TU Delft
Aerospace Engineering
AE4-ASM-511
Stability and Analysis of Structures II (SAS II)
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Assignment 2

Author: Saullo G. P. Castro

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1 Introduction to the assignment

Change proposed optimization

Reduce time to evaluate and decrease time step

Specify number of modes and keep in mind that the natural frequency is in rad/s

This is an individual assignment, meaning that you must to develop your own solutions. You are encouraged, however to work and discuss the necessary topics with your colleagues and professors.

The assignment consists on analysing and optionally optimizing a landing gear structure of an small airplane after it hits the ground and runs through a rough soil until it stops. The landing gear structure is shown in Figure 1. Material properties are given in Table 1. The landing gear's geometry that has to be modelled is a quarter of an ellipse, the left-hand side landing gear does not have to be modelled due to symmetry.

The students will learn:

- Add discrete masses into the finite element model
- Convert prescribed displacements into prescribed forces
- Convert acceleration field (gravity) into forces
- Do modal analysis in systems with forced linear vibration using the convolution method that is applicable to any general loading condition

The finite element formulation is the same one used in Assignment 1. Note that in the current finite element model the wheel and the airplane contribute to the mass matrix as concentrated masses. The airplane's mass discounting the landing gear is 250 kg. The wheel's mass moment of inertia about z axis assumed to be calculated simply by $I_0 = \frac{1}{12}m(2r)^2$. The design variables are those defining the landing gear's cross section at A and B: h_A, b_A, h_B, h_B. These variables defining the cross section geometry vary linearly between A and B.

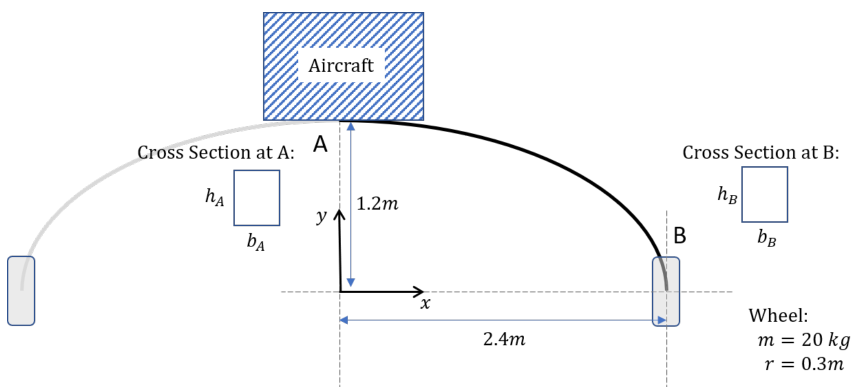

Figure 1: Landing Gear

Assumptions:

- Airplane:

    Has only translational motion

    Has no lift after touching the ground

    Does not bounce after touching the ground

Table 1: Material and Geometric Properties

$$E = 203\ GPa$$
$$\rho = 7.83 \times 10^3\ kg/m^3$$
$$h_A = b_A = 0.07\ m$$
$$h_B = h_B = 0.05\ m$$

- Wheels:

  Friction loads and loads due to rolling motion can be ignored

  Loads due to soil roughness must be considered

  Do not bounce back after touching the ground

- Structure:

  Damping can be neglected in all simulations

  At least 300 nodes are required do discretize the landing gear structure

Only one load case will be used in this assignment, which is described below:

- The airplane has an initial vertical velocity of 3 m/s downwards at the moment of impact with the ground
- Assume horizontal displacement and rotation at A to be zero; and vertical displacements at B to be zero. Set up correctly the velocities and accelerations corresponding to this boundary condition. When calculating the forced vibration the vertical displacement at B is imposed and calculated as a function of time
- Add the effect of gravity using $g = -9.81\ m/s^2$
- The airplane hits the ground with an initial horizontal velocity of 20 m/s, but this is only used to calculate the distance followed by the airplane on the ground as a function of time, since our model is in 2D only
- Consider a soil roughness with the following function: $v_B(s) = a_0 \sin\left(2 n_b \pi \frac{s}{L_r}\right)$;
  - s is the distance followed by the aircraft along the landing path
  - use $n_b = 1000$ as the number of bumps along L_r
  - $L_r = 100\ m$ is the distance followed by the aircraft before stopping. Assume constant deceleration rate
  - $a_0 = 0.01\ m$ is the amplitude of the soil roughness
  - Note that the vertical displacement at B given by $v_B(s)$ is currently a function of distance. To use in modal analysis v_B must become a function of time, which can be easily achieved finding a function of $s(t)$ using constant deceleration rate.
- Consider that the wheels keep in contact with the ground, meaning that $v_B(s)$ can be perfectly imposed at B.

2 Mandatory Tasks / Questions

All the analysis should be done from the time the airplane touches the ground until it stops.

1. Find the first 5 natural frequencies of the landing gear system. Plot each corresponding eigenmode.
2. How much time does it take for the airplane to reach zero vertical velocity for the first time?
3. Plot the vertical displacements at A as a function of time.
4. Plot the stresses at the lower and upper faces of the beam at point A as a function of time.

To solve, use the following:

- $\Delta t = 0.001\ s$ in the convolution method
- Determine the number of modes to be used to approximate the displacement in the modal analysis using the following rule of thumb:
  - The number of modes is selected such that the last mode has about 2 times the frequency of the applied load.
  - Hint: in the present example the maximum load frequency happens at the moment the aircraft starts running on the runway, going down as the aircraft decelerates. The maximum loading frequency is 200 Hz (horizontal velocity of 20 m/s with 10 bumps per meter).
  - Selecting more modes than necessary require smaller Δt in the convolution method, otherwise weird results will start to appear.

3 Optional bonus Tasks / Questions

These are are not required but are closer to real life experiences that you might encounter.

1. Find the minimum landing gear weight that would keep a maximum vertical displacement (measured from the original position) of the airplane after impact within 0.2m. Describe (very) briefly which optimization method you decided to use.
2. For LC1, find the first 5 natural frequencies of the optimized landing gear. Plot each corresponding eigenmode. Make an animation of each eigenmode.
3. How much time does it take for the airplane to reach zero vertical velocity for the first time with the optimized landing gear?
4. Plot the vertical reaction force at B as a function of time t. Comment on the validity of the assumed boundary conditions at point B.
5. Make an animation of the time response of the optimized landing gear from landing until the aircraft stops
6. Add modal damping to the simulation using $\zeta = 0.01$ and check how that affects the optimized response. Can the structure be further optimized when damping is taken into account?

A Working with prescribed displacements and gravity

Repeating for convenience the matrix decomposition shown previously, already ignoring the damping matrix $[C]$:

$$\begin{bmatrix}[M_{uu}] & [M_{uk}] \\ [M_{ku}] & [M_{kk}]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}_u\} \\ \{\ddot{u}_k\}\end{Bmatrix} + \begin{bmatrix}[K_{uu}] & [K_{uk}] \\ [K_{ku}] & [K_{kk}]\end{bmatrix}\begin{Bmatrix}\{u_u\} \\ \{u_k\}\end{Bmatrix} = \begin{Bmatrix}\{f_u\} \\ \{f_k\}\end{Bmatrix}$$

If prescribed displacements $\{u_k\}$ are known, the solution consists on finding $\{u_u\}$, which can be done with the following subsystem:

$$[M_{uu}]\{\ddot{u}_u\} + [M_{uk}]\{\ddot{u}_k\} + [K_{uu}]\{u_u\} + [K_{uk}]\{u_k\} = \{f_u\}$$

Since $\{u_k\}, \{\ddot{u}_k\}$ are known, the corresponding terms can be moved to the right-hand side:

$$[M_{uu}]\{\ddot{u}_u\} + [K_{uu}]\{u_u\} = \{f_u\} - [M_{uk}]\{\ddot{u}_k\} - [K_{uk}]\{u_k\} \tag{1}$$

Hence, $\{u_u\}$ can be calculated considering the right-hand side as the total equivalent applied force. In modal analysis, remember to transform this force in modal forces.

The force vector $\{f_u\}$ has forces applied to the unknown DOFs. For our problem, those come from gravity acceleration only:

$$\{f_u\} = [M_{uu}]\{\ddot{u}_{ug}\} + [M_{uk}]\{\ddot{u}_{kg}\}$$

where $\{\ddot{u}_{ug}\}$ and $\{\ddot{u}_{kg}\}$ are vectors where gravity is added only to the DOFs corresponding to vertical acceleration.

B Modal analysis with generic modal force

For a impulse load with duration ε and amplitude P_0, the solution of an undamped SDOF in modal coordinates, with natural frequency ω_n, is:

$$h(t_c - t_n) = \frac{1}{\omega_n}\theta(t_c - t_n)\sin\omega_n(t_c - t_n)$$
$$r(t) = \int_{-\infty}^{t} f(\tau)h(t-\tau)\mathrm{d}\tau$$
$$r(t_c) \approx \sum_{n=1}^{c} f_{i,n}\Delta t_n h(t_c - t_n)$$

Function $\theta(t-\tau)$ is a heaviside step function being $\theta(t \geq \tau) = 1$ and $\theta(t < \tau) = 0$, conveniently used to guarantee that forces happening at τ will only affect instants after τ. Function $f(\tau)$ is the modal force calculated from the right-hand side of Eq. 1. Use $\Delta t = 0.001s$ for the current problem.

C Calculating reaction forces (optional)

Considering there are two groups of degrees-of-freedom:

- With prescribed displacements: $\{u_k\}$
- With prescribed loads: $\{u_u\}$

Solving the dynamic system of equations consists on finding $\{u_u\}$, with the boundary conditions being represented by $\{u_k\}$. At $\{u_k\}$ the reaction forces are unknown and can be computed using the equations of motion. Writing them with matrices $[M], [C], [K]$ partitioned according to $\{u_k\}$ and $\{u_u\}$:

$$\begin{bmatrix} [M_{uu}] & [M_{uk}] \\ [M_{ku}] & [M_{kk}] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_u\} \\ \{\ddot{u}_k\} \end{Bmatrix} + \begin{bmatrix} [C_{uu}] & [C_{uk}] \\ [C_{ku}] & [C_{kk}] \end{bmatrix} \begin{Bmatrix} \{\dot{u}_u\} \\ \{\dot{u}_k\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{uk}] \\ [K_{ku}] & [K_{kk}] \end{bmatrix} \begin{Bmatrix} \{u_u\} \\ \{u_k\} \end{Bmatrix} = \begin{Bmatrix} \{f_u\} \\ \{f_k\} \end{Bmatrix}$$

When solving the dynamic system using modal analysis, $\{u_u\}, \{\dot{u}_u\}, \{\ddot{u}_u\}$ can be calculated with:

$$\{u_u\} = [S]\{r_u\}$$
$$\{\dot{u}_u\} = [S]\{\dot{r}_u\}$$
$$\{\ddot{u}_u\} = [S]\{\ddot{r}_u\}$$

where $[S]$ is the matrix to change from modal to actual displacements. Given that the expressions for $\{r_u\}, \{\dot{r}_u\}, \{\ddot{r}_u\}$ are known, this process becomes straightforward. Thus, the reaction forces $\{f_k\}$ corresponding to the degrees of freedom of known displacements (and rotations) can be readily calculated as:

$$\{f_k\} = [M_{ku}]\{\ddot{u}_u\} + [M_{kk}]\{\ddot{u}_k\} + [C_{ku}]\{\dot{u}_u\} + [C_{kk}]\{\dot{u}_k\} + [K_{ku}]\{u_u\} + [K_{kk}]\{u_k\}$$