A Short Disproof of the Riemann Hypothesis

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The **Riemann Hypothesis** is one of the most important unsolved problems in Mathematics and its validity will have a great consequence on the precise calculation of the number of primes. Riemann developed an explicit formula relating the number of primes with the hypothesized *non-trivial zeros* of the Riemann zeta function. Riemann hypothesis states that all the non-trivial zeros of the zeta function have real part equal to one-half.

Despite many attempts to solve it for about 150 years, no one have, so far succeeded. The Riemann hypothesis is based on the **existence of the zeros o**f the zeta function. If it can be shown, that such zeros do not exist, then the Riemann Hypothesis is false.

The Riemann zeta function (or zeta function) shown below is central to the Riemann Hypothesis,

(1) $$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad s = \sigma + \omega i,$$

where s is a complex variable with real part σ and imaginary part ω. The modulus of $\zeta(s)$, denoted by $|\zeta(s)|$, is a positive number associated with it. The quantity σ has a damping effect on $\zeta(s)$ while ω acts as a filter that can remove some of its components. Thus, σ and ω can have a great effect on the convergence of the infinite series in (1).

The infinite series in (1) converges absolutely for $\sigma > 1$,

$$|\zeta(\sigma+\omega i)| = \left|\sum_{n=1}^{\infty} n^{-\sigma+\omega i}\right| \leq 1+2^{-\sigma}+3^{-\sigma}+4^{-\sigma}+5^{-\sigma}+\cdots+n^{-\sigma} = \sum_{n=1}^{\infty} n^{-\sigma}.$$

The infinite series in (1) can also be expressed as

$$(2) \qquad \zeta(s) = \prod_{p}^{\infty} \frac{1}{1-p^{-s}}.$$

The infinite product in (2) runs through all the prime numbers *p* and is widely known as the Euler product. Formula (2) can only be used for $\sigma > 0$ and its modulus is given by

$$(3) \qquad |\zeta(\sigma+\omega i)| = \prod_{p}^{\infty} \frac{1}{\sqrt{1 - 2p^{-\sigma}\cos(\omega \log p) + p^{-2\sigma}}}.$$

For $\sigma > 0$ and, for all ω and *p*

$$(4) \qquad 1 - 2p^{-\sigma}\cos(\omega \log p) + p^{-2\sigma} > 0,$$

since the least value of (4) is attained when cos(ωlog *p*) = 1 resulting in (4) still greater than zero,

$$(1 - p^{-\sigma})^2 > 0.$$

Each individual term in (3) converges absolutely for $\sigma > 0$ while their product converges conditionally if $0 < \sigma \le 1$, and their product converges absolutely for $\sigma > 1$. In fact, the divergent nature of $\zeta(s)$ at $0 < \sigma \le 1$ proves the existence of the infinity of primes but at $\sigma \le 0$ it is completely invalid. Also, as a consequence of (4), the zeta function has no zeros in its domain and its modulus is always greater than zero,

$$(5) \qquad \zeta(s) \ne 0 \quad \text{and} \quad |\zeta(s)| > 0, \qquad \sigma > 0.$$

Each term in (3) can be either greater than one if cos(ωlog *p*)>0 or less than one if cos(ωlog *p*)<0 and as *p* becomes larger and larger the succeeding term approaches unity which prevents $\zeta(s)$ from being zero.

The following conclusions can be made as a consequence of (5):

(a) Riemann's functional equation $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$ is not valid.

(b) The equality $\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s) = \pi^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)$ is invalid.

(c) Any approximation on $\zeta(s)$, say *Z*(s), tends to have a lower value of *σ* at which it converges than $\zeta(s)$. But $\zeta(s)$ stays the same because $Z(s)\approx\zeta(s)$ is not the same as $Z(s)=\zeta(s)$. All the pseudo-zeta functions, therefore, are *only* approximations on $\zeta(s)$ and are *not* its analytic continuations and the lowering of the value of *σ* is due to the approximation methods used.

(d) The Riemann zeta function $\zeta(s)$ is defined only on the the right half-plane of the *s*-domain.

(e) The Riemann zeta function $\zeta(s)$ has no zeros in its domain and its modulus is always greater than zero.

(f) The Riemann hypothesis is not true, since it is based on a false presupposition: the existence of the zeros of the zeta function.

Conclusions (a), (b), and (c) dispense of the notion that the zeta function $\zeta(s)$ can still be extended beyond the right half-plane.

REFERENCE

Riemann, Bernhard (1859). *On the Number of Prime Numbers less than a Given Quantity.*