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COMPARATIVE ANALYSIS OF METHODS OF CALCULATION IN TRANSIENT AND WAVE PROCESSES IN ELECTRIC CIRCUITS

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Abstract. The problem of calculating the transient and wave processes in the circuits with the concentrated and distributed parameters is examined. A comparative-qualitative analysis of the analytical and numerical methods used for these purposes was carried out, indicating the advantages and disadvantages of their application. It is presented algorithms for applying the examined methods. It is found that numerical calculation methods have many advantages in studying stationary and dynamic processes in the natural sequence of processes course in the circuit. It is recommended to use the finite difference method. It is presents the main phases of realization of the application of a numerical calculation method and results of the comparative and qualitative analysis of the analytical methods (classical, state variables, frequency, stationary waves, Fourier transform, operational, traveling waves) and numerical (finite difference, finite element, finite volume).

Keywords: *distributed parameters, traveling waves, numerical methods, finite difference method, algorithm, advantages.*

Introduction

The transmission and distribution lines of the electrical energy are a very important functional element of power systems. In these structures are manifested most frequently of the particularities of the characteristic regimes for the circuits with the distributed parameters. As a consequence, is the necessary to solving a wide range of technical problems are required for the design of overhead power lines and cable lines, such as the determination of the reasonable level of insulation, requirements for protection systems, power flow management, static stability and dynamic stability, loss calculation on permanent and transients modes etc.

Solving this problem cluster requires new approaches and application of new computational calculation methods of both the permanent regime and the dynamic regime to perform various operative maneuvers in the power system. These suggestions also refer to non-stationary and dynamic regimes. The quality and completeness of solving the problems of designing and operating power systems depends on the depth of knowledge of all aspects of operation in both permanent and non-stationary regimes.

1. Structural and functional characteristics of electrical circuits. Investigation issue

Traditionally, calculation of the operating regimes of the power systems is done separately for permanent and transient regimes. The methods currently used operate with the equivalent scheme of electrical lines with generalized parameter values. The equivalent calculation scheme has the structure diagram of the phase of the electric line which has the m phases. For this situation, it applies a hypothesis that considers that the phases of a polyphase system, for example with m-phases, are symmetrical.

In fact, electrical processes in power systems are continuous and uninterrupted. Based on this finding it can be considered rationally to examine the electrical and electromagnetic processes in their natural sequence of development, because any transient process ends with the attainment of the status of the permanent regime, and each permanent regime is the beginning of a transitional regime. In circuits with distributed parameters, the fast transient process has a waveform. This mechanism is conditioned by the high velocity of propagation of the potential (voltage) and current waves in power lines.

The study of wave processes in power equipment (electrical machines, transformers, suspended cables and multiple cables, overhead power lines) has been the focus of many researchers and the scientists [1-13]. As a significant indicator of the functioning of the power systems, the energy losses are considered. The reduction of losses is possible based on the deep knowledge of the essence of process developments at all the phases of transformation and transport of power flows through the circuits of the power system. Consumption is an independent and random variable, and system operator interventions have to be executed based on algorithms that can ensure optimization of the system, for example, based the diminishing of losses of the controlled object [14].

Structural inhomogeneity is a frequent feature of power systems. The inhomogeneities significantly influence processes and operating regimes, including in the power system [11]. Electronic computers and the development of numerical computational software allow a new approach to the problem of the research of the regimes in the electric circuits with concentrated and distributed parameters compared to the known classical methods [12, 13, 15].

The analysis of scientific publications in this field indicates that there is currently no single and strictly formulated wording on the methodology of calculating the transient and waveforms of energy indicators that are used to characterize power transmission processes in distributed parameter circuits and variables. For the description of the waveforms and the calculation of these phenomena in the electric lines, the telegraph equations are most frequently used [4, 5, 16]. It can be mentioned that the reasoned analytical solution is known only for voltage and current waveforms with rectangular slope when propagating them in the semi-finite homogeneous line without losses [4].

The inhomogeneity of distributed parameter circuits is another major factor that creates great difficulty in calculating the operating modes of the long line with distributed parameters, including the transient regimes in these circuits. The approximation of the long line through the quadriplets allows the processes to be calculated in the non-homogeneous lines with a satisfactory accuracy [17, 18]. This finding is correct in the case of solving the direct wave propagation problem in circuits with distributed parameters, for example waves generated by partial discharge in the stator winding of high power electric machines [19] and high voltage cables [20]. The establishment of the stationary short circuit in electric networks can also be seen as a non-homogeneity of the circuit. Often, the short circuit mode is an unbalanced short-circuit, which makes it difficult to analyze the operation of the electrical networks. In the case of unsymmetrical short-circuits it is necessary to use calculation methods that operate with the direct, indirect and monopolar sequences of the currents, which leads to the increase of the volume of calculations even for the stationary short-circuit [9, 21].

The electrical lines are constructed with several phases and circuits. This creates difficulties in investigating electromagnetic processes in lines with many conductors. In the this lines is the effect is noticed of the mutual influence between the phases and the circuitry of the line, which is not taken into account when analyzing the particularities of the operation of the three-phase lines. This leads to different loading of the phases, even case at symmetrical load and in the granted mode of the polyphase line with the load [22]. For these reasons, the substitution of the three-phase electrical line with its one-conductor equivalent is an approximation, which has a methodological error from the start.

The inhomogeneities caused by the constructional particularities and the topology of the physical infrastructure of the electrical networks (branching, connection of the electric energy receptors at different points of the circuit or the connection of the distributed generation sources) are the causes of additional methodological errors in obtaining the sought solutions. The broadband deviation of geometric dimensions, the jump change of electrophysical parameters at the marginal boundaries of the construction elements, present real difficulties and limit the fields of use of traditional methods of calculating the transient and wave modes in non-homogeneous circuits with distributed parameters.

The use of Maxwell equations to study the interaction of electromagnetic phenomena in electrodynamics presents a difficult mathematical problem due to the need to consider the interaction of the electromagnetic field with the non-homogeneous environment [22] and the particularities of changing the direction of energy transfer in networks with smart grid properties [2, 3]. The development and use of computational methods and mathematical models to study dynamic processes in heterogeneous environments is a complex and non-trivial problem. The theoretical bases for solving these problems are the Maxwell equations in differential and integral form. The telegraph equations are an approximation of the Maxwell equations. The community of these equations systems presents a unique methodological platform for studying the various problems characteristic of contemporary power systems.

No deductive procedure used to calculate or predict a process does not accept large gradients evolution of unknown variables and calculated functions (time and space). As an example of this type of circuit, it is possible to indicate the circuit formed by overhead lines when they include cable lines, Franklin's lightning arrestor with a vertical earthling electrode, lightning current leakage circuit, stator windings of electric machines great power, etc. Different portions of these circuits have a large jump ratio of parameter values, for example, the characteristic impedance of the airline and cable sections, which can reach the value of the 8 ... 12 ratio.

In breakdown modes (phase conductor rupture, ground fault), these phenomena may be equivalent to a change in the line load at that point from the idle mode (IM) to the short circuit (SC). We will mention that this change also has the transition phase from one regime to another marginal regime, ie from IM to SC and vice versa. The development and use of mathematical models to study dynamic processes in non-homogeneous environments is a complex and non-trivial problem.

This paper analyzes the advantages and disadvantages of some methods of calculating the transient and wave processes in the long lines, arguing the utility and advantages of applying the apparatus of mathematical physics and the numerical methods of calculus, which adapted to solving problems in the field of the modern of the power systems and electrical engineering.

2. Calculation methods of the waves and transitory processes

The development of the computing technique has opened new horizons in the research of the transient and wave processes in the non-homogeneous circuits with lumped and distributed parameters. Computational methods use mathematical models of electrical circuits that are equivalent to the respective physical infrastructure. So, the first step of any study in this field is to formulate the problem, the mathematical description of the studied object, the selection of the method of solving the equation or the system of equations, and the analysis of the obtained results.

The mathematical model presents a system of equations with a variable or several independent variables. In all cases, the coefficients in equations are parameters of the equivalent circuit, which are determined experimentally [24, 25], by calculation based on the design data [26] or as a result of a synthetic approach, using the results of the measurements of the instantaneous electrical values for calculating the parameters of the equivalent circuits schematics [27].

The permanent mode is characterized by constant values in time of currents and voltages in the electric circuits. This is also true if currents and voltages are regular functions, mostly harmonic functions. In the case of transient regimes, currents and voltages are non-periodic functions in the circuit elements.

If the circuit satisfies these conditions then this circuit is in a transient or non-stationary state.

Methods of calculating regimes in electrical circuits are divided into two large groups: methods operating in the time domain and in the frequency domain. These particularities determine the method of calculating the mode of the analyzed circuit. Table 1 presents succinct information that characterizes the basic methods used to calculate the transient regimes in lumped - parameter circuits [28].

Table 1

Basic methods used to calculate the transient processes in circuits [28].

Name of the method	Additional conditions limiting the scope	The conditions for estimating the accuracy of the solution	Simplicity of use	Development opportunities on future
Classic method	1) Equations must be single-function functions. 2) The number of integration constants should not be high. 3) In the equations containing integral the determined functions must have derivatives.	There are, but they are little known. They can be determined by using methods known in mathematics.	The method is not simple. It is need to determine the integration constants.	The possibilities are great, but their realization is facing great difficulties.

Continuation Table 1.

The Cauchy-Hevis-aide method	1) The initial and boundary values of the searched functions must be zero. Otherwise, it is necessary to use artificial operations. 2) The product's function t (time) is missing. 3) The method does not indicate estimates of marginal values the domain use.	The method does not allow determination of these conditions. In part, conditions can be determined by analogy with the Laplace method.	The method is very simple.	The possibilities are very limited. The exception - method development in the analogous direction to the Laplace transform
Fourier transform method	1) It is not robust for initial and limit conditions other than zero. 2) Unknown functions must have the Fourier image (functions with breaks of type 1 are admitted, but many useful functions are excluded).	Exact conditions are known.	The method is excellent.	The possibilities are limited because it can only be used for functions with Fourier image.
The method of the Laplace transform	The determined and sought functions must have the Laplace image (It is useful for all functions for which the Cauchy-Hevisaide and Fourier methods are robust). It is robust for many functions that can not be determined by the classic method.	Exact conditions are known.	The method is excellent.	The possibilities are great.

2.1. Analytical methods

Classic method. This method is based on the direct integration of the differential equations describing the electromagnetic state of the circuit. The solutions obtained are time functions of currents and voltages in the elements and portions (chains) of the analyzed electric circuit. The currents for the transient phase until the process is completed for this mode are determined, so finally, the characteristic values for the stationary phase are obtained.

The method is used to calculate the mode in linear electrical circuits. The solution has two components: the forced component and the free component, which overlap simultaneously in the run-through of the transient process. The time length of the transient process is determined by the attenuation time of the free component to zero value or very close to zero. The state equations of the circuit are compiled according to Ohm and Kirchhoff's laws for instantaneous voltages and currents.

The process of obtaining the solution of the analyzed problem includes the following steps: define the independent variables; the initial conditions (determination of voltage values on capacities and currents in inductances for time $t = 0$) are formulated; is describes the electromagnetic state of the circuit with differential equations; we determine the relations of the general solution, which includes the solution of the homogeneous differential equation and the particular solution of the non-homogeneous differential equation; the root values of the characteristic equation are calculated; is determined from the initial conditions the integration constants. Obtaining the solution is not a problem if the differential equation order describing the equilibrium state of the voltages is not great.

The Duhamel integral method. It is used for the case that the signals applied to the circuit terminals have an arbitrary form of evolution over time. The method is robust for linear electrical circuits. The general solution is obtained by summing the circuit reactions to disturbance with type signals, which are presented of the voltage jump at discreet moments of the time in the arbitrary shape definition range. For this it is necessary to know the circuit response to the perturbation formed by the unitary value signal. The circuit response for this type of stress is described by the transient conductivity function g (t) of the current and the transient response function of the voltage h(t) [9, 29]. The general solution is obtained using the concept of superposition of the circuit reactions to the perturbations conditioned by the signals with the form in the step, which are a way of approximating the original function with the arbitrary shape, applied to the terminals of the studied circuit. Duhamel integral is used. The independent variable is denoted by τ and the duration of the time interval for determining the current evolution in the circuit is denoted by t.

The execution order of the calculation option of the transient process when using the Duhamel integral: the transient function g (t) with the classical method or the operational method of calculating the transient processes is determined; we calculate the derivative of the function under the sign of the Dhuamel integral (function that integrates with the independent variable τ). This is done by determining the relation of the derivative of the known function with the independent variable t (for each voltage-perturbed disturbance which approximating the arbitrary shape signal). In the obtained function of the derivative is substituted the size denoted by t with the symbol τ; the relationship of the Duhamel integral is drawn from the moment t = 0 up to the defined time limit t; the solution of the Duhamel integral for the timeframe defined by the respective integration limits is obtained.

The state variable method. The state variable method is an orderly method for determining the electromagnetic state of a circuit based on solving a system of first order differential equations. For this it is necessary to draw up a system of first order differential equations. The number of system equations is determined by the number of independent energy storage elements indicated in the equivalent scheme of the circuit. These equations must satisfy two conditions: the equations must be independent relations and ensure the possibility of retrieval according to the known values of the state variables, the state of other arbitrary variables. Practice indicates that as state variables it is reasonable to select the currents in inductances or the total magnetic flux in these elements, the load or the voltage of the capacitors. Knowledge of the temporal evolution laws of these physical quantities allows substitution of these elements with voltage and current sources with known parameters. The portions of the circuit without energy storage elements are represented by passive elements in which the current can be calculated according to the known values of the voltage and current source parameters.

When applying of the state variable method, an algebraic system of equations, which includes unknown state variables and external sources of disturbance, is developed. The system of equations is in the form of a matrix. It may be mentioned that for complex circuits there may be some difficulty in making algebraic equations, using Kirchhoff laws. In order to overcome this difficulty, a formalized method for the elaboration of state equations has been proposed.

The method includes the following steps: compiling the directional graph of the circuit with the shaft selection, which includes all the capacitors and the voltage sources of circuit. The resistances (passive elements) are used to ensure the connection of all the nodes by the tree; the chain graphs are assigned; start assigning the chain numbers to the capacitor circuit, then to the resistor chains, and the latter to the chains with inductive elements; the table reflecting the connection of the circuit elements is completed.

The first row of the table is completed with the capacitive, resistive elements and the graph tree's voltage sources. The first column of the table includes the resistive and inductive elements of the communication (link) and power sources. Completing the free fields of the table consists in alternating the tree branch closing by means of the communication branches until the circuit loop is formed. The '+' sign marks the graph chains, the orientation of which coincides with the circular direction of the loop and with the sign '-' chains that have a counter-orientation in the direction of movement. The final result consists of filling in all the fields in the table. The fields that form the columns of the table correspond to the first Kirchhoff law, the lines that form the rows of the table correspond to Kirchhoff's second law. It is necessary that when converting the data forming the rows of the connection status table into algebraic equations, the opposite sign is taken for passive elements as compared to the one indicated in the status table. Using the completed table in the order described allows obtaining the equation system of the analyzed circuit using a rigid and formalized algorithm for drawing the equilibrium voltage equations the circuit.

The operational method. The operational (symbolic) method consists of solving a system of algebraic equations for the images of the desired variables, followed by a transition from the functions of the operational solution images to the original functions. This method is applicable to linear equations with variable coefficients, so it can be used to determine the characteristics of the transient process and in the circuits with distributed parameters.

The essence of the operational method consists in the substitution of the original function and its derivatives from the time domain with an associated function $F(p)$, whose independent variable is a complex number $p = a \pm j\ b$ [9]. This associated function $F(p)$ is called the original function image f (t). Derivatives and integers in the integral-differential equations are substituted with the respective images. The derivative option is substituted by multiplication to the operator p, and the integration operation by division to the operator p. As a consequence of these substitutions the transformation of the integral-differential equations in the algebraic equations, in which the operator p presents the independent variable, is ensured.

In the [30] the following algorithm for the application of the operational calculation method of the regime in the electric circuits is proposed: calculate the permanent regime before the transition state ($t < t_0$), to obtain the initial values for the currents in inductances i_{Lm} and the voltage on the capacitances of the circuit u_{Cn}, where m and n are the respective inductive and capacitive elements. If time $t_0 \neq 0$ (the start time of the transient process in the circuit) a change of the origin of time $t' = t - t_0$ is made because the unilateral Laplace transformation is defined for t > 0; it is built the diagram of operational circuit, the operational circuit diagram, corresponding to its structure at time t = 0+, which will contain the operational impedances of the passive elements of circuit , the Laplace images of the voltage and current independent sources and of the "fictitious" sources corresponding to the non-zero initial conditions $i_{Lm}(0_-)$ and $u_{Cn}(0_-)$; depending on the structure and complexity of the equivalent operational scheme obtained in step 2, the more appropriate method of analysis (Kirchhoff operational formulas, loop current method in operational formulas, classical nodal method or operational nodal method, superposition theorem, transfigurations methods, picture-type methods etc.) to determine Laplace images of unknown functions (circuit variables - I_{Lm} (p) and U_{Cn} (p)); the determined of the original unknown functions with applied by one of the inverse transformation methods, for example, using the Heaviside formulas.

Frequency method based on Fourier transform. This method is widely used in solving synthesis problems. The frequency method is used in circuits whose input characteristic is a non-periodic function. The non-periodic function is represented by the sum of an infinite set of sinusoidal functions with infinitely small amplitudes and frequencies, which have all possible values from -∞ to + ∞ [31]. The decomposition of non-sinusoidal function into sinusoidal components allows the use of known calculation methods of linear electrical circuits. The currents in the circuit are calculated from the action of the individual voltage components, after which the resulting current is determined using the overlapping method. Any non-sinusoidal function that has a finite number of extreme values (min. and max.), and discontinuities of the first type over a complete period can be represented as a Fourier series. The non-periodic function is characterized by a continuous frequency spectrum.

Transient process calculation algorithm by frequency method: based on the initial data, the equivalent scheme of the circuit is elaborated; it is determine the harmonic spectrum of the signal applied to the analyzed circuit by determining the current and voltage values for the circuit reference points; it is determine the current and voltage frequency characteristic and with the application of images or of the decomposition theorem it is determine the time function of the unknown variable, eg the current $i(t)$. The advantage of the frequency method is that this method can be applied to any circuit regardless of its degree complexity of circuit.

The method of stationary waves. When solving engineering problems, it is reasonable to use the Fourier method. In this case the solution presents itself as an infinite array of harmonics with the stationary waveform. The disadvantage of this method is the fact that for some problems, non-orthogonal own functions are obtained. This does not allow the determination of Fourier coefficients independently of each other. The particularities of the execution of the non-static processes calculations in the with high gradient circuits of the distributed parameter values lead to the Gibbs [32] phenomenon in the solutions obtained since the initial voltages and current functions fulfilling the pre-commutation limit conditions must be arranged in series, based on its own functions, which satisfy the limit conditions of switching. Practically, calculations in the presence of the Gibbs phenomenon can be made by summing the arithmetic mean values of the series [32] or the Lanczos multipliers method is used [33].

An advantage of the frequency method is the possibility of relatively fast determination of the spectrum of the natural frequencies of the analyzed system, the knowledge of which is necessary for the subsequent calculation of the surges amplitude. Knowing the frequency characteristics of the system allows the formulation of recommendations and the determination of solutions to exclude resonance phenomena and disturbance filtering that may affect the operation of protection systems and control and measurement circuits. It can be mentioned that, compared to the frequency method [34], the use of the Fourier method in calculating the overvoltage values only requires knowledge of the discreet spectrum of the circuit's own frequencies and characteristic impedance values for these own frequencies (according to the Dirichlet Theorem [35]).

Wave way method (d'Alembert waves). This method in the best way corresponds to the physical essence of the transient processes in the circuits with distributed parameter [4]. At the same time, the direct application of the d'Alembert method to solving mixed problems is typically used for lossless lines and the lines that do not distort the shape of the traveling waves. In other cases, it is impossible to talk about the propagation of current and potential waves in the exact sense of the waveform definition, since the wave way form changes very much during their movement from the input point to the circuit [36, 37]. This method allows to be calculated of the wave amplitude for the first wave propagation in the circuit. Accounting for wave reflections and refractions for subsequent time intervals, leads to complications of the computational algorithms and a rapid increase in the amount of information that is needed for the memorize [38].

2.2. Numerical calculation methods

The use of the theory and classical methods of calculating regimes in electrical circuits, including the transient regimes, may face several difficulties. Analytical methods have limited applicability and are not capable of covering a wide range of problems characteristic of modern power systems. As a result of the development of the computing technique, the scope of numerical calculation methods has been extended. The computers allow overcoming many barriers conditioned by necessary to work with the large volumes information in the time during the numerical calculations.

The numerical analysis of the transient processes in the circuits with concentrated parameters and the circuits with distributed parameters it is based on the use of the mathematical models of the studied objects, which as a rule are presented by systems of integral - differential equations or differential equations [39], which including the distributed parameters. These models are developed by ensuring steady state for both permanent and transient regimes. In the latter case, it is necessary to operated with instantaneous values, which it is the functions of the time and space.

There are approved algorithms that are used to compile and present state equations of electrical circuits in the form of canonical equations. This ensures the possibility of solving these equations with approved numerical methods, which ensures the credibility and precision of the obtained numerical solutions. However, the application of these algorithms and of test methods is not possible for some non-homogeneous electrical circuits with jump of the parameter values at the boundaries points of connection of different circuit portions. They lead to the necessity to modify both the algorithms and the numerical methods of calculation, the decomposition of the studied problem and its solving in several phases, for example the determination of the initial and limit conditions at the boundaries points of connection of different circuit portions.

The application of the traditional methods and numerical methods for the integration of the differential state equations faces difficulties in investigating with slow and fast speed electromagnetic processes in electrical networks, characterized by: initial rapid change and a slow change of the dynamic process in the circuit; the jump change of parameters in the circuit, including in circuits with distributed parameters and the jump change of the electrical parameters that can change under the influence of disturbances from the outside of the circuit as well as internal switches in circuit; concomitantly running multiple processes with different frequencies.

Numerical methods of calculation are from the beginning approximate calculation methods. The methods allow formalization of the solution obtaining procedure because by introducing a set of basic functions (frequently continuous functions on continuous function mesh intervals) can be excluded from the differential equation solving algorithm in the process of searching for complex functions, which satisfy this differential equation and marginal conditions [39]. Basic functions allow determination derivatives of differential equation and relationships describing the energy (mechanical or electrical) of the studied object. As a result of these approximations, the continuous function with a single variable or more independent variables it is presented through the discrete values in the some landmarks (characteristic nodes). In this context, continuous function analysis is done by executing a simple set of algebraic operations with the known values of functions in the discrete points (nodes) of the numerical computation network. Thus, the use of numerical computational methods to solve problems in the field of mathematical physics is reduced to the use of simple calculation methods, which are executed with the use of electronic computing machines.

2.3. Algorithm of obtained the numerical solution

The process of solving complex problems with the use of numerical calculation methods is reduced to the following steps [40]:

Physical formulation of the problem. At this stage it is necessary to correctly formulate the question of the investigation, taking into account the particularities of the studied object, as well as the physical essence of the processes that unfold in this object. In order to ensure the correctness of the formulation of the problem it is necessary to study deeply the issue in question.

Mathematical formulation of the problem. Physical problem formulation is executed in a mathematical language (mathematical model). So, the physical problem is presented in the form of mathematical equations (algebraic, differential, integral, integral-differential or equation systems). The mathematical model must correspond to the fundamental laws of the studied physical process.

Mathematical analysis continues. This phase is operated with functions, with physical sizes presented in general form. So, we are looking for a solution to the problem presented in a general form, which is described by mathematical formulas without introducing concrete values of the independent coefficients or variables and the variables sought into this solution.

Numerical methods. The problem solution is presented as mathematical finite operations - addition, multiplication. Numerical methods make it possible to reduce the solution of a problem to executing a finite number of arithmetic operations with numerical values. The results are obtained as numerical values.

Algorithmization. Algorithmization serves to simplify actions in the form of an accurate description of the process. The algorithm can be described as a diagram (scheme)or described in another way.

Programming. The algorithm for obtaining the solution is made as a computing software in some high-level programming languages.

Adjusting the computing software. At this stage, a search is made of the errors that occurred during the implementation of the previous steps. The software is tested by solving the standard problems whose solutions, often analytical, are known and credible. This ensures the credibility and reliability of the numeric solutions obtained with this software. In case of failure, it is necessary to review the formulation of the investigation problem and the steps described above.

Making calculations. At this stage, the initial data needed to execute the programmed calculations are prepared. The calculations are based on a work program, which is elaborated, based on the purpose and objectives of the investigation.

Analysis of numerical results. The analysis of the results is necessary to estimate the correctness and veracity of the numerical solution obtained, as well as the subsequent refinement of the mathematical model based on these results.

2.4. Quality criteria of numeric solution

When using the algorithm to achieve the numerical computation process it is necessary to satisfy the requirements for ensuring the accuracy of the numerical solution obtained and the convergence of the calculation method. The convergence of discrete methods (methods based on the substitution of continuous functions with discrete values for fixed points) is defined as the tending of the values of the solution obtained with the numerical method of calculation to the values of the original solution of the problem, when decreasing the step size of discretization of the numeric grid.

The problem formulation can be considered correct if the solution is unique and stable for any set of initial data that corresponds to the problem under consideration. Incorrect formulations will greatly increase errors in the process of numerical computing, which is unacceptable. The solution obtained by a numerical method is usually approximate, so it contains an error. As sources of errors can be indicated: inappropriate formulation of the mathematical problem for the studied phenomenon; initial data error; the error of the method used; rounding errors in arithmetic operations and other number operations. The method must have a low sensitivity to the initial data errors. For this, small errors in the initial data should lead to small errors in the numerical solution. In another case, the instability of the calculation process and the inadequate result of the numerical analysis are obtained.

The numerical method can be considered correctly selected if its error is several times less than the inevitable error, and the error due to rounding, called calculated error, is several times smaller than the method error. If there is no fatal error, the method error should be somewhat less than the specified inaccuracy. Thus, in order to obtain a solution with the required precision, the problem formulation must be correct and the numerical method used must be convergent and ensure the stability of the calculation process, and therefore the correctness of the respective numerical solution.

2.5. Characteristic of numerical methods

Finite difference method. Numerical method for solving differential equations is based on the option of replacing derivatives with components presented in the form of differences in function values.

The essence of the method consists in building a computing network (s) on that surface or a linear structure formed from discrete portions (which may have different dimensions or lengths). It is select the calculation scheme for finite differences and for each node of the network and a difference equation is elaborated (a relationship analogous to the initial equation). It is necessary to formulate the limit conditions for the structure of the built-up network. This leads to system of linear algebraic equations.

Solving this system assures us the determination of the approximate values of the variables searched in the nodes of the computation network (linear or plane). The difficulty in applying the method is determined by the correctness of the computation scheme construction in differences to ensure the convergence of the calculation process for the solution of the equation system. The construction of the calculation scheme is based on the properties of the initial differential operator.

The finite difference method is robust in examining variable time processes (non-static). In this case, the iterative calculation method applies. At each iteration we find a solution to a new layer of time. To solve such problems, explicit, implicit and predictor-correction schemes (a pair of explicit and implicitly chosen schemes) are used. Explicit schemes and predictor-correction schemes simply recalculate the calculated magnitude value using information from the earlier layers of time [11]. Using a default scheme leads to the solution of the respective equation (or of a system of equations). For parabolic and hyperbolic equations, methods are often used concurrently because the time variable derivatives is approximated by the scheme in differences, and the space operator is approximated using a finite element formula [41].

Finite element method. The finite element method was developed due to the needs of the field of construction mechanics and the elasticity theory, including for the purpose of using solutions of differential equations (calculation of dam structures). The finite element method is currently used for modeling diffusion, thermal conductivity, hydrodynamics, mechanics, and electrodynamics [39, 42]. Based on this method, powerful software for calculating physical fields, including electromagnetic fields in limited volumes [43], has been developed.

The essence of the method is to divide the domain into a finite number of subdomains (elements). In each element, the approximation function is arbitrarily selected, for example, it is a polynomial. În afara elementului, această funcție este zero. At the boundaries of the finite elements (nodes) the selected function is considered as a solution to an investigated problem, the value of which is not known in advance. The values of coefficients of the approximated function are determined by the equation of the values of neighboring functions at the boundaries of the finite elements (at the nodes), with the expression of these coefficients by the values of the functions in the element nodes. A system of linear algebraic equations is compiled. The number of equations is equal to the number of unknown values in the nodes looking for the original system solution. Each element is associated with a limited number of neighbors and the system of linear algebraic equations has a low aspect. This greatly simplifies the process of obtaining of solution of the system of algebraic equations.

Finite volume method. The numerical method is used to integrate equation systems with partial-derivative. It is used to calculate physical fields in a limited volume. Processes in the selected domain can be generated by fluid, gas, and electromagnetic fields. Macroscopic parameters are presented as unknown sizes: speed fields, pressure, electric field and / or magnetic field distribution. The fields are described by equations that satisfy mathematically formulated laws, eg Maxwell equations [9, 44, 45].

It is considered that for any value at any point in the space that is limited by a closed volume, at the moment the sum of the physical size in this selected volume can only be changed as a result of the following processes: transmission of the quantity of this physical quantity through the surface limiting the selected volume (the process is characterized as flow) and the generation or annihilation of a quantity of the size that characterizes the physical state within the selected and controlled volume (source or leakage).

If the problem is formulated in terms of the finite volume method, the physical interpretation of the studied quantity is used. For example, when solving heat transfer problems, the heat conservation law is used in each control volume.

3. Comparative analysis of methods of calculation a transient processes

The description of the transient processes is done using differential equations. The problem of studying the transient processes is reduced in this case to obtaining the solution of the equations or the system of differential equations. If the differential equations are of order I or order II, it is quite easy to obtain the analytical solution. To this end, knowing the initial and limit conditions, it is necessary to calculate the roots of the characteristic equation of the differential equation. These calculations can be performed using any of the methods previously reviewed.

Classic method. Method is effective is when the differential equations are of order I or order II using the classical method of obtaining the solution of the transient process. Increasing the complexity of the topology of the analyzed electric circuit, which as a result is described by high-order differential equations (three, four etc.), leads to the situation as it becomes increasingly difficult to determine the analytical function of the transient process. This results not only in the difficulty of determining the values of the roots of the characteristic equation but also in the necessity of determining the integration constants.

Thus, if the order of the characteristic equation is greater than the fourth, the classical method is less suitable for use. As a result of this finding, it becomes obvious the need to use the operational method.

The operational method. The use of the operational method is reasonable for circuits that are described by integral-differential equations. In this case, it is simpler to obtain the characteristic equation for determining the roots, which are necessary both for estimating the stability of the solution of these equations and for obtaining the solution in the time domain (the original function).

The advantage of the operational method lies in the fact that this method does not impose the condition on the necessity to determine the integration constants from the initial conditions by obtaining the solution of the permanent regime described by a system of equations. When calculating the image function values for equivalent schemes whose elements are presented by image functions, the whole set of methods of calculating permanent modes in the electric circuits can be used. These particularities show the great advantage of the operational method in calculating the transient processes in the concentrated parameter circuits. This approach can also be used for circuits with distributed parameters (long electric lines, windings of high power electric machines in the mode of propagation of voltage and current waves generated by the partial discharges or atmospheric phenomena) [17, 36].

The disadvantages of the operational method include the complexity of calculating the terms by the decomposition theorem of the $Q(p) / D(p)$ polynomial ratio. This difficulty arises as a result of the need to take into account the voltage sources outside the circuit. The structure of these polynomials depends on the equivalent sources (calculation) the electromotive voltage, conditioned by the inertia of the energy storage elements of the circuit, i.e. inductivity and capacitors. The values of these electromotive voltages are determined from the initial conditions, so it is necessary to know the state of equilibrium of the circuit at the moment t0 = 0 of the initiation of the transient process. These components are determined as follows, knowing the functions of the images: pi_L (0) and u_C (0) / p.

When applying the operational method for calculating the free component of the transient current, the equivalent scheme is excluded in the components describing the electromotive voltage of the external power supplies of the circuit, which leads to the simplification of the imaginary functions of current and voltages. In order to determine the internal electromotive voltage values, as mentioned above, it is necessary to know the regime up to the moment of the circuit switching and the forced-mode parameters after switching. These nuances lead to recommendations for the application of the operational method in the case of cases when the external electromotive tensions have a simple form of evolution over time (DC source, harmonic, exponential form). For these conditions, it is relatively easy to calculate the forced component values of circuit currents.

Frequency method. The calculation of the transient processes using the frequency method (Fourier transform) according to the essence and application technique is close to the operational method. This method is useful to apply in cases where Fourier transform based frequency methods have been applied for the analyzed circuit.

The method is also effective for approximate calculations of the transient processes when the amplitude, frequency and phase characteristics of the input resistance are obtained experimentally. In these cases, the Fourier method has advantages over the operational method. After the experimental acquisition of the frequency characteristics at the input of the circuit or the mutual conductivity and the determination of the frequency spectrum $E(j\omega)$, the frequency spectrum of the current $i(j\omega)$ can be determined graphically. As a result, the frequency characteristics for the real or imaginary component frequency function of the current can be built. In these cases, the Fourier integral method has advantages compared to the operational method. Based on these data, the transient process can be approximated.

The Duhamel method. If an arbitrary voltage signal is applied to the circuit terminals and this signal can be approximated with an analytical-graph curve, it is reasonable to use the Duhamel integration method. Transition conductivity g(t) or transition function h (t) = y (t) is determined based on known methods and procedures. The advantage of the Duhamel integral method is determined by the possibility of studying the transient processes in the linear electric circuits when feeding them from voltage sources with the arbitrary shape of the output voltage.

The state variables method is reasonable to use in investigating work processes in power converters made on the basis of electronic power devices. This is due to the nature of the operation of these convertors, which, during operation, has circuits with variable topology. The processes in these circuits can be described of the differential equations with order I and order II for time intervals established by the circuit particularities. These ranges are determined by the control system of the converter.

The stationary wave method allows the values of the overvoltages to be determined based on the known discrete spectrum of the circuit's own frequencies and the characteristic impedance values for these own frequencies at the respective points of the circuit. From the point of view of the physics of the propagation process, this regime is possible if in the circuit are established where waves with the equal amplitudes and frequencies. This mode is possible in circuits with distributed parameters. For these reasons, this method can be presented as a particular variant of solutions obtained by the method of traveling waves (d'Alembert's waves). The stationary wave is the result of overlapping two progressive waves of equal amplitudes and frequencies that propagate in opposite directions. In the nodes the resulting oscillation amplitude is null. The wavelength of a stationary wave is equal to two distances between two adjacent nodes (zeros - zero amplitudes) of the wave values in the propagation space examined.

The d'Alembert wave method is useful for the qualitative analysis of processes in loss-free circuits (ideal) in order to establish marginal (degraded) regime laws such as idle and short-circuiting modes. In this context, this method has a methodological value. In fact, any circuit has losses and the direct application of the d'Alembert wave method is followed by errors. In this context, it is actuality the problem of elaboration of the methods to calculation the traveling wave processes, which take into account the real parameters of these circuits, including the phenomena of energy dissipation and dispersion.

Numerical methods in one way or another uses analytical calculation methods by transferring them to the area of approximate solutions. For the calculation of the transient processes it is considered as the most attractive - the finite difference method. This method can be used to obtain solutions of partial derivative equations (telegraph equations) describing processes in long lines with distributed parameters. Applying the finite difference method requires knowing the values of the coefficients in the telegraph equations. Determination of the values of these coefficients can be done by using the finite element method and the finite volume method. In this context, the numerical methods can complement each other in solving the problems of calculating the transient processes in the non-homogeneous circuits with distributed parameters in which the wave modes are manifested, including taking into account the multiple reflections of the waves on the inhomogeneities of these circuits.

Conclusions

The application and use of the methods of calculating the transient and wave processes is a non-trivial problem because the realization of the calculations is influenced by several factors, including the impact of the selected and applied calculation methodon the correctness and veracity of the numerical solution. In order to obtain accurate results it is necessary to formulate the question of the study, its mathematical form and the initial and limit conditions.

The problem of calculating non-stationare processes with non-zero initial condition can be reduced at the problem with a zero-initial conditions. The operational method and the Fourier integration method are widely used in the theory of automated control and in the calculation of transient processes in electric machines, in the calculation of transient processes in circuits with distributed parameters (transmission and distribution lines) in order to estimate overvoltages and current shocks. Metoda clasică în toate aceste cazuri aproape că nu găsește aplicații.

Numerical computational methods have a high potential for application as research tools of the transient and solution regimes of a wide range of problems in the field of electrical network design, protection systems and monitoring power systems and the of operating of the power equipment (rotating electric machines, transformers, electric lines), following simplifications through approximations and formalizations of numerical calculation procedures.

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