The Schrodinger Equation in Terms of Flux

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In previous notes, it has been argued that the time-independent Schrodinger equation may be written in terms of complex conditional probability, namely:

[Sum over p p*p/2m fp exp(ipx)] / W(x) = .5m v(x)v(x) where v(x) is the classical velocity and W(x) the wavefunction. ((1))

In this note, we try to understand the Schrodinger equation only in terms of flux momentum flow. We note that:

d/dx density(x) = 2 density(x) u(x) where u(x)=[d/dx W(x)]/ W(x) ((2))

d/dx u(x) = -m v(x)v(x) - u(x)u(x)/m ((3))

d/dx entropy density(x) = 2 u(x) entropy density(x) - 2 density(x) u(x) ((4)) and

d/dx (.5m v(x)v(x) density(x)) = Force(x) density(x) + .5mv(x)v(x) 2 density(x) u(x) ((5))

Equations ((2)) and ((4)) are equivalent. If u(x)=0 in ((2)), density should still change due to the second derivative of density (and higher derivatives). In this note, we argue the time-independent Schrodinger equation follows from flux considerations if one writes d/dx density(x) = 2 density(x) u(x) or u(x)=.5[d/dx density]/density and uses integral dx density(x) u(x)/m u(x)/m= integral dx density(x) v(x)v(x) to show that d/dx d/dx density(x) = 2 density u(x)u(x)/m - 2 density(x) m (v(x)v(x). Here v(x) is the classical velocity given by sqrt[(2m) (E-V(x))].

Flux Considerations

Consider a system which has a certain density(x). Next, imagine that on average one has classical conservation of energy at each point, namely:

.5mv(x)v(x) + V(x) = E. ((7))

Given the density changes from point to point, to first order:

density(x+b) = density(x) + b d/dx density(x) ((8))

Imagine next that:

d/dx density(x) = 2 density(x) u(x) ((9))

where u(x) is a flux velocity times m. The 2 is added for convenience. One may immediately ask whether u(x)=v(x), where v(x) is the classical root mean square velocity, because classically v(x) is the only velocity in the problem. Imagine, however, that u(x) is not v(x). In fact, consider the density has “humps” i.e. points for which u(x)=0 even though there is no classical turning point. In such a case, u(x)/m is a second velocity in the problem. One may ask whether there is any relationship between u(x) and v(x). It seems:

Integral dx 1/2m u(x)u(x) density(x) = Integral dx .5m v(x) v(x) density(x) ((10))

The RHS of ((10)) is the average kinetic energy. If u(x) is the leading velocity term in ((9)) one might expect it would yield the same overall average kinetic energy, even though at each point u(x) is not equal to v(x). If it did not, one would have to account for extra kinetic energy density from the LHS versus the classical RHS.
(This is in fact the case in quantum mechanics for bound states as: d/dx (W(x) d/dx W(x)) = [d/dx W(x)] * [d/dx W(x)] + W(x) d/dx d/dx W(x). Integrating, the LHS is zero.)

One may already write u(x)= .5 [d/dx density(x)]/ density(x).

Next, consider a point at which u(x)=0. In such a case, there should still be flow as it was argued that this is not a classical turning point.
Consider:

d/dx d/dx density(x)= 4 density(x) u(x) u(x) + 2 density(x) d/dx u(x) ((11))

where we have used ((9)).

The objective is to determine d/dx u(x). Note:

Integral dx d/dx d/dx density(x) = d/dx density(x) evaluated at endpoints = density(x) u(x) evaluated at endpoints = 0 if the density dies down very rapidly.

In such a case, a solution is

d/dx u(x) = -m v(x)v(x) - u(x)u(x)/m   ((12))

[Note: In quantum mechanics, d/dx density(x) = d/dx W(x)W(x)= 2 density(x) [d/dx W(x)]/W(x) so u(x)= 2 [d/dx W(x)]/W(x) . Taking the derivative of this will lead to a form like ((12)).]

Thus:

d/dx d/dx density(x) = 2 density u(x)u(x)/m - 2 density(x) m (v(x)v(x) ((13))

Integrating the RHS of ((13)) should yield zero.

(13) may be linked to the time independent Schrodinger equation.

The time-dependent Schrodinger equation may be written as:

(-1/2m) W(x) d/dx d/dx W(x) = .5m v(x)v(x) W(x)W(x) where density = W(x)W(x) ((14))

Using W(x)=sqrt(density), it may be shown that ((14)) is equivalent to:

-1/4m (1/density(x)) d/dx d/dx density(x) = -1/2m [u(x) $]^2$ + .5mv(x)v(x)

If u(x)= [d/dx W(x)] / W(x) where W(x) = sqrt(density(x)). This is equivalent to d/dx density(x)= 2density(x) u(x).

Relationship to Entropy

The relationship d/dx density(x) = density(x) u(x) can be expressed as a Shannon's spatial entropy equation because:

u(x) = [d/dx W(x)] / W(x) = .5 d/dx ln(density(x)) ((15))

This leads to:

d/dx entropy density(x) = 2 u(x) entropy density(x) - 2 density(x) u(x) ((16))

Thus, the flux velocity u(x)/m carries not only density, but entropy density. In addition, one may write:

d/dx [.5mv(x)v(x) density(x)] = F(x) density(x) + .5mv(x)v(x) density(x) u(x) ((17))

Where F(x) is the force.

Conclusion

It seems one may make use of d/dx density(x) =2 density(x) u(x), where u(x)/m is a flux velocity, to define a theory which allows u(x) to be zero at certain points which are not classical turning points. The classical root mean velocity v(x) (where classical kinetic energy is .5mv(x)v(x)) defines one velocity and u(x) another. If u(x) is zero at certain points, there should still be a velocity flow.

In this note, we argue the time-independent Schrodinger equation follows from flux considerations if one writes d/dx density(x) = 2 density(x) u(x) or u(x)=.5[d/dx density]/density and uses integral dx density(x) u(x)u(x)= integral dx density(x) v(x)v(x) to show that d/dx d/dx density(x)= 2density(x) u(x)u(x)/m - 2density(x) v(x)v(x). Using u(x)=.5 [d/dx density]/density leads to the time-independent Schrodinger equation.

We also link the d/dx density(x) equation to entropy density flow.