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Neutrosophic Triplet Metric Topology

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Abstract: Topology is a branch of mathematic that deals with the specific definitions given for spatial structure concepts, compares different definitions and explores the connections between the structures described on the sets. Also, neutrosophic triplet metric and neutrosophic triplet topology are a new concept in neutrosophy and they are completely different from classical structures. In this paper, we firstly study neutrosophic triplet metric topology. Furthermore, we give some new definitions and properties for neutrosophic triplet metric space, neutrosophic triplet topology and neutrosophic triplet metric topology. Thus, we obtain neutrosophic triplet metric topology using the neutrosophic triplet metric and neutrosophic triplet topology. Also, we show relationship between neutrosophic triplet metric space and neutrosophic triplet topology.

Keywords: topology, metric space, neutrosophic triplet metric space, neutrosophic triplet topology, neutrosophic triplet metric topology

1 Introduction

In 1980 Smarandache first introduced the concept of neutrosophy. In neutrosophy, there are neutrosophic set, neutrosophic logic and neutrosophic probability. Neutrosophic sets are, in fact, the generalized state of the previously described fuzzy sets [2] and intuitionistic fuzzy sets [3]. Because, unlike fuzzy sets and intuitionistic fuzzy sets, in neutrosophic sets, truth value (t), falsity value (f) and indeterminacy value (i) are completely independent of each other. Therefore, neutrosophic sets are more useful in coping with uncertainties. For this reason, many researchers have done many studies on neutrosophic structures [3 - 15].

Smarandache and Ali defined neutrosophic triplet (NT) sets and neutrosophic triplet (NT) groups [16] in 2016 and gave the properties of these structures. In order for a set to be a NT set, for each "n" element in this set must have a neutral element and an anti element. The neutral element does not have to be just one for all elements as in the classical group and must be different from the classical identity element. So there may be more than one neutral element in NT sets. Also, a "n" element of a NT set is shown in the form of <n, neut(n), anti(n)>. Therefore, NT structures are different from classical structures. Also, many researchers have introduced NT structures.

Recently, Ali and Smarandache studied neutrosophic triplet ring and neutrosophic triplet field [17]; Şahin and Kargın obtained neutrosophic triplet normed space [18]; Şahin and Kargın introduced neutrosophic triplet inner product space [19]; Smarandache, Şahin and Kargın studied neutrosophic Triplet G- Module [20]; Bal, Shalla and Olgun obtained neutrosophic triplet cosets and quotient groups [21]; Şahin, Kargın and Çoban introduced fixed point theorem for neutrosophic triplet partial metric space [22]; Şahin and Kargın neutrosophic triplet v – generalized metric space [23]; Çelik, Shalla and Olgun studied fundamental homomorphism theorems for neutrosophic extended triplet groups [24].

Topology is a branch of mathematic that deals with the specific definitions given for spatial structure concepts, compares different definitions and explores the connections between the structures described on the sets. In mathematics it is a large area of study with many more specific subfields. Subfields of topology include algebraic topology, geometric topology, differential topology, manifold topology.

Topology has many different application areas in mathematic. For example, a curve, a surface, a family of curves, a set of functions or a metric space can be a topological space. Also, the topology has been studied on neutrosophic set, fuzzy set, intuitionistic fuzzy set and soft set. Many researchers have introduced the topology in [25-32]. Furthermore, Şahin, Kargın and Smarandache obtained NT topology [33].

In this paper, we obtain NT metric topology. In Section 2; we give definitions of NT set [16], NT metric space [18] and NT topology [33]. In Section 3, we obtain some properties of NT topology. We define base for NT topology. In Section 4, we obtain some properties of NT metric space. We define NT open balls for NT metric space and isometric NT metric spaces. We define NT metric topological space. Also, we show relationship between NT metric spaces and NT topology. In Section 5, we give conclusions.

2 Preliminaries

In this section, we give definition of NT sets [16], NT metric space [18], NT topology [33] and NT open sets [33]. We give new properties and definitions for NT topology and NT metric space using these definitions. Also, we firstly obtain NT metric topology using the NT metric space and NT topology.

In this paper, we show neutrosophic triplet briefly with NT.

Definition 2.1: [16]: Let # be a binary operation. A NT set (X, #) is a set such that for x ∈ X,

i) There is a neutral of "x" = neut(x) such that x#neut(x) = neut(x)#x = x, for every x∈X;

ii) There is an anti of "x" = anti(x) such that x#anti(x) = anti(x)#x = neut(x), for every x∈X;

Also, an element "x" is showed with (x, neut(x), anti(x)).

Furthermore, neut(x) must different from classical identity element.

Definition 2.2: [18] A NT metric on a NT set (N, *) is a function d:NxN→ $\mathbb{R}$ such for every n, m, s ∈ N,
i) n * m ∈ N
ii) d(n, m) $\geq$ 0
iii) If n = m, then d(n, m) $=$ 0
iv) d(n, m) = d(n, m)
v) If there is at least an element s ∈ N for each n, m ∈ N pair such that d(n, m) ≤ d(n, m*neut(s)), then d(n, m*neut(s)) ≤ d(n, s)+ d(s, m).

Definition 2.3: [33] Let (X, *) be a NT set, P(X) be set family of each subset of X and $\mathscr{T}$ be a subset family of P(X). If $\mathscr{T}$ and X are satisfied the following conditions, then $\mathscr{T}$ is called a NT topology on X.
i) A*B ∈ X, for every A, B ∈ X
ii)∅, X ∈ $\mathscr{T}$
iii) For ∀ i ∈ K, If A_i ∈ X, then $\bigcup_{i\in K} A_i \in \mathscr{T}$
iv) For ∀ i ∈ K (K is finite), If A_i ∈ X, then $\bigcap_{i\in K} A_i \in \mathscr{T}$
Also, ((X, *), $\mathscr{T}$) is called NT topological space.

Definition 2.4: [33] Let ((X, *), $\mathscr{T}$) be a NT topology. For every A∈$\mathscr{T}$, A is called a NT open set.

3 Some Properties for Neutrosophic Triplet Topology

Definition 3.1: Let ((X, *), $\mathscr{T}$) be a NT topological space and $\mathscr{B}$ ⊂ P(X) be a set family. If $\mathscr{B}^{\#}$ = $\mathscr{T}$ such that $\mathscr{B}^{\#}$ ={A⊂X: A = ∪C, C⊂$\mathscr{B}$}, then it is said that $\mathscr{B}$ is a base of $\mathscr{T}$.

Theorem 3.2: Let (X, *) be a NT set and $\mathscr{B}$ ⊂ P(X) be a base of NT topology. If the following conditions are satisfied, then ((X, *), $\mathscr{B}^{\#}$) is a NT topological space such that $\mathscr{B}^{\#}$ ={A⊂X: A = ∪C, C⊂$\mathscr{B}$ }.
c_1) x*y ∈ X, for every x, y∈ X
c_2) X = ∪C (C ⊂ $\mathscr{B}$)
c_3) For every C_1, C_2 ∈ $\mathscr{B}$ and x ∈ $C_1 \cap C_2$; there is at least a set C_3 ∈ $\mathscr{B}$ such that x ∈ $C_3 \subset C_1 \cap C_2$.

Proof: We suppose that conditions c_1, c_2 and c_3 are satisfied. We show that ((X, *), $\mathscr{B}^{\#}$) is a NT topological space.

In Definiton 2.3,

i) is equal to condition c_1.

ii) It is clear that ∅∈ $\mathscr{B}^{\#}$ since $\mathscr{B}^{\#}$ = {A⊂X: A = ∪C, C⊂ $\mathscr{B}$}. Also, we obtain X∈ $\mathscr{B}^{\#}$ since condition c_2,

iii) We take $A_i \in \mathcal{B}^\#$ (i ∈I). ∪ A_i = ∪∪ C_i ($C_i \subset \mathcal{B}$). Thus, we obtain ∪ $A_i \in \mathcal{B}^\#$.

iv) We take $A_1, A_2 \in \mathcal{B}^\#$. If x ∈ $A_1 \cap A_2$, then x ∈ A_1 and x ∈ A_2. Thus, there is at least a pair element C_1, $C_2 \in \mathcal{B}$ such that x ∈ $C_1 \subset A_1$ and x ∈ $C_2 \subset A_2$ since $\mathcal{B}^\#$ ={A⊂X: A = ∪C, C⊂ $\mathcal{B}$ }. Then from c_3, there are an element $C_3 \in \mathcal{B}$ such that x ∈ $C_3 \subset C_1 \cap C_2 \subset A_1 \cap A_2$. Thus, we obtain $A_1 \cap A_2 \in \mathcal{B}^\#$ since $\mathcal{B}^\#$ = {A⊂X: A = ∪C, C⊂ $\mathcal{B}$}.

4 Neutrosophic Triplet Metric Topology

Definition 4.1:

a) Let ((N, *), d) be a NT metric space and a∈N. The set B(a, r) = {x∈N:d(a, x) < r} is called open ball centered at a with radius r (r> 0).

b) Let ((N, *), d) be a NT metric space and a∈N. The set B[a, r] = {x∈N:d(a, x) ≤ r} is called closed ball centered at a with radius r (r > 0).

b) Let ((N, *), d) be a NT metric space and a∈N. The set S(a, r) = {x∈N:d(a, x) = r} is called sphere centered at a with radius r (r > 0).

Definition 4.2: Let ((N, *), d) be a NT metric space and x∈N.Then an open ball neighbourhood of x is a B(x, ε), for some ε > 0.

Theorem 4.3: Let ((N, *) , d) be a NT metric space and B(a, ε) be an open ball in this space. If there is at least an element x ∈ N for each a, y ∈ N pair such that d(a, y) ≤ d(a, y*neut(x)), then there is an open ball such that B(x, r) ⊂ B(a, ε) for all x ∈ B(a, ε).

Proof: We suppose that there is at least an element x ∈ N for each a, y ∈ N pair such that

$$d(a, y) \leq d(a, y*neut(x)). \quad (1)$$

Then, we take an element x ∈ B(a, ε). Thus, from Definition 4.1 we obtain d(a, x) < ε. Also, we take a real number r such that

$$0< r < \varepsilon - d(a, x). \quad (2)$$

Now, we take an element y ∈ B(x, r). Thus, from Definition 4.1 we obtain d(y, x) < r. From (1) and Definition 2.2, we obtain

$$d(a, y) \leq d(a, x) + d(x, y). \quad (3)$$

From (2) and (3), we can write d(a, y) < d(a, x) + r < ϵ. Thus, we obtained y ∈ B(a, ε) and B(x, r) ⊂ B(a, ε).

Theorem 4.4: Let ((N, *), d) be a NT metric space and $\mathcal{B}$ be set of all open ball of ((N, *), d)). Then $\mathcal{B}$ is a base of NT topology on (N, *).

Proof: We show that $\mathcal{B}$ = {B(a, ε): a ∈ N, $\varepsilon > 0$} is a base of a NT topology such that $\mathcal{B}^\#$ ={A⊂N: A = ∪C, C⊂ $\mathcal{B}$}. Thus, we show that B satisfies the conditions in Theorem 3.2.

c_1) It is clear that since ((N, *), d) is a NT metric space.

c_2) For every ε >0 and a ∈ N, we can write N = ∪ B(a, ε) since a∈ B(a, ε) ⊂ N.

c_3) Let C_1, $C_2 \in \mathcal{B}$ and x ∈ $C_1 \cap C_2$. From Theorem 4.3,

if x ∈ C_1, then there exits at least a B(x, r_1) open ball such that B(x, r_1) ⊂ $\mathcal{B}$ and r_1> 0. Similarly, if x ∈ C_2, then there exits at least a B(x, r_2) ball such that B(x, r_2) ⊂ ℬ and r_2> 0. If we take r = min{r_1, r_2}, then

x ∈ B(x, r)⊂ $C_1 \cap C_2$.

Thus, ℬ is a base of NT topology on (N, *) such that $ℬ^{\#}$ ={A⊂N: A = ∪C, C⊂ ℬ}.

Corollary 4.5: Let ((N, *), d) be a NT metric space and B be set of all open ball of ((N, *), d)). From Theorem 4.4 and Definition 3.1, $ℬ^{\#}$ ={A⊂N: A = ∪C, C⊂ ℬ}. is a NT topology on (N, *).

Corollary 4.6: Let ((N, *), d) be a NT metric space and B(x, r) be an open ball in this space. From Corollary 4.5 and Definition 2.4, B(x, r) is an open set.

Definition 4.7: Let ((N, *), d) be a NT metric space and B be set of all open ball of ((N, *), d)). Then, ((N, *), $\mathcal{T}_d$ = $ℬ^{\#}$) is called NT metric topological space such that $ℬ^{\#}$ ={A⊂N: A = ∪C, C⊂ℬ}.

Example 4.8: Let N = {0, 2, 5, 6} be a set. (N, .) is a NT set under multiplication module 10 in ($\mathbb{Z}_{10}$, .). Also, NT are (0, 0, 0), (2, 6, 2), (5, 5, 5) and (6, 6, 6).
Then we take that d: NxN→N is a function such that d(k, m) = ($|2^k - 2^m|$)/8.
Now we show that d is a NT metric.
i) It is clear that k, m ∈ N, for every k, m ∈ N.
ii) If k = m, then d(k, m) = ($|2^k - 2^m|$)/8 = ($|2^k - 2^k|$)/8 = 0. Also, d(k, m) = ($|2^k - 2^m|$)/8 ≥ 0.
iii) d(k, m) = ($|2^k - 2^m|$)/8 =($|2^k - 2^m|$)/8= d(m, k).
iv)

It is clear that d(0, 0) ≤ d(0, 0.2) = d(0, 0). Also, d(0, 0) = 0 and d(0, 2) = 3/8 . Thus, we obtain
d(0, 0) ≤ (0, 2) + (2, 0).

It is clear that d(0, 2) ≤ d(0, 2.8) = d(0, 6). Also, d(0, 2) = 3/8 and d(0, 6) = 63/8. Thus, we obtain
d(0, 6) ≤ d(0, 8) + d(8, 2).

It is clear that d(0, 4) ≤ d(0, 4.6) = d(0, 4). Also, d(0, 4) = 15/8, d(0, 6) = 63/8 and d(6, 4) = 48/8 = 6. Thus, we obtain
d(0, 4) ≤ d(0, 6) + d(6, 4).

It is clear that d(0, 5) ≤ d(0, 5.5) = d(0, 5). Also, d(0, 5) = 31/8 and d(5, 5) = 0. Thus, we obtain
d(0, 5) ≤ d(0, 5) + d(5, 5).

It is clear that d(0, 6) ≤ d(0, 6.8) = d(0, 8). Also, d(0, 6) = 63/8, d(8, 6) = 192/8 = 24 and d(0, 8) = 255/8. Thus, we obtain
d(0, 8) ≤ d(0, 8) + d(8, 6).

It is clear that d(0, 8) ≤ d(0, 6.8) = d(0, 8). Also, d(0, 6) = 63/8, d(8, 6) = 192/8 = 24 and d(0, 8) = 255/8. Thus, we obtain
d(0, 8) ≤ d(0, 6) + d(6, 8).

It is clear that d(2, 2) ≤ d(2, 2.5) = d(2, 0). Also, d(2, 0) = 3/8 and d(5, 2) = 28/8 = 7/2. Thus, we obtain
d(2, 0) ≤ d(2, 5) + d(5, 2).

It is clear that d(2, 4) ≤ d(2, 4.6) = d(2, 4). Also, d(2, 6) = 60/8 = 15/2 and d(4, 6) = 48/8 = 6. Thus, we obtain
d(2, 4) ≤ d(2, 6) + d(6, 4).

It is clear that d(2, 5) ≤ d(2, 5.5) = d(2, 5). Also, d(2, 5) = 28/8 = 7/2 and d(5, 5) = 0. Thus, we obtain
d(2, 5) ≤ d(2, 5) + d(5, 5).

It is clear that d(2, 6) ≤ d(2, 6.8) = d(2, 8). Also, d(2, 8) = 254/8 = 127/4 and d(6, 8) = 192/8 = 24. Thus, we obtain
d(2, 8) ≤ d(2, 8) + d(8, 6).

It is clear that d(2, 8) ≤ d(2, 6.8) = d(2, 8). Also, d(2, 8) = 254/8 = 127/4 and d(8, 8) = 0. Thus, we obtain
d(2, 8) ≤ d(2, 8) + d(8, 8).

It is clear that d(4, 4) ≤ d(4, 4.5) = d(4, 0). Also, d(4, 5) = 16/8 = 2 and d(4, 0) = 15/8. Thus, we obtain
d(4, 0) ≤ d(4, 5) + d(5, 4).

It is clear that d(4, 5) ≤ d(4, 5.4) = d(4, 0). Also, d(4, 5) = 16/8 = 2 and d(4, 0) = 15/8. Thus, we obtain
d(4, 0) ≤ d(4, 4) + d(4, 5).

It is clear that d(4, 6) ≤ d(4, 6.8) = d(4, 8). Also, d(4, 8) = 240/8 = 30 and d(8, 6) = 192/8 = 24. Thus, we obtain
d(4, 8) ≤ d(4, 8) + d(8, 6).

It is clear that d(4, 8) ≤ d(4, 8.6) = d(4, 8). Also, d(4, 8) = 240/8 = 30 and d(8, 8) = 0. Thus, we obtain
d(4, 8) ≤ d(4, 8) + d(8, 8).

It is clear that d(5, 5) ≤ d(5, 5.0) = d(5, 0). Also, d(0, 5) = 31/8 and d(5, 5) = 0. Thus, we obtain
d(5, 0) ≤ d(5, 0) + d(0, 5).

It is clear that d(5, 6) ≤ d(5, 6.8) = d(5, 8). Also, d(5, 8) = 224/8 = 28 and d(8, 6) = 192/8 = 24. Thus, we obtain
d(5, 8) ≤ d(5, 8) + d(8, 6).

It is clear that d(5, 8) ≤ d(5, 8.6) = d(5, 8). Also, d(5, 6) = 32/8 = 4 and d(8, 6) = 192/8 = 24. Thus, we obtain
d(5, 8) ≤ d(5, 8) + d(8, 6).

It is clear that d(6, 6) ≤ d(6, 6.8) = d(6, 8). Also, d(6, 8) = 192/8 = 24. Thus, we obtain
d(6, 8) ≤ d(6, 8) + d(6, 8).

It is clear that d(6, 8) ≤ d(6, 8.6) = d(6, 8). Also, d(6, 8) = 192/8 = 24 and d(8, 8) = 0. Thus, we obtain
d(6, 8) ≤ d(6, 8) + d(8, 8).

It is clear that d(8, 8) ≤ d(8, 8.2) = d(8, 6). Also, d(6, 8) = 192/8 = 24. Thus, we obtain
d(8, 6) ≤ d(8, 6) + d(6, 8).

Therefore, ((N, .), d) is A NT metric space.

Now, we show that open balls in ((N, .), d).

For 0 ∈ N,

B(0, 3/8) = {0}
B(0,15/8) = {0, 2}
B(0, 31/8) = {0, 2, 4}
B(0, 63/8) = {0, 2, 4, 5}
B(0, 255/8) = {0, 2, 4, 5, 6}
B(0, 32) = {0, 2, 4, 5, 6, 8}

For 2 ∈ N,

B(2, 3/8) = {2}
B(2,3/2) = {0, 2}
B(2, 7) = {0, 2, 4}
B(2, 15/2) = {0, 2, 4, 5}
B(2, 63/2) = {0, 2, 4, 5, 6}
B(2, 32) = {0, 2, 4, 5, 6, 8}

For 4 ∈ N,

B(4, 15/8) = {4}
B(4, 3/2) = {0, 4}
B(4, 2) = {0, 2, 4}
B(4, 6) = {0, 2, 4, 5}
B(4, 30) = {0, 2, 4, 5, 6}
B(4, 31) = {0, 2, 4, 5, 6, 8}

For 5 ∈ N,

B(5, 2) = {5}
B(5, 31/8) = {4, 5}
B(5, 4) = {0, 4, 5}
B(5, 7) = {0, 4, 5, 6}
B(5, 31/2) = {0, 2, 4, 5, 6}
B(5, 16) = {0, 2, 4, 5, 6, 8}

For 6 ∈ N,

B(6, 4) = {6}
B(6, 63/8) = {5, 6}
B(6, 15/2) = {0, 5, 6}
B(6, 8) = {0, 2, 5, 6}
B(6, 24) = {0, 2, 4, 5, 6}
B(6, 25) = {0, 2, 4, 5, 6, 8}

For 8 ∈ N,

B(8, 31/2) = {8}
B(8, 24) = {5, 8}
B(8, 30) = {5, 6, 8}
B(8, 63/2) = {4, 5, 6, 8}
B(8, 255/8) = { 2, 4, 5, 6, 8}

B(8, 32) = {0, 2, 4, 5, 6, 8}.

Thus,
in ((N, .), d), we obtain set of every open ball $\mathcal{B}$ = {{0}, {2}, {4}, {5}, {6}, {8}, {0, 2}, {0, 4}, {4, 5}, {5, 6}, {5, 8}, {5, 6, 8}, {0, 5, 6}, {0, 4, 5}, {0, 2, 4}, {4, 5, 6, 8}, {0, 2 ,5, 6}, {0, 2, 4, 5}, {0, 4, 5, 6}, {2, 4, 5, 6, 8}, {0, 2, 4, 5, 6}, {0, 2, 4, 5, 6, 8}}.

Also, from Definition 3.1, it is clear that $\mathcal{B}$ is a base of $\mathcal{T}$= $\mathcal{B}^{\#}$ = {A⊂N: A = ∪C, C⊂ $\mathcal{B}$}. Furthermore, B and $\mathcal{B}^{\#}$ satisfy the conditions in Theorem 3.2. Therefore, ((N, .), $\mathcal{B}^{\#}$) is a NT topological space. Also, every open ball A ∈ $\mathcal{B}$ is satisfies the conditions in Theorem 4.3. Finally, from Definition 4.7, ((N, .), $\mathcal{T}_d$ = $\mathcal{B}^{\#}$) is a NT metric topological space.

Corollary 4.9: Let ((N, *), d) be a NT metric space and ((N, *), $\mathcal{T}_d$) be a NT metric topology. Then, there is a unique ((N, *), $\mathcal{T}_d$) NT topology.

Definition 4.10: Let ((N, *), d) be a NT metric space, A ⊂ N and $\mathcal{B}$ be set of all open ball of ((N, *) , d)). If A = ∪ C_i (i ∈I and C_i ⊂ $\mathcal{B}$) is satisfied, then it is called that A is an open set in ((N, *), d).

Theorem 4.11: Let ((N, *), d) be a NT metric space. Then ((N, *), $\mathcal{T}_d$) is a NT metric topology such that

$\mathcal{T}_d$ = {C_i⊂N: for every i ∈I and a∈ C_i, there exits at least a ε >0 such that a∈ B(a, ε) ⊂ C_i}.

Proof: We show that $\mathcal{T}_d$ = {C_i⊂N: for every i ∈I and a∈ C_i, there exits at least a ε >0 such that a ∈ B(a, ε) ⊂ C_i} is a NT topology.

i) It is clear that since ((N, *), d) is a NT metric space.

ii) For every x ∈ N, it is clear that B(x, ε) ⊂ N. Thus, N ∈ T_d. Also, if we choose the ε large enough, then we can write ∅ ∈ $\mathcal{T}_d$.

iii) Let C_i ∈ T_d, for every i ∈ I. We take an element x ∈ $\bigcup_{i\in I} C_i$. Thus, there is a j ∈ I such that x∈ C_j ∈ $\mathcal{T}_d$. Where, there is a ε > 0 such that B(x, ε) ⊂ C_j. Hence,

B(x, ε) ⊂ C_j ⊂ $\bigcup_{i\in I} C_i$ and $\bigcup_{i\in I} C_i$ ∈ $\mathcal{T}_d$.

iv) We suppose that $C_1, C_2, \dots, C_n \in T_d$. If $C_1 \cap C_2 \cap \dots \cap C_n = \emptyset$, then we obtain $C_1 \cap C_2 \cap \dots \cap C_n = \emptyset$ ∈ $\mathcal{T}_d$ since ∅ ∈ $\mathcal{T}_d$. We suppose that $C_1 \cap C_2 \cap \dots \cap C_n \neq \emptyset$. Where, there is an element x ∈ $C_1 \cap C_2 \cap \dots \cap C_n$. Then,

if x ∈ C_1, then there is at least r_1> 0 such that B(x, r_1)⊂ C_1.

if x ∈ C_2, then there is at least r_2> 0 such that B(x, r_2)⊂ C_2.

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if x ∈ C_n, then there is at least r_n> 0 such that B(x, r_n)⊂ C_n.

If we take r = min{$r_1, r_2, \dots, r_n$}, then

B(x, r)⊂ B(x, r_1)∩ B(x, r_2)∩ ... ∩ B(x, r_n)⊂ $C_1 \cap C_2 \cap \dots \cap C_n$.

Thus, we obtain $C_1 \cap C_2 \cap \dots \cap C_n$ ∈ $\mathcal{T}_d$.

Definition 4.12: Let ((N_1, *), d_1) and ((N_2, *), d_2) be NT metric spaces, T_{d_1} and T_{d_2} be NT metric topologies. If T_{d_1}= T_{d_2}, then it is called that ((N, *), d_1) is equal to ((N, *), d_2).

Definition 4.13: Let ((N_1, *), d_1), ((N_2, *), d_2) be NT metric spaces and f: $N_1 \to N_2$ be a function. If d_1(f(a), f(b)) = d_2(a, b), for every a, b ∈ N_1, then f is called a NT isometry. Also, if f is one to one and surjective, then it is called that ((N_1, *), d_1) and ((N_2, *), d_2) are NT isometric spaces.

Example 4.14: From Example 4.8,

N_1 = {0, 2, 5, 6}. (N_1, .) is a NT set under multiplication module 10 in ($\mathbb{Z}_{10}$, .). Also, NT are (0, 0, 0), (2, 6, 2), (5, 5, 5) and (6, 6, 6). Thus, d_1(k, m) = ($|2^k - 2^m|$)/8 is a NT metric such that d_1: N_1x$N_1 \to N_1$.
Now, if we take that
N_2 = {10, 12, 15, 16}. (N_2, .) is a NT set under multiplication module 10 in ($\mathbb{Z}_{10}$, .).
Actually, 10 ≡ 0, 12 ≡ 2, 15 ≡ 5 and 16 ≡ 6 in $\mathbb{Z}_{10}$. Thus, NTs are (0, 0, 0), (2, 6, 2), (5, 5, 5) and (6, 6, 6). Therefore, it is clear that d_2(k, m) = ($|2^k - 2^m|$)/2^{13} is a NT metric such that d_2: N_2x$N_2 \to N_2$.
Now, we define function f: $N_1 \to N_2$ such that f(x) = x + 10. We show that f is a NT isometry.

d_2(f(a), f(b)) = d_2(a + 10, b + 10) = ($|2^{a+10} - 2^{b+10}|$)/2^{13} =
2^{10}.$|2^a - 2^b|$)/2^{13} =$|2^a - 2^b|$)/2^3 = $|2^a - 2^b|$)/8 = d_1(a, b).
Thus, from Definition 4.13, f is a NT isometry. Also, it is clear that f is one to one and surjective. Therefore, ((N_1, .), d_1) and ((N_2, .), d_2) are NT isometric spaces.

Definition 4.15: Let ((N_1, *), d_1) and ((N_2, *), d_2) be NT metric spaces, $x_0 \in N_1$ and f: $N_1 \to N_2$ be a function. f is continuous at point x_0 if and only if for every B(f(x_0), ε), there is a B(x_0, δ) such that
f(B(x_0, δ)) ⊂ B(f(x_0), ε).

Definition 4.16: Let ((N_1, *), d_1) and ((N_2, *), d_2) be NT metric spaces, $x_0 \in N_1$ and f: $N_1 \to N_2$ be a function. f is continuous at point x_0 if and only if

There is a $\delta > 0$ depending on x_0 and ε such that it is $d_1(x_0$,x) < $\delta \Rightarrow d_2$(f(x_0), f(x)) < ε.

Conclusion

Topology has many different application areas in classical mathematic. Also, NT structures are a new concept in neutrosophy. In this paper, we introduce NT metric topology. We give some properties and definitions for NT topology, NT metric and NT metric topology. Also, we obtain that in a NT metric space, all NT open sets are a NT based for a NT topology. Hence, we can obtain a NT topology using each NT metric topology. Thus, we add NT metric topology to NT structures which is a new concept.

Furthermore, by utilizing NT metric topology, researcher can obtain new structure and properties. For example, researcher can define NT quasi – metric topology, NT Hausdorff metric topology, NT partial metric topology, NT v-generalized metric topology, NT b-metric topology.

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