Diskerfery and the Alignment of the Four Main Giza Pyramids

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Abstract

The curious alignment of the three main pyramids at Giza has puzzled many people over the ages. Various theories have been proposed, either based on site geometry or stellar alignments coupled with known ancient Egyptian religious beliefs. This paper shows that the precise alignment of the pyramids can be explained with mathematics alone, using diskerfery and other geometry. We then use the same techniques to identify the most-probable location of the now-dismantled fourth pyramid at Giza. The techniques and discoveries in this paper provide the basis for dating Giza, as discussed in the companion paper "55,550 BCE and the 23 Stars of Giza" (Douglas, 2019).

Keywords: *Giza, pyramids, alignment, archaeogeometry.*

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1. Introduction

1.1 Prologue

After watching a video[1] about the Nebra disk showing a circle divided in the golden ratio φ, I started examining the Giza plateau, trying to find a similar construction to explain the non-linear alignment of the pyramids. I initially failed, while finding other things, but after some months, I "thunk differently" and suddenly there φ was, all over the place. This and further research eventually led to an understanding of the thinking behind the layout.

I used the technique of dividing a circle shown in the video, and have labelled it "diskerfery[1]," meaning "the art of dividing circles," typically by an irrational like π, φ or √2.

1.2 Thesis

This paper and its companion (*55,550 BCE and the 23 Stars of Giza*, Douglas 2019 [2]), propose that the Giza pyramid site layout was *broadly* modelled on a stellar arrangement. At the same time, the actual precise alignments between the main pyramids were determined mathematically, rather than trying to exactly match the stars. Instead, the clear close matching with the stars is assumed to indicate a date for the construction, since one of the stars is fast-moving and would only align in a relatively small window of time.

This implies a prior intelligent and far-sighted civilization, which had advanced skills.

2. Notation, accuracy and methodology

2.1 Notation

The main pyramids at Giza are usually designated as G1 for Khufu, G2 for Khafre, or G3 for Menkaure, alternatively as P1 to P3. I have used the P1 to P3 notation, as the grid reference introduced later also uses letters, and using G for the pyramids will be confusing.

The corners and centre are abbreviated respectively as NW, NE, SW, SE and C, for North West, North East, South West, South East, and Centre, following the cardinal directions. We can then refer to Pyramid 1, North West corner as P1 NW without confusion.

1 Arithmetic geometry (arithmeometry ?) is already used for other things, as is "kemetry.". Cirscilery (circle slicing) sounds like an evil character from The Game of Thrones. Cirdivery is at least a noble Sir but still sounds suspicious. Disk is a synonym for "circle," kerf for "cutting." Diskerfery also channels "discovery" and "curve."

I have also used the traditional names associated with P1, P2 and P3 as a convenience, although I don't think those people had anything to do with the original construction, only maintenance.

I take the royal cubit (henceforth Ͼ) as π/6 metres (*The Beautiful Cubit System*, Douglas 2019 [3]).

2.2 Accuracy

How accurate must things be? We have no idea what tools or technologies the builders had, what they considered "accurate" or "good enough," nor exactly how earthquakes or tectonic shifts have affected the relative positions over time. We can not assume that their standards were the same as ours. There is no such thing as perfect accuracy in building construction, despite which, we can demonstrate alignments generally to within 0.5° of that calculated mathematically.

Note that "close" in context of this discussion refers to practical measurements on a large-scale building project using unknown instruments, not something like modern micro-electronics. My ballpark for angular measurements is preferably less than 0.5° difference from true.

I am indebted to Glen Dash [4] and the Giza Plateau Mapping Project for their work on providing accurate measurements for the pyramids at Giza. Note that the co-ordinates for Menkaure and Khafre are not as accurate as for Khufu. Co-ordinates are given accurate to the nearest tenth of a metre.

2.3 Algorithmic Mathematics and Dialectic Mathematics.

We refer the reader to papers by Man-Keung Siu [5] and P. Henrici [6] on the topic of Algorithmic and Dialectic mathematics, the following extract from Henrici is relevant:

> *Dialectical mathematics* is a rigorously logical science, where statements are either true or false, and where objects with specified properties either do or do not exist. *Algorithmic mathematics* is a tool for solving problems. Here we are concerned not only with the existence of a mathematical object, but also with the credentials of its existence. *Dialectical mathematics* is an intellectual game played according to rules about which there is a high degree of consensus. The rules of the game of *algorithmic* mathematics may vary according to the urgency of the problem on hand. We never could have put a man on the moon if we had insisted that the trajectories should be computed with dialectic rigor. The rules may also vary according to the computing equipment available. *Dialectic* mathematics invites contemplation. *Algorithmic* mathematics invites action. *Dialectic* mathematics generates insight. *Algorithmic* mathematics generates results.

In general, I interpret the mindset as the difference between mathematics and engineering, where in mathematics π is 3.14159265359... and in engineering, 3.14 or 3.14159 is good enough for practical purposes.

2.4 Variance from true North-South-East-West alignment

As is well known, the pyramids are well aligned with the cardinal points, but not with 100% accuracy. Using the published co-ordinates [4], we can calculate the slope of each side as in Table 1:

Name	Point 1	Point 2	Calculated °	Desired °	Difference °	Absolute Difference°
P1 top	P1 NW	P1 NE	0.0497	0	-0.0497	0.0497
P1 bottom	P1 SW	P1 SE	0.0497	0	-0.0497	0.0497
P1 right	P1 SE	P1 NE	90.0746	90	-0.0746	0.0746
P1 left	P1 SW	P1 NW	90.0746	90	-0.0746	0.0746
P2 top	P2 NW	P2 NE	0.0798	0	-0.0798	0.0798
P2 bottom	P2 SW	P2 SE	0.0798	0	-0.0798	0.0798
P2 right	P2 SE	P2 NE	90.1064	90	-0.1064	0.1064
P2 left	P2 SW	P2 NW	90.1064	90	-0.1064	0.1064
P3 top	P3 NW	P3 NE	-0.3255	0	0.3255	0.3255
P3 bottom	P3 SW	P3 SE	-0.2170	0	0.2170	0.2170
P3 right	P3 SE	P3 NE	89.7826	90	0.2174	0.2174
P3 left	P3 SW	P3 NW	89.7830	90	0.2170	0.2170

Table 1: Skewness of the pyramids

The slight skewness creates a problem for measuring angles ... do we measure along the side of the pyramid, ignoring the 8-side problem, or as per the Cartesian plane? In general, I have measured as per the Cartesian plane, when one of the edges was aligned to a side of a pyramid, and used relative angles when not aligned to a side. As shown in the table 1, the maximum error is around 0.3° for measurements involving the northern edge of Menkaure.

The Giza site is large, with the distance from P1 NE to P3 SW being 1,172.3m, and maintaining accuracy on a building site over that distance is very difficult.

2.5 Methodology: Diskerfery: The art of dividing circles

The general methodology is based on measuring angles between the pyramids, and some other points on the ground, which will present themselves in due course. The general idea was a result of watching the Nebra disk video[1], which shows a circle divided by the golden ratio.

The technique involves splitting a circle in a given ratio, and then using the resultant angles to measure between various points. The ratios are mostly irrational numbers like π, φ, e, √2, etc. This sounds a bit contrived, but once seen in action, it leads to an understanding of what the site builders were trying to convey.

The technique is also an elegant method of displaying knowledge of arithmetic and geometry at the same time.

We start with the golden ratio, phi. I have some shorthand notation so that we don't need to keep says "360 divided by."

Let ⦣ mean "360 divided by," so that ⦣φ means $\frac{360}{\varphi}$, measuring angles in the conventional Cartesian matter, with 0° to the right, and measuring anti-clockwise.

If we divide a circle by φ, as ⦣φ, we get the angles of 222.49° and 137.51°, which divide the circle as in Figure 1.

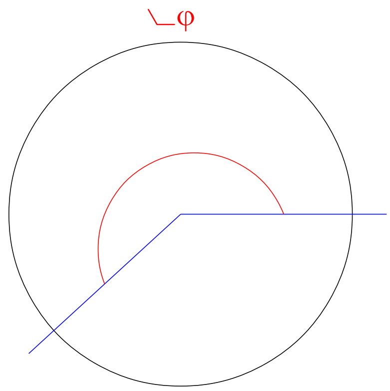


Figure 1: Dividing a circle in φ ratio

This division appears in various places in ancient artefacts, for example the Nebra disk as show in Figure 2.

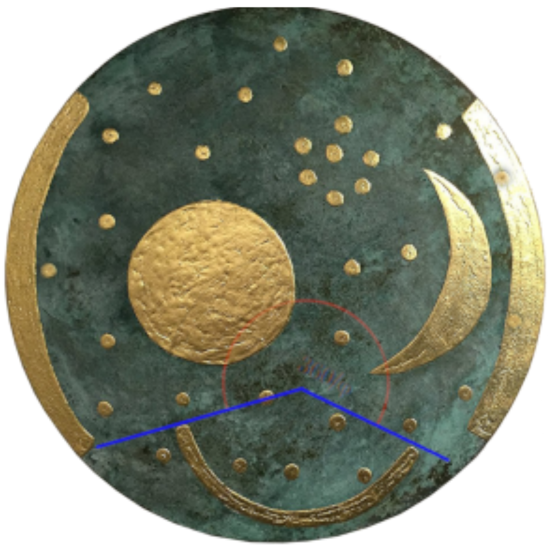


Figure 2: The Nebra disk divided in φ ratio

Or Stonehenge, as show with the original outer banks in Figure 3.

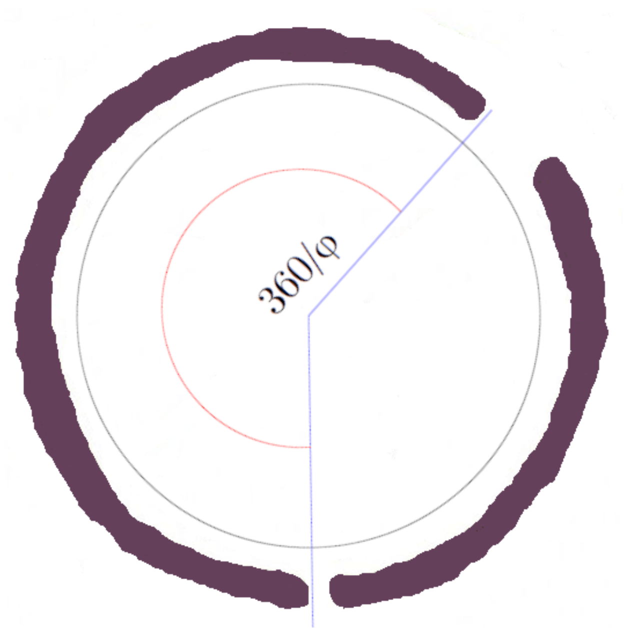


Figure 3: Original Stonehenge outer banks showing φ ratio

We could also measure from zero in the other direction, which we can indicate using ⟋‾φ instead of ⟍_φ. The last refinement we need at the moment is to specify a different origin to zero. For example, we can start measuring at 270°, or, if we were going the other way, at 90°, which would be the same starting point, as in Figure 4.

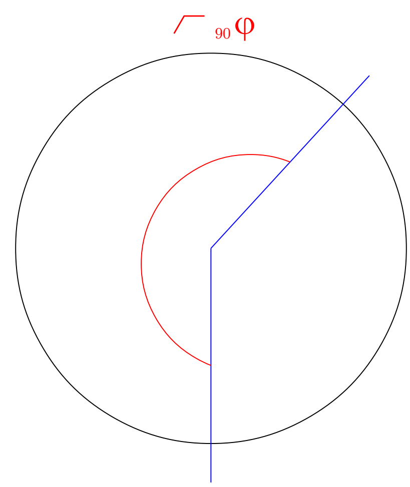


Figure 4: Dividing a circle in φ ratio, negative direction, non-zero start. ⟋‾$_{90}$φ

The φ angle occurs frequently at Giza, for example in these six places (there are more) in Figure 5.

In fact, the Giza architects used φ so often I have started thinking there is a whole section of mathematics revolving around φ that we have not rediscovered yet. It can't just be for aesthetically-pleasing design.

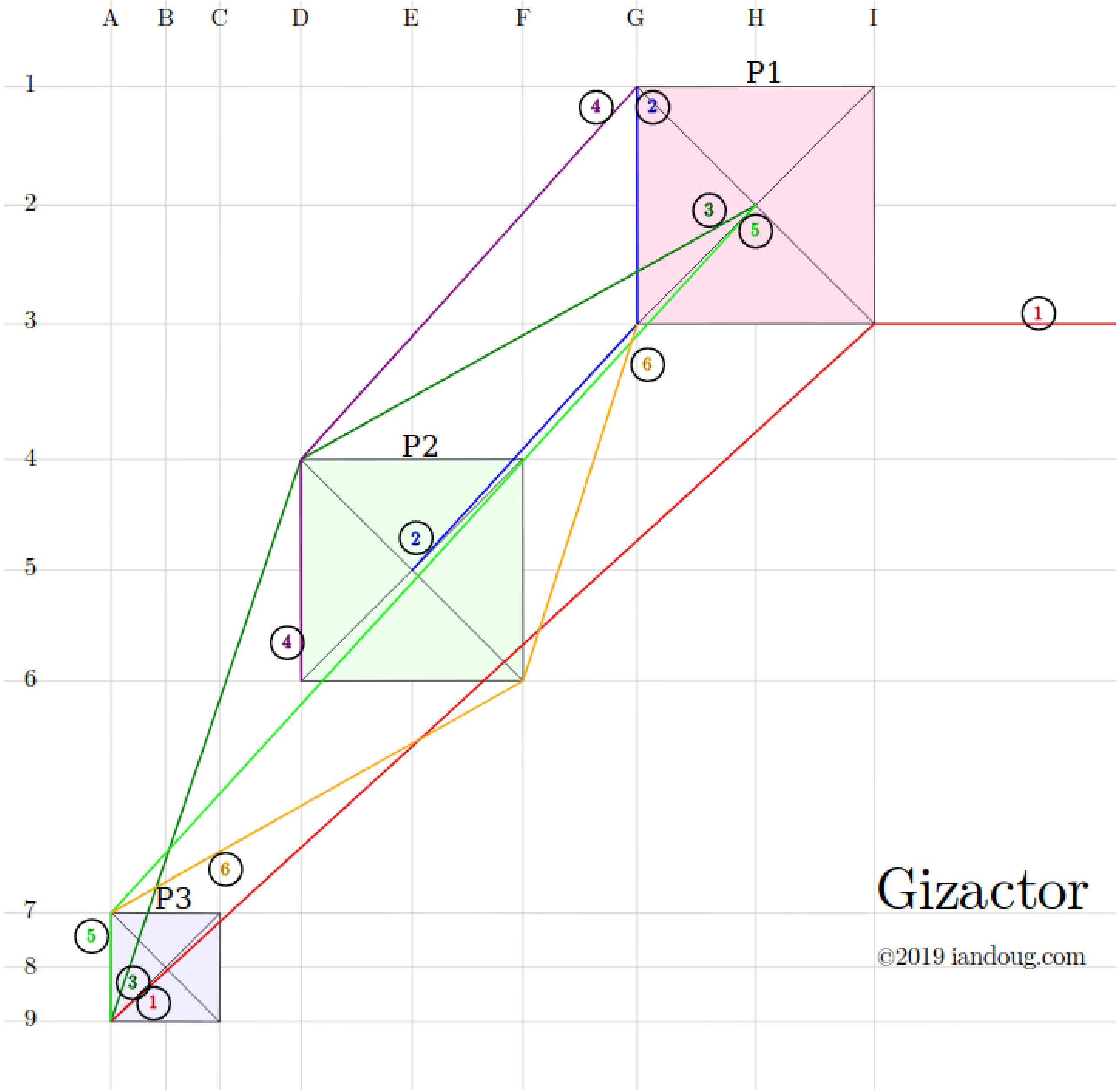


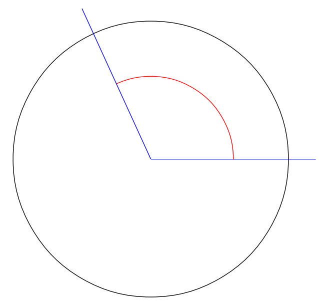
Figure 5: Various φ angles in Giza layout

Line	Points of Angle	Calculated °	Desired °	Absolute delta °
1	P3 SW : P1 SE : right	222.41	222.49	0.08
2	P1 NW : P1 SW : P2 C	222.55	222.49	0.06
3	P3 SW : P2 NW : P1 C	222.11	222.49	0.38
4	P2 SW : P2 NW : P1 NW	222.15	222.49	0.34

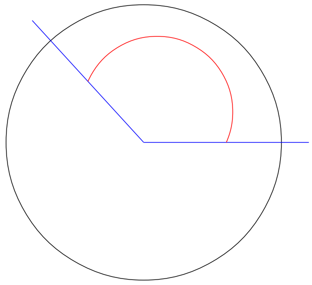
Line	Points of Angle	Calculated °	Desired °	Absolute delta °
5	P3 SW : P3 NW : P1 C	222.12	222.49	0.37
6	P3 NW : P2 SE : P1 SW	222.86	222.49	0.37

Table 2: Analysis of φ ratios in Figure 5

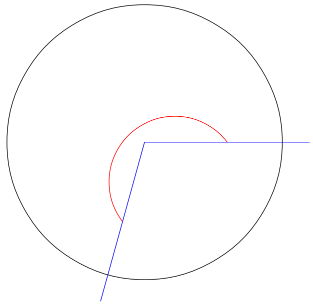
Similarly, we can also divide a circle by π, e, √2, √3, π/φ, πφe or other ratios. Table 3 has some examples to illustrate, other circle divisions used but not shown here work in the same way.



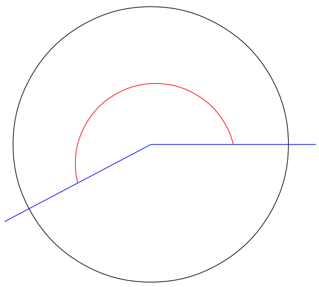
∟π = 114.59°



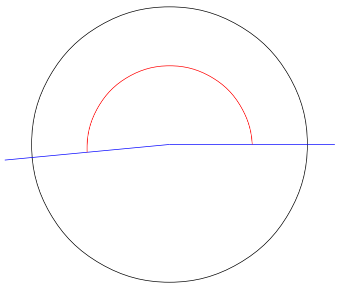
∟e = 132.44°



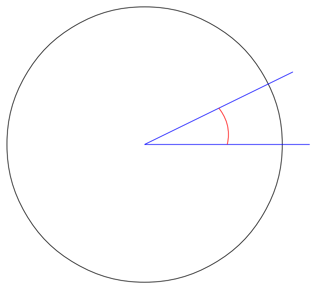
∟√2 = 254.56°



∟√3 = 207.85°



φ∟π = 185.41°



∟φπe = 26.05°

Table 3: Dividing a circle in various ratios

Note that ∟φ (222.492°) is close to ∟e + 90 (222.437°). If we put both on the same diagram back to back, they form an almost perfect 90° angle.

These angles and others are used in the analysis that follows.

3. Overview of existing geometric explanations

Numerous researchers have done mathematical analysis of Giza, and of Khufu in particular. I am only concerned with the site layout rather than Khufu, so we will only give an overview of selected investigations.

1. Samuel Laboy has an analysis of the site plan construction using rectangles and circles.[7]

2. Gary B. Meisner has thorough analysis of the site plan and the relationship to φ.[8]

3. J.A.R Legon has a geometric analysis with a lot of calculations.[9]

4. Edward Nightingale has another geometric analysis, using circles and φ spirals, based on a centre location equivalent to my grid reference G5.[10]

5. R.J Cook presents a geometric analysis, extended to Khufu's internals[11]

6. R.J Cook also has a trans-generational analysis of Giza[12]

7. Scott Creighton presents a theory for the layout of the three pyramids based on the centroids of carefully constructed triangles.[13]

8. Eckart D. Schmitz has an algebraic and geometric analysis of the Great Pyramid and Giza.[14]

9. Richard C. Mercier has an analysis based on pentacles.[15]

10. Douglas M. Keenan maps the Giza layout to the planets Venus, Earth and Mars.[16]

11. H.P.M. Klaassen has an analysis based on circles, squares, pentagrams and hexagrams, which links to the precessional cycle. [17]

12. Jiří Mrůzek has detailed instructions for drawing the Giza plan from scratch. [18]

13. Jim Alison has an analysis of both the site plan and Great Pyramid.[19]

14. Hans Jelitto has an analysis linking the three pyramids, and internal structure of Khufu, to the three inner planets. [20]

All of these analyses are only for the three extant pyramids, sometimes including the Sphinx and minor pyramids.

Astronomical alignments are covered in the companion paper.

4. The geometry behind the layout

4.1 Existing diskerfery alignments between the first three pyramids

The Holy Grail of alignment theories has always been to explain the off-linear alignment of the three centres. Before we get to that, let us first examine a few other relations between the pyramids, where the site planners display their great skill. They were, for example, able to simultaneously satisfy multiple alignment conditions, as in these linkages between P1, P2 and P3, using π, φ, √2, √3 and 7 in Figure 6.

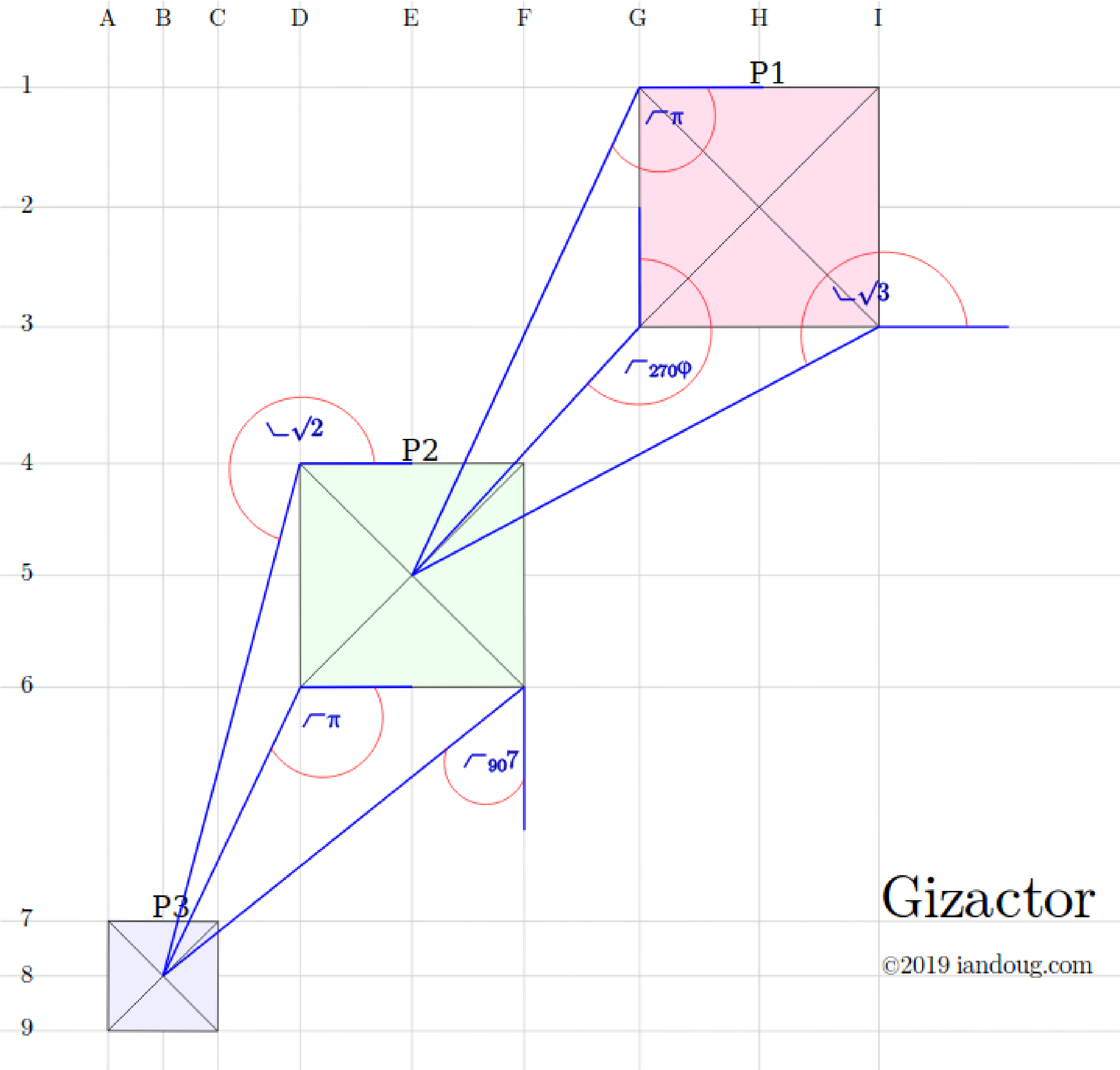


Figure 6: π, φ, √2, √3 and 7 in Giza layout

Source	Target	Angle	Calculated °	Desired °	Absolute delta °
P1 NW	P2 C	⟋‾π	114.97	114.59	0.38
P2 SW	P3 C	⟋‾π	115.42	114.59	0.83
P1 SE	P2 C	⟍_$\sqrt{3}$	208.04	207.85	0.19
P2 NW	P3 C	⟍_$\sqrt{2}$	254.52	254.56	0.04
P1 SW	P2 C	⟋‾$_{270}\varphi$	222.55	222.49	0.06
P2 SE	P3 C	⟋‾$_{90}7$	51.42	51.43	0.01

Table 4: Analysis of mathematical linkages in Figure 6

These angles are a function of the size of each pyramid and so do not explain the alignment of the centres relative to each other.

Note that the π angle between P2 and P3 is slightly off, which may reflect issues with the co-ordinates, or because other alignments were considered more important. A more interesting symmetrical alignment follows later.

It may seem odd to see 7 on the list with things like π and $\sqrt{2}$, but it shows an ability to create the heptagon.

The above is an example of the site planners repeating themselves to make a point, something they did on several occasions, not just here. For example:

“The connection here from P1 to P2 is related to π.”

“That’s just a fluke.”

“Well, it is also here between P2 and P3.”

“Mm....”

Or...

“The connection here from P2 to P3 is related to $\sqrt{2}$.”

“That’s just a fluke.”

“Would you like $\sqrt{3}$ with that?”

4.2 Analysis of straight-line alignments between the first three pyramids

There are straight-line alignments between the three pyramids, as shown in Figure 7:

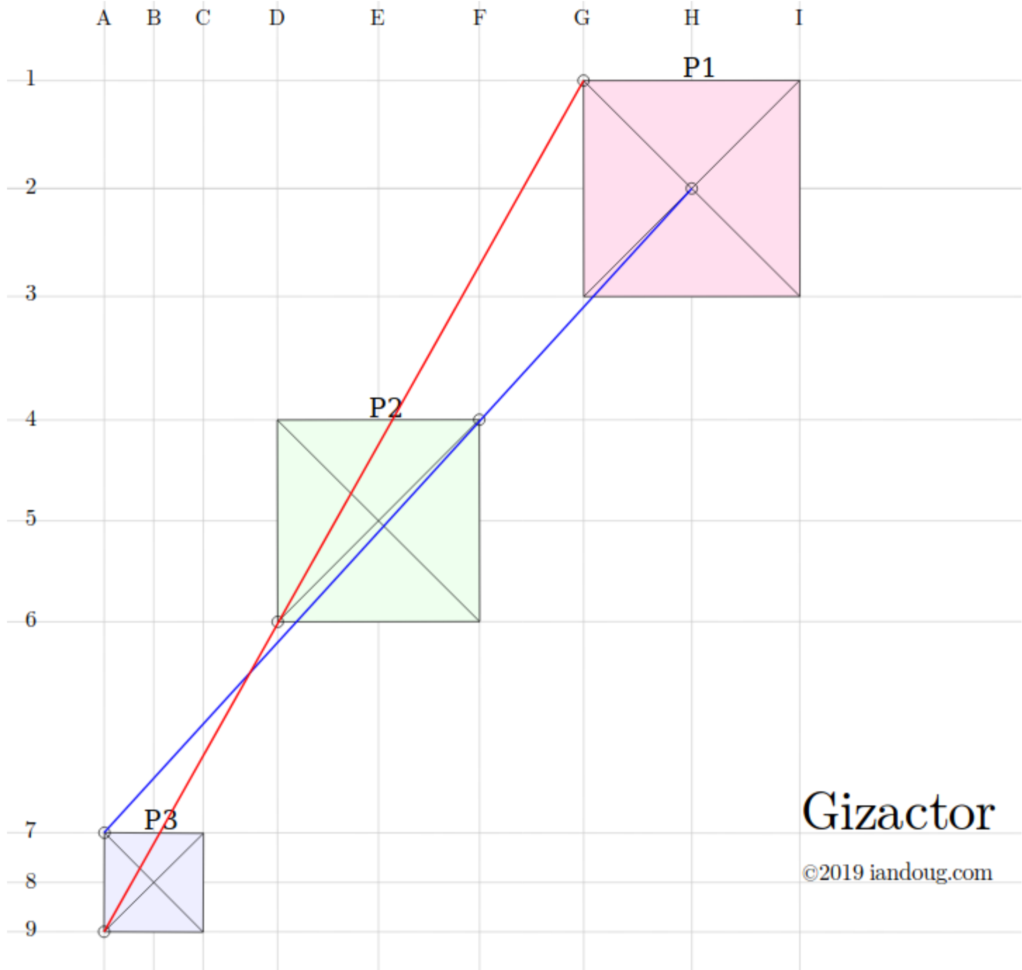


Figure 7: Two straight-line connections between the three pyramids

These straight lines are at important angles. The blue line from P3 NW to P1 C is at an angle of $\angle_{180}e$, as in Figure 8:

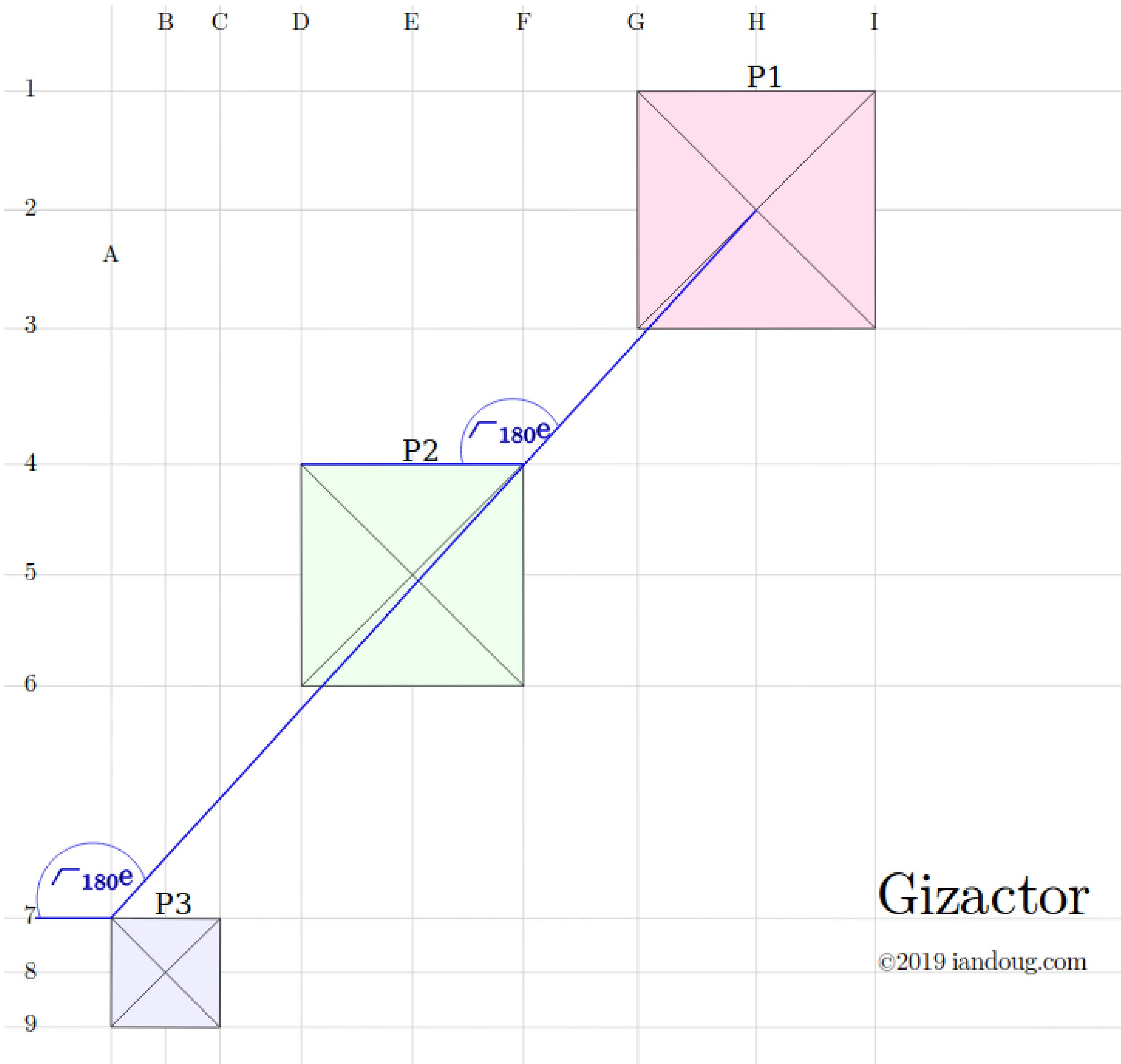


Figure 8: ⦣$_{180}$e angle between the pyramids

Source	Target	Calculated °	Desired °	Absolute delta °
P3 NW	P2 NE	132.21	132.44	0.23
P2 NE	P1 C	132.57	132.44	0.13
P3 NW	P1 C	132.34	132.44	0.10

Table 5: Analysis of ⦣$_{180}$ angle in Figure 8

Given the relationship between ⦣e and ⦣φ, we could have measured off the vertical instead (90° difference), but we shall use ⦣e because of what comes next.

The red line from P3 SW to P1 NW is more interesting, because it maps to ⦣(e-1).
The ratio (e-1):1 is the G:foot ratio, which we shall return to later.

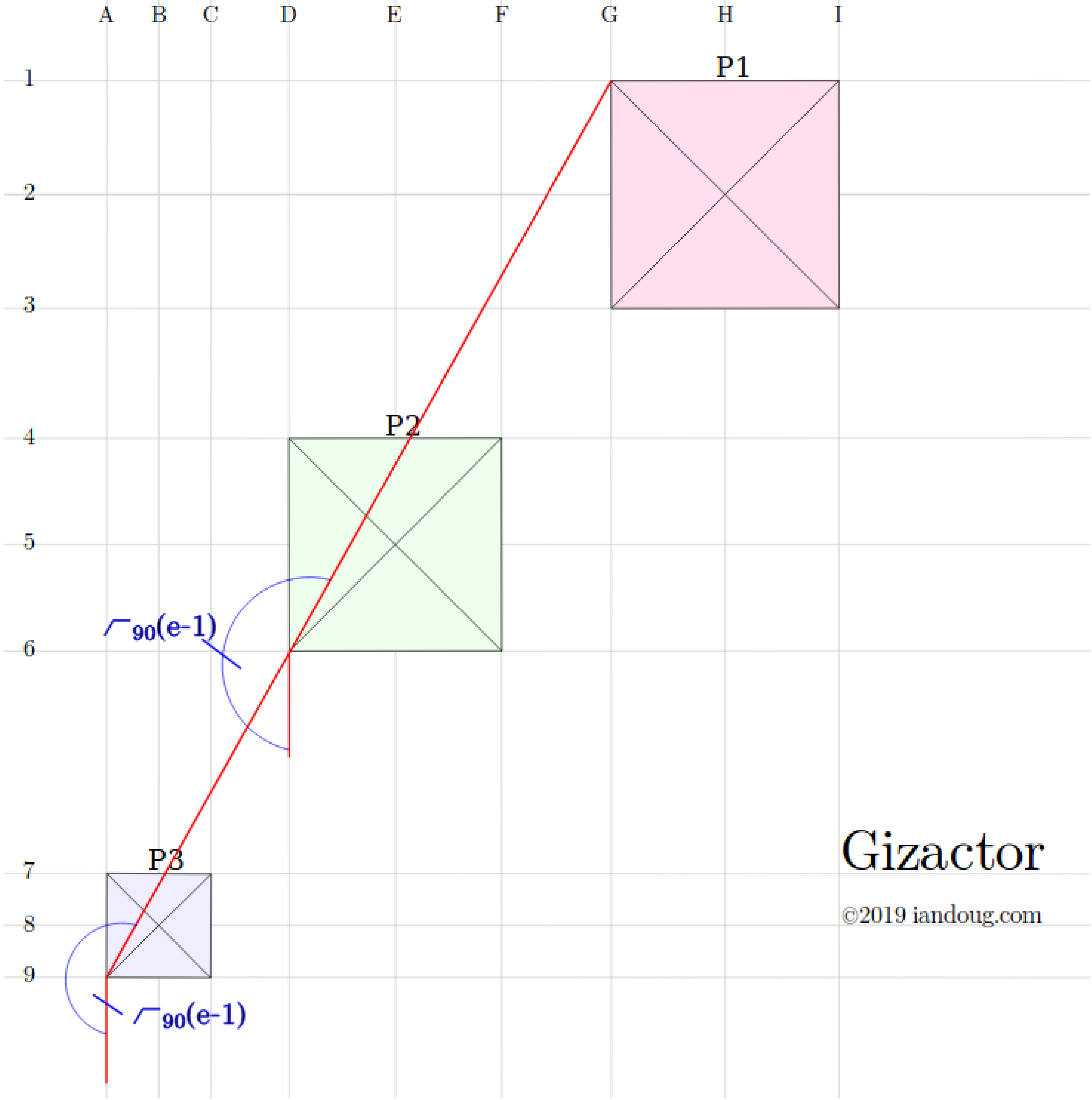


Figure 9: ⦣$_{90}$(e-1) *straight line between the pyramids*

Source	Target	Calculated °	Desired °	Absolute delta °
P3 SW	P2 SW	209.25	209.51	0.26
P2 SW	P1 NW	209.45	209.51	0.06
P3 SW	P1 NW	209.38	209.51	0.13

Table 6: Analysis of ⦣$_{90}$(e-1) *angles in Figure 9*

4.3 The key to the alignment of the three main pyramids

However, those straight lines do not link the three centres as we would like.

The key to **that** alignment is the ratio π/φ.

If we split the circle as ∟(π/φ), that equals φ∟π, which is 185.41°.

It may be easier to visualise it as in Figure 9a.

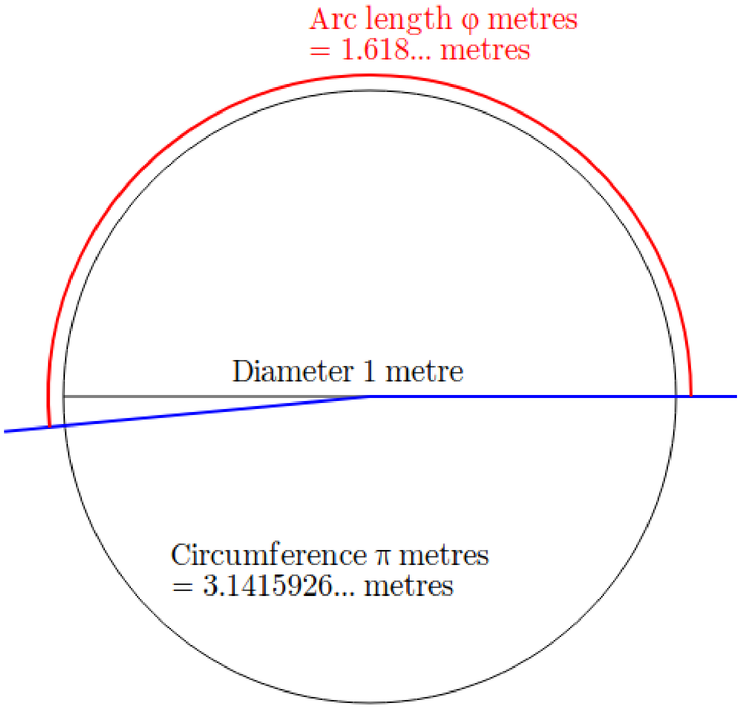


Figure 9a: The π/φ angle, visualised.

This allows us to find the angle between P1 and P3 as in Figure 10. The 46.7° comes from the fact that the diagonal between P2C and P1C is not at 45° but at 46.6974°.

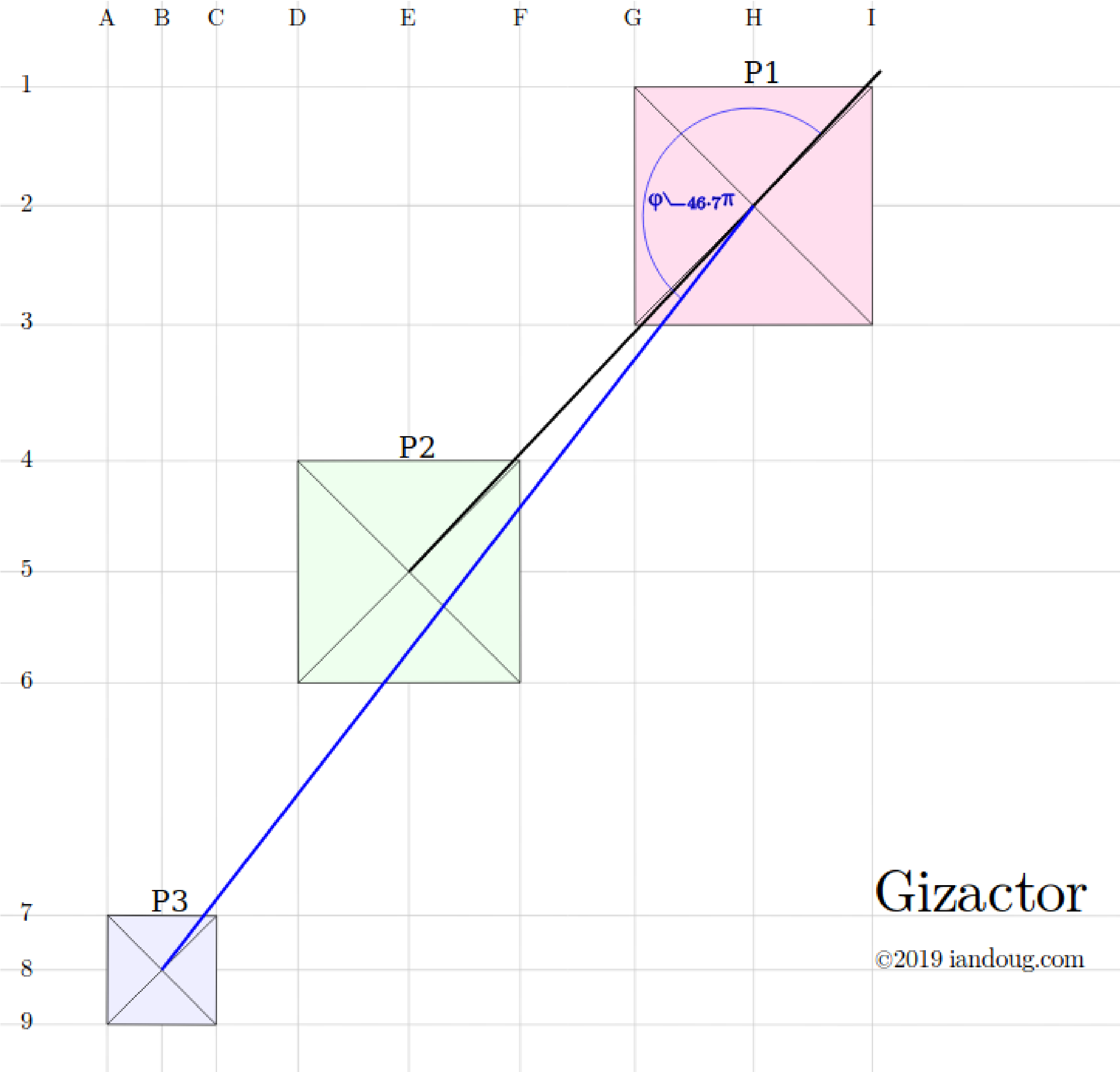


Figure 10: $\varphi\llcorner_{46\cdot7}\pi$ angle shows the offset between P2C and P3C

Angle	Calculated °	Desired °	Absolute delta °
P2 C –> P1 C extended, P3 C	185.53	185.41	0.12

Table 7: Analysis of $\varphi\llcorner_{46\cdot7}\pi$ angle in Figure 10

The $\varphi\llcorner\pi$ angle also explains the deviation between the diagonal of P1, and the furthest corner of P3. An alternative way of looking at this is that the line defines the size of P3 by the intersection of the 45° diagonals from P3C, and the $\varphi\llcorner\pi$ angle of P1 diagonal, as shown in Figure 11.

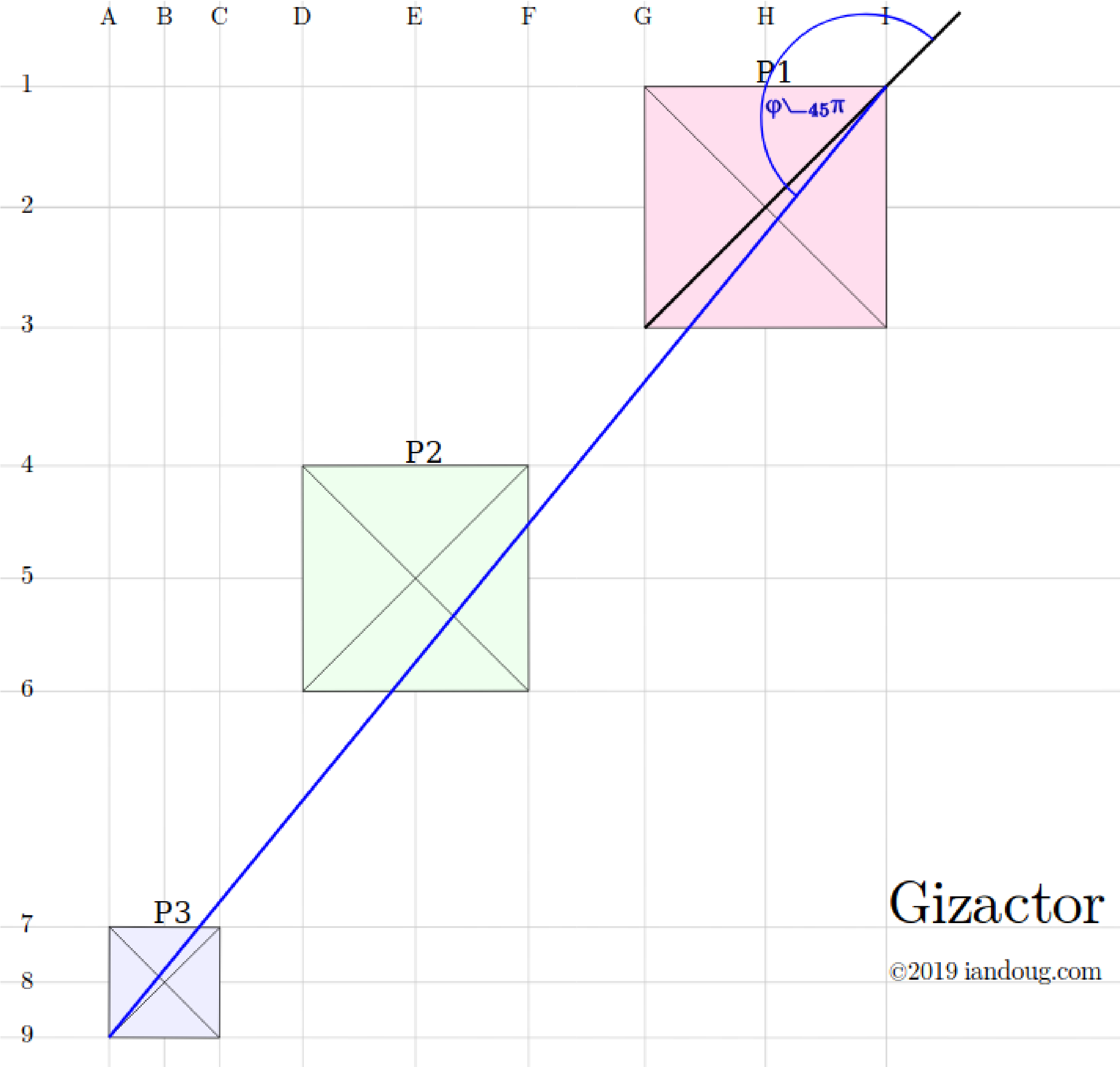


Figure 11: $\varphi\llcorner_{45}\pi$ angle shows the offset between P1 diagonal and P3SW

Angle	Calculated °	Desired °	Absolute delta °
P1 SW –> P1 NE extended, P3 SW	185.70	185.41	0.29

Table 8: Analysis of $\varphi\llcorner_{46.7}\pi$ angle in Figure 11

These mathematical alignments, and the ones that follow, are an elegant mathematical explanation for the arrangement of the pyramid centres.

We now introduce the ∟πφe angle, tentatively labelled as the "pyramid angle", for lack of a better name, and as an easy way to refer to it. The angle is 26.054°, which is close to

26.57°, one of the angles resulting from drawing the diagonal in half a square. Compare that to the angle of the descending passage in Khufu, given as 26.52°. [21]

There is a nice symmetrical arrangement with ∟πφe as shown in Figure 12:

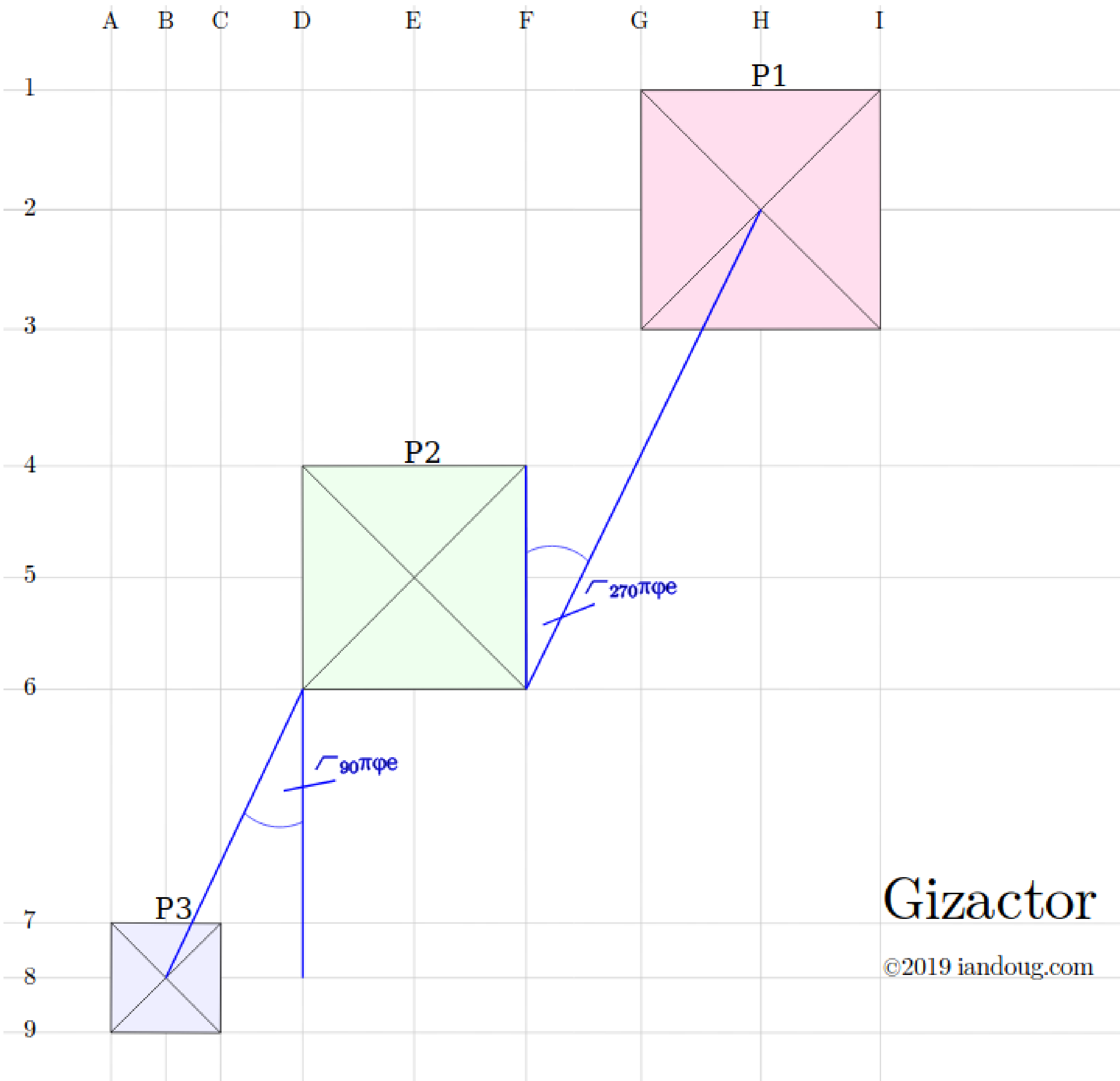


Figure 12: Symmetrical πφe angles in Giza plan

Source	Target	Calculated °	Desired °	Absolute delta °
P2 SW	P3 C	25.88	26.05	0.17
P2 SE	P1 C	26.08	26.05	0.03

Table 9: Analysis of symmetrical πφe angles in Figure 12

The occurrence of multiple simultaneous alignments, some symmetrical, using famous mathematical numbers (π, φ, e, $\sqrt{2}$, $\sqrt{3}$, and products and ratios of these) can not be ascribed to "chance." They must be seen as evidence of intelligent and deliberate design, and a demonstration of mathematical knowledge and skill.

For many more such mathematical alignments, see [22]

5. The Fourth Pyramid

Frederic Ludvig Norden (Travels in Egypt and Nubia. 1755/7) provided the following relevant descriptions of the pyramids:

> "*The two most northerly pyramids are the greatest, and have five hundred feet perpendicular height. The two others have much less, but have some particularities which occasion their being examined and admired.*
>
> *"The three other great pyramids, as I have already remarked above, are situated almost on the same line as the preceding (i.e. the Great Pyramid) and may be about five or six hundred paces one from another.*
>
> *"The third pyramid is not so high as the two former, by an hundred feet; but in other respects it resembles them entirely as to the structure.*
>
> *"As to the fourth pyramid, it is still one hundred feet less than the third. It is likewise without coating, closed, and resembles the others, but without any temple like the first. It has however one particular deserving remark; which is, that its summit is terminated by a single great stone, which seems to have served as a pedestal. It is, moreover, situated out of the line of the others, being a little more to the west.*
>
> *[After critiquing classical authors and their lack of mention of the 4th pyramid, he directs some comments at earlier explorer Greave and his "Pyramidographia," and the confusion between the 4th pyramid and the satellite pyramids at Menkaure.]*
>
> *"If our learned author had taken the trouble to go near it, he would have seen, that the fourth pyramid as been made, towards the middle, of a stone more black than the common granite, and at least as hard. I dare not, however, ascertain, that is is the basaltis; for it differs from the material of which the beautiful vase is made, that I have seen in Rome, in the palace of the cardinal Alexander Albani, and which they give out for the basalto.*

> *"The stones, that are wanting to this pyramid lye upon the ground, at the north east corner. They there make a very great heap. Mr. Greaves, however, is in some measure excusable, for not having observed this pyramid. It is situated in such a manner, that, if you do not see it at a certain distance, you do not easily perceive it, even though you are near, because the others conceal it. Its summit is of a yellowish stone, and of the quality of that of Portland; and it is likewise the same kind of stone, that the other pyramids are built with. I shall speak elsewhere of its top, which terminates in a cube.*
>
> *"The existence of this fourth pyramid is, moreover, indubitable. It makes a series with the three others; this is a matter I can aver. My lord Sandwich has very justly observed it, and my designs (i.e. illustrations) attest the same truth."*

I have been unable to find where he speaks further of this pyramid.

There is a ruin of a wall or compound next to Menkaure, and Tony Bushby [23] sites the fourth pyramid there. He also states that British Freemasons dismantled the pyramid in 1759, looking for objects said to be hidden inside it. Note this diagram is orientated with North at the top.

Figure 13: Menkaure and empty plot. Image credit: Google Earth

Norden supplied 3 different diagrams showing the four pyramids, of which the 2D/3D map view is the most useful. This diagram (Figure 14), which is orientated with West at the top, clearly does not suggest the above location for the fourth pyramid.

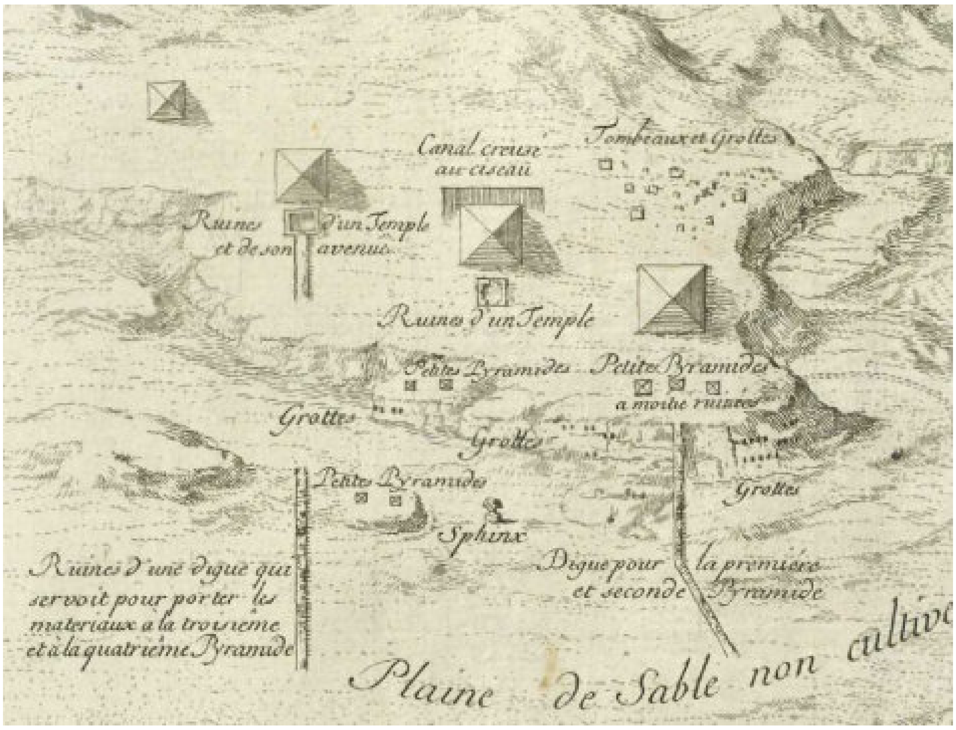


Figure 14: Norden's sketch of the Giza plateau (French version)

Examining the area with Google Earth suggested another area, which looked like it had a square outline. Comparison with the star map showed that it wasn't bad, but not ideal either.

My guides (see appendix) then came to the rescue, and suggested that probably the designers used the same φ⦣ π angle as in the alignment between the first three pyramids, but measured in the negative direction as φ/‾ π. So I drew that line in.

Then I accepted that there would be at least one ⦣π or ⦣φ alignment as well, so tried some, and found one, which put the location in a better spot. The co-ordinates of the intersection were calculated mathematically using the known starting points, and angle of the lines.

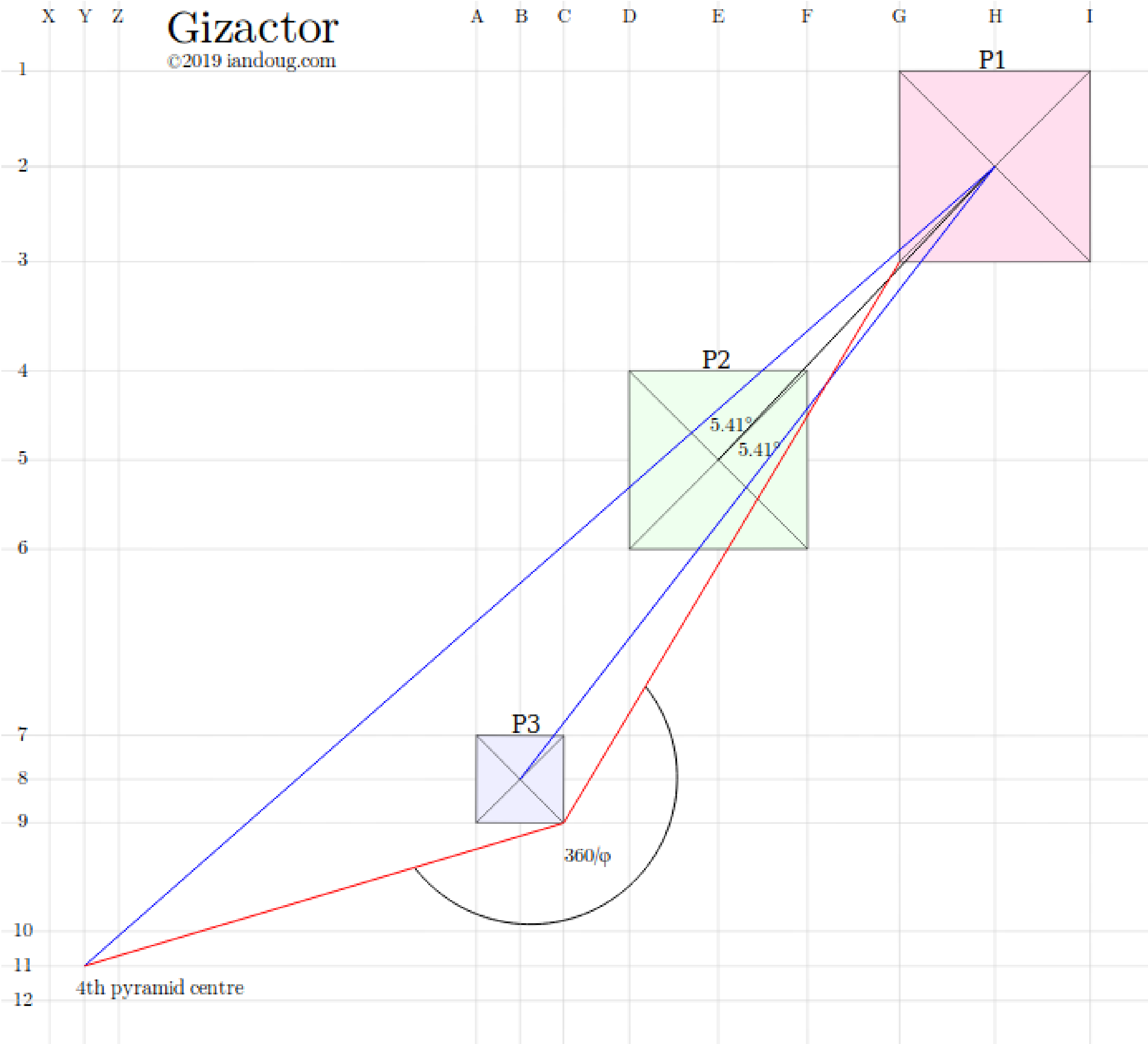


Figure 15: Locating the possible centre of P4

However, the co-ordinates we have for P3 are not perfect, so some fine-tuning is necessary.

Alan Green [18] points out that the ratios between foot, cubit and metre can be computed from various measurements in and of the Khufu pyramid.

Length	Value m	Formula	Value Ͼ	Ratios as m	Abs delta m
A foot	0.3048	Half base + height	500.000	**A/C** = 0.3052	0.0004
B royal cubit	0.5236	Base + corner slope	858.569	**B/C** = 0.5240	0.0004
C metre	1.000	Diagonal + base + half base + side slope	1638.344	**C/C** = 1.0000	

Table 9: Metre, cubit, foot ratios in Khufu

0.0004m is less than the average diameter of a grain of sand.

Green also points out that the cubit/foot ratio is 1.718, which maps to (e-1) correctly to 3 decimal places.

If we compare the east-west distances between the centres of P1, P2 and P3, we notice something interesting.

East - West distance	**Value m**
P1 C to P2 C	334
P1 C to P3 C	573.5
ratio (P1 C to P3 C) / (P1 C to P2 C)	1.7171
ratio ₢/foot	1.7178

Table 10: Analysis of east-west distances between the pyramids

This strongly hints at a foot-cubit relationship in the east-west distances between the pyramids.

If we simply extend the logic to the metre, we can reverse-calculate what a metre distance would be:

East - West distance	**Value m**
P1 C to P2 C, 334m, scaled foot to metre	1095.8
P1 C to P3 C, 573.5m, scaled ₢ to metre	1095.3
P1 C to P4 C distance calculated above	1099.8

Table 11: Comparing theoretical East-West location of P4 to provisional calculation

So it does indeed appear that the pyramids were in a metre-cubit-foot ratio distance apart.

We need to move P4 C between 4 and 4.5m east. I decided on 4m, the difference between the two options is just on one ₢, but the measurements for Menkaure are not as good as for Khafre, so I decided to go with the Khafre measurement. I calculated the intersection of a North-South line from the correct East-West location, and the φ⁄‾ π line from P1 C. This is then the centre of P4.

To get the size of the base, I again followed the same technique using the φ⁄‾π angle, this time aligning it along the NE-SW diagonal of P1 instead of between the centres of P1 and P2. Where this line intersected the 45° line from the identified P4 centre, became the top left corner. This is shown in Figure 16.

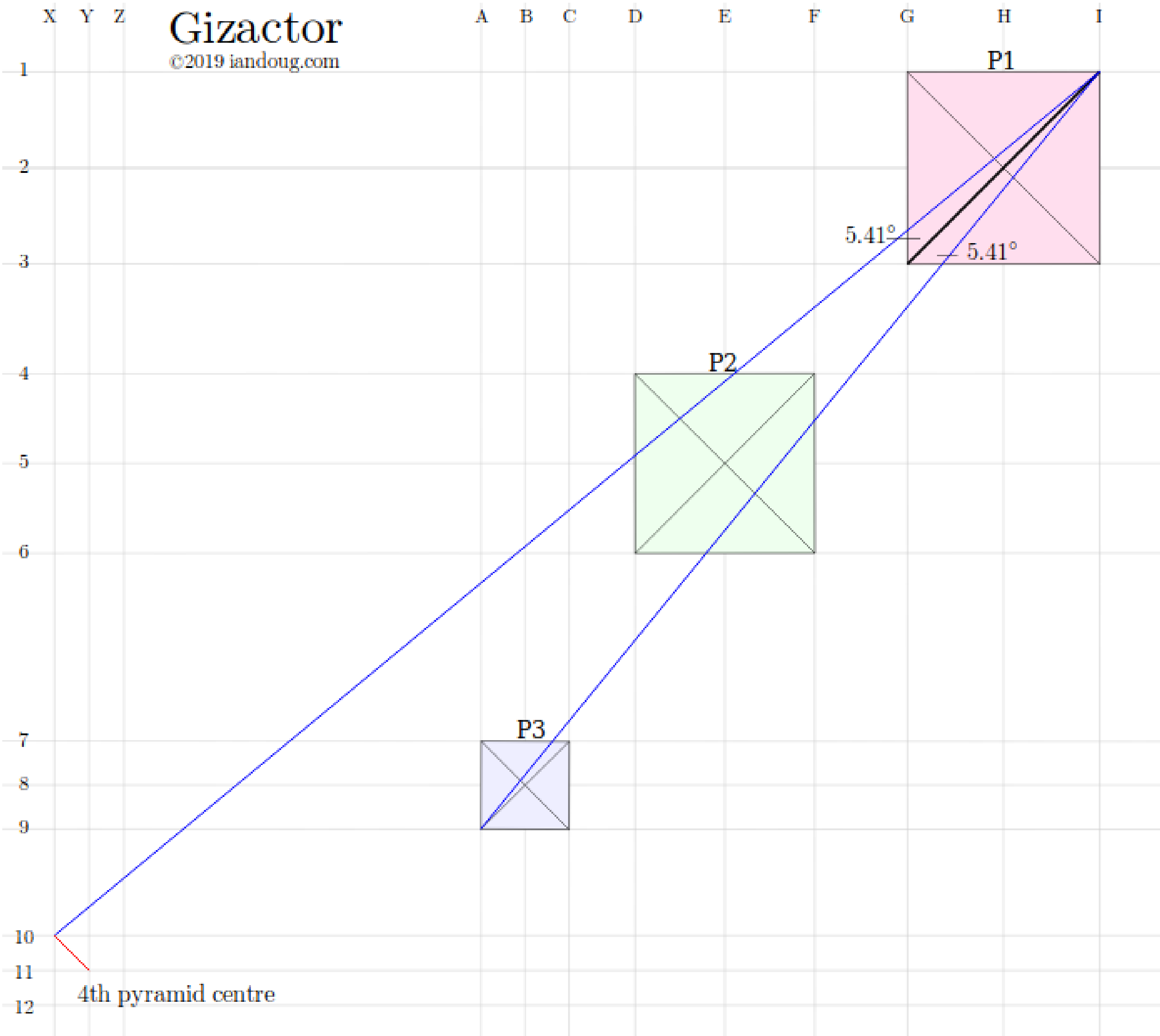


Figure 16: Calculating the NW corner of P4 after calculating correct centre.

From this we could determine the other three corners. The corner co-ordinates were calculated as before using basic trigonometry.

The actual spot might be a little to the south, where there is a faint outline of a square as shown in Figure 17. However, that square is too small to be the calculated size.

Figure 17: Calculated vs alternate possible location of 4th pyramid. Image credit: edited from Google Earth.

In another example of "do it twice," there is a similar relationship in the North - South distances between the pyramid centres.

North - South distance	Value m	Ratio	Value as m	Maps to
A P1 C to P2 C	354.4	**A/B**	0.48	1 metre - 1 ℭ
B P1 C to P3 C	740.1	**B/B**	1.0000	1 metre
C P1 C to P4 C	965.7	**C/B**	1.3048	1 metre + 1 foot
D P2 C to P4 C	611.3	**D/B**	0.83	1 ℭ + 1 foot

Table 12: Analysis of North-South distances between pyramid centres

The accuracy here for ratios A and B is only good to 2 decimal places, while C is correct to 4 places. Ratio D maps to the purported Megalithic Yard [24], I feel that ℭ + foot is a simpler way of getting it than that proposed by the originators.

We can now show the genius that went into planning Giza, in the metre-cubit-foot ratio relationships, noting that the North-South and East-West ratios are from different metre bases. Figure 18 should probably come with some sort of warning.

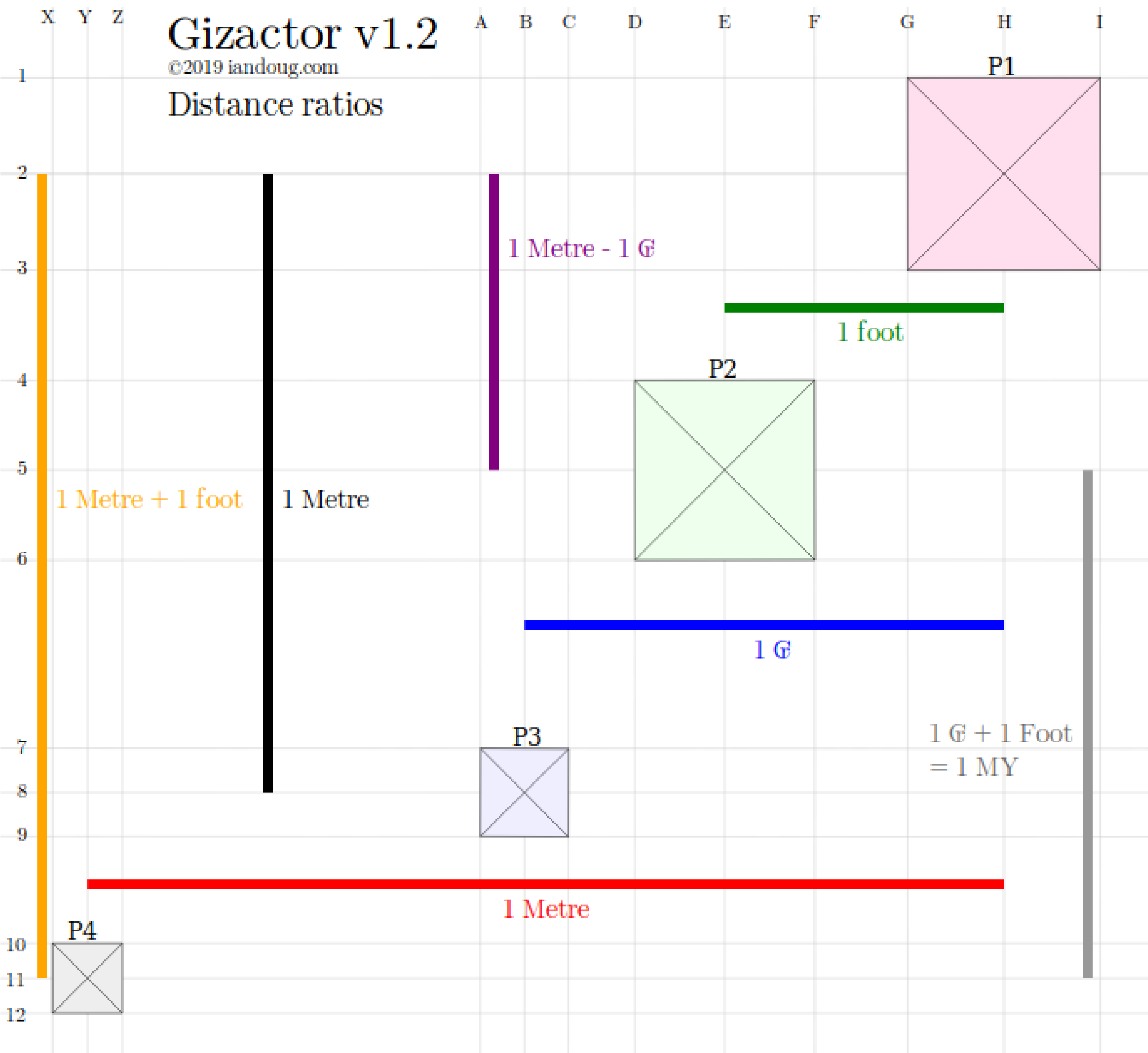


Figure 18: The Metre-Cubit-Foot relationships between the four pyramids. Metre scale is different for North-South and East-West. It is the relative ratios that are important.

These results act as confirmation for the centre of P4. Further confirmation for the location and size of P4 comes from the stellar alignment, as well as some other diskerfery angles. For example, Figures 19 and 20 with ⦜φ:

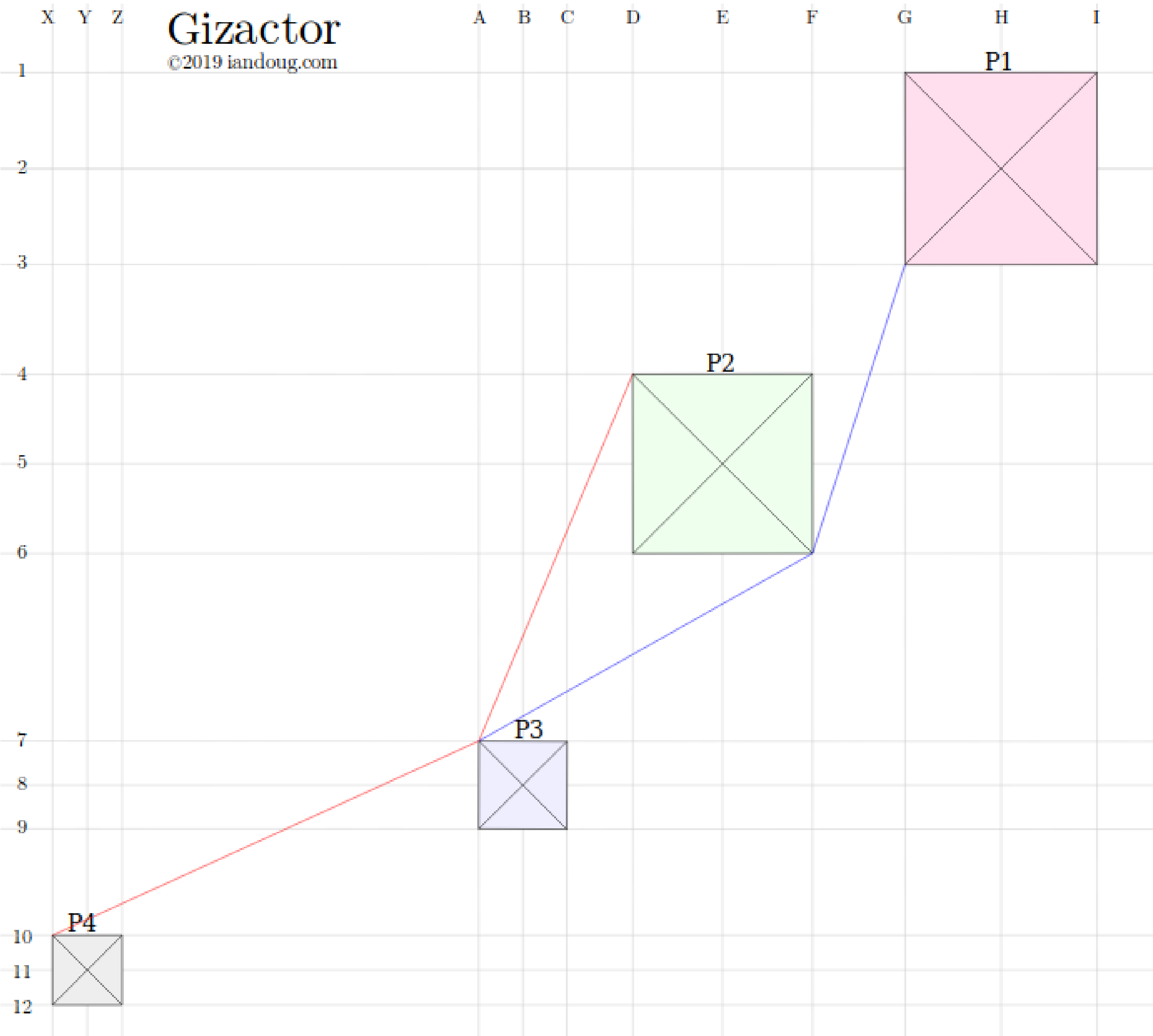


Figure 19: Balanced φ angles in Giza layout

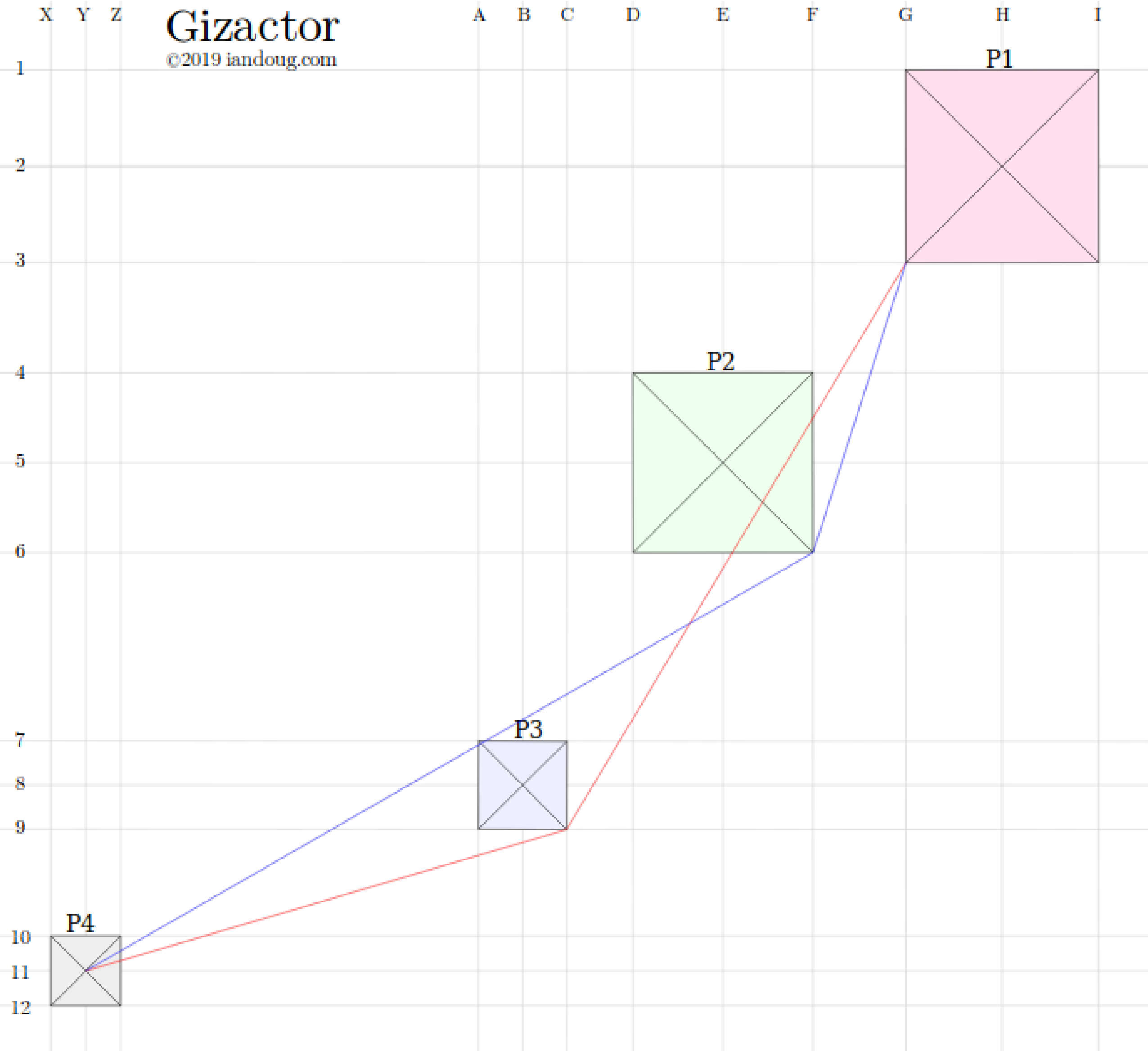


Figure 20: Further balanced φ angles in Giza layout

Angle	Calculated °	Desired °	Absolute delta °
P4 NW : P3 NW : P2 NW	222.760	222.492	0.268
P3 NW : P2 SE : P1 SW	222.863	222.492	0.371
P4 C : P3 SE : P1 SW	222.704	222.492	0.212
P4 C : P2 SE : P1 SW	222.334	222.492	0.158

Table 13: Analysis of φ angles in Figures 19 and 20

There are multiple other alignments with $\sqrt{\varphi}$, as well as other constants.

The centre location of P4 is at about 29.970399° N, 31.122788° E. This was done visually on Google Earth using a transparent printout.

The co-ordinates on Glen Dash's system are in table 14.

Point	**X**	**Y**
North West	498862.458	99079.587
North East	498945.942	99079.587
South West	498862.458	98996.103
South East	498945.942	98996.103
Centre	498904.200	99037.845

Table 14: Co-ordinates for P4 on Glen Dash's system

Due to everything being calculated, all sides are equal and orientated with the cardinal directions.

We can compare Norden's sketch with the result:

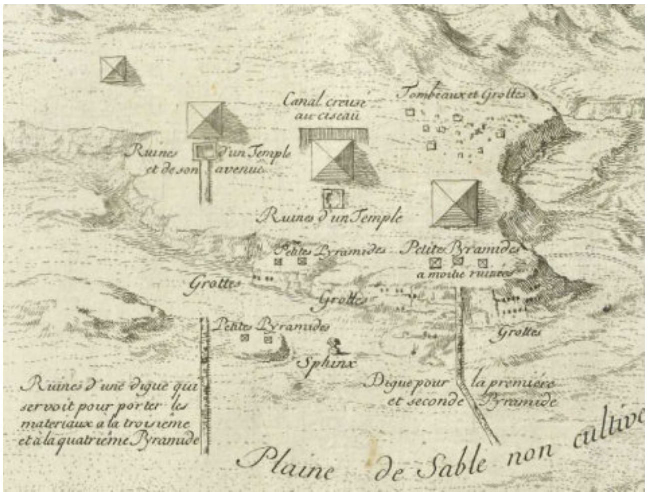


Figure 21: Norden's 2D/3D sketch of Giza

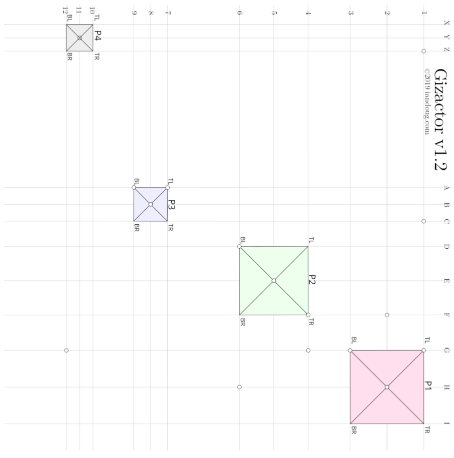


Figure 22: SVG plot of Giza with four pyramids

Norden clearly got the East-West distances wrong in his sketch, making the west edge of P1 almost in the same line as the east edge of P2, and similarly with P2 - P3. So we need to take that into account. He did put in a clear east-west gap between P3 and P4.

The base size calculates out at 83.484 metres, which converts to 159.443 ₲, so I accept that the design size was a more likely base of 160 ₲. In Norden's drawing, the fourth pyramid side is about ¾ of Menkaure's, so 160 versus 202 is not too bad.

Determining the height

Norden provided height estimates in feet for the four pyramids, summarised in Table 15:

Pyramid	Feet	Royal cubits (rounded)	Actual Royal cubits
Khufu	500	291	280
Khafre	500	291	274
Menkaure	400	233	125
Fourth	300	175	? 100 ?

Table 15: Norden's estimates of the heights of the four pyramids

Norden clearly made reasonable estimates for the first two, but almost doubled the actual height of Menkaure. Thus, the estimate for the Fourth pyramid is also likely incorrect.

For the height, we can look for clues from the other pyramids. "Equivalent" should be read in terms of "as close as you're going to get when using whole cubit dimensions." I take Menkaure at 202 ₢ rather than the normally-quoted 200 ₢, based on Glen Dash's co-ordinates. Using 202 gives a better figure of φ as well.

Pyramid	Base ₢	2 x base ₢	Height ₢	2x base / height	Equivalent	Base/ height	Equivalent	Slope °
Khufu	440	880	280	3.1429	π			51.8428
Khafre	411	822	274	3	3	1.5	1.5	53.13
Menkaure	202	404	125			1.616	φ	51.34
Fourth	160	320	99 ?			1.61616	φ	51.059
Fourth	160	320	100 ?			1.6	φ	51.34
Fourth	160	320	102 ?	3.1373	π			51.892

Table 16: Comparisons of bases and heights for the four pyramids

So it was likely to have been either 99, 100 or 102 ₢. Given the angles between P3 and P4 centre shown further below, I would pick 99 ₢.

On the other hand, given that Norden describes the top as being a great stone, which looks like a pedestal, the pyramid may in fact have had three "heights." For example, the base pyramid itself, say up to 99 ₢, then a 1₢ solid pedestal (now 100 ₢), topped by a 2 ₢ pyramidion, for a total of 102 ₢.

Note that if it had been 102, the average of Khufu and Fourth π ratios is 3.140056022.

With four points, we can plot a curve. The equation

$$y=\left(\frac{1000\,\pi}{e}-\left(\frac{\frac{1000\,\pi}{e}}{1+\left(\frac{x}{27.322^2}\right)^{(\pi+1)}}\right)\right)$$

Gives a reasonable fit.

27.322 is the number of days in a lunar sidereal or tropical month, to three decimal places.

The 1000π/e is basically just to scale it correctly. See the next formula below.

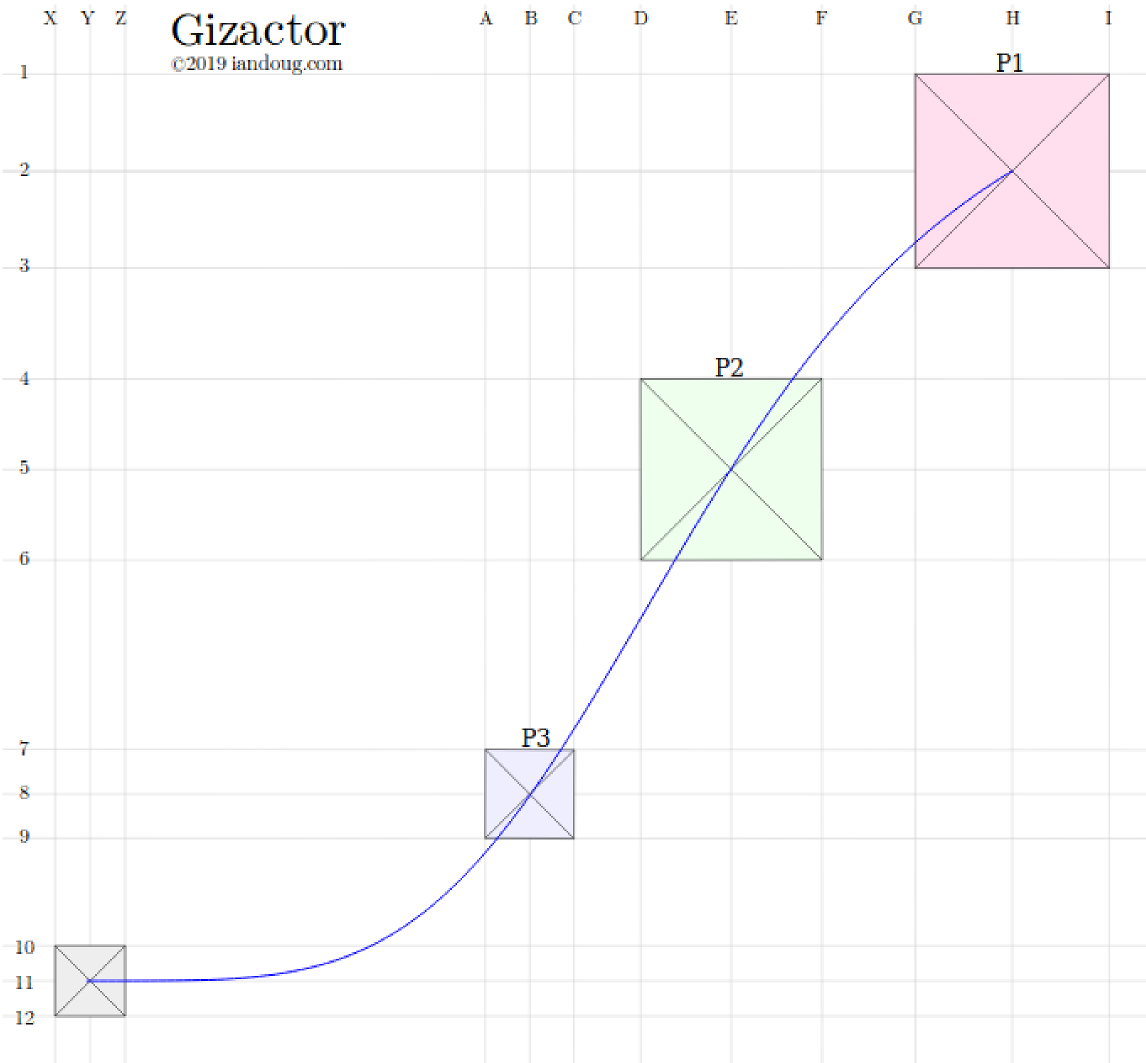


Figure 23: Curve fitted to the four Giza pyramids

The exact formula as calculated by http://mycurvefit.com when fed the pyramid centre locations on the SVG drawing, was

$$y=\left(1164.32+\left(\frac{6.040874\cdot 10^{-14}-1164.32}{1+\left(\frac{x}{746.6309}\right)^{4.079892}}\right)\right)$$

This assumes that the centre points of the pyramids are 100% correct, which is not necessarily true. My equation is a simplified version using more logical numbers. The 1164 part is related to scaling since P4 is around that number of pixels vertically from the top in the SVG diagram.

I used a program to generate the (x,y) points for the curve, which were then added to the SVG diagram. The relevant piece of code is in Listing 1. It works backwards because of the way SVG co-ordinates work. The x and y points were moved appropriately to fit the centres.

```
for ($x = 1100; $x >=0; $x=$x-5)  # for each 5 pixels, from P1 to P4
     {
       $y = (1000 * $pi / $e) - ((1000 * $pi / $e) / (1 +
($x / (27.322**2))**($pi+1)));
       $x1 = $x + 100;
       $y1 = -1 * ($y - 1162.2);
       fwrite ($fpout, "$x1,$y1 ");
   }
```

Listing 1: Code snippet to generate curve

P4 C is actually at (104.2, 1162.2) so there is a slight under-adjustment of 4 pixels in the x direction to get the fit.

We can now add the fourth pyramid to the “chain”, and highlight the angles between P3 and P4 centre. Given that the co-ordinates of P3 are already not perfect, the alignments are not as good as previously, but the intent and mathematical knowledge is clear.

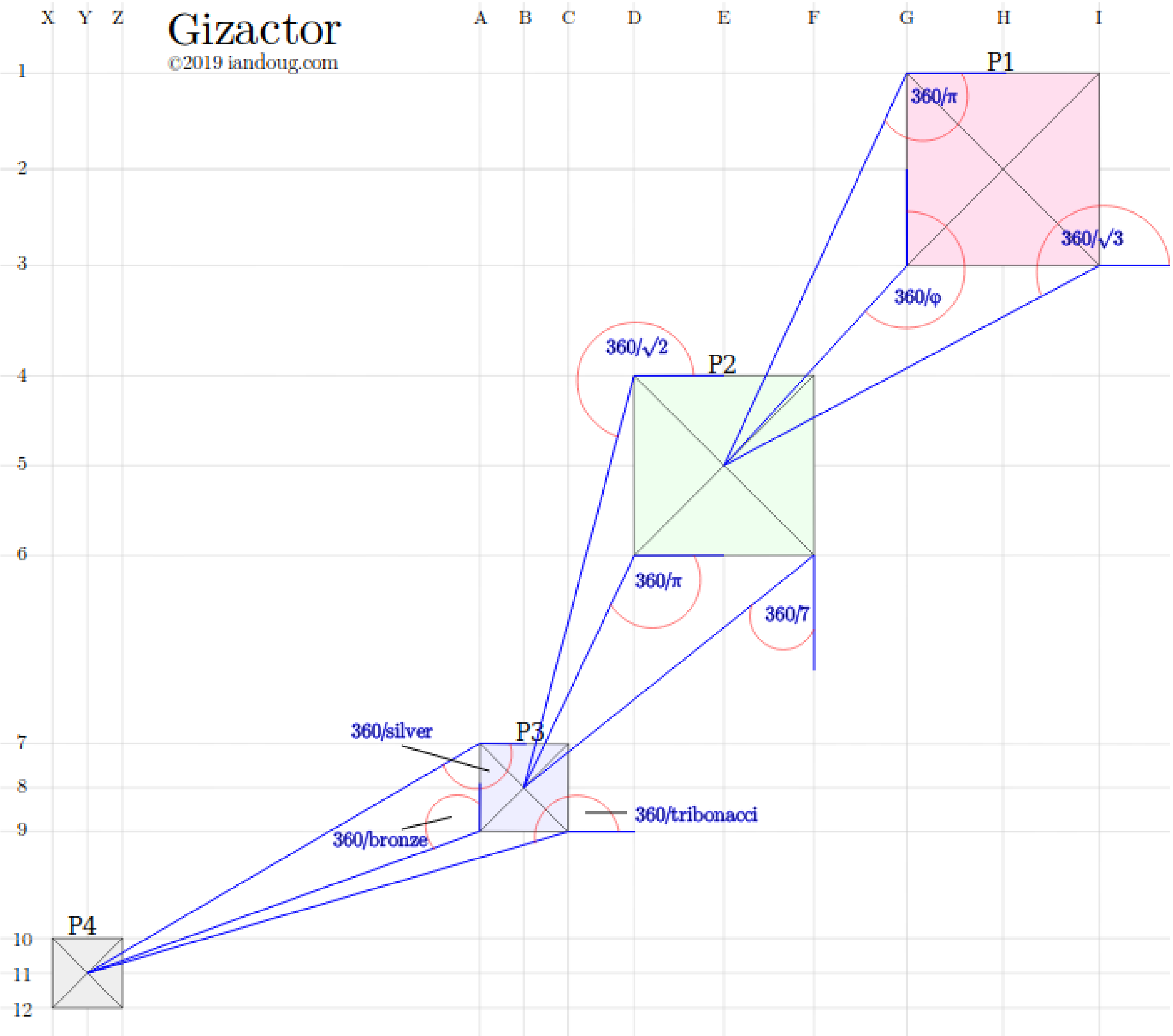


Figure 24: Chain of angles linking the four pyramids

The silver, bronze and tribonacci ratios are relatives of the Golden Ratio.

Source	Target	Angle	Calculated °	Desired °	Absolute delta °
P3 NW	P4 C	⌐silver ratio	149.32	149.12	0.20
P3 SW	P4 C	∟$_{90}$bronze ratio	110.07	109.00	1.07
P3 SE	P4 C	∟tribonacci ratio	196.39	195.73	0.66

Table 17: Analysis of P3 to P4 angles in Fig. 24

As a reminder, we have the following formulas:

Golden ratio $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618033989$

Silver ratio $= \frac{2+\sqrt{8}}{2} \approx 2.414213562$

Bronze ratio $= \frac{3+\sqrt{13}}{2} \approx 3.302775638$

Tribonacci ratio $T = \frac{1+\sqrt[3]{19+3\sqrt{33}}+\sqrt[3]{19-3\sqrt{33}}}{3} \approx 1.839286755214$

These results were unexpected but made me smile (it's one of their mathematical jokes), and suggest a sly sense of humour on the part of the site planners. The base/height ratio of Menkaure is 1.616, and the fourth pyramid base dimensions are almost 100 times that, also hinting at φ, although 162 would have been better.

Joining two pyramids related to the golden ratio with the angles of the silver, bronze AND tribonacci ratios, is one way to make your point. The results act as another confirmation of the location of the fourth pyramid.

6. Discussion

I started looking at the alignment of the pyramids of Giza in September 2018. Now, about 9 months later, I have come to the following conclusions about the site plan:

1. The site was planned as a coherent whole.
2. The planners were very, very smart. The more I discover, the greater my awe. It's one thing to try to figure out the puzzles, it's quite another to think them up and put them there in the first place. They succeeded in multiple different alignments simultaneously, while matching up with the stars as well (see companion paper). This feat makes me wonder if they had a computer or similar to crunch the numbers. This raises other questions about the human timeline.
3. The planners were familiar with the metre, (royal) cubit and foot.
4. There is evidence they were also familiar with π, φ and e, as well as square roots, variants of the golden ratio, and other interesting numbers.
5. The latitude of Khufu's centre echoes the speed of light in metres/second, as does the difference between the inscribed and circumscribed circles of Khufu's base. We must accept that they knew the speed of light. By extension, also the length of a

second. From the One Second Pendulum, you can get the metre. From a circle with diameter 1 metre, you can get the ₢.

6. They were capable of conceiving, planning and executing a building project that even today, we are unable to duplicate. It clearly took a lot of time and money, and we are still unable to figure out what it was for.
7. The above pre-supposes written language and mathematical notation, plus a considerable time to develop and discover these things.
8. If the dating in the companion paper is correct, then we need to rethink our timeline, as the Dynastic Egyptians had absolutely nothing to do with the original construction.
9. As to who built it, I have no idea. I doubt it was aliens or gods. It must have been a variant of human, now long gone, along with other physical evidence of their presence.

9. Acknowledgements

In no particular order, thanks to

1. Andrew Collins, author and researcher, for assorted inputs and feedback.

2. Glen Dash and the GPMP for the precise co-ordinates.

3. George Douros for the Symbola font.

Bibliography

[1] 'The Nebra Sky Disc the Most Astounding Ancient Artifact Ever Discovered - YouTube'. [Online]. Available: https://www.youtube.com/watch?v=azSrG0GaiCI. [Accessed: 03-Jun-2019].
[2] Douglas, Ian, '55,550 BCE and the 23 Stars of Giza'. .
[3] Douglas, Ian, 'The Beautiful Cubit System'. 30-Jun-2019.
[4] rubenbristian.com, 'Location, Location, Location: Where, Precisely, are the Three Pyramids of Giza? | Glen Dash Foundation for Archaeological Research'. .
[5] M.-K. Siu, '"Algorithmic Mathematics" and "Dialectic Mathematics": The "Yin" and "Yang" in Mathematics Education.', 2002.
[6] J. P. L. Salle, *The Influence of Computing on Mathematical Research and Education*. American Mathematical Soc., 1974.
[7] 'New Page 1'. [Online]. Available: http://www.samuellaboy.com/English/chap14.htm. [Accessed: 05-Jun-2019].

[8] 'Golden ratios in Great Pyramid of Giza site topography', *The Golden Ratio: Phi, 1.618*, 14-May-2016. [Online]. Available: https://www.goldennumber.net/great-pyramid-giza-complex-golden-ratio/. [Accessed: 05-Jun-2019].
[9] 'Giza Site Plan'. [Online]. Available: http://www.legon.demon.co.uk/gizaplan.htm. [Accessed: 05-Jun-2019].
[10] E. G. Nightingale, 'The Geometry of Giza - Graham Hancock Official Website'. .
[11] 'Digital Giza | Discussions in Egyptology 31 (1995), pp. 35-45, "The Elaboration of the Giza Site-Plan"'. [Online]. Available: http://giza.fas.harvard.edu/pubdocs/642/full/. [Accessed: 05-Jun-2019].
[12] R. Cook, 'A Trans-generational plan at Giza'.
[13] S. Creighton, 'The Giza Oracle: A New Theory Concerning the Design of the Pyramids of Giza - Graham Hancock Official Website'. .
[14] 'The Great Pyramid of Giza - Decoding The Measure of a Monument | Online Version | The Great Pyramid of Giza'. [Online]. Available: http://thegreatpyramidofgiza.ca/content/index.html#the-golden-ratio. [Accessed: 05-Jun-2019].
[15] R. C. Mercier, 'THE ENIGMA OF GIZA: PLANNING & SCALE ON THE GIZA PLATEAU', p. 29.
[16] D. M. Keenan, 'Our Solar System in the Layout at Giza', p. 36.
[17] 'The Geometry of Giza.' [Online]. Available: http://members.home.nl/peregrine/The%20Geometry%20of%20Giza.html. [Accessed: 07-Jun-2019].
[18] 'Exact Reconstruction of the Ground Plan of the Great Giza Pyramids'. [Online]. Available: http://www.vejprty.com/gizaplan.htm. [Accessed: 07-Jun-2019].
[19] 'Untitled Document'. [Online]. Available: http://home.hiwaay.net/~jalison/Art3.html. [Accessed: 07-Jun-2019].
[20] H. Jelitto, *Planetary Correlation of the Giza Pyramids - P4 Program Description*. 2015.
[21] 'The Descending Passageway', *The Khufu Pyramid*. [Online]. Available: http://khufupyramid.dk/inside-dimensions/the-descending-passageway. [Accessed: 15-Jun-2019].
[22] 'The Irrational Mathematicians of Giza, Part 1 | IanDoug'. [Online]. Available: https://iandoug.com/?p=258. [Accessed: 07-Jun-2019].
[23] T. Bushby, *The Secret in the Bible*, 1st edition. Maroochydore, Qld.: Stanford Publishing Group, 2003.
[24] 'Megalithic Yard', *Wikipedia*. 15-May-2019.
[25] BARDCODE, *CPAK Alan Green Trailer*. .

9. Appendices

9.1 The spooky stuff

This section properly has no place in an academic paper, but in the interests of full disclosure and giving credit where credit is due, I have included it.

Kekulé, who discovered the ring structure for benzene, said the answer came to him in a dream. Robert Bauval sometimes refers to "the spooky stuff" when discussing the Great Pyramid. In the interests of openness, I have experienced my own version of the spooky stuff while investigating the layout of Giza. There were several times where ideas just popped into my head, as if someone was whispering in my ear, which lead me to new discoveries. This was either my intuition on overdrive, or some other source. I don't entertain sky gods, aliens, spooks or re-incarnation. The only other thing I can think of beyond native intuition is that my ancestors were involved in the Giza project and that this knowledge has somehow been passed down through their descendants. Bizarre, yes, but I can't think of any other explanation. That idea is deeply disturbing in multiple ways.

The hints I received include the "join the dots" idea discussed in the companion paper, which lead to the astronomical alignment, and hence the date, as well as the clues to finding the fourth pyramid. I was not even aware of the fourth pyramid until a few months ago.

There were also times when new discoveries surfaced, and after I recovered my composure, it felt like those ancestors were smiling at me ... we shared the mathematical joke and joy together.

Like I said, spooky stuff.

Let me give one example in detail, relating to the two straight line connections between the first three pyramids as shown in Figure 5. I figured out the ⦣e angle quite easily, but had a lot of trouble with the other line. I was focused on the acute angle, which was just over 60°. The best divider I could find was (360/tau) + π, which produced a reasonable number, but the logic made no sense. Nevertheless, I could not find anything better so reluctantly used that.

Then I watched the video from Alan Green [25] which spoke about the relationship between foot and 𝔊, versus e and e-1. Little did I realise at the time that that was a major hint.

Then one night, the issue of (360/tau) + π was bothering me again, and I decided to have another go at solving it. It felt like those ancestors were nagging and/or encouraging me to do this. So I played around with the calculator, seeing what happens if I measured from different starting points instead of Cartesian zero. The process is basically like this:

1. Measure / calculate an angle.
2. Divide 360 by angle in (1).
3. Does the answer have meaning, or can I manipulate easily it to have meaning?

At some point in this process the number 1.718 popped up, and I thought, "Hey, wait a minute... I know that number..." Indeed, there was the answer, as 360/(e-1). Then I remembered that the other line was 360/e and I had to laugh in amazement at the genius of the design. Even worse, I didn't realise that this discovery was a major hint for the metre-cubit-foot ratios in the East-West alignments, until after I found it by other means, again as the result of some unknown guidance.

I know this sounds rather "woo-woo" but that is how this whole process has been. I suppose it actually started when I saw the Nebra disk video.

What I like about these discoveries is how everything "works together" ... from the mathematics of the layout to the alignment with the stars. That is why I risk ridicule by publishing.

9.2 The Gizactor, 4 pyramid version

SVG code for the Gizactor. I used Symbola font (for its wide glyph coverage) and Noto Sans because it's freely and easily available. Feel free to use your own fonts. You can open this file in a browser or dedicated image editor. If the image in the browser is too large, zoom out.

```
<svg  xmlns="http://www.w3.org/2000/svg" version="1.1" width="1400"
height="1400" style="background:white">

<!-- includes 4th pyramid -->

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  <font-face font-family="Symbola" font-weight="normal" font-style="normal"
units-per-em="1000">

    <font-face-src>

      <font-face-name name="Symbola" />

    </font-face-src>

  </font-face>

</font>

<font id="NotoSans" horiz-adv-x="1000">

  <font-face font-family="Noto Sans" font-weight="normal" font-style="normal"
units-per-em="1000">

    <font-face-src>

      <font-face-name name="Noto Sans" />

    </font-face-src>

  </font-face>

</font>
```

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<!-- p1 -->

<line x1="0" y1="84.9" x2="1400" y2="84.7" style="stroke:#cccccc;stroke-
width:1" />

<line x1="0" y1="315.1" x2="1400" y2="315.3" style="stroke:#cccccc;stroke-
width:1" />

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<line x1="1315" y1="0" x2="1315.3" y2="1400" style="stroke:#cccccc;stroke-
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width:1" />

<line x1="1200" y1="0" x2="1200" y2="1400" style="stroke:#cccccc;stroke-
width:1" />

<polygon points="1084.6,84.9 1315,84.7 1315.3,315.1 1084.9,315.3"
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<line x1="1084.6" y1="84.9" x2="1315.3" y2="315.1" style="stroke:black;stroke-
width:1" />

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width:1" />

<!-- p2 -->

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width:1" />

<line x1="0" y1="662.2" x2="1400" y2="661.9" style="stroke:#cccccc;stroke-
width:1" />

<line x1="758.2" y1="0" x2="758.6" y2="1400" style="stroke:#cccccc;stroke-
width:1" />

<line x1="973.5" y1="0" x2="973.9" y2="1400" style="stroke:#cccccc;stroke-
width:1" />

<line x1="0" y1="554.4" x2="1400" y2="554.4" style="stroke:#cccccc;stroke-
width:1" />

<line x1="866" y1="0" x2="866" y2="1400" style="stroke:#cccccc;stroke-
width:1" />

<polygon points="758.2,446.9 973.5,446.6 973.9,661.9 758.6,662.2"
style="fill:#eeffee;stroke:black;stroke-width:1" />

<line x1="758.2" y1="446.9" x2="973.9" y2="661.9" style="stroke:black;stroke-
width:1" />
```

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<line x1="973.5" y1="446.6" x2="758.6" y2="662.2" style="stroke:black;stroke-width:1" />

<!-- p3 -->

<line x1="0" y1="887.1" x2="1400" y2="887.7" style="stroke:#cccccc;stroke-width:1" />

<line x1="0" y1="992.7" x2="1400" y2="993.1" style="stroke:#cccccc;stroke-width:1" />

<line x1="573.9" y1="0" x2="573.5" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<line x1="679.5" y1="0" x2="679.1" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<line x1="0" y1="940.1" x2="1400" y2="940.1" style="stroke:#cccccc;stroke-width:1" />

<line x1="626.5" y1="0" x2="626.5" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<polygon points="573.9,887.1 679.5,887.7 679.1 993.1 573.5 992.7"
style="fill:#eeeeff;stroke:black;stroke-width:1" />

<line x1="573.9" y1="887.1" x2="679.1" y2="993.1" style="stroke:black;stroke-width:1" />

<line x1="679.5" y1="887.7" x2="573.5" y2="992.7" style="stroke:black;stroke-width:1" />

<!-- p4 -->

<line x1="0" y1="1120.4" x2="1400" y2="1120.4" style="stroke:#cccccc;stroke-width:1" />

<line x1="0" y1="1162.2" x2="1400" y2="1162.2" style="stroke:#cccccc;stroke-width:1" />

<line x1="0" y1="1203.9" x2="1400" y2="1203.9" style="stroke:#cccccc;stroke-width:1" />

<line x1="62.5" y1="0" x2="62.5" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<line x1="104.2" y1="0" x2="104.2" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<line x1="145.9" y1="0" x2="145.9" y2="1400" style="stroke:#cccccc;stroke-width:1" />

<polygon points='62.5,1120.4 145.9,1120.4 145.9,1203.9 62.5,1203.9'
style='fill:#eeeeee;stroke:black;stroke-width:1' />

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<!-- labels -->

<text x="200" y="50" font-family="Symbola" font-size="56">Gizactor v1.2 </text>

<text x="200" y="80" font-family="Symbola" font-size="24">©2019 iandoug.com</text>

<text x="1180" y="81" font-family="NotoSans" font-size="28">P1</text>

<text x="846" y="444" font-family="NotoSans" font-size="28">P2</text>

<text x="611" y="884" font-family="NotoSans" font-size="28">P3</text>

<text x="80" y="1115" font-family="NotoSans" font-size="28">P4</text>

<!-- grid labels -->

<text x="566" y="25" font-family="Symbola" font-size="24">A</text>

<text x="620" y="25" font-family="Symbola" font-size="24">B</text>

<text x="672" y="25" font-family="Symbola" font-size="24">C</text>

<text x="750" y="25" font-family="Symbola" font-size="24">D</text>

<text x="858" y="25" font-family="Symbola" font-size="24">E</text>

<text x="966" y="25" font-family="Symbola" font-size="24">F</text>

<text x="1076" y="25" font-family="Symbola" font-size="24">G</text>

<text x="1192" y="25" font-family="Symbola" font-size="24">H</text>

<text x="1309" y="25" font-family="Symbola" font-size="24">I</text>


<text x="48" y="20" font-family="Symbola" font-size="24">X</text>

<text x="92" y="20" font-family="Symbola" font-size="24">Y</text>

<text x="134" y="20" font-family="Symbola" font-size="24">Z</text>

<text x="20" y="90" font-family="Symbola" font-size="24">1</text>

<text x="20" y="206" font-family="Symbola" font-size="24">2</text>

<text x="20" y="320" font-family="Symbola" font-size="24">3</text>

<text x="20" y="453" font-family="Symbola" font-size="24">4</text>

<text x="20" y="560" font-family="Symbola" font-size="24">5</text>

<text x="20" y="668" font-family="Symbola" font-size="24">6</text>

<text x="20" y="893" font-family="Symbola" font-size="24">7</text>

<text x="20" y="946" font-family="Symbola" font-size="24">8</text>

<text x="20" y="998" font-family="Symbola" font-size="24">9</text>
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<text x="5" y="1130" font-family="Symbola" font-size="24">10</text>
<text x="5" y="1172" font-family="Symbola" font-size="24">11</text>
<text x="5" y="1214" font-family="Symbola" font-size="24">12</text>
</svg>
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