{e}\ne\bar{e}_\cup\}$$

These respectively show that is had $\{\bar{e}\equiv\bar{e}_\cup\}$ only if $\bar{e}\ne\bar{E}$ (i.e. only if is ignored $\bar{E}$) and $\{\bar{e}\equiv\bar{E}\}$ only if $\bar{e}\ne\bar{e}_\cup$ (i.e. only if is ignored $\bar{e}_\cup$). Nevertheless, as said in occasion of (17), the ignore an event is not a logical error. Thus (51) and (61) are both valid and differ only because deduced with different argumentations.

So ultimately, the appear in both the (51) and (61) the same true $\mathfrak{P}\langle e\rangle$ and $\kappa=\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\kappa)/\Sigma_{m=1,\not{m}}(\not{n}_m)$ imply $\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\rho\langle e_M \mid \bar{e}_M\rangle)\le\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\rho\langle e_m \mid \bar{e}_m\rangle)\le\Sigma_{m=1,\not{m}}(\not{n}_m\cdot\rho\langle e_M \mid \bar{e}_M\rangle)$ which, being evidently true, confirms (51) inasmuch *vice versa* would be erroneous some part of the previous argumentation and hence it could be erroneous also the same (51).

4 THE CALCULATION OF THE CONFIDENCE INTERVAL

The set $\underline{\mathfrak{R}}$, of which the $e\equiv\{X\in\underline{\mathfrak{R}}\}$ treated in section 3, has been defined by the only $\underline{\mathfrak{R}}\subseteq\mathfrak{R}$, therefore is had its $\underline{\mathfrak{R}}\equiv\cup_{i=1,\not{i}}(\underline{\mathfrak{I}}_i)$ of which $\underline{\mathfrak{I}}_i\equiv[\underline{\mathfrak{A}}_i,\underline{\mathfrak{B}}_i]$ where "[$\underline{\mathfrak{A}}_i$" and "$\underline{\mathfrak{B}}_i$]" can be substitued by the respective "($-\infty$" and "∞)". Such $\underline{\mathfrak{R}}$ is a zone of the real line (an interval if $\not{i}=1$) of confidence $\mathfrak{P}\langle e\rangle$ (expressed in (47)) for the unknown constant X.

The calculation of $\mathfrak{P}\langle e\rangle$ can take place by means of (47) only if it is known every $\rho\langle e_t \mid \bar{e}_t\rangle$ of which $\bar{e}_t\in\underline{I}$. In order to achieve this necessary condition it is sufficient to know the functions

$$\{a_t(\mathfrak{A},\mathfrak{B}),b_t(\mathfrak{A},\mathfrak{B}),\mathfrak{p}\langle s_t\rangle(x);\, t=1,\not{t}\} \tag{62}$$

such as to verify

$$\{ \mathscr{a} \leq \mathrm{X} \leq \mathscr{B} \}_t \equiv \{ a_t(\mathscr{a},\mathscr{B}) \leq \mathrm{s}_t \leq b_t(\mathscr{a},\mathscr{B}) \} \qquad \bar{\mathrm{e}}_t \equiv \{ \mathrm{s}_t \in \underline{\mathfrak{R}} \} \tag{63}$$

that, in conformity to (41) and (40), have as a necessary condition

$$\{\neg\{\underline{\mathbb{M}}\langle \mathrm{s}_A \in \underline{\mathfrak{R}}\rangle \subseteq \underline{\mathbb{M}}\langle \mathrm{s}_B \in \underline{\mathfrak{R}}\rangle\}; \forall\{(A,B) \mid \{A,B\} \subseteq \{t=1,\mathfrak{t}\}\}\} \tag{64}$$

Indeed from: (24), $\underline{\mathrm{e}}_t \subseteq \bar{\mathrm{e}}_t$; $\underline{\mathfrak{R}} \equiv \cup_{i=1,\mathfrak{i}}(\underline{\mathfrak{I}}_i)$, second of (37); first of (20), second of (12), $\underline{\mathfrak{I}}_i \equiv [\mathscr{a}_i,\mathscr{B}_i]$; (63); (25); follows (coherently with (4.2.19) of [1])

$$\rho\langle \mathrm{e}_t \mid \bar{\mathrm{e}}_t\rangle = \mathfrak{N}\langle \underline{\mathrm{e}}_t\rangle / \mathfrak{N}\langle \underline{\bar{\mathrm{e}}}_t\rangle = \mathfrak{N}\langle \underline{\mathbb{M}}\langle \cup_{i=1,\mathfrak{i}}(\{ \mathrm{X} \in \underline{\mathfrak{I}}_i \}_t)\rangle\rangle / \mathfrak{N}\langle \underline{\bar{\mathrm{e}}}_t\rangle = \Sigma_{i=1,\mathfrak{i}}(\mathfrak{N}\langle \underline{\mathbb{M}}\langle\{ \mathscr{a}_i \leq \mathrm{X} \leq \mathscr{B}_i \}_t\rangle\rangle / \mathfrak{N}\langle \underline{\bar{\mathrm{e}}}_t\rangle) =$$

$$\Sigma_{i=1,\mathfrak{i}}(\mathfrak{N}\langle \underline{\mathbb{M}}\langle a_t(\mathscr{a}_i,\mathscr{B}_i) \leq \mathrm{s}_t \leq b_t(\mathscr{a}_i,\mathscr{B}_i)\rangle\rangle / \mathfrak{N}\langle \underline{\mathbb{M}}\langle \mathrm{s}_t \in \underline{\mathfrak{R}}\rangle\rangle) = \Sigma_{i=1,\mathfrak{i}}(\int\langle a_t(\mathscr{a}_i,\mathscr{B}_i), b_t(\mathscr{a}_i,\mathscr{B}_i)\rangle(\mathfrak{P}\langle \mathrm{s}_t\rangle(x)\cdot dx)) \tag{65}$$

and thus (62) allows to know each $\rho\langle \mathrm{e}_t \mid \bar{\mathrm{e}}_t\rangle$ by means of (65).

This and the deduce $\bar{\mathrm{e}}_\cup \equiv \hat{\mathrm{s}}_\cup$ of which $\hat{\mathrm{s}}_\cup \equiv \cup_{t=1,\mathfrak{t}}(\mathrm{s}_t \in \underline{\mathfrak{R}})$, from $\bar{\mathrm{e}}_\cup \equiv \cup_{t=1,\mathfrak{t}}(\bar{\mathrm{e}}_t)$ (in (42)) and second of (63), allow to write (47) as

$$\mathfrak{C}\langle \hat{\mathrm{s}}_\cup\rangle \underline{\rightarrow} \{\mathfrak{P}\langle \mathrm{e}\rangle = \mathfrak{t}^{-1}\cdot\Sigma_{t=1,\mathfrak{t}}(\Sigma_{i=1,\mathfrak{i}}(\int\langle a_t(\mathscr{a}_i,\mathscr{B}_i), b_t(\mathscr{a}_i,\mathscr{B}_i)\rangle(\mathfrak{P}\langle \mathrm{s}_t\rangle(x)\cdot dx)))\} \tag{66}$$

for which is sufficient to know (62) of which (63) which is worth only if subsists (64).

The $\mathfrak{P}\langle \mathrm{e}\rangle$ is the true probability of e in front of its alternatives merely conventional, but its calculation by means of (66), as is found at least in the cases considered, is prevented by the excessive greatness of $\mathfrak{t}$ i.e. $\mathfrak{N}\langle \underline{I}\rangle$, following that in practice (66) must be used replacing its $\underline{I}$ with a conventional $\underline{I}_C$ of which $\underline{I}_C \subset \underline{I}$ and then being able to evaluate not $\mathfrak{P}\langle \mathrm{e}\rangle$ but a its conventional approximation $\mathbb{P}_C\langle \mathrm{e}\rangle$ that improves with increasing of $\mathfrak{N}\langle \underline{I}_C\rangle$. In what follows this substitution operative of $\underline{I}$ and $\mathfrak{P}\langle \mathrm{e}\rangle$ with $\underline{I}_C$ and $\mathbb{P}_C\langle \mathrm{e}\rangle$ is implicit, noting in particular that, in this use of (66), the greatness of $\mathfrak{N}\langle \underline{I}_C\rangle$ and the treat numbers *floating point* make it convenient appropriate precautions as the *Kahan summation algorithm* which may be written as follows in a pseudolanguage derived from the *Visual Basic*

```
Function SOMMA(Ai; i=1,i)
Dim S, C, T, Y As Double
C = 0
S = 0
 For i = 1 To i
   Y = Ai – C
   T = S + Y
   C = T – S  –  Y
   S = T
 Next
 Return S
End Function
```

with this **Function** that returns $\Sigma_{i=1,\mathfrak{i}}(A_i)$.

For such a use of (66) specifically inherent the cases (of great importance in the experimental sciences) that X is the mean o variance of a normal (i.e. Gaussian) random variable, are below reported some PDF functions that specify the $\mathfrak{P}_S(x)$ of (25) and whose analytical deduction is referred in section 6 of [1]. In that regard is had $\{\mathfrak{P}\langle a\rangle \equiv \mathfrak{P}\langle b\rangle\} \equiv \{a \equiv b\}$.

A normal random variable g, with mean $\mathrm{M_g}$ and variance V^2, and the standard normal random variable Z have

$$\mathfrak{P}\langle \mathrm{g}\rangle(x) \equiv G\langle \mathrm{M_g},\mathrm{V}^2\rangle(x) \equiv (2\cdot\pi\cdot\mathrm{V}^2)^{-0.5}\cdot\exp\langle -0.5\cdot(x-\mathrm{M_g})^2/\mathrm{V}^2\rangle \qquad \mathfrak{P}\langle Z\rangle(x) \equiv Z(x) \equiv G\langle 0,1\rangle(x) \equiv (2\cdot\pi)^{-0.5}\cdot\exp\langle -0.5\cdot x^2\rangle$$

of which $\underline{\mathfrak{R}}\langle x\rangle = \underline{\mathfrak{R}}\langle \mathrm{g}\rangle = \underline{\mathfrak{R}}\langle Z\rangle = \underline{\mathfrak{R}}$, $\mathrm{V} > 0$ and $\exp\langle \mathrm{S}\rangle \equiv \mathrm{e}^{\mathrm{S}}$ (with e the Napier's or Euler's constant).

In relation to these g and Z is had

$$\int_{a,b}(G\langle \mathrm{M_g},\mathrm{V}^2\rangle(x)\cdot dx) = \int\langle (a-\mathrm{M_g})/\mathrm{V}, (b-\mathrm{M_g})/\mathrm{V}\rangle(Z(x)\cdot dx)$$

whose second member is calculable specifying the last equation of (25), using the relation said in [18] between a $\int_{-\infty,c}(Z(x)\cdot dx)$ and the *incomplete gamma function*, and calculating this with the algorithm exposed in [19].

With reference to section 4.1 of [1], a sample $\underline{X}$ of a population $\underline{\mathbb{X}}$ is random if each $\mathfrak{E}\langle \underline{X}\rangle$ is determined when each $\mathfrak{E}\langle \underline{\mathbb{X}}\rangle$ has the same probability to have such determination.

Is intended that $\mathbb{I}\langle \underline{x}\rangle$, with $\underline{x}$ a set of $\mathfrak{k}$ quantities of which $\underline{x} \equiv \{x_k; k=1,\mathfrak{k}\}$, means that such quantities are independent i.e. that $\underline{\mathfrak{R}}\langle x_k\rangle$ is not modified by any $(\mathfrak{k}-1)$-tuple of values that can respectively have the remaining $\{x_k; k \neq k; k=1,\mathfrak{k}\}$.

A $\underline{\mathrm{s}} \equiv \{\mathrm{s}_k; k=1,\mathfrak{k}\}$, of which $\mathfrak{P}\langle \mathrm{s}_k\rangle \equiv \mathfrak{P}\langle \mathrm{s}\rangle$, implies that $\underline{\mathrm{s}}$ can be indifferently considered a set of $\mathfrak{k}$ random variables that have as

PDF the same $\wp\langle s\rangle(x)$ or $\not{k}$ values of the same s. And if in the second case is had $\mathbb{I}\langle \underline{s}\rangle$, $\underline{s}$ becomes evident as a random sample of the population of all values of s (which is obviously different from its subset $\underline{\mathfrak{R}}\langle s\rangle$).

By placing $\underline{g} \equiv \{g_a;\, a=1,\not{a}\}$, $\mathbb{I}\langle \underline{g}\rangle$, $\wp\langle g_a\rangle \equiv G\langle M_g, V^2\rangle$ and intending $m\langle \underline{g}\rangle = \Sigma_{a=1,\not{a}}(g_a)\,/\,\not{a}$, is had the random variable z of which

$$\wp\langle z\rangle \equiv Z \qquad z = \not{a}^{0.5}\cdot(m_{\underline{g}} - M_g)\,/\,V \tag{67}$$

A χ^2 (chi-square) random variable with ν degrees of freedom has

$$\wp\langle \chi^2\rangle(x) \equiv X\langle\nu\rangle(x) \equiv (2^{\nu/2}\cdot\Gamma(\nu/2))^{-1}\cdot x^{\nu/2-1}\cdot \exp\langle -0.5\cdot x\rangle$$

where $\underline{\mathfrak{R}}\langle x\rangle = \underline{\mathfrak{R}}\langle \chi^2\rangle = [0,\infty)$, ν is a natural number greater than 0, $\Gamma(\alpha)$ is the *gamma function* defined by $\Gamma(\alpha) \equiv \int_{0,\infty}(t^{\alpha-1}\cdot e^{-t}\cdot dt)$, $\underline{\mathfrak{R}}\langle\alpha\rangle = (0,\infty)$. In order to calculate a $\int_{-\infty,a}(X\langle\nu\rangle(x)\cdot dx)$ (and make (25) useful as just said) is indicated the algorithm in [18].

A $\mathcal{T}$ (Student's t) random variable with ν degrees of freedom is defined by a $\mathcal{T} = Z\,/\,(\chi^2/\nu)^{0.5}$ of which $\mathbb{I}\langle Z,\chi^2\rangle$ and has

$$\wp\langle \mathcal{T}\rangle(x) \equiv T\langle\nu\rangle(x) \equiv (\pi\cdot\nu)^{-0.5}\cdot\Gamma^{-1}(\nu/2)\cdot\Gamma((\nu+1)/2)\cdot(1 + x^2/\nu)^{-(\nu+1)/2} \tag{68}$$

of which $\underline{\mathfrak{R}}\langle x\rangle = \underline{\mathfrak{R}}\langle \mathcal{T}\rangle = \underline{\mathfrak{R}}$. To calculate a $\int_{-\infty,a}(T\langle\boldsymbol{\nu}\rangle(x)\cdot dx)$ is referred the algorithm in [18].

The number of all the different partitions of a set of $\not{k}$ elements is equal to the $\not{k}$-th Bell number (of which [9], [10], [20]) that is indicated $\mathfrak{B}\langle\not{k}\rangle$. For the determination of such partitions is referred the algorithm in [20].

Is placed $\underline{g} \equiv \{g_a;\, a=1,\not{a}\}$ of which $\not{a} > 1$, $\mathbb{I}\langle \underline{g}\rangle$, $\wp\langle g_a\rangle \equiv G\langle M_g, V^2\rangle$, thus is had $\{\underline{g} \equiv \underline{g}\} \;\dot\vee\; \{\underline{g} \not\equiv \underline{g}\}$ and

$$\{\underline{g} = \{\underline{g}_{ph};\, h=1,\not{h}_p\};\, p=1,\mathfrak{B}\langle\not{a}\rangle\} \tag{69}$$

where $\{\underline{g}_{ph};\, h=1,\not{h}_p\}$ is the p-th partition of $\underline{g}$ with $\underline{g}_{ph} \equiv \{g_{phk};\, k=1,\not{k}_{ph}\}$, and of which is placed $\not{h}_1 = \not{a}$, $\not{h}_{\mathfrak{B}\langle\not{a}\rangle} = 1$.

In this regard is had (coherently with (6.3.26) of [1]), for $\not{h}_p > 1$ i.e. $p < \mathfrak{B}\langle\not{a}\rangle$, $\wp\langle D^2_p/V^2\rangle \equiv X\langle\not{h}_p - 1\rangle$ of which $D^2_p = \Sigma_{h=1,\not{h}\langle p\rangle}(\not{k}_{ph}\cdot(m_{\underline{g}\langle p,h\rangle} - m_{\underline{g}})^2)$ and, for $\not{h}_p < \not{a}$ i.e. $p > 1$, $\wp\langle D^2_p/V^2\rangle \equiv X\langle\check{N}_p\rangle$ of which $D^2_p = \Sigma_{h=1,\not{h}\langle p\rangle}(\Sigma_{k=1,\not{k}\langle p,h\rangle}((g_{phk} - m_{\underline{g}\langle p,h\rangle})^2))$, $\check{N}_p = \Sigma_{h=1,\not{h}\langle p\rangle}(\not{k}_{ph} - 1)$. This is written

$$\{\wp\langle D^2_p/V^2\rangle \equiv X\langle\not{h}_p - 1\rangle, \wp\langle D^2_p/V^2\rangle \equiv X\langle\check{N}_p\rangle;\, p=2,\mathfrak{B}\langle\not{a}\rangle - 1\} \qquad \wp\langle D^2_1/V^2\rangle \equiv \wp\langle D^2_{\mathfrak{B}\langle\not{a}\rangle}/V^2\rangle \equiv \Sigma_{a=1,\not{a}}((g_a - m_{\underline{g}})^2) \equiv X\langle\not{a} - 1\rangle$$

i.e.

$$\{\wp\langle \mathbf{D}^2_q/V^2\rangle \equiv X\langle\boldsymbol{\nu}_q\rangle;\, q=1,2\cdot\mathfrak{B}\langle\not{a}\rangle - 3\} \tag{70}$$

of which

$$\{\mathbf{D}^2_1,\boldsymbol{\nu}_1\} \equiv \{\Sigma_{a=1,\not{a}}((g_a - m_{\underline{g}})^2), \not{a} - 1\} \qquad \{\{\mathbf{D}^2_q,\boldsymbol{\nu}_q\} \equiv \{D^2_q, \not{h}_q - 1\};\, q=2,\mathfrak{B}\langle\not{a}\rangle - 1\}$$
$$\{\{\mathbf{D}^2_q,\boldsymbol{\nu}_q\} \equiv \{D^2_{p\langle q\rangle}, \check{N}_{p\langle q\rangle}\};\, q=\mathfrak{B}\langle\not{a}\rangle, 2\cdot\mathfrak{B}\langle\not{a}\rangle - 3\} \qquad p_q \equiv q - \mathfrak{B}\langle\not{a}\rangle + 2 \tag{71}$$

4.1 The mean of a normal random variable

From the previous definitions of random variables is deduced (with particular reference to (68), (67) and (70))

$$\wp\langle t_q\rangle \equiv T\langle\boldsymbol{\nu}_q\rangle \qquad t_q = z\,/\,((\mathbf{D}^2_q/V^2)/\boldsymbol{\nu}_q)^{0.5} = (m_{\underline{g}} - M_g)\,/\,w_q \qquad w_q = (\mathbf{D}^2_q/(\boldsymbol{\nu}_q\cdot\not{a}))^{0.5} \tag{72}$$

Is called $\mathbb{G}$ the set of all combinations of the elements of $\underline{g}$ and so is placed $\mathbb{G} \equiv \{\mathbb{G}_u;\, u=1,\not{u}\}$ of which $\mathbb{G}_u \equiv \{\mathbb{G}_{ua};\, a=1,\not{a}_u\}$, $\not{u} = \Sigma_{k=1,\not{a}}(Б\langle\not{a},k\rangle)$. Is called $\mathcal{G}$ the set of all combinations of class greater than 1 of the elements of $\underline{g}$ and so is placed $\mathcal{G} \equiv \{\underline{\mathcal{G}}_u;\, u=1,\not{u}\}$ of which $\mathcal{G}_u \equiv \{\mathcal{G}_{ua};\, a=1,\not{a}_u\}$, $\not{u} = \Sigma_{k=2,\not{a}}(Б\langle\not{a},k\rangle) = \not{u} - \not{a}$.

As (69) is had also

$$\{\underline{\mathcal{G}}_u = \{\underline{\mathcal{G}}_{uph};\, h=1,\not{h}_{up}\};\, p=1,\mathfrak{B}\langle\not{a}_u\rangle\}$$

where $\{\underline{\mathcal{G}}_{uph};\, h=1,\not{h}_{up}\}$ is the p-th partition of $\underline{\mathcal{G}}_u$ with $\underline{\mathcal{G}}_{uph} \equiv \{\underline{\mathcal{G}}_{uphk};\, k=1,\not{k}_{uph}\}$, and of which is placed $\not{h}_{u1} = \not{a}_u$, $\not{h}_{u\mathfrak{B}\langle\not{a}\langle u\rangle\rangle} = 1$.

The (72) remains valid also if its $\underline{g}$ and $\underline{g}$ are replaced by respective $\mathbb{G}_u$ and $\underline{\mathcal{G}}_u$ of which $\{\mathbb{G}_u \equiv \underline{\mathcal{G}}_u\} \;\dot\vee\; \{\mathbb{G}_u \not\equiv \underline{\mathcal{G}}_u\}$. Such a substitution in (72) entails the replacement of $\{m_{\underline{g}}, \not{a}\}$ with one of the $\{\{m_{\mathbb{G}\langle u\rangle}, \not{a}_u\};\, u=1,\not{u}\}$ and the replacement of $\{\mathbf{D}^2_q, \boldsymbol{\nu}_q\}$ with a $\{\mathbf{D}^2_{uq}, \boldsymbol{\nu}_{uq}\}$ where u refers $\underline{\mathcal{G}}_u$ and is had $q \in \{q=1,2\cdot\mathfrak{B}\langle\not{a}_u\rangle - 3\}$ analogously to $q \in \{q=1,2\cdot\mathfrak{B}\langle\not{a}\rangle - 3\}$ of (70).

Therefore the set of all these substitutions can be indicated $\{\{m_{\mathbb{G}\langle u\rangle}, \not{a}_u, \mathbf{D}^2_{uq}, \boldsymbol{\nu}_{uq}\};\, q=1,\not{q}_u;\, u=1,\not{u};\, u=1,\not{u}\}$ of which $\not{q}_u = 2\cdot\mathfrak{B}\langle\not{a}_u\rangle - 3$, and the (q,u,u)-th element of such set of $N_{\underline{g}}$ substitutions, of which $N_{\underline{g}} = 2\cdot\not{u}\cdot\Sigma_{u=1,\not{u}}(\mathfrak{B}\langle\not{a}_u\rangle) - 3\cdot\not{u}\cdot\not{u}$, gives rise to

$$\wp\langle t_{uuq}\rangle \equiv T\langle\boldsymbol{\nu}_{uq}\rangle \qquad t_{uuq} = (m_{\mathbb{G}\langle u\rangle} - M_g)\,/\,w_{uuq} \qquad w_{uuq} = (\mathbf{D}^2_{uq}/(\boldsymbol{\nu}_{uq}\cdot\not{a}_u))^{0.5} \tag{73}$$

of which is had, as (71),

$$\{\mathbf{D}^2_{u1},\boldsymbol{\nu}_{u1}\} \equiv \{\Sigma_{a=1,a\langle u\rangle}((G_{ua} - \mathsf{m}\langle \underline{G}_u\rangle)^2), a_u - 1\} \qquad \{\{\mathbf{D}^2_{uq},\boldsymbol{\nu}_{uq}\} \equiv \{\mathrm{D}^2_{uq}, h_{uq} - 1\}; q{=}2,\mathscr{B}\langle a_u\rangle - 1\}$$

$$\{\{\mathbf{D}^2_{uq},\boldsymbol{\nu}_{uq}\} \equiv \{D^2_{up\langle uq\rangle}, \check{N}_{up\langle uq\rangle}\}; q{=}\mathscr{B}\langle a_u\rangle, 2{\cdot}\mathscr{B}\langle a_u\rangle - 3\} \qquad p_{uq} \equiv q - \mathscr{B}\langle a_u\rangle + 2 \tag{74}$$

where

$$\mathrm{D}^2_{uq} = \Sigma_{h=1,h\langle uq\rangle}(k_{uqh}\cdot(\mathsf{m}_{\underline{G}\langle uqh\rangle} - \mathsf{m}_{\underline{G}\langle u\rangle})^2) \qquad D^2_{up} = \Sigma_{h=1,h\langle up\rangle}(\Sigma_{k=1,k\langle uph\rangle}((G_{uphk} - \mathsf{m}_{\underline{G}\langle uph\rangle})^2)) \qquad \check{N}_{up} = \Sigma_{h=1,h\langle up\rangle}(k_{uph} - 1)$$

The second of (73) entails that $\mathscr{A} \le \mathsf{M}_g \le \mathscr{B}$ is equivalent to $\mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{B} \le \mathtt{t}_{uuq}\cdot \mathtt{w}_{uuq} \le \mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{A}$. Therefore is had

$$\{\mathscr{A} \le \mathsf{M}_g \le \mathscr{B}\}_{uuq} \equiv \{\alpha_{uuq}(\mathscr{A},\mathscr{B}) \le \mathtt{t}_{uuq} \le \beta_{uuq}(\mathscr{A},\mathscr{B})\} \qquad \{\mathsf{M}_g \in \underline{\mathfrak{R}}\}_{uuq} \equiv \{\mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\} \tag{75}$$

of which $\alpha_{uuq}(\mathscr{A},\mathscr{B}) \equiv (\mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{B}) / \mathtt{w}_{uuq}$, $\beta_{uuq}(\mathscr{A},\mathscr{B}) \equiv (\mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{A}) / \mathtt{w}_{uuq}$.

Placing $\mathscr{A} = \mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{K}$ and $\mathscr{B} = \mathsf{m}_{\underline{G}\langle u\rangle} + \mathscr{K}$ with $\mathscr{K} > 0$, is had $\alpha_{uuq}(\mathscr{A},\mathscr{B}) \equiv -\mathscr{K} / \mathtt{w}_{uuq}$ and $\beta_{uuq}(\mathscr{A},\mathscr{B}) \equiv \mathscr{K} / \mathtt{w}_{uuq}$. This implies the

$$\{\,|\mathsf{M}_g - \mathsf{m}_{\underline{G}\langle u\rangle}| \le \mathscr{K}\} \equiv \{\mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{K} \le \mathsf{M}_g \le \mathsf{m}_{\underline{G}\langle u\rangle} + \mathscr{K}\} \equiv \{-\mathscr{K}/\mathtt{w}_{uuq} \le \mathtt{t}_{uuq} \le \mathscr{K}/\mathtt{w}_{uuq}\} \tag{76}$$

$$\{\mathsf{m}_{\underline{G}\langle u\rangle} - \mathscr{K}\cdot\mathtt{w}_{uuq} \le \mathsf{M}_g \le \mathsf{m}_{\underline{G}\langle u\rangle} + \mathscr{K}\cdot\mathtt{w}_{uuq}\} \equiv \{-\mathscr{K} \le \mathtt{t}_{uuq} \le \mathscr{K}\} \tag{77}$$

which, by means of (27), (25) and second of (75), allow respectively, when $\mathfrak{C}\langle \mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\rangle$, to calculate the probability of $\{\,|\mathsf{M}_g - \mathsf{m}_{\underline{G}\langle u\rangle}| \le \mathscr{K}\}$ (where $|\mathsf{M}_g - \mathsf{m}_{\underline{G}\langle u\rangle}|$ can be considered the error that occurs in replacing M_g with $\mathsf{m}_{\underline{G}\langle u\rangle}$) and to determine an interval that contains M_g with probability arbitrarily established through $\mathscr{K}$.

Intending $Æ\langle vvz \,/\, uuq \,/\, \{v \ne u\} \vee \{v \ne u\} \vee \{z \ne q\}\rangle$, from: þ; (25); follows

$$\{\{\mathtt{t}_{vvz} \in \underline{\mathfrak{R}}\} \equiv \{\mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\}\} \equiv \{\{a \le \mathtt{t}_{vvz} \le b\} \equiv \{a \le \mathtt{t}_{uuq} \le b\}\} \underline{\rightarrow} \{\textstyle\int_{a,b}(\mathfrak{D}\langle \mathtt{t}_{vvz}\rangle(x)\cdot dx) = \int_{a,b}(\mathfrak{D}\langle \mathtt{t}_{uuq}\rangle(x)\cdot dx)\}$$

but the last member of this is false and thus, by (3), is such also the first member. This implies $\underline{\mathbb{M}}\langle \mathtt{t}_{vvz} \in \underline{\mathfrak{R}}\rangle \cap \underline{\mathbb{M}}\langle \mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\rangle = \varnothing$ because *vice versa* there would be an impossibility to justify such as that of the x-th paragraph of page x, hence is had

$$\neg\{\underline{\mathbb{M}}\langle \mathtt{t}_{vvz} \in \underline{\mathfrak{R}}\rangle \subseteq \underline{\mathbb{M}}\langle \mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\rangle\} \tag{78}$$

Is placed $\mathtt{t} \equiv \cup_{u=1,u}(\cup_{u=1,u}(\cup_{q=1,q\langle u\rangle}(\mathtt{t}_{uuq} \in \underline{\mathfrak{R}})))$. The last two of (73) imply $\{\mathtt{t}_{uuq} \in \underline{\mathfrak{R}}\} \equiv \{\mathsf{m}_{\underline{G}\langle u\rangle} \in \underline{\mathfrak{R}}\} \wedge \{\mathbf{D}^2_{uq} \in [0,\infty)\}$. These $\{\mathsf{m}_{\underline{G}\langle u\rangle} \in \underline{\mathfrak{R}}\}$ and $\{\mathbf{D}^2_{uq} \in [0,\infty)\}$ happen if are known $\underline{g}$ and $\underline{\mathscr{g}}$ (i.e. are known their a and a elements). Therefore this condition and the intention of consider with equal probability one of the $\mathsf{N}_{\underline{g}}$ events that define $\mathtt{t}$ are sufficient for $\mathfrak{C}\langle \mathtt{t}\rangle$.

The $\{\alpha_{uuq}(\mathscr{A},\mathscr{B}), \beta_{uuq}(\mathscr{A},\mathscr{B}), \mathfrak{D}\langle \mathtt{t}_{uuq}\rangle(x); q{=}1,q_u; u{=}1,u; \mathrm{u}{=}1,\mathrm{u}\}$, (75), (78) and M_g are specifications of (62), (63), (64) and $\mathtt{X}$, following that (66) can, coherently with x-th paragraph of page x, be specified by the second relation of the

$$\{\underline{g} \text{ and } \underline{\mathscr{g}} \text{ are known}\} \underline{\rightarrow} \mathfrak{C}\langle \mathtt{t}\rangle \underline{\rightarrow} \{\mathscr{P}_C\langle \mathsf{M}_g \in \underline{\mathscr{R}}\rangle = \mathsf{N}_{\underline{g}}^{-1}\cdot\Sigma_{u=1,u}(\Sigma_{u=1,u}(\Sigma_{q=1,q\langle u\rangle}(\Sigma_{i=1,i}(\textstyle\int\langle \alpha_{uuq}(\mathscr{A}_i,\mathscr{B}_i), \beta_{uuq}(\mathscr{A}_i,\mathscr{B}_i)\rangle(T\langle \boldsymbol{\nu}_{uq}\rangle(x)\cdot dx)))))\} \tag{79}$$

of which $Æ\langle \mathsf{N}_{\underline{g}} \,/\, \mathfrak{N}\langle \underline{I}_C\rangle\rangle$ and whose first relation is due to consider implicit the intention said in the penultimate paragraph.

As (47) is related to (79), (48) is inherent to

$$\{\underline{g} \text{ and } \underline{\mathscr{g}} \text{ are known}\} \underline{\rightarrow} \mathfrak{C}\langle \mathtt{t}\rangle \underline{\rightarrow} \dot{\forall}_{u=1,u}(\dot{\forall}_{u=1,u}(\dot{\forall}_{q=1,q\langle u\rangle}(\rho\langle\{\mathsf{M}_g \in \underline{\mathscr{R}}\}_{uuq} \,|\, \{\mathsf{M}_g \in \underline{\mathfrak{R}}\}_{uuq}\rangle = \mathscr{P}\langle\{\mathsf{M}_g \in \underline{\mathscr{R}}\}_{uuq}\rangle \le \mathscr{P}\langle \mathsf{M}_g \in \underline{\mathscr{R}}\rangle))) \tag{80}$$

where each $\mathscr{P}\langle\{\mathsf{M}_g \in \underline{\mathscr{R}}\}_{uuq}$ is known and which shows as, in absence of (79), it would only possible replace $\mathscr{P}\langle \mathsf{M}_g \in \underline{\mathscr{R}}\rangle$ with a $\mathscr{P}\langle\{\mathsf{M}_g \in \underline{\mathscr{R}}\}_{uuq}\rangle$ or choose a $\mathscr{P}\langle\{\mathsf{M}_g \in \underline{\mathscr{R}}\}_{uuq}\rangle \le \mathscr{P}\langle \mathsf{M}_g \in \underline{\mathscr{R}}\rangle$ among the many, but, so, having to make, in both cases, a choice unjustifiable.

Indeed such a choose might follow from considering that (74) shows that

$$\underline{G}_u \equiv \underline{g} \qquad \{\mathbf{D}^2_{uq},\boldsymbol{\nu}_{uq}\} \equiv \{\Sigma_{a=1,a}((g_a - \mathsf{m}_{\underline{g}})^2), a - 1\} \tag{81}$$

entails a greater $\boldsymbol{\nu}_{uq}\cdot a_u$, and that third of (73) and (75) show that a greater $\boldsymbol{\nu}_{uq}\cdot a_u$ implies generally a greater $\mathscr{P}\langle\{\mathscr{A} \le \mathsf{M}_g \le \mathscr{B}\}_{uuq}\rangle$. However a probability is not more reliable just because is greater and thus there is no reason to prefer the $\mathscr{P}\langle\{\mathscr{A} \le \mathsf{M}_g \le \mathscr{B}\}_{uuq}\rangle$ identified by (81).

Instead (74) and third of (73) show (81) convenient when is not about choose (as just said) between several probability of a same event, but between the events defined by (76) and (77), since it is clear that (81) generally in these cases entails respectively the greater $\mathscr{P}\langle\,|\mathsf{M}_g - \mathsf{m}_{\underline{G}\langle u\rangle}| \le \mathscr{K}\rangle$ and the wider interval between those which have equal probability of containing M_g. This is confirmed by the *law of large numbers* (of which also in section 5.3.1 of [1]) that affirms

$$\lim_{a\to\infty}(\mathscr{P}\langle\,|\mathsf{m}_{\underline{g}} - \mathsf{M}_g| > 0\,\rangle) = 0$$

for which generally the increase of $\mathtt{a}$ entails a $\mathsf{m}_{\mathtt{g}}$ more approximate to $\mathsf{M}_{\mathtt{g}}$ and thus a greater $\mathbb{P}\langle\,|\,\mathsf{M}_{\mathtt{g}} - \mathsf{m}_{\mathtt{G}\langle u\rangle}\,|\, \le \mathcal{K}\rangle$.

Intending $\{\mathtt{s}_n;\, n=1,\mathrm{N}_{\underline{g}}\} \equiv \{\mathtt{s}_{uuq};\, q=1,\mathtt{q}_u;\, u=1,\mathtt{u};\, u=1,\mathtt{u}\}$, for a C-th linear combination $\mathtt{T}_C$ of $\{\mathtt{t}_n;\, n=1,\mathrm{N}_{\underline{g}}\}$ defined by non negative arbitrary constants $\{\lambda_{Cn};\, n=1,\mathrm{N}_{\underline{g}}\}$, is had $\mathtt{T}_C \equiv \Sigma_{n=1,\mathrm{N}\langle\underline{g}\underline{g}\rangle}(\lambda_{Cn}\cdot \mathtt{t}_n) = h_C - k_C\cdot \mathsf{M}_{\mathtt{g}}$ of which $h_C \equiv \Sigma_{n=1,\mathrm{N}\langle\underline{g}\underline{g}\rangle}(\lambda_{Cn}\cdot \mathsf{m}_{\mathtt{G}\langle n\rangle} / \mathtt{w}_n)$, $k_C \equiv \Sigma_{n=1,\mathrm{N}\langle\underline{g}\underline{g}\rangle}(\lambda_{Cn} / \mathtt{w}_n)$. A linear combination of random variables is a further random variable whose PDF is always calculable with general methods like those of section 5.2 of [1] or more efficiently with methods which are specific to the given random variables. Is deduced $\{\mathcal{A} \le \mathsf{M}_{\mathtt{g}} \le \mathcal{B}\}_C \equiv \{h_C - k_C\cdot\mathcal{B} \le \mathtt{T}_C \le h_C - k_C\cdot\mathcal{A}\}$ and hence relations analogous to (75). Moreover for two random variables $\mathtt{T}_A$ and $\mathtt{T}_B$, also they linear combinations as $\mathtt{T}_C$, is deduced a relation analogous to (78). Finally what has been just said can be reiterated adding variables of type $\mathtt{T}_C$ to variables of which are considered the linear combinations. It is therefore evident how, without having to consider other variables, the number of variables of (79) (i.e. the specification of $\mathfrak{N}\langle \underline{I}_C\rangle$) can be increased unlimitedly, following evident also the necessity (said in the x-th paragraph of page x) of replace the true $\mathfrak{P}\langle \mathsf{M}_{\mathtt{g}} \in \underline{\mathfrak{R}}\rangle$ with a probability conventional as the $\mathbb{P}_C\langle \mathsf{M}_{\mathtt{g}} \in \underline{\mathfrak{R}}\rangle$ of (79).

The numerosity of population of all values of $\mathtt{g}$ is unlimited in consequence (with reference to section 4 of [1]) of the continuity of $\mathfrak{p}\langle \mathtt{g}\rangle$, following that is unlimited also the number of samples of such population. This and the being $\underline{\mathtt{g}}$ one of these samples imply that is unlimited the number of cases as (75) and among which there are relations as (78). Is therefore evident an other reason which makes unlimited the numerosity of the present specification of $\underline{I}$ and that makes of consequence necessary the substitution of $\mathfrak{P}\langle \mathsf{M}_{\mathtt{g}} \in \underline{\mathfrak{R}}\rangle$ with a probability conventional.

Coherently with what just said, generally (79) makes a better approximation of $\mathfrak{P}\langle \mathsf{M}_{\mathtt{g}} \in \underline{\mathfrak{R}}\rangle$ if has a greater $\mathfrak{N}\langle \underline{\mathtt{g}}\rangle$ that implies a greater $\mathrm{N}_{\underline{g}}$ (of which $\text{Æ}\langle \mathrm{N}_{\underline{g}} \,/\, \mathfrak{N}\langle \underline{I}_C\rangle\rangle$) and thus a better substitution of type said in the x-th paragraph of page x.

Concluding this section is noted incidentally that $a \ne b$ (which implies $\underline{\mathtt{G}}_a \ne \underline{\mathtt{G}}_b$) and $\mathfrak{E}\langle \underline{\mathtt{G}}_u\rangle \in \underline{\mathfrak{R}}$ (due to $\mathbb{I}\langle \underline{\mathtt{g}}\rangle$) show $\mathbb{I}\langle \mathsf{m}_{\mathtt{G}\langle a\rangle}, \mathsf{m}_{\mathtt{G}\langle b\rangle}\rangle$ and thus $\mathbb{I}\langle \mathtt{t}_{acd}, \mathtt{t}_{bcd}\rangle$, and that instead $\underline{\mathfrak{R}}\langle \mathtt{w}_{uuq}\rangle = [0,\infty)$ shows that from $\mathtt{t}_{uuq} \ge 0$ follows $\mathtt{t}_{uef} \ge 0$, following $\neg\mathbb{I}\langle \mathtt{t}_{uuq}, \mathtt{t}_{uef}\rangle$ because $\underline{\mathfrak{R}}\langle \mathtt{t}_{uef}\rangle = \underline{\mathfrak{R}}$.

4.2 The variance of a normal random variable

In relation to $\hat{\underline{g}} \equiv \{\hat{\underline{g}}_a;\, a=1,\hat{\mathtt{a}}\}$, $\mathbb{I}\langle \hat{\underline{g}}\rangle$, $\mathfrak{p}\langle \hat{\underline{g}}_a\rangle \equiv G\langle \mathsf{M}_{\hat{g}}, \mathsf{V}_{\hat{g}}^2\rangle$, $\hat{\underline{g}} \equiv \{\hat{\underline{g}}_a;\, a=1,\hat{\mathtt{a}}\}$, $\mathbb{I}\langle \hat{\underline{g}}\rangle$, $\mathfrak{p}\langle \hat{\underline{g}}_a\rangle \equiv G\langle \mathsf{M}_{\hat{g}}, \mathsf{V}_{\hat{g}}^2\rangle$, is had, coherently with (6.2.29) of [1],

$$\mathbb{I}\langle \mathsf{m}\langle \hat{\underline{g}}\rangle, \mathsf{m}\langle \hat{\underline{g}}\rangle\rangle \underline{\rightarrow} \{\mathfrak{p}\langle (\mathsf{m}_{\hat{g}} - \mathsf{m}_{\hat{g}} + \mathsf{M}_{\hat{g}} - \mathsf{M}_{\hat{g}}) / (\mathsf{V}_{\hat{g}}^2 / \hat{\mathtt{a}} + \mathsf{V}_{\hat{g}}^2 / \hat{\mathtt{a}})^{0.5}\rangle = Z\}$$

This, considering that in relation to $\underline{\mathtt{G}} \equiv \{\mathtt{G}_u;\, u=1,\mathtt{u}\}$ of section 4.1 is had $\mathbb{I}\langle \mathtt{G}_u\rangle$ (due to $\mathbb{I}\langle \underline{\mathtt{g}}\rangle$), $\mathfrak{p}\langle \mathfrak{E}\langle \mathtt{G}_u\rangle\rangle \equiv G\langle \mathsf{M}_{\mathtt{g}}, \mathsf{V}^2\rangle$ and (as said in section 4.1) $\mathbb{I}\langle \mathsf{m}_{\mathtt{G}\langle a\rangle}, \mathsf{m}_{\mathtt{G}\langle b\rangle}\rangle$, implies $\mathfrak{p}\langle (\mathsf{m}_{\mathtt{G}\langle a\rangle} - \mathsf{m}_{\mathtt{G}\langle b\rangle}) / (\mathsf{V}^2 / \mathtt{a}_a + \mathsf{V}^2 / \mathtt{a}_b)^{0.5}\rangle \equiv Z$. This, $\{\mathfrak{p}\langle \mathtt{s}\rangle \equiv Z\} \equiv \{\mathtt{s} \equiv Z\}$ and $\mathfrak{p}\langle Z^2\rangle(\mathrm{x}) = \mathrm{x}^{-0.5}\cdot Z(\mathrm{x}^{0.5})$ (of which $\underline{\mathfrak{R}}\langle \mathrm{x}\rangle = \underline{\mathfrak{R}}\langle Z^2\rangle = [0,\infty)$) affirmed by (6.2.7) of [1], give rise to

$$\mathfrak{p}\langle \mathrm{z}^2{}_{ab}\rangle(\mathrm{x}) = \mathrm{x}^{-0.5}\cdot Z(\mathrm{x}^{0.5}) \equiv X\langle 1\rangle(\mathrm{x}) \qquad \mathrm{z}^2{}_{ab} = \mathrm{y}^2{}_{ab} / \mathsf{V}^2 \qquad \mathrm{y}^2{}_{ab} = (\mathsf{m}_{\mathtt{G}\langle a\rangle} - \mathsf{m}_{\mathtt{G}\langle b\rangle})^2 / (\mathtt{a}_a^{-1} + \mathtt{a}_b^{-1})$$

whose second entails that $\mathcal{A} \le \mathsf{V}^2 \le \mathcal{B}$ (of which is implicit $\mathcal{A} \ge 0$) equates to $\mathrm{y}^2{}_{ab} / \mathcal{B} \le \mathrm{z}^2{}_{ab} \le \mathrm{y}^2{}_{ab} / \mathcal{A}$. Therefore is had

$$\{\mathcal{A} \le \mathsf{V}^2 \le \mathcal{B}\}_{ab} \equiv \{\mathrm{y}^2{}_{ab} / \mathcal{B} \le \mathrm{z}^2{}_{ab} \le \mathrm{y}^2{}_{ab} / \mathcal{A}\} \qquad \{\mathsf{V}^2 \in [0,\infty)\}_{ab} \equiv \{\mathrm{z}^2{}_{ab} \in [0,\infty)\} \tag{82}$$

of which $\{a,b\} \in \{\{a,b\};\, b=a+1,\mathtt{u};\, a=1,\mathtt{u}-1\}$ (with $\mathtt{u} = \Sigma_{k=1,\mathtt{a}}(\text{Б}\langle \mathtt{a},k\rangle)$, $\mathtt{a} = \mathfrak{N}\langle \underline{\mathtt{g}}\rangle$ as in section 4.1) because $\{a,b\}$ is one of the $\text{Б}\langle \mathtt{u},2\rangle$ combinations of class 2 of $\{u=1,\mathtt{u}\}$.

From $\text{Æ}\langle \{\mathbf{D}^2{}_{uq}, \boldsymbol{\nu}_{uq}\},(74) \,/\, \{\mathbf{D}^2{}_q, \boldsymbol{\nu}_q\},(71) \,/\, (70)\rangle$ is deduced

$$\mathfrak{p}\langle z^2{}_{uq}\rangle \equiv X\langle \boldsymbol{\nu}_{uq}\rangle \qquad z^2{}_{uq} = \mathbf{D}_{uq}{}^2 / \mathsf{V}^2$$

whose second gives rise to

$$\{\mathcal{A} \le \mathsf{V}^2 \le \mathcal{B}\}_{uq} \equiv \{\mathbf{D}_{uq}{}^2 / \mathcal{B} \le z^2{}_{uq} \le \mathbf{D}_{uq}{}^2 / \mathcal{A}\} \qquad \{\mathsf{V}^2 \in [0,\infty)\}_{\mathrm{v}} \equiv \{z^2{}_{uq} \in [0,\infty)\} \tag{83}$$

of which $\{u,q\} \in \{\{u,q\};\, q=1,\mathtt{q}_u;\, u=1,\mathtt{u}\}$ with $\mathtt{q}_u$ and $\mathtt{u}$ expressible as said in section 4.1 and being therefore $\{u,q\}$ element of a set of numerosity $\mathrm{N}_{\underline{g}} / \mathtt{u}$.

The (82) and (83) give rise to

$$\{\mathcal{A} \le \mathsf{V}^2 \le \mathcal{B}\}_{\mathrm{v}} \equiv \{\alpha_{\mathrm{v}}(\mathcal{A},\mathcal{B}) \le \mathrm{r}^2{}_{\mathrm{v}} \le \beta_{\mathrm{v}}(\mathcal{A},\mathcal{B})\} \qquad \{\mathsf{V}^2 \in [0,\infty)\}_{\mathrm{v}} \equiv \{\mathrm{r}^2{}_{\mathrm{v}} \in [0,\infty)\} \tag{84}$$

of which $\alpha_{\mathrm{v}}(\mathcal{A},\mathcal{B}) \equiv \psi^2{}_{\mathrm{v}} / \mathcal{B}$, $\beta_{\mathrm{v}}(\mathcal{A},\mathcal{B}) \equiv \psi^2{}_{\mathrm{v}} / \mathcal{A}$,

$$\{\mathrm{r}^2{}_{\mathrm{v}}, \psi^2{}_{\mathrm{v}};\, \mathrm{v}=1,\mathtt{v}\} \equiv \{\{\mathrm{z}^2{}_{ab}, \mathrm{y}^2{}_{ab};\, b=a+1,\mathtt{u};\, a=1,\mathtt{u}-1\}, \{z^2{}_{uq}, \mathbf{D}_{uq}{}^2;\, q=1,\mathtt{q}_u;\, u=1,\mathtt{u}\}\}$$

with $\mathtt{v} = \text{Б}\langle \mathtt{u},2\rangle + \mathrm{N}_{\underline{g}} / \mathtt{u}$. This allows for V^2 results analogous to those obtained for $\mathsf{M}_{\mathtt{g}}$ at the x-th paragraph of page x.

As (78) is deduced also

$$\{\neg\{\mathbb{M}\langle r^2{}_a\in\Re\rangle\subseteq\mathbb{M}\langle r^2{}_b\in\Re\rangle\};\ \forall a\neq b\}\qquad(85)$$

Is placed $r\equiv\cup_{v=1,¥}(r^2{}_v\in[0,\infty))$. As for $\mathfrak{C}\langle t\rangle$ in (79), also for $\mathfrak{C}\langle r\rangle$ are deduced sufficient the knowledge of $\underline{g}$ and $\underline{g}$ and the intention of consider with equal probability one of the ¥ events which define r.

The $\{\alpha_v(\mathcal{A},\mathcal{B}),\beta_v(\mathcal{A},\mathcal{B}),\mathfrak{p}\langle r^2{}_v\rangle(x);\ v=1,¥\}$, (84), (85) and V^2 are specifications of (62), (63), (64) and X, following that (66) can, coherently with the x-th paragraph of page x, be specified by the second relation of

$$\{\underline{g}\ \text{e}\ \underline{g}\ \text{sono noti}\}\ \underline{\rightarrow}\ \mathfrak{C}\langle r\rangle\ \underline{\rightarrow}\ \{\mathfrak{P}_C\langle V^2\in\Re\rangle=¥^{-1}\cdot\Sigma_{v=1,¥}(\Sigma_{i=1,i}(\int\langle\alpha_v(\mathcal{A}_i,\mathcal{B}_i),\beta_v(\mathcal{A}_i,\mathcal{B}_i)\rangle(\mathfrak{p}\langle r^2{}_v\rangle(x)\cdot dx)))\}\qquad(86)$$

of which $\Re\subseteq[0,\infty)$ because $\mathfrak{p}\langle r^2{}_v\rangle(x)$ is not defined for $x\notin[0,\infty)$, and whose first relation is due to the just said intention.

As (47) is related to (86), (48) is inherent to

$$\{\underline{g}\ \text{e}\ \underline{g}\ \text{sono noti}\}\ \underline{\rightarrow}\ \mathfrak{C}\langle r\rangle\ \underline{\rightarrow}\ \dot\forall_{v=1,¥}(\mathfrak{P}\langle\{V^2\in\Re\}_v\rangle\leq\mathfrak{P}\langle V^2\in\Re\rangle)\qquad(87)$$

where each $\mathfrak{P}\langle\{V^2\in\Re\}_v\rangle$ is knowable and which shows as, in absence of (86), it would only possible replace $\mathfrak{P}\langle V^2\in\Re\rangle$ with a $\mathfrak{P}\langle\{V^2\in\Re\}_v$ or choose a $\mathfrak{P}\langle\{V^2\in\Re\}_v\rangle\leq\mathfrak{P}\langle V^2\in\Re\rangle$ among the many, but, so, having to make, in both cases, a choice unjustifiable. Indeed also in this case a $\mathfrak{P}\langle\{V^2\in\Re\}_v\rangle$ would not be made more reliable from the being generally greater.

For a C-th linear combination $R^2{}_C$ of $\{r^2{}_v;\ v=1,¥\}$ defined by non negative arbitrary constants $\{\lambda_{Cv};\ v=1,¥\}$, is had $R^2{}_C\equiv\Sigma_{v=1,¥}(\lambda_{Cv}\cdot r^2{}_v)=h_C{}^2/V^2$ of which $h_C{}^2\equiv\Sigma_{v=1,¥}(\lambda_{Cv}\cdot\psi^2{}_v)$, $\{\mathcal{A}\leq V^2\leq\mathcal{B}\}_C\equiv\{h_C{}^2/\mathcal{B}\leq R^2{}_C\leq h_C{}^2/\mathcal{A}\}$. Hence also in this case, as at x-th paragraph of page x, are deduced the possibility of increase unlimitedly the number of variables of (86) and the following necessity of replace $\mathfrak{P}\langle V^2\in\Re\rangle$ with a probability conventional as the $\mathfrak{P}_C\langle V^2\in\Re\rangle$ of (86). In this regard are immediate the further considerations analogous to those of the previous section and in particular how a greater $\mathfrak{N}\langle\underline{g}\rangle$ entails generally a better approximation of $\mathfrak{P}\langle V^2\in\Re\rangle$.

CONCLUSION

The utility of a probability consist ultimately in the being a measure of the possibility of happen an event and is obviously prevented when coexist different probabilities of a same event among which is not possible identify one as the only totally reliable.

A such impediment is typical in treating a confidence interval, as ascertained in sections 4.1 and 4.2 where is clear that, in absence of (79) and (86), there would be, in both cases and coherently with (80) and (87), an number unlimited of different and equally reliable confidences, i.e. probabilities, of a same event.

However in the usual treatments these difficulties are irrelevant because, among many equally reliable and generally different confidences, are considered only those deducible by the whole sample and is chosen one of these arbitrarily or because it is the only contingently calculable. At this regard is noted that the (8.5) of [1] does not constitute a definitive progress because of its character substantially conventional.

In consequence of this the essential purpose of this work has been contextualize and circumstantiate concepts and procedures with which to define and calculate a confidence as the only totally reliable.

This aim has been achieved satisfyingly, because has been reached the (47) (i.e. (66)) where, as said in the x-th paragraph of page x, the searched confidence is expressed so that it, although not exactly calculable, is however unlimitedly approximable.

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