Spatial Distribution of Survey Controls and Effect on Accuracy of Geometric Geoid Models (Multi-quadratic and Bicubic) in FCT, Abuja

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Abstract: Spatially distributed studies may follow two broad approaches: points that are randomly located and points that are clustered or clumped together. They are discussed within a discipline described as geo-statistics. Geo-statistics is applied in order to interpolate geoid undulation (N) and hence generate through a geoid model orthometric heights from scattered/random or closely/clustered located controls. Control points are coordinated points within a primary/secondary geodetic survey network. Kriging method was adopted to produce topographical maps of the both scattered and closely scenarios. Accuracy computed revealed that standard deviation (σ) of multiquadratic and bicubic models in the study area are respectively 11cm and 14cm in lopsided control study area while over clustered distributed located controls are respectively 12cm and 15cm. Standard deviation with the lowest values among the determined geometric geoid models is at all times preferable scattered/random than closely/clumped scenario. This implies that the multiquadratic models can be applied across the entire study area with high accuracy/reliability irrespective of spatial distribution of the points. Hence, the accuracy of the models are better when the total number of points distributed within the entire study area was used than when a limited number of points within a particular part of the study area was used.

Keywords: spatial, distribution, multiquadratic, bicubic, geometric, geoid, model, accuracy

INTRODUCTION

In accordance with Tobler's first law of geography introduced by Waldo R Tobler (1969), all geographic objects share certain similarities, but their similarity decreases with an increase in distance between these objects opined Longley et al. (2005). The need for homogeneous distribution of controls in geometric geoid modelling has been stressed by several researchers including Erol and Celik (2004), Kaloop *et al* (2008), among several others. However, circumstances in practical reality may make homogeneity impossible/infeasible (location of various classes of controls) and therefore the need to investigate the implication of such occurrences is desirable by computing accuracy/standard deviations (σ). Geoid modelling by Nwilo (2013), Eteje et al (2018), Oluyori (2019), and others may be applied to modernization of heights to produce orthometric heights by reference to vertical datum when adopting Global Positioning System (GPS) technology. GPS technology produces three dimensional coordinates (N, E, h) relative to earth ellipsoid. To obtain orthometric height (H) of a point from GPS observations, a geoid undulation (N) of the point is needed. Eteje *et al.* (2018) and Oluyori (2019) gave the model for the transformation of the GPS ellipsoidal height (h) to orthometric height (H) if the geoid height is known as:

$$H = h - N \tag{1}$$

In applications requiring mapping, cadastral, engineering, etc., the above relationships, equation (1) is necessary and desirable. Vertical datum is referred to as geopotential reference frame capturing gravity, geoid undulation, orthometric height and deflection of the vertical while geometric reference frame is used to replace horizontal datum to include geocentric X, Y, Z; latitude, longitude and ellipsoid height (h) according to Hansar (2016). Nwilo (2013) as well as Oluyori (2019) can fit into modern categorization of height modernization. One very important factor to be stressed in accuracy determination in geometric geoid modelling according to Kaloop *et al* (2008) is the spatial distribution and number of controls available within study area. If observation points do not reliably represent the analyzed area or represent only its part, the resulting model will be burdened by significant errors, which will contribute to high prediction errors opined Cellmer (2014). Issues related to number of controls and spatial distribution may lead to loss of surface reality and hence suitability of surface and in fact Ning (2015) pointed to reduction of “internal precision” arising from increase to more observations with attendant costs.

The main goal of this study is to find out if the spatial arrangements of the location of the controls have an impact on the accuracy achieved on the geoid models within the study area by computing and comparing their standard deviation/accuracy.

Scope of Study

This study is limited to comparison of the accuracy of geometric geoid models of Federal Capital Areas of Abuja as depicted in Figures 3 and 4 using orthometric heights of concentrated and widely separated controls within the study area. Two hours Relative GPS observations was used for data acquisition over primary and secondary controls.

Study Area

Federal Republic of Nigeria consists of 36 states and Federal Capital, the FCT, Abuja. Details of study are given in Oluyori *et al*. (2018). Figures 1 and 2 show the maps of the study area.

Figure 1: Map of Nigerian States and FCT Abuja
Source: Arcinfo Shapefile 2010 (ESRI)

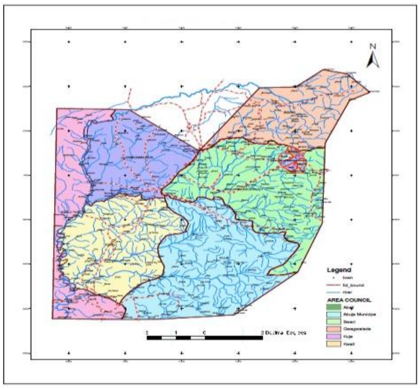
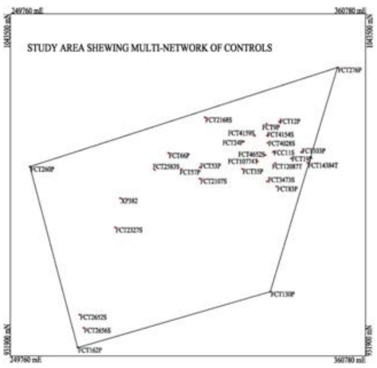


Figure 2: Map of FCT Six Area Councils
Source: Survey and Mapping Dept., FCDA, Abuja

The scenarios considered in the study in Figure 2 are:

i. Lopsided location of controls within the whole FCT i.e. 24 controls as shown in figure 3
ii. Dense or closely located controls in the FCT i.e. 14 controls as shown in figure 4.



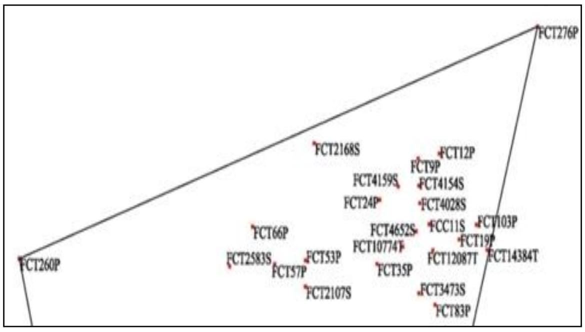


Figure 3: Plot of Lopsided Controls Within FCT.

Figure 4: Plot of Densely Located Controls Within FCT.

Geometric Geoid Models

Geometric geoid model are determined by finding the differences between the ellipsoidal and orthometric heights of selected points to obtain the geoid heights of the points. A geometric geoid surface is then fitted to the computed geoid heights of the points to enable geoid heights of new points within the study area to be computed. Polynomial surfaces are used to represent continuous surfaces over study areas as stated by Oluyori *et al.* (2018). Oluyori *et al.* (2018) respectively gave the Multiquadratic and Bicubic models as:

$$N = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 Y^2 + a_5 XY + a_6 X^2 Y + a_7 XY^2 + a_8 X^2 Y^2 \qquad (2)$$

$$N = a_o + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 x^3 + a_9 y^3 \qquad (3)$$

Where,

$$Y = ABS\,(y - y_o)$$
$$X = ABS\,(x - x_o)$$

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y = Northing coordinate of observed station
x = Easting coordinate of observed station
y_o = Northing coordinate of the origin (average of the northing coordinates)
x_o = Easting coordinate of the origin (average of the easting coordinates)
N is geoidal undulation at the point of interest
$a_0, a_1, a_2, \ldots, a_n$, unknown parameters

To enable redundancies for robustness of least squares solution, geoidal undulation (N) of at least eleven points must be known within the study area.

Geo-spatial Prediction Methods

Geo-statistics was originally developed as a self-contained methodology for spatial prediction opined Matheron (1963). One of the fundamental premises in geo-statistics is the similarity in the values of the analyzed variable as a function of distance opined Cellmer (2014). Geo-statistical prediction includes two stages:

i. Identification and modelling of spatial structure where continuity, homogeneity and spatial structure of a given variable is studied using a variogram.
ii. Geo-statistical estimation using kriging technique which depends on the properties of the fitted variogram which affects all stages of the process.

Since not all points can be observed or visited physically on the ground, the need for prediction to obtain acceptable data/information is very important for decision making and analysis. One of the methods of prediction is by Kriging. Kriging was developed by Krige (1951, 1962) who pursued the moving average concept in statistical interpolation methods to remove the effect of regression in estimating mineral resources.

METHODOLOGY

The methodology adopted in this study was divided into data acquisition, data processing and results presentation and analysis. Figure 5 shows the flow chart of the adopted methodology.

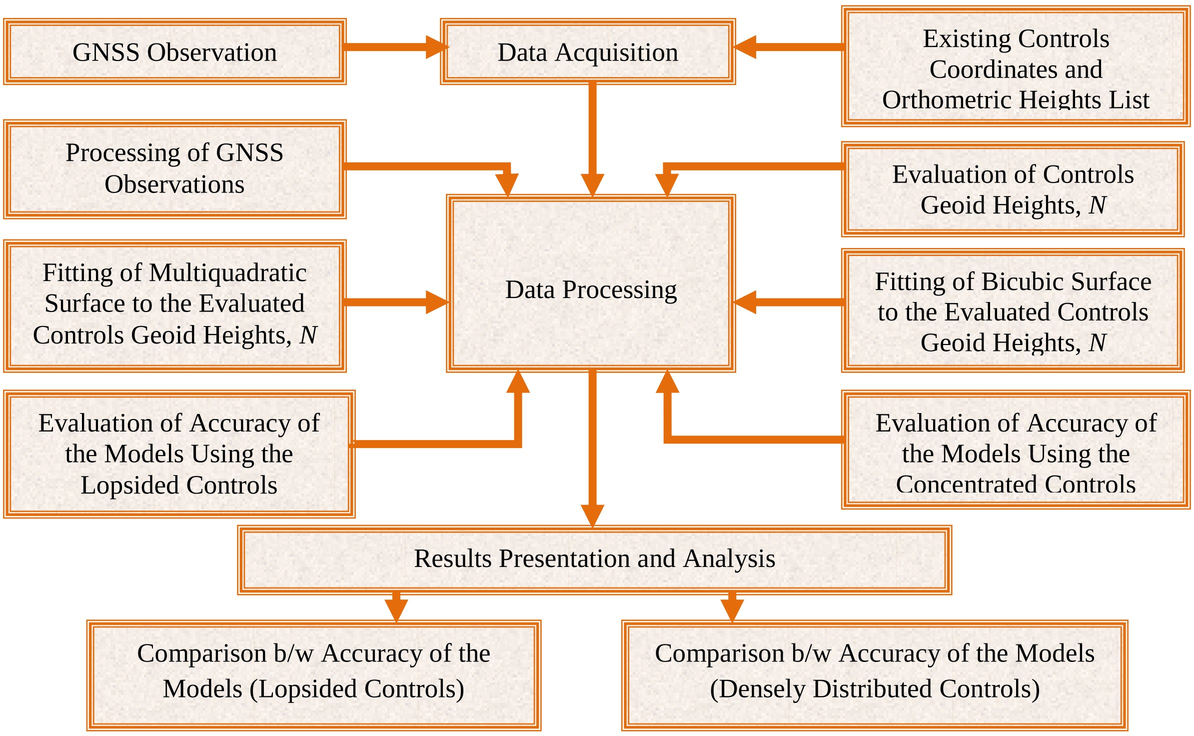

Figure 5: Adopted Methodology Flow Chart

DGPS receivers (dual frequency Hi-Target V30 Pro GNSS) were used to acquire data for two hours in static mode on controls (primary and secondary order) in FCT. The northing, easting and ellipsoidal heights, h of the control points were obtained from the processing of the DGPS observations. The secondary data, northing, easting and orthometric heights, H of the controls were obtained from the Surveying and Mapping Department of FCDA, Abuja. The geoid heights of the occupied points were computed

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using the ellipsoidal heights obtained from DGPS observation and the existing orthometric heights of the control stations with equation (1). The multi-quadratic and bicubic polynomial surfaces, equations (2) and (3) were fitted to the geoid heights of the controls to enable the geoid heights of new points within the study area to be computed using the determined geoid models. Microsoft Excel programs were written using equations (2) and (3). The orthometric heights of the points were computed from the two determined geometric geoid models, multiquadratic and bicubic models as given in table 1.

Table 1: Eastings, Northings and Models Orthometric (H) heights

Controls	Eastings(m)	Northings (m)	H(m)Multiquadratic	H(m) Bicubic
FCC11S	331888.114	998442.043	485.161	485.149
FCT260P	255881.175	993666.807	201.963	201.956
FCT103P	340639.766	998375.578	532.681	532.710
FCT12P	333743.992	1008308.730	735.826	735.913
FCT19P	337452.408	996344.691	635.703	635.704
FCT2168S	308926.908	989748.256	431.087	431.097
FCT24P	310554.927	1009739.930	453.807	453.684
FCT276P	322719.776	1001884.850	625.58	625.506
FCT4154S	351983.716	1025998.314	476.896	476.911
FCT4159S	329953.882	1003831.280	452.269	452.228
FCT66P	326124.422	1003742.860	296.925	296.858
FCT9P	299148.035	998114.283	497.334	497.394
FCT35P	329821.512	1007612.091	427.252	427.276
FCT57P	322183.380	992926.363	323.747	323.768
FCT4028S	303234.270	992916.402	449.642	449.635
FCT53P	330164.634	1001388.240	351.944	352.011
FCT4652S	308943.361	993406.773	462.916	462.876
FCT162P	329441.767	997474.808	189.694	189.789
FCT130P	270791.291	934625.533	695.579	695.596
FCT2327S	330982.584	952889.869	183.221	183.283
FCT2652S	282526.612	973821.470	138.960	139.091
FCT2656S	271370.273	945385.429	204.715	204.503
FCT83P	272644.591	941062.460	568.910	568.872
XP382	332954.205	987231.606	274.399	274.401

The two geometric geoid models (Multiquadratic and Bicubic models) orthometric heights were compared with their respective known orthometric heights to obtain the standard errors as well as the accuracy of each of the determined models considering both lopsided and clustered located controls. In the lopsided located controls, all the control stations (24 controls) orthometric heights were considered while in the clustered located controls, only 14 control stations that were closely located orthometric heights were considered.

RESULTS PRESENTATION AND ANALYSIS

Table 2 and figure 6 present comparison between the accuracy of Multiquadratic and Bicubic models considering only the lopsided controls. This was done to determine which of these models is better in terms of accuracy for application in the study area. Standard deviation is one of the indicators of how the model fits the FCT surface. The smaller the Standard deviation, the better the geoid model. It can be seen from table 2 that the standard deviation (σ) of multiquadratic and bicubic models are respectively 0.109959231m and 0.135719119m. That is, 11cm as against 14cm accuracy which implies that multiquadratic model is better for application in the study area than bicubic model. It can also be seen from figure 6 that the bar of the multiquadratic model is smaller than that of the bicubic model, which also implies that the multiquadratic model is better for application in FCT than the bicubic model. Though the two models can be applied throughout the FCT as the difference between their standard errors is very small.

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Table 2: Comparison between the Accuracy of Multiquadratic and Bicubic Models (Lopsided Controls)

Controls	DIFF B/W EXISTING AND MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHTS (A)	DIFF B/W EXISTING AND BICUBIC MODEL ORTHOMETRIC HEIGHTS (B)	A^2	B^2
FCC11S	0.285841605	0.298482551	0.0817054233	0.0890918334
FCT260P	0.019112060	0.012296746	0.0003652708	0.0001512100
FCT103P	0.122675646	0.151970694	0.0150493142	0.0230950917
FCT12P	0.119230612	0.205745694	0.0142159389	0.0423312904
FCT19P	0.059281596	0.060457902	0.0035143076	0.0036551580
FCT2168S	0.000472599	0.009592057	0.0000002233	0.0000920076
FCT24P	0.002586221	0.119698876	0.0000066885	0.0143278209
FCT276P	0.008313477	0.066451743	0.0000691139	0.0044158342
FCT4154S	0.085268640	0.069715646	0.0072707410	0.0048602712
FCT4159S	0.039248531	0.001560435	0.0015404472	0.0000024350
FCT66P	0.185538642	0.252681189	0.0344245876	0.0638477831
FCT9P	0.081473227	0.140784399	0.0066378867	0.0198202470
FCT35P	0.080509242	0.105236756	0.0064817381	0.0110747748
FCT57P	0.096904403	0.075769114	0.0093904633	0.0057409586
FCT4028S	0.049988498	0.043316085	0.0024988499	0.0018762832
FCT53P	0.000778872	0.068313577	0.0000006066	0.0046667448
FCT4652S	0.204730669	0.165304325	0.0419146470	0.0273255200
FCT162P	0.001755336	0.092805825	0.0000030812	0.0086129211
FCT130P	0.028904350	0.011997291	0.0008354615	0.0001439350
FCT2327S	0.066385996	0.003844076	0.0044071005	0.0000147769
FCT2652S	0.007916644	0.139298723	0.0000626733	0.0194041341
FCT2656S	0.009012951	0.221467692	0.0000812333	0.0490479387
FCT83P	0.157723116	0.120082336	0.0248765813	0.0144197674
XP382	0.186634409	0.184536082	0.0348324025	0.0340535656
STANDARD DEVIATION σ (SQRT OF AVERAGE OF A^2 or B^2) =			**0.109959231m**	**0.135719119m**

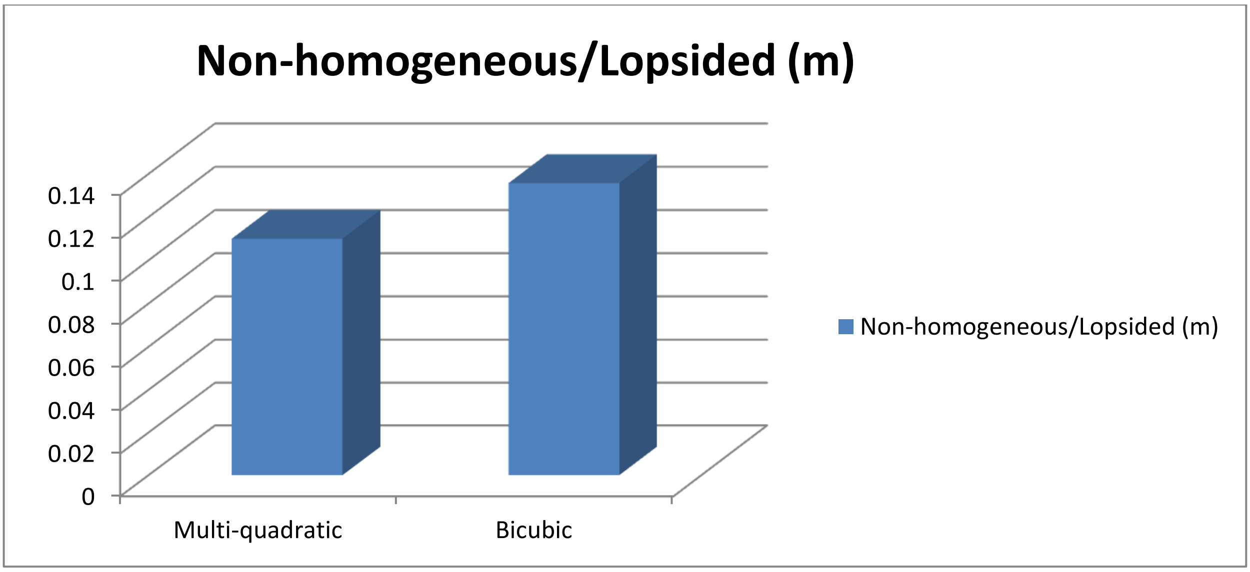

Figure 6: Plot of Standard Deviation of Multiquadratic and Bicubic Models Using the Non-homogeneous Controls

Also, tables 3 and 4, and figure 7 present the accuracy/standard deviation for multiquadratic and bicubic models using densely distributed points within the study area. This was done to compare the accuracy of the models using the total number of points (24 points) distributed within the entire study area and the densely distributed points (14 points) in a particular part of the study area. From table 3 it can be seen that the accuracy of the models, multiquadratic and bicubic are respectively 0.122391029m and 0.153398946m. Comparing these accuracy of the models using tables 2, 3 and 4, and figure 7, it can be seen that the accuracy of

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the models were better when the total number of points distributed within the entire study area was used than when a limited number of points within a particular part of the study area was used. This implies that the models can be applied across the entire study area with high accuracy/reliability irrespective of spatial distribution of the points.

Table 3: Accuracy of the Two Models (Multiquadratic and Bicubic Models) Using Densely Distributed Points in a Particular Part of the Study Area.

CONTROL POINTS	**DIFF B/W EXISTING AND MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHTS (A)**	**DIFF B/W EXISTING AND BICUBIC MODEL ORTHOMETRIC HEIGHTS (B)**	**A2**	**B2**
FCC11S	0.285841605	0.298482551	0.081705423	0.089091833
FCT103P	0.122675646	0.151970694	0.015049314	0.023095092
FCT12P	0.119230612	0.205745694	0.014215939	0.042331290
FCT2168S	0.000472599	0.009592057	0.000000223	0.000092008
FCT24P	0.002586221	0.119698876	0.000006689	0.014327821
FCT66P	0.185538642	0.252681189	0.034424588	0.063847783
FCT9P	0.081473227	0.140784399	0.006637887	0.019820247
FCT35P	0.080509242	0.105236756	0.006481738	0.011074775
FCT57P	0.096904403	0.075769114	0.009390463	0.005740959
FCT4028S	0.049988498	0.043316085	0.002498850	0.001876283
FCT53P	0.000778872	0.068313577	0.000000607	0.004666745
FCT2327S	0.066385996	0.003844076	0.004407101	0.000014777
FCT2652S	0.007916644	0.139298723	0.000062673	0.019404134
XP382	0.186634409	0.184536082	0.034832403	0.034053566
STANDARD DEVIATION σ (SQRT OF AVERAGE OF A2 or B2) =			**0.122391029**	**0.153398946**

Table 4: Lopsided and Dense Location Accuracy

Model type	**Non-homogeneous/Lopsided (m)**	**Concentrated/Dense (m)**
Multi-quadratic	0.109959231	0.122391029
Bicubic	0.135719119	0.153398946

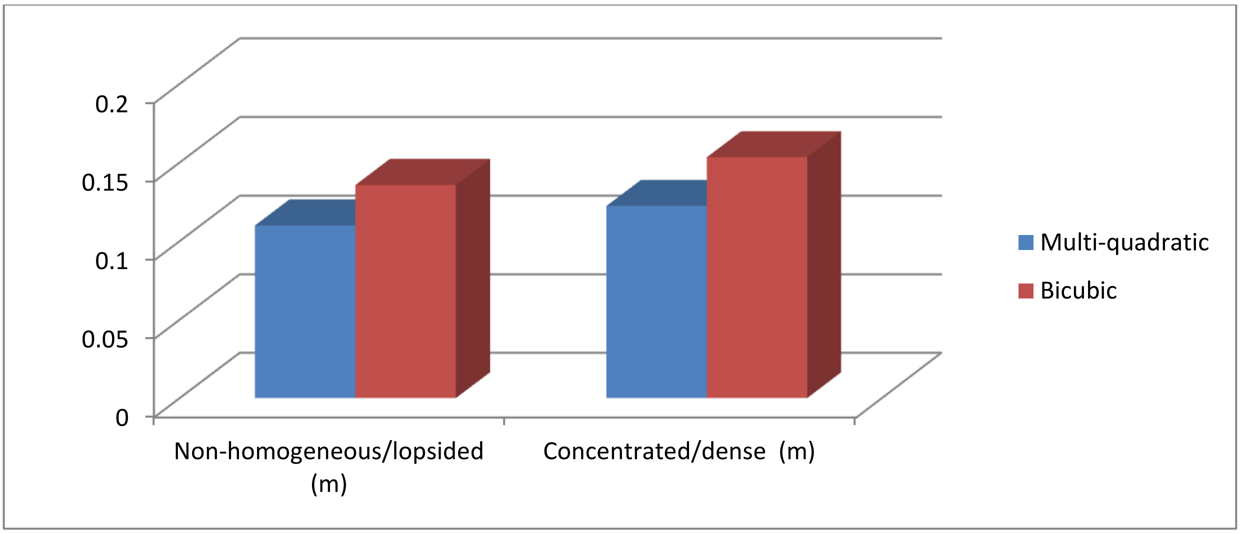

Figure 7: Plot of Standard Deviation of Multiquadratic and Bicubic Models Using Both Lopsided and Dense Controls

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The geographic location and size of a project area may point to the need for adoption of different polynomial interpolation surfaces used for different regions instead of single surface model to cover all. The ideal scenario is however, for a single geoid model to cover a study area which was achieved by Oluyori (2019).

Where control points represent only concentrated part of study area, the resulting model may have significant errors, which will lead to high prediction errors.

CONCLUSION AND RECOMMENDATIONS

i. Multiquadratic model is confirmed to take good care of lack of non-homogeneous distribution of selected controls in geoid modelling observed Oluyori (2019), Doganalp and Sevi (2015) and as well has the capacity to generate reliable geometric geoid model.

ii It is apparent that the standard deviation, 11cm obtained over the whole study area for multiquadratic model is better than that of the concentrated part (12cm) of the study area.

iii. Thus multiquadratic model is very appropriate for non-homogeneous distributed controls encountered in this study for geoid modelling.

iv. Spatial integrity and consistency enhances reliability of data for various applications within the study area.

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