Journal of Engineering Science *Vol. XXVI, no. 2 (2019), pp. 35-39*
Fascicle *Industrial Engineering* *ISSN 2587-3474*
Topic *Applied Engineering Sciences and Management* *eISSN 2587-3482*

DOI: 10.5281/zenodo.3249178
CZU 621.321

CALCULATION THE SERVICE WAITING PROBABILITY WITH SELF-SIMILAR NETWORK TRAFFIC

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Received: March, 09, 2019
Accepted: June, 23, 2019

Abstract. Calculation of the service quality characteristics in a single-channel system with queue for the packet network is often reduced to the determination of the Hurst exponent for self-similar traffic, after which using the known Norros formula calculated average number of packets in the system. However, this method does not allow for the set value of the Hurst exponent calculated yet very important characteristics of quality of service, such as the average delay time of packets in the storage buffer and the service waiting probability of packet. In this work we propose a method for approximating the distribution function of the states of the system and on its basis, a formula for calculating the service waiting probability in a single-channel system with a self-similar traffic.

Key words: *self-similar traffic, service waiting probability, service quality characteristics, single-channel system with queue.*

I. Introduction

In packet networks, the flow of packets is formed by a plurality of requests sources for the provision of a network of services and network applications that provide video, data, speech and other services. The sources of requests involved in the process of creating a packet stream differ significantly in values of the specific intensity of the load. The intensity of the load of the resulting packet stream at any given time depends on what applications are served by query sources and what is the ratio of their number to different applications. Therefore, packet flows (traffic) significantly differ from the Poisson flow model where the exponential distribution function of the time interval between the moments of packet arrival.

The structure of traffic is also influenced by the technological features of the used service algorithms. For example, if the service is provided by multiple applications or in the used protocols have the repeated transfer of incorrectly accepted packets, then the moments of packet requests are much correlated. Because of this, in the process of service, the output streams vary considerably and in the resultant traffic there are long-term dependencies in the intensity of the arrival of packets. In this case, traffic is no longer a mere sum of the number of independent stationary and ordinary streams, such as Poisson flows of telephone networks. In multiservice packet switched networks, traffic is heterogeneous, and streams of different applications require a certain level of service quality. In these conditions, the flows of all applications are provided by a single multiservice network with shared protocols and management laws, even though the sources of each application have different rates of information transmission or change it during the communication session (maximum and average speed). As a result, the combined packet stream is characterized by the so-called "burstiness" of traffic with random frequency and duration of peaks and recessions. For such packet traffic is characterized by strong unevenness of the packets arrival intensity. Packets are not smoothly dispersed on different time intervals but grouped in "packets" on the same intervals and are absent or very small at other intervals of time [1].

For packet networks, a mathematical model of self-similar traffic is used, but there is no reliable and recognized methodology for calculating the parameters and characteristics of the quality in mass-servicing systems in the context of servicing such traffic. With the growth of the degree of self-similarity of packet traffic, the quality characteristics in the system significantly deteriorate compared with the maintenance of traffic of similar intensity, but without the effect of self-similarity.

The estimation of service quality characteristics (*QoS*) in a one-channel system with an infinite queue for self-similar traffic (model fBM/D/1/∞) often reduces to the estimate of the Hurst exponent H of self-similar traffic, after which according to the known Norros formula, the calculation of average number of packets in the system N [2] Other characteristics such as the average number of packets Q in the queue, the average packet time in the system T, and the average delay time of packets in the system of W are then calculated based on their known functional relationships from the calculated mean N [3]. However, such an algorithm from the Hurst exponent H does not allow to be calculated such characteristics as the service waiting probability for packet and the average packet delay time of t_q in the buffer memory.

The purpose of this work is to establish an approximating function for the distribution of states in a one-channel system with an infinite queue and self-similar traffic at the moment of packets receipt, and on the its basis made receiving the formulas for calculating the service waiting probability for packet and the average delay time of packets in the cumulative buffer.

II. Calculation method

In the mathematical models of the Queuing System (QS), the type of input stream, the scheme of QS and service rule are considered. In this case, an input stream with self-similar properties is considered, in which, for example, Pareto or Weibull distributions [1] are used to describe the distribution of the time interval between the moments of packets arrival. The service rule of packets in the flow is without losses but with the possibility of waiting in the infinite queue, and the rule of servicing packets from the queue - according to the rule of FIFO (firs input - firs output). The QS scheme is single-channel.

The calculation of the service quality characteristics in the QS is always performed based on a mathematical description of the system response to the input packet stream. Under the reaction of the system, they understand the states that, due to the random nature of the packets flow, are mathematically described by the probabilistic distribution function of the number of occupied channels and waiting places Pi, where i is the number of packets in the system (in channels and in the queue). This function coincides with the distribution function of the number of packages in the system (serviced and waiting in the queue), since each packet occupies one channel in system or one place in a queue at the waiting.

In the case of the simplest Poisson model of flow in a QS with a loss or waiting (queue), the states of the system are described by one of the known Erlang distributions (i.e., the first or second distribution of Erlang, respectively) [3]. Finding the system state distribution function for more complex stream models is a very difficult task, and therefore, for this flow model, there are not of similar solutions.

The utilization factor of ρ is defined as the ratio of the intensity of the input flow of requirements λ to the service intensity μ. For a single-channel system in any packet stream (arbitrary distribution G of the time interval between the arrival times of packets) $\rho = 1 - p_0$, where p_0 is the probability of a system's freedom or the state of the system p_0 (system have 0 packets). Thus, ρ coincides with the probability of the employment of the system or $P_e = \rho$.

For the Poisson flow of packets, the service waiting probability of P_w coincides with the probability of employment P_e [3, p. 49] of the system and therefore for a single-channel model, for example, M/G/1/∞ (for any law of service distribution) we get $P_w = P_e = \rho$.

Taking into account packets in queue in stationary mode there is a stationary distribution of system states or number of packets in the system p_k, where *k* is the number of packets (state p_0 - in the system 0 packets, state p_1 - busy single channel, state p_2 – occupied channel and one place in a queue, etc). Distribution p_k *does not depend on the moments of the packets arrival* into the system (does not depend on whether *the packet arrives or does not arrive in the system*). For the Poisson flow of packets this distribution is sufficient to calculate the service waiting probability P_w, since

$$P_w = \sum_{k=1}^{\infty} p_k = 1 - p_0 . \quad (1)$$

For arbitrary packet flows, for example, the G/G/1/∞ system, $P_w \neq P_e$ and this formula can only be used if the known distribution r_k of the number of packets in the system at the moment of receipt of new packets, where *k* is the number of packets. The p_k distribution differs from the r_k distribution by the fact that $p_0 = 1 - P_e$ (or $p_0 = 1 - \rho$), while $r_0 = 1 - P_w$. From this it follows that the packet should expect service with the probability $P_w = 1 - r_0$. For the M/G/1/∞ system, the equation $p_k = r_k$ is executed and therefore the p_k distribution [3] is used instead of r_k distribution.

Consequently, in the case of a self-similar packet flow model with time interval distribution between the moments of packet arrival according to Pareto or Weibull's laws, the waiting probability calculation for service is possible if it is known system states distribution or the distribution r_k of packets number in the system *at the moment of receipt of new packages*.

III. Results and discussions

In Figure 1 for a one-channel system with an infinite queue by a dashed line shows the distribution function of the number of packets in the system p_k, which does *not depend on the moments of the arrival of packets* into the system, and a continuous broken line shows the distribution function r_k of the number of packets in the system *at the moment of receipt of new packets*. These functions were obtained using a computer simulation program of self-similar traffic [4].

It should be noted that in the self-similar traffic of packet communication networks there are large breaks (pauses) in the arrival of packets into the system [3], and therefore the probability p_0 (for this example p_0 = 0,495) is the largest in the distribution function of the system states.

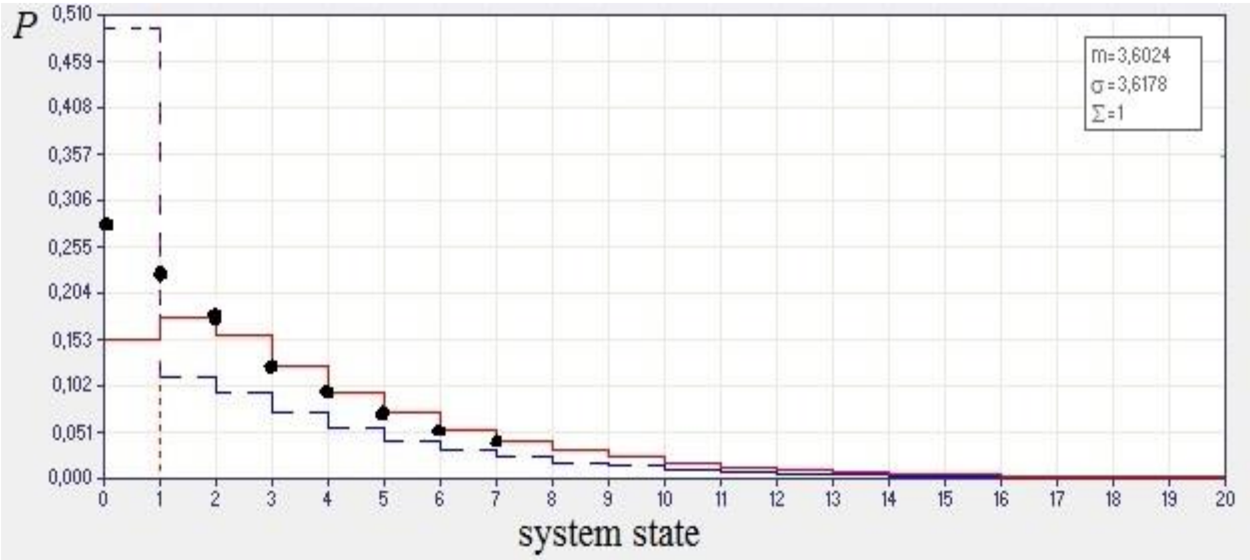

Figure 1. – Distribution functions of the system states and its approximation.

From Figure 1, we see that the bulk of the distribution function of the number of packets in the system at the moment of new packets receipt r_k without probabilities r_0, r_1 and r_2 is sufficiently qualitatively consistent with the approximating function B_i (shown by the points), as proposed by the following expression:

$$B_i = \frac{1}{T}\exp\left(-\frac{1}{T}i\right), \tag{2}$$

where *T* – the average stay length of packets in the system.

In formula (2), the approximating function B_i is an exponential function with a distribution parameter 1 / T, ρ – is load of the system or utilization factor (0 < ρ < 1).

In the non-Poisson flow with a Generalized distribution G of the time interval between the moments of arrival of packets (for example, the self-similar flow of type fBM), the service waiting probability in a single-channel system is calculated by formula (1), but necessarily with the use of the distribution function r_k of the number of packets in the system *at the moment of new packets receipt*:

$$P_w = \sum_{k=1}^{\infty} r_k = 1 - r_0 . \tag{3}$$

But, as it can be seen from Figure 1, if the probability B_0 from the approximating functions (2) is directly calculated instead of the true r_0, then a big error will be obtained. Therefore, the error of calculating the service waiting probability by the formula $P_w = 1 - B_0$ will be the same large error. Consequently, according to expressions (3) and (2), the service waiting probability in a one-channel system with an infinite queue of type fBM/G/1/∞ will be defined as follows:

$$P_w = \sum_{k=1}^{\infty} r_k \approx \sum_{k=1}^{\infty} B_k = \sum_{k=1}^{\infty} \frac{1}{T}\exp\left(-\frac{1}{T}k\right). \tag{4}$$

Thus, if it is possible to set the average stay length of packets in the system *T* or after determining the Hurst exponent using the Norros formula [2] to calculate the upper limit of the possible average *N*, then using the approximation (2) and using formula (4), one can calculate the waiting probability P_w of the packet. Since in the approximating distribution (2) parameter $1 / T = \rho / N$ [3], where N is the average number of packets in the system, then for practical calculations in the distribution (2) we can specify not $1 / T$ but ρ / N, where ρ – is load of the system or utilization factor ($0 < \rho < 1$).

Conclusions

In the conclusions, it should be noted that imitation modeling [4] confirmed the correctness of this calculation method of service quality characteristics in the system fBM/G/1/∞ with self-similar traffic. At the same time, the difference between the simulation and calculation results does not exceed 5% when the system loads in the range $0.3 < \rho < 1$ (with $\rho \geqslant 0{,}6$ error less than 2%) and the change in the Hurst exponent values in the range $0.5 < H < 0.9$ [5].

At that, as it can be seen from Figure 1, the result of calculating the service waiting probability P_w will always be somewhat overestimated, since the approximating function (2) also gives somewhat inflated results relative to the real probabilities r_1 and r_2, which are included in the sum of the calculation formula B_k (4). For example, Figure 1 shows that the probability $r_0 = 0.153$ and therefore the real service waiting probability $P_w = 0.847$. The calculation of this probability by the formula (4) gives the value $P_w = 0.885$, which is only 4.7 % higher than the real value of the service waiting probability. This is the case when $\rho = 0.5$, with $\rho \geqslant 0.6$ the error less than 2% and so on.

From the known formula $W = T - 1$ the average delay time of packets in the system W is calculated, after which one can calculate the average delay time of packets in the cumulative buffer $t_q = W / P_w$.

References:

1. Lozhkovsky A.G. Traffic Model in Packet Switched Multiservice Networks / A.G. Lozhkovsky // Naukovi pratsi ONAZ im. O.S. Popov. - 2010. - № 1. - p. 63–67 (in Russian).
2. Norros Ilkka. A storage model with self-similar input / Ilkka Norros // Queuing Systems, 1994. – Vol. 16.
3. Lozhkovsky A.G. Theory of mass service in telecommunications / Lozhkovsky AG - Odessa: ONAZ im. O.S. Popov. - 2012. - 112 p.(in Russian)
4. Computer program "Simulation of telecommunication networks' self-similar traffic". Certificate of registration of copyright in the work of Ukraine No. 61946 / AG Lozhkovsky, O.V. Verbanov // State Service for Intellectual Property dated 02.10.2015 (in Ucrainian)
5. Dependence approximation of the Hurst coefficient on the traffic distribution parameter // Lozhkovskyi A.G., Levenberg Ye.V. / Information & Telecommunication Sciences. – 2017. – № 2. – P.18–22.