

Adaptive Loudness Compensation in Music Listening

Leonardo Fierro

Signals and Communication Lab,
Dept. Information Engineering,
University of Brescia, Brescia, Italy
lfuegofierro@gmail.com

Jussi Rämö and Vesa Välimäki

Acoustics Lab,
Dept. Signal Processing and Acoustics,
Aalto University, Espoo, Finland
jussi.ramo@aalto.fi
vesa.valimaki@aalto.fi

ABSTRACT

The need for loudness compensation is a well known fact arising from the nonlinear behavior of human sound perception. Music and other sounds are mixed and mastered at a certain loudness level, usually louder than the level at which they are commonly played. This implies a change in the perceived spectral balance of the sound, which is largest in the low-frequency range. As the volume setting in music playing is decreased, a loudness compensation filter can be used to boost the bass appropriately, so that the low frequencies are still heard well and the perceived spectral balance is preserved. The present paper proposes a loudness compensation function derived from the standard equal-loudness-level contours and its implementation via a digital first-order shelving filter. Results of a formal listening test validate the accuracy of the proposed method.

1. INTRODUCTION

Loudness compensation is based on the equal-loudness-level contours first reported by Fletcher and Munson in the 1930s [1] and different approaches to loudness compensation have been discussed since then [2–5]. It is well known how perceived bass and sub-bass ranges are much more affected than high frequencies when the sound level goes down. As a consequence, it is beneficial to adapt the compensation based on the listening level of the audio track.

Recently, Prasad described a compensation based on and approximation of the difference in sensitivity, which can be implemented using a filterbank or fast convolution based on the FFT (Fast Fourier transform), which causes some processing latency [6]. Hawker and Wang proposed the use of a scalar function describing the change in SPL required to effect a change of 1 Phon. Their method is implemented using FIR (Finite Impulse Response) filters with 1024 coefficients, which are also best to implement using fast convolution [7].

According to Katz [8], music is nowadays usually mixed and mastered with loudspeakers at the sound pressure level (SPL) of 83 dB, or more generally at SPL between 80 to

85 dB. In such a range, human loudness perception is the closest to be flat while avoiding painful levels. However, those are quite high levels and a prolonged exposure can tire the listener or even damage the hearing [9] [10]. Safer listening levels for consumers (in particular using headphones) lie in the 60–75 dB SPL range.

Inevitably, when the sound reproduction level is changed, the perceived spectral balance is altered as well and the fidelity to the original master is lost. The ultimate goal of a loudness compensation method is not to provide the best subjective bass compensation according to the listener, but to recover such lost fidelity by regaining the spectral balance of the playback sound. Consumer audio equipment sometimes offered a “loudness” switch, whose action was merely a constant bass boost regardless of the playback level or a variable analog shelving filter control without calibration [4, 11]. More recent devices have removed this feature, leaving the user to manually change the volume controls.

This paper proposes a computationally efficient compensation technique using a first-order IIR (Infinite Impulse Response) digital filter to improve the listening experience, based on the equal-loudness-level contours (ELLC) provided by the ISO226:2003 standard [12]. The proposed method is highly accurate approximating the ELLC curves within ± 1 dB. Furthermore, the low-order IIR filter does not introduce practically any processing latency. This is similar to the best analog loudness control circuits with the addition that it allows level calibration.

The rest of this paper is organized as follows. Section 2 briefly illustrates the ELLC, reporting the contour function and data interpolation useful for the proposed compensation method described in Section 3. Section 4 is related to the filter design, and Section 5 to the optimization of filter parameters. Description and results of conducted listening tests are shown in Section 6. Section 7 concludes this paper.

2. EQUAL-LOUDNESS-LEVELS CONTOURS

The ISO226 standard is used as reference for the work described in this paper. The standard specifies the sound pressure levels of a pure tone, as function of frequency, perceived as equally loud by human listeners in free space [12]. Polynomial function for the contours is given by:

Copyright: © 2019 Leonardo Fierro et al. This is an open-access article distributed under the terms of the [Creative Commons Attribution 3.0 Unported License](https://creativecommons.org/licenses/by/3.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

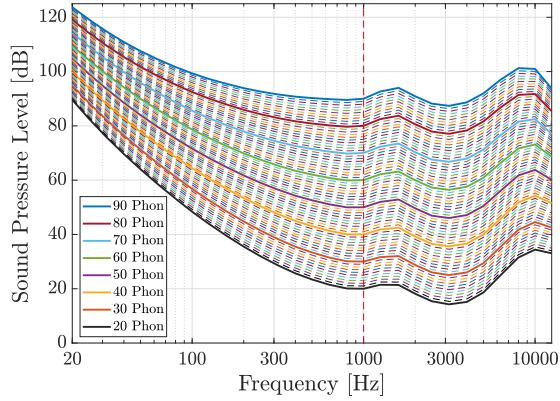


Figure 1: Interpolated ELLC for 20–90 Phon range between 20 Hz and 12.5 kHz.

$$L_p = \frac{10}{a_f} \log_{10} A_f - L_u + 94, \quad (1)$$

where

$$A_f = \frac{4.47}{10^3} \left(10^{\frac{L_n}{40}} - 1.15 \right) + \left(0.4 \cdot 10^{\frac{T_f + L_u}{10} - 9} \right) a_f; \quad (2)$$

L_p is the sound pressure level (in dB SPL) of a pure tone; L_n is the loudness level (in Phon); f is the frequency of the pure tone; T_f is the hearing threshold (in dB SPL); a_f is the exponential factor, accounting for loudness perception; L_u is the magnitude (in dB) of the frequency response, normalized at 1 kHz. The data range provided by the standard is 20 to 90 Phon, whereas the frequency spans from 20 Hz to 12.5 kHz, and it is shown in Fig. 1.

From the ELLC, it is easy to see how the sensitivity of human perception changes nonlinearly with frequency and how low-frequency range is the most heavily affected part of the spectrum. Data from ISO226 was linearly interpolated with 1-Phon steps to provide intermediate curves (Fig. 1).

3. COMPENSATION METHOD

The main idea behind the proposed method is quite straightforward: derive a *sensitivity function* from the ELLC, relative to the listening level, then find an inverse function to be used as a trace-guide for the design of a digital filter that can then correct the spectral balance.

It is possible to normalize each curve in Fig. 1, with respect to its SPL value at 1 kHz, in order to evaluate the sensitivity of human hearing for different SPLs, i.e. to obtain a sensitivity function S :

$$S(f, L_n) = -L_p(f, L_n) + L_p(1000, L_n). \quad (3)$$

For each sensitivity curve shown in Fig. 2, the difference in perception with respect to SPL at 1 kHz corresponds to the gain (or attenuation) to be introduced in order to have a flat response.

The mastering level (L_M) and the listening level (L_L) for music are usually different, as the latter is quieter than the

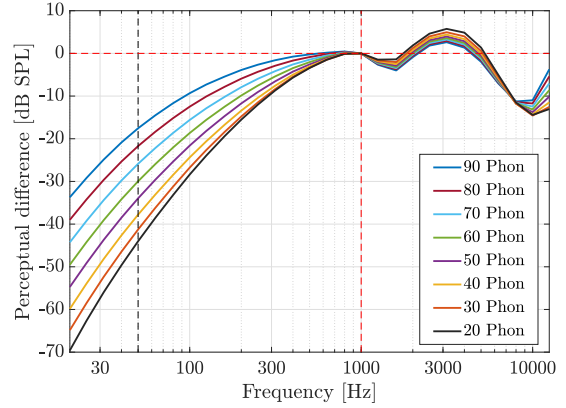


Figure 2: Sensitivity function, for different levels.

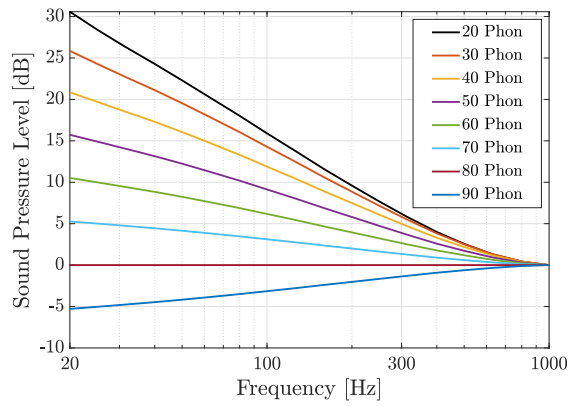


Figure 3: Derived filtering trace-guides at $f \leq 1$ kHz for certain listening levels, $L_M = 80$ dB SPL.

first one. The goal is to compensate the perceived spectral balance at L_L in such a way that it matches the perceived spectral balance at L_M . A relationship between the two levels is derived, identifying a *difference curve* ΔL_p :

$$\Delta L_p(f, L_M, L_L) = L_p(f, L_M) - L_p(f, L_L) - N_f, \quad (4)$$

where N_f is the normalization factor for the sensitivity function, according to (2):

$$N_f = L_p(1000, L_M) - L_p(1000, L_L) = L_M - L_L. \quad (5)$$

Inverting (3), a balancing curve is finally obtained. It corresponds to the magnitude response that perfectly balances the perception of spectral components as intended by the mastering:

$$\begin{aligned} H(f, L_M, L_L) &= -\Delta L_p(f, L_M, L_L) \\ &= -L_p(f, L_M) + L_p(f, L_L) + N_f. \end{aligned} \quad (6)$$

Notice how bass reduction instead of boost is required for $L_L > L_M$ in Fig. 3. This is significant for situations like live concerts or discos, where music can be played at high levels [11].

4. FILTER DESIGN

Since a set of curves—referred as *trace-guide* from here on—has been obtained, the next step is to identify a type of digital filter whose magnitude response is sufficiently close to the trace-guide and that can easily adapt to a change in listening level; low order and low complexity are desired, in order to have minimum impact on the reproduction system and allow a real-time implementation.

IIR filters are a natural choice in terms of efficiency for many audio DSP applications [13]. Their processing is typically low demanding in terms of operations and memory, enabling the implementation of such filters in low cost architectures and products hitting the markets. FIR filters allow linear-phase processing, but require generally a larger number of operations per output sample and more memory on the DSP interface than IIR filters. Furthermore, FIR filters can be limited in resolution when working with low frequencies, affecting the quality of the filter coefficients [13]. For those reasons, FIR filters are not considered in this paper.

Digital filters can be derived from analog filters, converting a transfer function with analog poles/zeros from the Laplace-domain to the z-domain and obtaining a difference equation, using common transformations such as the bilinear transform, the matched Z-transform, the pole-zero mapping method, or the impulse-invariant method [14]. Digital filters can also be designed by choosing appropriate locations for the poles and zeros on the unit circle in the z-domain, thus imposing desired corner frequencies and slope [15].

Depending on the quality of the back-end application, IIR filters can be applied on both fixed-point and floating-point architectures. Consequently, either Direct Form I, II or Transpose II can be chosen for implementation: DF2 and DF2T require less delay elements but are prone to overflows with high-order filters, so DF1 may be a simpler and better solution for fixed-point architectures [15].

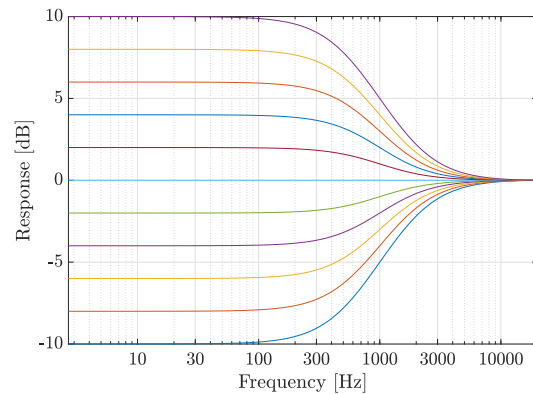
4.1 Fractional order filters

From Fig. 3, it can be observed that the target magnitude response slope is less than 20 dB/decade, for every curve. This suggests to look for filters with an order smaller than one—the so called *fractional order filters*—and with a low-pass behaviour.

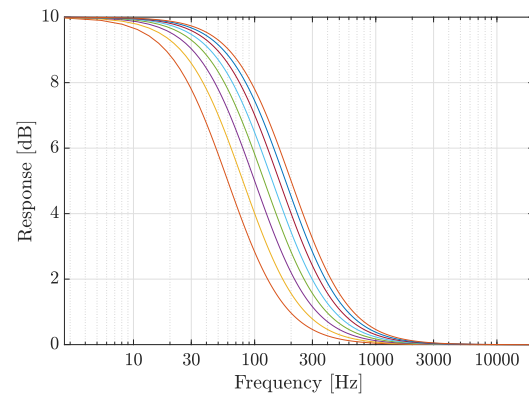
Fractional order filters (FOF) are neither well documented nor well described in DSP literature at the moment of writing [16], but a design process for digital FOFs has been proposed by Nielsen [11]. Although providing similar results as shelving filters (Section 4.2), the latter should eventually be preferred to FOFs, due to the wider use and lower complexity (always a good thing when discussing real-time audio applications). For this reason, FOFs are not considered in this paper. Future studies might provide interesting results, also relevant for this work.

4.2 Shelving filters

A shelving filter boosts or attenuates the magnitude of an input signal in a certain frequency band—either the lowest



(a) Variable G , constant $\omega_c = 1$ kHz



(b) Variable ω_c , constant $G = 10$ dB

Figure 4: Effects of gain and crossover frequency on the magnitude response of a first-order low shelving filter.

frequency band or the highest frequency band—without cutting out the harmonics in that band as a typical low-pass/high-pass filter would do [13, 17].

Depending on whether it affects the bass or the *treble* (high frequencies), it will be referred to as either a *low-shelving* or *high-shelving* filter, respectively.

This type of filter is largely used in parametric equalizers, due to the smooth transition of the response between affected and unaffected regions and the simple implementation. A classic parametric equalizer presents two knobs to the user, one for bass and one for treble¹, through which it is possible to alter the filter shape and its effect on the playback sound.

Simplicity comes from the fact that the behavior of a shelving filter is completely described by just the gain G and the crossover frequency f_c (often also called *corner* or *cut-off* frequency). As can be seen from Fig. 4, the gain parameter affects the gain at low frequencies and the slope (Fig. 4a), while the crossover frequency parameter affects the width of the response, i.e. its frequency span (Fig. 4b).

Transfer functions for both the first and second-order low-shelving digital filters have been derived by Välimäki and Reiss [13]. The transfer function of the first-order shelf can

¹ Typically, one or more knobs adding peaks/notches in the mid-frequencies range are also available in common music equipment.

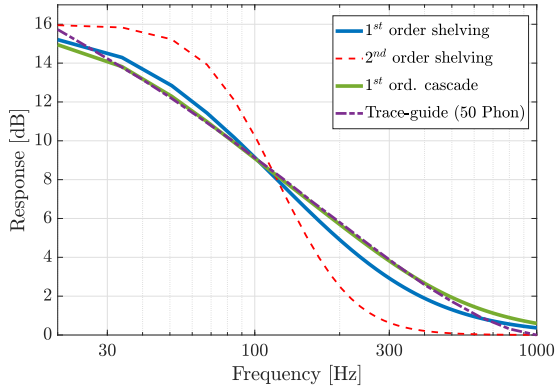


Figure 5: Comparison between trace-guide and shelving filters, $L_M = 80$ dB SPL, $L_L = 50$ dB SPL.

be written as

$$H_{LS}(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}, \quad (7)$$

where

$$b_0 = \frac{G\Omega + \sqrt{G}}{\Omega + \sqrt{G}}, \quad b_1 = \frac{G\Omega - \sqrt{G}}{\Omega + \sqrt{G}}, \quad a_1 = \frac{\Omega - \sqrt{G}}{\Omega + \sqrt{G}},$$

$\Omega = \tan(\omega_c/2)$, and $\omega_c = 2\pi f_c/f_s$.

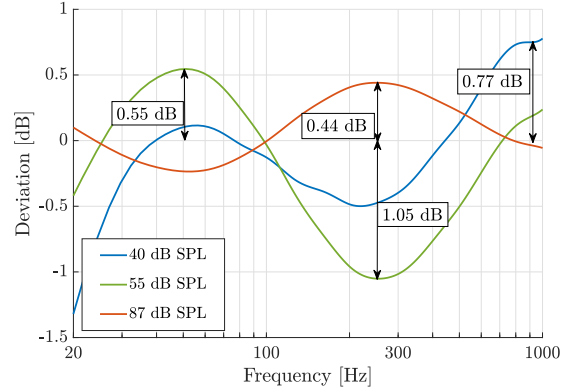
Fig. 5 shows a comparison of different low-shelving filter responses, where the trace-guide is interpolated with a spline function to provide more frequency points. As can be seen, the first-order shelving filter presents a fairly good approximation of the trace-guide; an even better result is achieved with a cascade of two first-order filters, but a second-order low-shelving returns curves which are too steep. Moreover, the flat response towards the lowest frequencies avoids unnecessarily boosting the frequencies close to DC, or 0 Hz.

5. OPTIMIZATION OF FILTER PARAMETERS

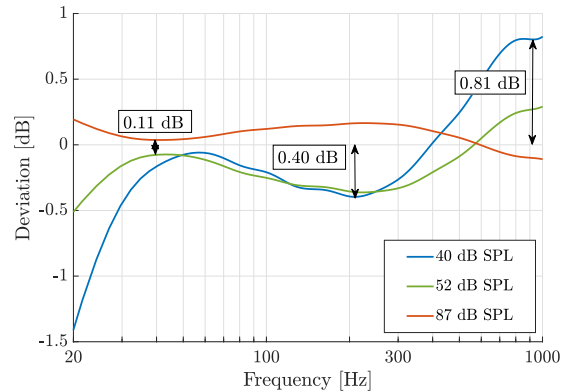
After choosing the filter type, the optimal parameters (G , ω_c) should be found. In case of a cascade of two first-order shelves, there are four parameters: (G_1 , G_2 , ω_{c1} , ω_{c2}). However, few things might be taken into account in order to simplify the optimization problem:

- L_M typically lies in a very limited interval (80–85 dB SPL), so trace-guides will be similar for levels in such a range;
- it is reasonable to choose trace-guide SPL at 20 Hz as G ; in case of a filter cascade, the product of the gains (the sum, in the log domain) should match such value.

Holding to these considerations, an optimization algorithm can be run to identify the optimal parameters. A genetic algorithm (GA) has been chosen for this task [18], due to its suitability to solving search problems and the high-quality solutions it is capable of generating in a reasonable time.



(a) Single first-order filter



(b) Cascade of first-order filters

Figure 6: Worst-case deviations from trace-guide given by first-order low-shelving filters, with peaks of max deviation.

5.1 Crossover frequencies

Initial GA runs over the 80–85 dB SPL range for L_M show that the optimal solution for gains in the shelving cascade is really close to an equal weighting. So it is safe to assume, in first approximation:

$$G_1 = G_2 = \frac{1}{2} G. \quad (8)$$

This way, the complexity of the optimization task has already been reduced by one degree. Of course, this does not concern the single filter case.

Then, the crucial step in optimization seems to be the choice of the poles, i.e. the crossover frequencies. It is easy to change filter parameters in real-time application. However, given the short range of considered mastering levels and the definite frequency span of the trace-guide, fixing the poles simplifies the problem even further without loss of generality, leaving only G to be modified as L_L changes.

Multiple GA runs return, as consistent optimal solution: $f_c = 122$ Hz for the first-order shelving filter; and $f_{c1} = 61.1$ Hz and $f_{c2} = 242$ Hz for the cascade of first-order shelving filters. Maximum deviations from trace-guide are plotted in Fig. 6. As can be seen, both cases provide inter-

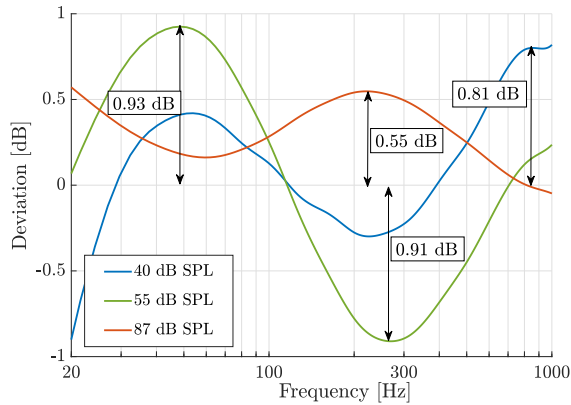


Figure 7: Deviation from trace-guide given by first-order low-shelving filter, with gain adjustment $\alpha = 0.485$ dB.

esting results, with small errors for all considered listening levels and minimum error on the mid frequencies. To achieve high-fidelity, a maximum deviation of ± 1 dB from the trace-guide is desired. As shown in Fig. 6b, the filter cascade error always lies inside such a range, while the single filter deviation slightly exceeds -1 dB around 250 Hz (Fig. 6a). Although, the cascade already satisfies requirements, the use of a single filter is desirable to further reduce complexity and computation time.

5.2 Gain adjustment

Since the crossover frequency has been fixed, a possible solution is to adjust the gain with a small bias term α in order to compensate for the deviation peak around 250 Hz, without exceeding the range somewhere else. The modified filter gain is then determined as:

$$G_{\text{dB}} = \Delta L_p(20, L_M, L_L) + \alpha. \quad (9)$$

Running the GA again, an optimal bias term $\alpha = 0.485$ dB was found. Fig. 7 shows that this small bias reduces the maximum deviation, while maintaining the error between ± 1 dB in the rest of the bass range.

Fig. 8 shows the magnitude responses of the first-order shelving filters using the modified gain and (7). The filter coefficients used for these curves are listed in Table 1.

6. LISTENING TEST

6.1 Design

A listening test was conducted on a selection of experienced listeners. No one reported any hearing impairments or medical conditions. The test was designed for this purpose and conducted in the MATLAB environment on a MacOS computer, using a pair of Sennheiser HD 650 dispatched inside a listening booth at the Aalto Acoustics Lab.

Audio samples were chosen from different genres for having a prominent bass line and other different spectral features. They consisted of short tracks (4 to 8 seconds) cut from the following songs:

1. Queen, “*Another One Bites The Dust*” (1980);

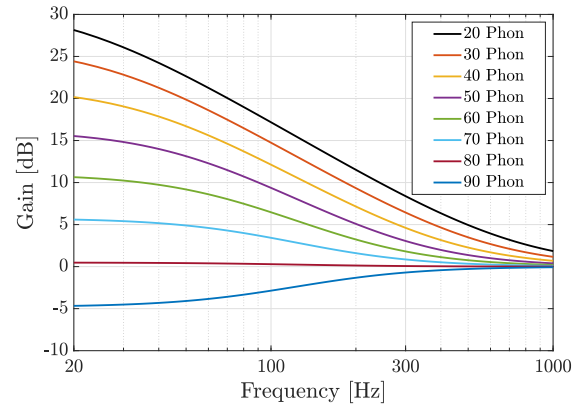


Figure 8: Magnitude responses of the first-order shelving filter at $f \leq 1$ kHz for certain listening levels, when $M_L = 80$ dB. Cf. Fig. 3.

L_L [dB]	Numerator		Denominator
	b_0	b_1	a_1
90	0.9952	-0.9821	-0.9773
80	1.0005	-0.9827	-0.9832
70	1.0058	-0.9818	-0.9876
60	1.0117	-0.9791	-0.9908
50	1.0186	-0.9746	-0.9932
40	1.0271	-0.9678	-0.9949

Table 1: Shelving filter coefficients for various choices of L_L , when $M_L = 80$ dB.

2. White Stripes, “*Seven Nation Army*” (2003);
3. Daft Punk, “*Around The World*” (1997).

From further on, each track will be identified with its number from the list above, e.g. Track 1, Track 2, Track 3. Track 2 is composed by just bassline and drumline, showing narrow spectral content concentrated in the bass range. Track 3 present a broader spectral content; same for Track 1, which also includes vocals.

Subjects under test were presented with 7 instances of each track, for a total of 21 stimuli pairs. Each step held a version of the track played at L_M (see Section 6.2), named *reference*, and an attenuated variant. Applied loudness reduction varied between 0 and 40 dB in steps of 10 dB, corresponding to five different listening levels in the 40–80 dB SPL range. 80 dB SPL (no reduction, same as reference) and 60 dB SPL (20 dB attenuation) were presented twice per each track: repeated reproductions were used during the screening phase to evaluate subject consistency and then discarded before statistical analysis of results.

Loudness compensation was applied to the attenuated variant using the single first-order low-shelving filter. The crossover frequency was fixed at 122 Hz (see Section 5.1), and the subjects modified the filter gain using a slider during the test. The slider selected a different gain for the filter based on the ELLC trace-guide (Section 5). Slider movements were discretized and each step corresponded

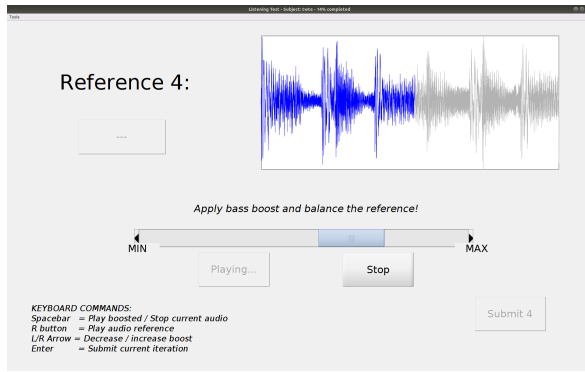


Figure 9: Screen-shot of the test GUI.

to a 2-dB variation in the selection of the curve.

The subjects were asked to focus on the bass of the played sounds, in particular on the balance between the overall loudness and the bass loudness of the reference, and to compensate the spectral balance of the variant in order to match the reference balance, but at a different L_L . Subjects had access to a horizontal slider (Fig. 9) that, it was told them, allowed to give “boost or reduction” to the bass of the variant. The range of the slider was hidden and slightly randomized, having only the labels “Min” and “Max” at its two extremes.

The audio samples were set to play in a continuous loop until they were manually stopped. While it was possible to reproduce the variant as many times and a long as as desired, the play count of the reference was limited to two, the first starting automatically at each new step of the test. This means that, after stopping the reference the first time, it was possible to play it again just one more time. This choice was made to avoid the listener to go “back and forth” from the reference to the variant and force them to pay extra focus on the task.

Given the fast decay of human memory of sounds, the subjects have been suggested to get a general idea of the frequency components of the reference during the first play, then to reproduce the variant and explore the amount of possible “boost” given by the slider, before getting back to the reference and gain a more clear sense of the spectral balance. After the second stop, a final choice for variant compensation should have been made.

Listeners were allowed a short training session before starting the actual test to get acquainted with the interface, the keyboard shortcuts and the task itself. The results of the training session were not included in the statistical analysis.

6.2 Level calibration

Having an accurate measurement of the loudness level was critical for the goodness of the test, so a calibration phase was performed. Used instrumentation involved a RME Fireface 800 and a G.R.A.S. 45CA Headphone Test Fixture in compliance with the IEC 60318-4:2010 occluded-ear simulation [19].

The different tracks, played through the headphones allocated on the ear simulator, were loudness matched by using

	Loudness [LUFS]	Max SPL [dBA]
Track 1	−11.3	83.4
Track 2	−11.1	77.0
Track 3	−11.0	81.2

Table 2: ITU-based loudness in loudness units relative to full scale (LUFS) according to ITU-R BS.1770-4 and the maximum measured SPLs (in dBA).

the ITU-R BS.1770-4 [20] loudness measure. Their playback level was then set to be close to 80 dBA (A-weighted dB). The actual measured dBA vary, since the levels depend on the contents of the considered track. The ITU-based loudness levels and maximum A-weighted dB levels are reported in Table 2.

6.3 Screening

In order to isolate inconsistent listeners, 80 dB SPL and 60 dB SPL cases were presented twice. For screening, it was not tested how accurate subjects were, but their degree of repeatability. For this reason, the absolute difference between the first and the repeated value was calculated at both levels, for each song and for every subject. A double-threshold method was implemented to evaluate consistency:

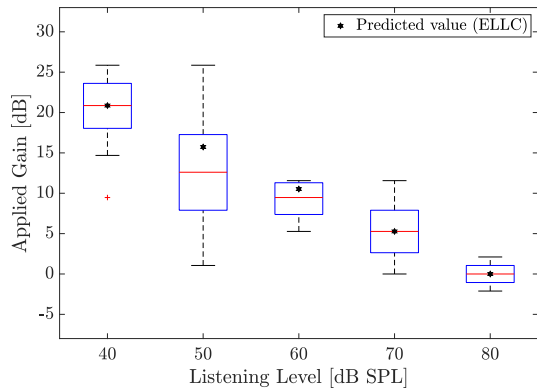
1. If $\forall i \Delta 80_i \leq 6$ dB, subject is consistent;
2. Otherwise, if $\Delta 80_i > 6$ dB for one track and $\Delta 60_i \leq 10$ dB for at least two tracks, subject is consistent;
3. Otherwise, subject is inconsistent and discarded.

Here $i = 1, 2, 3$ is the number of the track and $\Delta 80_i$ and $\Delta 60_i$ are the differences of the two instances. Since human perception was evaluated, it was reasonable to have a stricter threshold at the reference level (80 dB), where the spectral balance of two signals with the same level were matched, and a more relaxed threshold for the attenuated level (60 dB), which required a harder task of matching the spectral balance of two signals with different listening levels. A total of 18 subjects participated in the test; 11 of them passed the consistency screening and were included in the analysis.

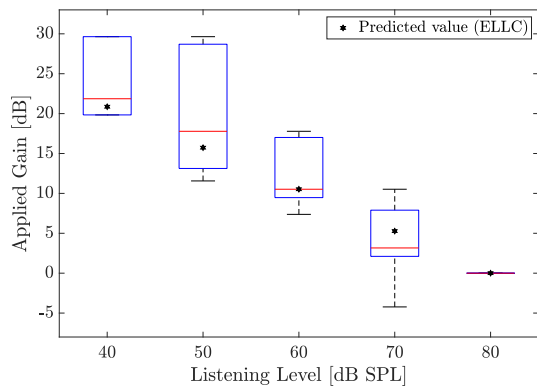
6.4 Results

Results from the listening test show that ELLC can reproduce the average response of the listeners. This is shown by the box plots in Figs. 10 and 11, where the box plots present the median (red line), the 25th and 75th percentiles (blue rectangle), the extension to the most extreme data points not considered outliers (black whiskers) and the outliers (red cross). Furthermore, the black markers represent the predicted correct compensation that matches the level of the ELLC.

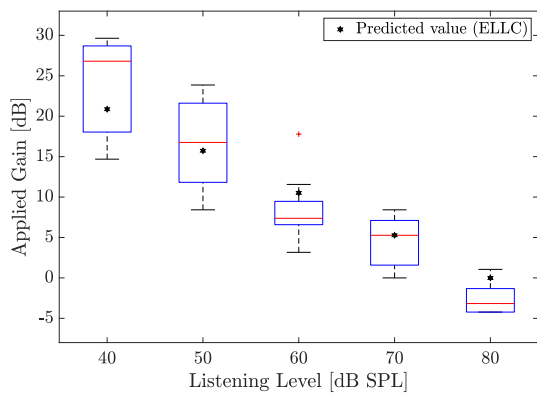
Fig. 10 plots the applied compensation versus the listening level of the track, showing how such compensation adapts to the level and increases towards the lowest levels.



(a) Track 1



(b) Track 2



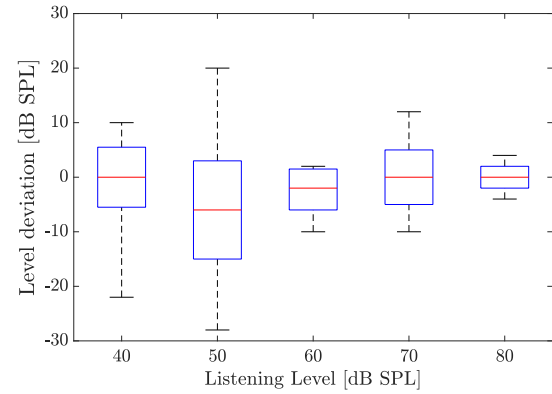
(c) Track 3

Figure 10: Test results, grouped by song. Box-plot of compensation introduced by subjects at each listening level.

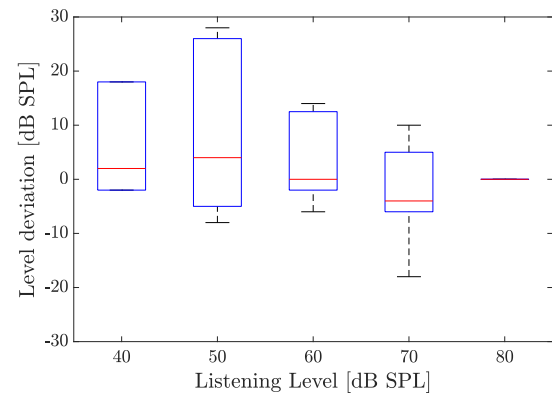
Fig. 11 shows the level deviations, so it is easier to see the goodness of the results and the cases of under or over compensation. As expected, data presents moderate variance, due to the difficulty of the task; nevertheless, the median of the error in level evaluation always lies in a close range near 0 dB (Fig. 11).

Analyzing the results, it is possible to state the following:

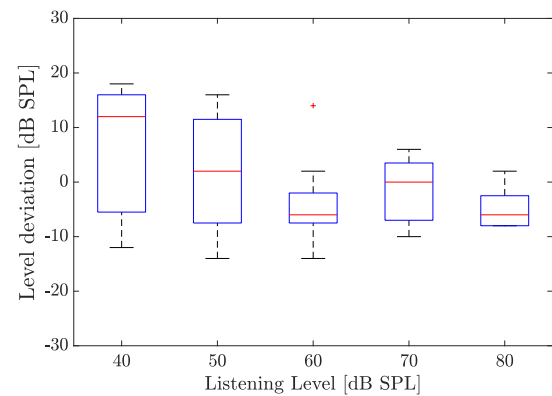
- The reference was matched quite well by almost all listeners for all samples, with slightly worst accuracy for Track 3 (Fig. 10c and 11c);



(a) Track 1



(b) Track 2



(c) Track 3

Figure 11: Test results, grouped by song. Box-plot of error in level evaluation versus the corresponding correct level.

- Fairly good results were obtained in typical music listening range (60 and 70 dB SPL);
- The variance increased towards the lowest levels (40 and 50 dB SPL), where sound was really quiet and the task of matching the perceived spectral balance became harder.

It is interesting to notice that, for Track 2 (Figs. 10b and 11b), the majority of the listeners tended to overcompensate when the music level went down. This makes sense, due to the sample having narrow spectral content and, as a

consequence, no “untouched” frequency components to be compared to, increasing the difficulty.

After taking the test, the subjects were asked for a feedback. They confirmed the difficulty of matching the spectral balance, when the level of reproduction went down. It was also difficult to notice the audible difference among small changes of the slider, since only the lowest frequencies were affected. They also stated that bass contribution was noticeable and pleasing.

7. CONCLUSION

A loudness compensation function derived from the equal-loudness-level contours and implemented via digital filters was proposed. This function introduces an adaptive contribution to the bass based on the listening level, in order to balance the perceived spectral variations given by the nonlinear response of the human hearing system.

Among different typologies, the first-order low-shelving filter with gain adjustment and fixed crossover frequency was shown to provide a high-fidelity approximation of the compensation function for a wide range of music listening levels. Its low computational complexity enables a real-time implementation.

A formal adaptive listening test was designed and conducted to validate the accuracy of the proposed compensation method, which was proved by the test results. Future work on this topic might include on-chip applications, customization for specific hardware or environments, and new listening tests conducted with a larger pool of non-trained listeners reflecting consumer market.

Acknowledgments

This work was conducted between October 4, 2018 and January 22, 2019, when L. Fierro was visiting the Aalto Acoustics Lab. The support of the University of Brescia and Prof. P. Migliorati is gracefully acknowledged. This research was partially supported by NordForsk’s Nordic University Hub “Nordic Sound and Music Computing Network – NordicSMC”, project no. 86892. Special thanks are due to the Aalto Acoustics Lab staff members participating in the listening test.

8. REFERENCES

- [1] G. H. Fletcher and W. A. Munson, “Loudness, its definition, measurement, and calculation,” *J. Acoust. Soc. Am.*, vol. 5, no. 2, pp. 82–108, Oct. 1933.
- [2] D. W. Robinson and R. S. Dadson, “A redetermination of the equal-loudness relations for pure tones,” *Brit. J. Appl. Phys.*, vol. 7, pp. 156–181, May 1956.
- [3] A. L. Newcomb Jr. and R. N. Young, “Practical loudness: An active circuit approach,” *J. Audio Eng. Soc.*, vol. 24, no. 1, pp. 32–35, Jan./Feb. 1976.
- [4] T. Holman and F. Kampmann, “Loudness compensation: Use and abuse,” *J. Audio Eng. Soc.*, vol. 26, no. 7/8, pp. 526–536, Jul./Aug. 1978.
- [5] A. Seefeldt, “Loudness domain signal processing,” in *Proc. Audio Eng. Soc. 123rd Conv.*, New York, NY, USA, Oct. 2007.
- [6] P. D. Prasad, “A low complexity approach for loudness compensation,” in *Proc. Audio Eng. Soc. 129th Conv.*, San Francisco, CA, USA, Nov. 2010.
- [7] O. Hawker and W. Wang, “A method of equal loudness compensation for uncalibrated listening systems,” in *Proc. Audio Eng. Soc. 139th Conv.*, New York, NY, USA, Oct.–Nov. 2015.
- [8] B. Katz, *Mastering Audio: The Art and the Science*, 2nd ed. Butterworth-Heinemann, 2014.
- [9] NIOSH-CDC, *Occupational Noise Exposure—Revised criteria for a recommended standard*, 1998. [Online]. Available: <https://www.cdc.gov/niosh/docs/98-126/pdfs/98-126.pdf>
- [10] S. Widén, S. Båsjö, C. Möller, and K. Kähäri, “Headphone listening habits and hearing thresholds in Swedish adolescents,” *Noise & Health*, vol. 19, no. 88, pp. 125–132, May–June 2017.
- [11] S. B. Nielsen, “A loudness function for analog and digital sound systems based on equal loudness level contours,” in *Proc. Audio Eng. Soc. 140th Conv.*, Paris, France, June 2016, paper no. 9559.
- [12] ISO 226:2003, “Acoustics – Normal equal-loudness-level contours,” ISO/TC 43 Acoustics, International Standard, Aug. 2003.
- [13] V. Välimäki and J. D. Reiss, “All about audio equalization: Solutions and frontiers,” *Applied Sciences*, vol. 6, no. 5/129, May 2016.
- [14] J. O. Smith III, *Physical Audio Signal Processing: For Virtual Musical Instruments and Audio Effects*. W3K Publishing, 2010.
- [15] ———, *Introduction to Digital Filters with Audio Applications*. W3K Publishing, 2007.
- [16] C. P. G. Tsirimokou and A. Elwakil, *Design of CMOS Analog Integrated Fractional-Order Circuits*. Springer, 2017.
- [17] U. Zölzer, *Digital Audio Signal Processing*, 2nd ed. Chichester, UK: Wiley, 2008.
- [18] S. U. Ahmad, “Design of digital filters using genetic algorithms,” Ph.D. dissertation, University of Victoria, Victoria, BC, Canada, Dec. 2008. [Online]. Available: <https://dspace.library.uvic.ca/handle/1828/1294>
- [19] IEC 60318-4, “Electroacoustics – Simulators of human head and ear – Part 4: Occluded-ear simulator for the measurement of earphones coupled to the ear by means of ear inserts,” IEC/TC 29 Electroacoustics, International Standard, Jan. 2010.
- [20] ITU-R BS.1770-4, “Algorithms to measure audio programme loudness and true-peak audio level,” ITU/Radiocommunications, standard, Oct. 2015.