Order in the particle zoo

Thomas Schindelbeck, Mainz, Germany,
schindelbeck.thomas@gmail.com

Abstract

The standard model of particle physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. The value of α can be approximated numerically by the gamma functions of the integrals for calculating particle energy. The series expansion of the energy equation provides quantitative terms for Coulomb, strong and gravitational interaction. The relationship of the model with gravitational effects is corroborated by the possibility to derive the basic equations directly from the Einstein field equations.
The only parameters of the model are the values of the speed of light and the elementary charge.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of particle physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of quarks.
Well hidden in the data of particle energies lies another ordering principle that may be derived directly from the principles of general relativity. A simple metric defined by a Ricci scalar of $R = -2/r^2$ yields a function Ψ(r) that will produce a quantization of energy states with powers of $1/3^n$ over the fine-structure constant α [1] and gives α itself as the product of two gamma functions, representing the energy terms in a photon and a point charge expression, suggesting that α may be understood as a characteristic coefficient for the curvature of spacetime implied by the corresponding difference in symmetry.
To conform to the correct absolute scale of energy will require to replace G/c_0^4 in the Einstein field equation (EFE) by an appropriate constant. The terms of this model suggest to use a natural unit system attributing the value of the speed of light, c_0, to the inverse value of electric and magnetic constant, ε_c and μ_c [2]. In addition the units will be chosen to yield the elementary charge, e_c, in units of energy. Using SI units this will result in:

$$c_0^2 = (\varepsilon_c \mu_c)^{-1} \qquad (1)$$

with
$\varepsilon_c = (2.998E+8\ [m^2/Jm])^{-1} = (2.998E+8)^{-1}\ [J/m]$
$\mu_c = (2.998E+8\ [Jm/s^2])^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$
From the Coulomb term [3] $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307E-28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.67E-36\ [J^2]$. In the following $e_c = 3.110E-18$ [J] and $e_c/\varepsilon_c = 9.323E-10$ [m] may be used as natural unit of energy and length.
The appropriate replacement for G/c_0^4 in the EFE will be $1/\varepsilon_c$.
In the development of this model originally Ψ(r) was introduced ad hoc and since this approach is elaborated in more detail it will be outlined first in chapter 2 while in chapter 3 it will be demonstrated that the basic equations can be derived from the Einstein field equations.
For both approaches it might be useful to visualize a particle as a rotating electromagnetic field with the E-vector constantly pointing towards the origin [4]. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity.
To focus on the more fundamental relationships the discussion of minor aspects of the model parameters is exiled to an appendix, related topics to be marked as [A]. The model may be used to calculate additional

1 The relation of the masses e, μ, π with α was noted in 1952 by Y.Nambu [2]. M.MacGregor calculated particle mass and constituent quark mass as multiples of α and related parameters [3]. There is an extended, slightly altered and more speculative working paper [4].
2 Subscript c will indicate the connection of the corresponding coefficients with c_0.
3 $b_0 = e^2/(4\pi\varepsilon)$ used as abbreviation, e=elementary charge, ε=electric constant
4 with B-field and propagation velocity perpendicular to it;

particle properties, magnetic moment is given as example in [A6]. Typical accuracy of the calculations presented is ~ 0.001 which would be in the order of magnitude of expectable QED corrections [5] .

2 Ad hoc approach

2.1 Basic calculations

The model may be essentially based on a single assumption:

Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.

An appropriate form of Ψ can be deduced from three boundary conditions:

1.) To be able to apply Ψ to a point charge Ψ(r = 0) = 0 is required

2.) To ensure integrability an integration limit is needed.

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy results in an exponent of 3 for r in the following equation (see (12)).

Condition 1.) to 3.) are met by an expression (corresponding differential equation see [A1]) :

$$\Psi_n(r) = \exp\left(-\left(\frac{\beta_n/2}{r^3} + \left[\left(\frac{\beta_n/2}{r^3}\right)^2 - 4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{2}$$

Up to the limit of the real solution of (2), r = r_n, with

$$r_n = (\sigma\, \beta_n/8)^{1/3}\ , \tag{3}$$

in all integrals over Ψ(r) given below equ. (4) may be used as approximation for (2)

$$\Psi_n(r<r_n) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \tag{4}$$

Phase will be neglected on this approximation level, properties of particles will be calculated by the integrals over $\Psi(r)^2$ (hence factor 2 in (2)ff) times some function of r which can be given by:

$$\int_0^{r_n} \Psi(r)^2 r^{-(m+1)} dr \approx \int_0^{r_n} \exp(-\beta/r_n^3) r^{-(m+1)} dr = \Gamma(m/3, \beta/r_n^3)\ \frac{\beta^{-m/3}}{3} = \int_{\beta/r_n^3}^{\infty} t^{\frac{m}{3}-1} e^{-t} dt\ \frac{\beta^{-m/3}}{3} \tag{5}$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, \beta/r_l^3)$) denotes the upper incomplete gamma function, given by the Euler integral of the second kind with β/r_n^3 as lower integration limit. For m ≥ 1 the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically.

Coefficient β_n may be given as partial product of a value for a reference particle, β_{ref}, carrying the dimensional term, β_{dim} , that might be given in the unit system defined in chpt. 1 as [see A4]:

$$\beta_{dim} = \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 = 5.131\text{E-}30\ [\text{m}^3] \tag{6}$$

times particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

$$\beta_n = \beta_{ref}\Pi_{k=1}^{n}\alpha_k = 2\sigma\alpha_{ref}\beta_{dim}\Pi_{k=1}^{n}\alpha_k = 2\sigma\alpha_{ref}\beta_{dim}\Pi_n \qquad n = \{1;2;..\} \tag{7}$$ [6]

Index n will indicate solutions of (2) and serve in the following as a radial quantum number. For the angular terms of Ψ(r, ϑ, φ), to be indicated by index l, only rudimentary results exist, their contribution has to be incorporated in parameter σ which according to (2) is related to the solution for a bound state and r_n. Coefficients r_n and σ determine the integration limit of the integrals over Ψ and thus are a crucial factor in particular for the (semi-classical) calculation of angular momentum J. For J = 1/2 [ħ] σ will have the following value, given here in various expressions useful in the following (see [A2,3]):

$$\sigma = 8\, r_n^3/\beta_n = (1.5133\ \alpha^{-1}\ 2/3\ |\Gamma_{-1/3}|)^3 = 1.5133^3\ \sigma^* = 8\left(\frac{4\pi|\Gamma_{-1/3}|^3}{3}\right)^3 = 1.772\text{E+}8\ [-] \tag{8}$$ [7]

5 errors due to approximation of Γ-functions may be in the same range;

6 Π_n denoting the sum of all particle coefficients except the one of the reference particle (electron, see below)

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$. To calculate energy, the integral over the electrical field E(r) of a point charge is used as a first approximation. Using (5) for m = 1 gives:

$$W_{pc,n} = \quad 2\varepsilon_0 \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3 r \quad = \quad 2 b_0 \int_0^{r_n} \Psi_n(r)^2 r^{-2} dr \quad = 2\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \qquad (9)$$

Using equation (5) for m = -1 to calculate the Compton wavelength, λ_C, in the expression for the energy of a photon, hc_0/λ_C , gives the following expression for λ_C :

$$\lambda_{C,n} \approx \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr \quad = \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3} e^{-t}\, dt \;\; \beta_n^{1/3}/3 \quad \approx \; 36\,\pi^2\; |\Gamma_{-1/3}|\; \beta_n^{1/3}/3 \qquad (10)$$

to be used in:

$$W_{Phot,n} = hc_0/\lambda_{C,n} \; = \; \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2\, dr} \; = \; \frac{3\, hc_0}{36\,\pi^2 |\Gamma_{-1/3}|\, \beta_n^{1/3}} \qquad (11)$$

The energy of a particle has to be the same in both photon and point charge description. Equating (9) with (11) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3} = 2.679$; $|\Gamma_{-1/3}| = 4.062$) gives (note: h => ħ):

$$\frac{4\,\pi\, \Gamma_{1/3} |\Gamma_{-1/3}|}{0.998} = \frac{9\, hc_0}{18\,\pi\, b_0} \; = \; \frac{\hbar\, c_0}{b_0} \; = \alpha^{-1} \qquad (12)$$

2.2 Quantization with powers of $1/3^n$ over α

Inserting (7) in the product of the point charge and the photon expression of energy, (9) and (11), gives for W_n^2:

$$W_n^2 \; = \; 2\, b_0\, hc_0 \; \frac{\int^{r_n} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \;\sim\; \frac{1}{\beta_n^{2/3}} \;\sim\; \frac{\alpha_1^{1/3}\, \alpha_2^{1/3} \ldots.. \alpha_n^{1/3}}{\alpha_1\, \alpha_2 \ldots. \alpha_n} \qquad (13)$$

The last expression of (13) is obtained by expanding the product $\Pi_n^{-2/3}$ included in $\beta_n^{-2/3}$ of (7) with $\Pi_n^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_1 only. Identifying α_1 as $\alpha_1 = \alpha_\mu = \alpha^3$ would give an expression using the muon as reference state:

$$\frac{\alpha^1 \alpha^{1/3} \ldots. \alpha \wedge (3/3^n)}{\alpha^3 \alpha^1 \ldots. \alpha \wedge (9/3^n)} \quad = \quad \alpha \wedge (3/3^n)/\alpha^3 \qquad n = \{1;2;..\} \qquad (14)$$

The corresponding term for particle energies would be (using (12)):

$$W_n \; = \; \left(\frac{4\,\pi\, b_0^2}{\alpha} \; \frac{\int^{r_n} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \right)^{0.5} = \; \left(\frac{4\, b_0^2\, \Gamma_{1/3}^2}{9 [\alpha\, 4\,\pi\, \Gamma_{1/3} |\Gamma_{-1/3}|]\, \beta_n^{2/3}} \right)^{0.5} \approx W_\mu \;\; \Pi_{k=1}^{n} \alpha\wedge(-1/3^k) \qquad n = \{1;2;..\} \qquad (15)$$

for spherical symmetry. Equation (15) requires a reference state though no state is singled out in particular in the equations. The partial product of (15) may be extended to include the electron by inserting *ad hoc* an additional factor ≈ 3/2 to represent the anomaly due to the energy ratio of e, μ, $W_\mu / W_e = 1.5088\ \alpha^{-1}$ (see [A3]).

Extending the terms of the partial product further, beyond the electron, yields a term for Planck energy, W_{Pl}, (see 2.6.1) which might be interpreted as a fundamental limit of the product, yielding the ground state term.

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7 Factor 1.5133 is also part of a minor term depending on the radial quantum number, n (see [A3,4]). Thus in the following β_n may be split into $\sigma^* = \sigma/1.5133^3 = 5.112E+7$ [-] and n-dependent terms containing factor 1.5133^3. The expression as cube of $|\Gamma_{-1/3}|^3$ results from a geometrical ansatz for σ ([A3], chpt. 4.2), numerically identical to the value from the angular momentum calculation.

The ratio of electron and Planck energy,

$$1.0006\frac{W_e}{W_{Pl}} = 1.5133^2\alpha^{10}/2 \;\; = 4.90\text{ E-22} \; = \alpha_o \tag{16}$$

may be used to define a ground state as:

$$\beta_{ref} = \beta_e = \sigma^*\alpha_0\;\beta_{\;dim} = \;\; \frac{\sigma*\alpha_0}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \tag{17}$$

indicating that the electron represents this ground state and enabeling to express particle energy “ab initio” by using equ. (16) (see [A4 (52)]).

2.3 Non-spherical symmetric states

Up to here only spherical symmetry, y_0^0, and Ψ(r) have been considered [8]. The ratio of the volume integrals attributed to spherical harmonic Y_1^0 and Y_0^0 gives a factor of 1/3. Assuming Y_1^0 to be a sufficient approximation for the next angular term and $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3}$ (V = volume) to be applicable for non-spherically symmetric states as well, will give $W_1^0/W_0^0 = 3^{1/3} = 1.44 = (y_1^0)^{-1/3}$. A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2. Such angular terms have to be attributed to the parameter σ (see 2.4). Results for particles assigned to y_0^0, y_1^0 are presented in table 1.
Relative to the energy of the electron, W_e, this gives in first approximation:

$$W_n/W_e \approx 3/2(y_l^m)^{-1/3}\Pi_{k=0}^{n}\alpha\text{^}(-1/3^k) \qquad n = \{0;1;2;..\} \tag{18}$$

2.4 Upper limit of energy

According to 2.3 higher angular terms will reduce the value of σ [9]. The variable part in σ is given by the term $(1.5133\;\alpha^{-1})^3$, leaving the minimum for σ, defined by $|\Gamma_{-1/3}|/3$, being required to appear in the integral expression for r, and the integers in the square bracket of (2), to be:

$$\sigma_{min} = (2/3\;|\Gamma_{-1/3}|)^3 \tag{19}$$

The maximum angular contribution to W_{max} would be :

$$\Delta W_{max,\;angular} = 1.5133\;\alpha^{-1} \tag{20}$$

According to (18) and (20) [10], the maximum energy will be $W_{max} = W_e\;1.5*1.5133\;\alpha^{-2.5} = 4.05\text{E-8}$ [J].
In the simple visualization sketched in the introduction the “rotating E-vector” might be interpreted to cover the whole angular range in the case of spherical symmetric states while a p-like state of an Y_1^0-analogue might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the angular limit case, l -> ∞, a state of minimal angular extension representing the original vector, however, extending in both directions from the origin and featuring parity p= -1. Considering only „half“ such a state, extending in one direction only and having p= +1, would notably feature an energy of 1.019 W_{Higgs}, suggesting the energy value of the Higgs boson as possible high energy end of (18).

2.5 Expansion of the incomplete gamma function $\Gamma(1/3, \beta_n/r^3)$

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (9) gives [7]:

$$\Gamma(1/3,\;\beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{21}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - \frac{2}{2}b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3}r} + 2b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3}r^4} = W_n - \frac{b_0}{r} + b_0\frac{\beta_n}{2r^4} \quad {}^{11} \tag{22}$$

The 2nd term in (22) drops the particle specific factor β_n and gives the electrostatic energy of two elementary

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8 y_l^m defined as $y_l^m = \int\int\Psi(\varphi,\vartheta)^2\sin(\vartheta)d\varphi\,d\vartheta\;/\;4\pi$

9 See 4.2 as well and the general expression of (2) for bound states;

10 and using the limit term of note 30 in place of 1.5133, i.e. using the exact terms;

11 Signs not adapted to conventional definition. The 2nd term may be divided by two since it represents only an electrostatic contribution, to be complemented by an equivalent magnetic term.

charges at distance r. The 3rd term is an appropriate choice for the 0th order term of the differential equation [A1 (38)ff]. It is thus supposed to be responsible for the localized character of a particle state and thus may be identified with the “strong force” of the standard model.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient (energy) equ (18)	α-coefficient in ß equ (7)	W_{calc}/W_{Lit}	J	r_n [fm]
Planck	(-1,∞)	1.0 E+21*	$(2/3\ \alpha^{-3})^3\ 3/2\ \alpha^{-1}\ 2$ source term, relative to e !		0.9994 rel. to e !	-	-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**$(3/2)^3\ \alpha^9$**	**1.0001**	**1/2**	**1412**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**$\alpha^9\alpha^3$**	**1.0000**	**1/2**	**6.83**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ 1.44	$\alpha^9\alpha^3/3$	1.0918	0	4.74
K		495				0	
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**$\alpha^9\alpha^3\alpha^1$**	**0.9933**	**0**	**1.32**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ 1.44	$\alpha^9\alpha^3\alpha^1/3$	1.0124	1	0.92
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ 1.44	$\alpha^9\alpha^3\alpha^1/3$	1.0028	1	0.92
K*		894				1	
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0016**	**1/2**	**0.76**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0003**	**1/2**	**0.76**
η'		958				0	
Φ^0		1019				1	
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}$**	**1.0106**	**1/2**	**0.63**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}$**	**1.0046**	**1/2**	**0.61**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**$\alpha^{27/2}$**	**1.0025**	**3/2**	**0.59**
Ξ		1318				1/2	
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ 1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9796	3/2	0.53
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ 1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9724	3/2	0.45
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0046	3/2	0.43
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$1.44	$\alpha^{27/2}/3$	1.0026	1/2	0.4
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}/2$	$(\alpha^{27/2})/(3/4\ \alpha^{-1})^3$	1.0192	0	0.006

Table 1: Particle energies for **y_0^0 (bold)**, y_1^0 [12]; col. 2: radial, angular quantum number; col. 3: energy values of [6] except* (see 2.6.1); col. 4: α-coefficient according to the energy terms of (18), including (2/3) α^{-3} of electron; col. 5: coefficients in β of (7); col. 6: W_{calc} calculated using the slightly more precise [A4 (50)f] in place of (18); ** see 2.4; [13]

2.6 Gravitation

2.6.1 Planck scale

Expressing energy/mass in essentially electromagnetic terms suggests to test if mass interaction i.e. gravitational attraction can be derived from the corresponding terms. Assuming the expansion of the incomplete Gamma function for the integral over r^{-2}, $\Gamma(1/3,\beta_n/r^3)$ (21)f, might be an adequate starting point for gravitational attraction as well, implies that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{G,e}/F_{C,e}$ = 2.41E-43, should be be a term that can be given as completely separate, self-contained expression.

This is equivalent to assume that gravitational interaction is a higher order, nonlinear effect of electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{23}$$

12 up to Σ'^0 all resonance states given in [6] as **** included; for residual gaps see [4]; Exponents of -9/2, 27/2 for Δ and tau are equal to the limit of the partial products in (7) and (18); r_n calculated with (3); 1.5133 rounded to 3/2;

13 Extending the model to energies below the electron with a coefficient of α^3 in (18) gives a state of energy ~ 0.2eV which is roughly in a range expected for a neutrino [5].

as definition for Planck terms, giving for the Planck energy W_{Pl}:

$$W_{Pl} = c_0^2 (b_0 /G)^{0.5} = c_0^2 (\alpha h c_0/G)^{0.5} \tag{24}$$

Expression (16) gives the ratio of W_e to W_{Pl} as 3rd power of the electron coefficient, $\alpha_e^3 = (1.5133^3\ \alpha^9)^3$, divided by two times the angular limit factor according to (20), 1.5133 α^{-1}. The latter factor may be interpreted as sum of minor factors of a more detailed analysis of β [A4 (50)ff]. The relationship is somewhat phenomenological, in chapter 4.2 a more fundamental interpretation of the position of W_{Pl} in the model will be discussed.

Using [A3 (48)] to express factor 1.5133 gives (F_G, F_C = gravitational, Coulomb forces):

$$\left(\frac{W_e}{W_{Pl}}\right)^2 = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} \approx \left(\frac{1.5133^3\alpha^9}{1.5133\,\alpha^{-1}2}\right)^2 = \left(\frac{(4\pi)^2|\Gamma_{-1/3}|^4\alpha^{12}}{2}\right)^2 = 1.0008^2\left(\frac{F_{G,e}}{F_{C,e}}\right)_{\exp} = \frac{G W_e^2}{c_0^4 b_0} = \alpha_0^2 \tag{25}$$

Using (12) and [A4 (52)] for calculating W_e would turn G into a coefficient based on electromagnetic constants:

$$G_{calc} \approx \frac{c_0^4}{4\pi\varepsilon_c}\left(\frac{1}{3\pi^{2/3}}\left(\frac{|\Gamma_{-1/3}|}{\Gamma_{1/3}}\right)^4\alpha^{12}\right)^2 \approx \frac{c_0^4}{4\pi\varepsilon_c}\,\frac{2}{3}\,\alpha^{24} = 1.001\,G_{\exp} \tag{26}$$

2.6.2 Virtual superposition states

Within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius ~ r_n, $\lambda_{C,n}$ etc. appropriate for energy of each virtual particle state (VS) [14], providing a source of energy at a distance r_{VS} from the primary particle and in turn contributing to the stress-energy tensor responsible for curvature of spacetime that manifests itself in gravitational attraction.

In general VS are not supposed to consist of analogues of e.g. spherical symmetric states covering the complete angular range of 4π but to be an instantaneous, short term extension of the (rotating) E-vector thus requiring the angular limit factor of (20).

A long range effect of the 3rd, the strong interaction term, of (22) may be exerted via virtual particle states. To estimate such an effect in first approximation the following will be used;

- the 3rd term of equ. (22) with β according to (7), (17),
- the angular limit state of σ^*_{min} according to (19), $\sigma^*_{min} \approx 1$,
- $\beta_{dim} = (4\pi)^{-2}\,(e_c/\varepsilon_c)^3 \approx (\alpha^{-1}\,r_e)^3$.

For any VS at $r = \alpha^{-1}\ r_{VS} = (4\pi)^{-2/3}\,\Pi_{VS}^{1/3}\ e_c/\varepsilon_c$, i.e. the radius of the VS in natural units, equ. (27) will hold:

$$W_{VS}(r) \approx b_0\frac{\beta_{VS}/2}{(\alpha^{-1}\ r_{VS})^4} \approx b_0\frac{\alpha_0\Pi_{VS}(\alpha^{-1}\ r_e)^3}{(\alpha^{-1}\ r_{VS})^3(\alpha^{-1}\ r_{VS})} \approx b_0\frac{\alpha_0\Pi_{VS}(\alpha^{-1}\ r_e)^3}{(\Pi_{VS}^{1/3}\alpha^{-1}\ r_e)^3(\alpha^{-1}\ r_{VS})} = b_0\frac{\alpha_0}{(\alpha^{-1}\ r_{VS})} = \frac{b_0}{(\alpha^{-1}\ r_{VS})}\left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} \tag{27}$$

Considering that the composition of the stress-energy tensor from virtual states is expected to be based on a much more complex mechanism requiring consideration of all possible virtual states at a particular point and appropriate averaging, (27) seems to be an acceptable first approximation. The crucial factor that turns the r^{-4} dependence of the strong interaction term into r^{-1} of gravitational interaction is the proportionality of β_n to the cube of any characteristic particle length, r_n, $\lambda_{C,n}$ etc. which is valid for each particle state subject to the relations of this model.

Equ. (27) is a representation of the gravitational potential of the electron, terms for other particles may be obtained by inserting values according to (18) in (27) which might be interpreted as the intensity/frequency of emergence of virtual states being proportional to the energy of the primary particle.

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14 The superposition states considered here would be not virtual in a Heisenberg sense, the energy is provided by the source particle.

3 Relationship with General Theory of Relativity

3.1 Basic model + order of magnitude

The model provides a quantitative expression for the constant of gravitation in electromagnetic terms as well as a mechanism for energy based curvature of spacetime at a distance from a primary particle, adding some features to the General Theory of Relativity (GR).

The minute factor G/c_0^4 in the Einstein field equation (EFE) is responsible for this equation not being particularly suited to attempt a calculation of particle energies based on this formalism. The interpretation of gravitation as a higher order effect with respect to electromagnetism suggests to replace G/c_0^2 [m/kg] or G/c_0^4 [m/J] by an equivalent electromagnetic term. A term of order $1/\varepsilon_c$ [m/J] may provide the appropriate units and the necessary order of magnitude, suggesting to use the substitution

$$G/c_0^4 \quad \Rightarrow \quad \frac{1}{\varepsilon_c} \quad ^{15} \qquad (28)$$

It is tempting to test if the equations of this model may be derived directly from the Einstein field equations. The central concept will be the "rotating E-vector" of the introduction, i.e. a photon is visualized as having its E-field vector component constantly oriented to a fixed point, the origin of the coordinate system used.

The basic question will be: What kind of metric will yield an undisturbed photon propagation according to Maxwell equations that manifests itself as a localized object in flat spacetime ? [16]

In a spherical coordinate system [17] the rotation of an object with extension in angular direction will result in some kind of self interaction increasing with r ->0 *unless* space(time) is curved in such a way as to prevent that. This will be the case if the r^2 - term in the angular coordinates is canceled, implying an expansion of curved spacetime with r^2 at any given r, i.e. the Ricci scalar should be $R(r) \sim -1/r^2$.

In a trivial version the metric will be given by

$$g_{\mu\nu}=(+1,-1,-r^2,-r^2\sin 2\Theta) \quad \Rightarrow \quad g_{\mu\nu}=(+1,-1,+r^2,+r^2\sin^2\Theta) \quad ^{18} \qquad (29)$$

All Christoffel symbols except Γ_{23}^3, Γ_{32}^3, Γ_{33}^2 will be zero, yielding $R_{22} = -1$, a Ricci scalar $R = -2/r^2$ and an Einstein tensor $G_{\mu\nu}$ of

$G_{\mu\nu} = (+1/r^2, -1/r^2, 0, 0)$

3.2 Differential equation + coordinate transformation

In the following the Ricci scalar will be required to be:

$$R = -2/r^2 \qquad (30)$$

and a general exponential ansatz will be used for $g_{00,11}$:

$g_{\mu\nu} = (+\exp(a\, v(r)), -\exp(b\, v(r)), +r^2, +r^2 \sin^2\theta)$

This will result in a Ricci scalar of (see [A5]):

$$R = \left(e^{-bv}\left[-av'' - \frac{a^2 v'^2}{2} + \frac{ab\, v'^2}{2} - \frac{(a-b)v'}{r} \right]_{00,11} + e^{-bv}\left[\frac{(b-a)v'}{r} - \frac{2}{r^2} \right]_{22,33} \right) - 2/r^2 \qquad (31)$$

To get $R = -2/r^2$ one has to set the term in curved brackets to zero.

The equation (31) refers to local coordinates and has to be solved for these or transformed to flat coordinates. The latter will be attempted by transforming the spherical object of a particle back into a photon of appropriate wavelength, assuming that

1.) for r ->0 the angular coordinates have to reflect the expansion $\sim 1/r^2$, while

2.) the energy-space-time relation of a photon, i.e. $W_{ph} \sim 1/r$, $\sim 1/T$ reflects a contraction of spacetime linear in coordinates ct, r .

15 It remains unclear if a factor of 4π, 8π or similar has to be included.

16 Or alternatively: How to transform an object of $C_{\infty,v}$ (O(2)) symmetry (considering for the photon the E-vector only) into one of K_h (O(3)) symmetry ? (considering phase / the rotational quality of the objects SO(2) and SO(3) may be more appropriate);

17 coordinates t, r, Θ, Φ = x_0, x_1, x_2, x_3; only diagonal elements considered, μ=ν;

18 The plus sign implies non-Lorentz invariance as expectable since a distinctly local coordinate system is dealt with.

A coefficient ρ [m] will be needed to obtain dimensionless terms [19]. This gives:

$$\left[- a v'' - \frac{a^2 v'^2}{2} + \frac{ab v'^2}{2} - \frac{(a-b) v'}{r} \right]_{00,11} \frac{\boldsymbol{r}}{\boldsymbol{\rho}} + \left[\frac{(b-a) v'}{r} - \frac{2}{r^2} \right]_{22,33} \frac{\boldsymbol{\rho^2}}{\boldsymbol{r^2}} = 0 \tag{32}$$

An equation of type (32) will in general feature solutions of type exp(v) = exp (+/- x/r³), which is a sufficient criterion to obtain equations (12), (13)f i.e. the numerical expression for α and the quantization of particle energies.

Setting a = b eliminates some terms, giving:

$$- av'' \frac{r}{\rho} - \frac{2}{r^2} \frac{\rho^2}{r^2} = 0 \qquad \Rightarrow \qquad a v'' = - \frac{2\rho^3}{r^5} \tag{33}$$

The solution of equ. (33) corresponds to equ. (4) with setting a = b = 1/3, $\rho^3 = 3/2\,\beta$:

$$e^v = \Psi(r) = \exp\left(\frac{-\rho^3}{3r^3}\right) \qquad \Rightarrow \qquad \frac{-4\rho^3}{r^5} + \frac{2\rho^3}{r^5} = - \frac{2\rho^3}{r^5} \quad {}^{20} \tag{34}$$

In order to reproduce the factors 4π included in (9) and (17) an appropriate term might be added to the angular terms of (31) giving

$$R = \left(e^{-v} [-a v'']_{00,11} \frac{r}{\rho} + e^{-v} \left[- \frac{2}{r^2} \right]_{22,33} \frac{\rho^2}{(4\pi r)^2} \right) - \frac{2\rho^2}{(4\pi r)^2 r^4} \tag{35}$$

The Einstein tensor component G_{00} will be:

$G_{00} = [- v''/2 - v'/r] + e^v \rho^2/((4\pi)^2 r^4) = e^v \rho^2/((4\pi)^2 r^4)$

Equating with the component of the stress-energy tensor, $G_{00} = T_{00}$, and using (28) will give (w = energy density):

$$e^v \frac{\rho^2}{(4\pi)^2 r^4} \approx \frac{w}{\varepsilon} \qquad \Rightarrow \qquad \frac{\varepsilon e^v \rho^2}{(4\pi)^2 r^4} \approx w \tag{36}$$

The volume integral over (36) gives the particle energy according to (9) if ρ in (36) is given by

$$\rho^3 = \frac{3}{2\sigma \alpha_e \Pi_n (4\pi)^2} \left(\frac{e_c}{\varepsilon_c} \right)^3 \quad {}^{21} \tag{37}$$

4 Discussion

4.1 Standard model of particle physics

The standard model of particle physics in spite of its numerous achievements is not particularly efficient in quantitative calculation of intrinsic particle properties such as mass / energy. Input parameters of lattice QCD calculations of hadron masses [8], [9], are quark masses, coupling constant and a reference particle for the absolute energy scale, i.e. typically about 4 parameters are needed to calculate mass of ~12 particles with an accuracy in the range of 1%. The model presented here achieves comparable results “ab initio” and in contrast to QCD methods includes all particles, leptons and hadrons. The standard model distinguishes quite rigidly between both types, postulating that a set of physical objects characterized by an almost identical set of experimental observables is based on completely different physical principles. The distinctive observable for both particle groups is assumed to be the strong force which is postulated to be zero for leptons, which per se is not verifiable beyond experimental accuracy. The three generation model, attributing a neutrino to each charged lepton, serves as supporting argument. However, the total number of neutrinos is not beyond doubt [5], [10], and neutrino oscillation obscures the earlier assumption of clearly distinct particles. Last not least, a distinctive interaction of neutrinos with the charged leptons might simply be due to a very weak strong interaction of the particles involved, not requiring any assumption beyond that.

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19 i.e. the factor r^2 in the angular terms will be canceled by ρ^2/r^2, restoring $C_{\infty,v}$ (O(2)) symmetry, while using factor r /ρ in the ct, r terms would assure all photons to obey $W_{ph} \sim 1/\rho$.

20 polar coordinates

21 The term $\sigma\, \alpha_e\, \Pi_n$ has to appear in the denominator since ρ^2 appears in the nominator of equ (36), not affecting the validity of the equations of this model. See [A1] as well.

According to this model weak strong interaction for leptons is expected in scattering events since the effect of strong interaction between particles is considered to be due to wave function overlap [11] depending on

1) comparable size and energy of wave functions,
2) sufficient net overlap: If regions with same and opposite sign balance to give zero net overlap, no interaction occurs.

From condition 1) it is obvious that the wave functions of neutrino and electron will not show effective interaction with hadrons [22]. In the case of the tauon the second rule is crucial. In this model the tauon is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign prohibiting net overlap and effective interaction with hadrons of higher symmetry, such as the proton.

In the standard model mass of elementary particles is generated by the "Higgs mechanism". In this model the Higgs boson is a candidate for the highest energy state indicating some qualitative relationship with the Higgs mechanism. A fundamental break of symmetry associated with the creation of a „localized photon" in this model is the generation of +/- charge due to the persistent orientation of the E-vector towards the origin.

4.2 Relation to General Relativity

The principles of GR [23] seem to fit particularly well to the concepts used:

- the basic equation of the model may be derived from the Einstein field equations,
- it provides an expression for the constant of gravitation using electromagnetic terms,
- it gives a mechanism for curvature of spacetime at a distance from a primary particle implying curvature to be in general identical to (the presence of) energy and spatial coordinate and energy to be intertwined inextricably,
- it suggests a close relationship of several mass/energy related phenomena - particle energy, elements of the Higgs mechanism, Planck energy - with GR.

As regarding to the last point, the interpretation of the 3[rd] power relationships for particle energies given above is not unambiguous.

One way to explain the concepts of this model is the "rotating E-vector". According to the considerations given above the state of maximum particle energy, that coincides with that of the Higgs boson, represents an archetype for such a "rotating localized photon".

Defining this object not by its energy but via possessing some well defined "maximum curvature of spacetime" this curvature may be spread over a larger volume [24] resulting in particles of less average curvature and thus less energy. In complete spherical symmetry "curvature" is spread out most evenly corresponding to the lowest energy. The coefficient $|\Gamma_{-1/3}|$, attributed to integrals over $\Psi(r)$ dr to yield lengths should appear as term $4\pi|\Gamma_{-1/3}|^3/3$ in the expressions for a spherical symmetric object (see [A3]). Obviously the state of the electron fits such a particle.

Alternatively one may start from assuming e_c to be a distinguished natural unit for energy and the fine-structure constant α^{-1}, that represents a relationship between different spatial symmetries (O(2) and O(3)), being a characteristic constant for "minimal curvature of spacetime", referring to one dimension [25]. The 3D equivalent of such an object would be characterized by α^{-3}. Spread over a volume $4\pi|\Gamma_{-1/3}|^3/3$ would give the energy of the electron [26]. W_{Higgs} would represent the corresponding maximum of energy / curvature according to (18).

Considering the electron to be equivalent to a spherical symmetric object "containing" *one* Higgs boson spreading out, one might ponder what the energy of a spherical object of "maximum curvature of spacetime" is, i.e. a spherical symmetric electron-type object containing not one Higgs boson but being filled up with Higgs boson-like curvature. Such a hypothetical object might be constructed by raising the energy ratio Higgs / electron to a power of 3, i.e. filling the whole 3D volume with Higgs particles [27]. The resulting

22 As for energy density ~ W_m/W_n^4 : e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, i.e. in case of μ/π some measurable effect might be expected; different symmetry may play an additional role.

23 as well as concepts connecting GR with electromagnetism based on 5-dimensional Kaluza type theories [12]

24 as seen from flat space

25 since originating from integrals over dr

26 Exact with the relationships given above, in 1st approximation: $(4\pi|\Gamma_{-1/3}|^3/3)^{-1}\alpha^{-3} e_c \approx 0.35\, W_e$.

27 implying a 4th power relationship between W_{Pl} and W_e, compare (25);

energy would be close to the Planck energy (≈1.8 W_{Pl}; Table 2 gives a corresponding estimate in terms of powers of α), implying both a Planck particle and a Higgs boson to represent some kind of "maximum curvature of spacetime" though in different symmetry.
This suggests to change the particle order in table 1 according to table 2:

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient (energy)	Calculated energy relative to electron
e_c	-	1.9 E-5	Reference	
e^{+-}	0, 0	0.51	$\alpha^{-3}/(4\pi\|\Gamma_{-1/3}\|^3/3) \approx \alpha^{-2}$	
μ^{+-}	1, 0	105.66	$\alpha^{-2}\alpha^{-1}$	
η^0	2, 0	547.86	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}$	
p^{+-}	3, 0	938.27	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	
n	3, 0	939.57	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	
Λ^0	4, 0	1115.68	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$	
Σ^0	5, 0	1192.62	$\alpha^{-2}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$	
Δ	∞, 0	1232.00	$\alpha^{-2}\alpha^{-3/2}$	
Higgs	∞,∞	1.25 E+5	$\alpha^{-2}(\alpha^{-5/2})$	$W_{Higgs, calc} = 0.90\ W_{Higgs, exp}$
Planck	$Higgs^3$	1.0 E+21	$\alpha^{-2}(\alpha^{-5/2})\ (\alpha^{-5/2})^3$	$W_{Pl, calc} = 1.14\ W_{Pl, def}$

Table 2: Table with particle energies, emphasizing a relationship between elementary charge and electron as well as Higgs boson and Planck energy; α-coefficients only, minor terms omitted ;

Conclusion

Constructing a metric for localizing a photon requires the Ricci scalar in the Einstein field equations to be $R = -2/r^2$. This yields the energy term for a point charge modified by an exponential function Ψ. Applying Ψ in the energy expression of the photon as well gives the following results:

- the fine-structure constant, α, being defined by the product of the Γ-functions in the integrals over Ψ(r) related to photon and point charge symmetry, $4\pi\ \Gamma_{+1/3}\ |\Gamma_{-1/3}| \approx \alpha^{-1}$,
- a quantization of energy levels given by a partial product of terms α^(-1/3^n)
- elementary charge and electron as lower limits and the Higgs boson and Planck energy as upper limits for particle energy,
- additional information about particle properties e.g. the lepton character of the tauon, magnetic moments,
- a series expansion for particle energy, including terms for rest energy, electromagnetic interaction and a 3^{rd} term which at short range yields effects associated with strong interaction, applied to virtual particle states gives a quantitative term for gravitational interaction.

The only parameters of the model are the values of the speed of light and the elementary charge.

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Appendix

[A1] Differential equation

The approximation $\Psi(r<r_n)$ of equation (4) provides a solution to a differential equation of type

$$-\frac{r}{6}\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{2r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{r^4}\Psi(r) = 0 \tag{38}$$

which corresponds approximately to the limit l -> ∞ while has to be amended by σ in the denominator of the last term for the general case.
With the 3rd term in (22) used for potential energy, V:

$$V(r) = b_0\,\beta_0/(2\,r^4) = b_0\,[\,\sigma^*\,\alpha_0\,(e_c/\varepsilon_c)^3\,/(4\pi)^2]\,/(2r^4) \tag{39}$$

and a corresponding expansion by $(\hbar c_0)^2\alpha^{-2}/b_0^2$ for the first term, equ. (38) may be given as:

$$-\frac{(\hbar c_0)^2 r}{\alpha^{-2} b_0}\frac{d^2\Psi(r)}{dr^2} + r\,V(r)\,\frac{d\Psi(r)}{dr} - \frac{V(r)}{\sigma}\Psi(r) = 0 \tag{40}$$

Equation (35) will produce a differential equation with non-vanishing terms of v' setting a=-i, b=4i, $v = -\rho^3/r^3$:

$$\left[+i\,v'' + 5/2\,v'^2 + \frac{5i\,v'}{r}\right]_{00,11} + \left[\frac{+5i\,v'}{r} - \frac{2}{r^2}\right]_{22,33}\frac{3i\rho^3}{2r^3} = 0 \tag{41}$$

giving

$$\left[\frac{-12i\rho^3}{r^5} + \frac{45\rho^3}{2r^8} + \frac{15i\rho^3}{r^5}\right]_{00,11} + \left[\frac{-45\rho^3}{2r^8} - \frac{3i}{r^5}\right]_{22,33} = 0 \tag{42}$$

[A2] Angular momentum

A simple relation with angular momentum J for spherical symmetric states will be given by applying a semi-classical approach using

$$J = r_2\,x\,p(r_1) = r_2 W_n(r_1)/c_0 \tag{43}$$

with $W_{kin,n} = 1/2\ W_n$, using term $2b_0$ of (9) as constant factor, integrating over a circular path of radius $|r_2| = |r_1|$ and setting the terms of (3), (8) as integration limits. This will give:

$$|J| = \int_0^{r_n}\int_0^{2\pi} J_n(r)\,d\varphi\,dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n}\Psi_n(r)^2 r^{-1}dr \tag{44}$$

From (5) follows for m = 0:

$$\int_0^{r_n}\Psi_n(r)^2 r^{-1}dr = 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}dt \approx 5.45 \approx \alpha^{-1}/8\pi \tag{45}$$

Inserting (45) in (44) would provide:

$$|J| = 4\pi\frac{b_0}{c_0}\frac{\alpha^{-1}}{8\pi} = 1/2\,[\hbar] \tag{46}$$

[A3] Coefficient ~1.5

The value of 1.51 α^{-1} in r_n, σ originates from the relationship with J through equ. (44) and is obviously close to the ratio $W_\mu/W_e = 206.8 = 1.5088\,\alpha^{-1}$. The exact value of 1.5133 for ≈ 1.51 has been chosen due to :
1. a possible geometrical interpretation (using(12)) of the term in σ:

$$1.51\,\alpha^{-1}\,|\Gamma_{-1/3}|\,/3 \approx |\Gamma_{-1/3}|\,/\Gamma_{1/3}\ \ 4\pi\,|\Gamma_{-1/3}|\,\Gamma_{1/3}/0.998\ \ |\Gamma_{-1/3}|/3 \approx \frac{4\pi|\Gamma_{-1/3}|^3}{3} = (\sigma/8)^{1/3} \tag{47}$$

providing a geometric definition for σ and giving 1.5133 as

$$1.5133 = 4\pi\Gamma_{-1/3}^2\ \alpha \tag{48}$$

2. Factor 1.5088 of the ratio W_μ/W_e being subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right) = \left(\frac{1.5133}{1.5}\right)^{1/3} \tag{49}$$

indicating that the radial terms of Π_n in β_n and the angular components of σ are not correctly separated yet or may not be separable even in the case of spherical symmetric states.

[A4] Particle parameter β

A more detailed expression for β than given in (17) will be attempted in the following.

The term (49) will be used within the particle specific factor (square brackets), thus coefficient 1.5133 of σ will be placed there, giving for the general term (i.e. excluding the electron):

$$\beta_n = \frac{2}{(2\pi)^3}\left(\frac{2}{3}\right)^3 \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \Pi_{k=0}^{n}\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]\wedge\left(\frac{3}{3^k}\right) \qquad n = \{1,2,...\} \tag{50}$$

[28]

for the electron:

$$\beta_e = \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \frac{\mathbf{2}}{(\mathbf{2\pi})^{\mathbf{3}}}\left(\frac{\mathbf{2}}{\mathbf{3}}\right)^{\mathbf{3}}\left[\frac{\mathbf{3}}{\mathbf{2}}\,\boldsymbol{\alpha}^{\mathbf{3}}\left(\frac{\mathbf{1.5133}}{\mathbf{1.5}}\right)\right]^{\mathbf{3}} = \boldsymbol{\alpha_0}\,\frac{\sigma}{1.5133^3}\,\frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \tag{51}$$

[29]

the particle specific factor is given in square brackets (α_0 in bold). The other factors are due to

- factor 2: Ψ appearing squared in the integrals,
- factor $1/(2\pi)^3$: representing 2π of the integral limit in (44),
- factor $(2/3)^3$: due to anomalous factor 2/3 in W_e/W_μ,
- $1/(4\pi)^2$: see 3.2 (35)ff.; b_0 appearing squared in (40).

Using (51) W_e may be given as:

$$W_e = 2b_0\frac{\Gamma_{+1/3}}{3}\left(\frac{9\pi^{5/3}\alpha}{|\Gamma_{-1/3}|}\left(\frac{\varepsilon_c}{e_c}\right)\left[\frac{\alpha^{-3}}{1.5133}\right]\right) = \frac{1.5\pi^{2/3}}{1.5133}\frac{\Gamma_{+1/3}}{|\Gamma_{-1/3}|}\frac{e_c}{\alpha^2} = 1.0001\, W_{e,exp} \tag{52}$$

[A5] Metric

$g_{\mu\nu} = (+\exp(a\, v(r)),\ -\exp(b\, v(r)),\ +r^2,\ +r^2 \sin^2\theta)$

$g^{\mu\nu} = (+1/\exp(av(r)),\ -1/\exp(bv(r)),\ +1/r^2,\ +1/r^2 \sin^2\theta)$

$\Gamma_{01}^{0} = \Gamma_{10}^{0} = a\, v'/2$ $\quad\Gamma_{00}^{1} = a\, v'\, e^{(a-b)v}/2$ $\quad\Gamma_{11}^{1} = b\, v'/2$

$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \Gamma_{13}^{3} = \Gamma_{31}^{3} = 1/r$ $\quad\Gamma_{22}^{1} = +r\, e^{-bv}$ $\quad\Gamma_{33}^{1} = \Gamma_{22}^{1}\sin^2\theta$

$\Gamma_{23}^{3} = \Gamma_{32}^{3} = \cot\theta$ $\quad\Gamma_{33}^{2} = -\sin\theta\cos\theta$

$R_{00} = e^{(a-b)v}\,[-a\,v''/2 - a^2v'^2/4 + ab\,v'^2/4 - a\,v'/r]$ $\quad R_{11} = [+a\,v''/2 + a^2v'^2/4 - ab\,v'^2/4 - b\,v'/r]$

$R_{22} = e^{-bv}\,[(b-a)\,v'r/2 - 1] - 1$ $\quad R_{33} = R_{22}\sin^2\theta$

$g^{00}R_{00} + g^{11}R_{11} = e^{-bv}\,[-a\,v'' - a^2v'^2/2 + ab\,v'^2/2 - (a-b)\,v'/r]$

$g^{22}R_{22} + g^{33}R_{33} = e^{-bv}\,[(b-a)\,v'/r - 2/r^2] - 2/r^2$

[A6] Magnetic moment

Within this model particles are treated as electromagnetic objects principally enabling a direct calculation of the magnetic moment M from the electromagnetic fields.

The magnetic moment M_e of the electron is given as product of the anomalous g-factor, g_a = 1,00116, Dirac-g-factor, g_D = 2, and the Bohr magneton, $\mu_B = e\,\hbar/(2m_e)$, times the quantum number for angular momentum, J = 1/2:

$$M_e = g_a g_D \mu_B/2 = g_a\frac{2ec_0^2}{2W_e}\,\frac{\hbar}{2} = g_a 9.274\text{E-}24\,[\text{Am}^2] \tag{53}$$

[30]

The factor g_a arises from the interaction of the electron with virtual photons as calculated in quantum electrodynamics and should not be part of a calculation of the magnetic moment from the field of the electron itself. Within this model the factor 2 of g_D originates from the fact that particle energy is supposed to be equally divided into contributions of the electric and magnetic field, $W_{el} = W_{mag} = W_n/2$ and only the magnetic field, i.e. W_{mag} contributes to the magnetic moment.

Inserting the term for particle energy of (9) in (53) gives:

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28 limit n -> ∞ for partial product 1.5133 $\Pi_{k=0}^{n}$ (1.5133/1.5)^1/3^k = 1.506645 = $(1.5*1.5133)^{0.5}$

29 Note: $2\,(2/3)^3/(2\pi)^3 \approx (1.5133\,\alpha^{-1}\,2)^{-1}$;

30 Note: to allow for comparison with tabulated values of M in units of [Am²] the calculations in this chapter use e [C] not e_c [J], conversion factor: $[m^2C/s]/[m^2 J/s] = e/e_c = 1/19.4$ [C/J].

$$\frac{M_e}{g_a} = \frac{e\hbar c_0^2}{2W_e} = \frac{e\hbar c_0^2}{2}\frac{3\beta_e^{1/3}}{2b_0\Gamma_+} = e c_0\beta_e^{1/3}\left(\frac{|\Gamma_{-1/3}|}{3}\frac{3}{|\Gamma_{-1/3}|}\right)\frac{3[\hbar c_0/b_0]}{4\Gamma_+} = e c_0\beta_e^{1/3}\frac{|\Gamma_{-1/3}|}{3}\left[\frac{9[\alpha^{-1}]}{4\Gamma_{1/3}|\Gamma_{-1/3}|}\right] \tag{54}$$

The relation of the values of E and B in an electromagnetic wave is given by B = E/c_0. This gives as first approximation for the value of M_n :

$$M_n \approx \frac{1}{\mu}\int_0^{r_n} B(r)\Psi_n(r)^2 d^3r = \varepsilon c_0\int_0^{r_n} E(r)\Psi_n(r)^2 d^3r = ec_0\beta_n^{1/3}\frac{|\Gamma_{-1/3}|}{3}\frac{1.5133}{\alpha} \tag{55}$$

Equation (55) neglects contributions to B(r) from other parts of the standing wave and requires an appropriate integration of those. The term 1.5133/α and that in square brackets of (54) consist of integral terms over $\Psi(r)^2$ indicating that it might be possible to get the exact solutions from a more elaborate version of this model.

	M_Lit [Am^2]	|M|_Calc [Am2]	|M|_Calc/|M|_Lit
e^{+-}	-9.28E-24	6.81E-23	-7.335
μ^{+-}	-4.49E-26	3.31E-25	-7.371
p^{+-}	1.41E-26	3.72E-26	2.638
n	-9.66E-27	3.72E-26	-3.850
Λ^0	-3.10E-27	3.10E-26	-10.014

Table 3: Absolute values calculated for magnetic moment approximated with (55) compared to literature [6]