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Neighbourly Pseudo Irregular Fuzzy Graphs

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Abstract: In this paper, neighbourly pseudo irregular fuzzy graphs and neighbourly pseudo totally irregular fuzzy graphs are defined. Comparative study between neighbourly pseudo irregular fuzzy graph and neighbourly pseudo totally irregular fuzzy graph is done. A necessary and sufficient conditions under which they are equivalent are provided. Also, few properties of neighbourly pseudo irregular fuzzy graphs and neighbourly pseudo totally irregular fuzzy graphs are discussed.

Key Words: 2-degree, pseudo degree of a vertex in a graph, neighbourly pseudo irregular fuzzy graph, neighbourly pseudo totally irregular fuzzy graph.

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§1. Introduction

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by $d(v)$. A graph G is regular if all its vertices have the same degree. The 2-degree of v is the sum of the degrees of the vertices adjacent to v and it is denoted by $t(v)$. A pseudo degree of a vertex v is denoted by $d_a(v)$ and defined as $\frac{t(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v.

A graph is called pseudo-regular if every vertex of G has equal pseudo (average) degree [3]. The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainly and vagueness [18]. The first definition of fuzzy graph was introduced by Haufmann in 1973. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [8]. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs plays a central role in combinatorics and theoretical computer science.

§2. Review of Literature

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in

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2008 [7]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [6]. Mathew, Sunitha and Anjali introduced some connectivity concepts in bipolar fuzzy graphs [16]. Akram and Dudek introduced the notions of regular bipolar fuzzy graphs [1] and also introduced intuitionistic fuzzy graphs [2]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graphs in [14].

N.R.S. Maheswari and C. Sekar introduced (2,k)-regular fuzzy graphs and totally (2,k)-regular fuzzy graphs [9]. N.R.S. Maheswari and C. Sekar introduced m-neighbourly irregular fuzzy graphs [13]. N.R.S. Maheswari and C. Sekar introduced neighbourly edge irregular fuzzy graphs [10]. N.R.S. Maheswariand C. Sekar introduced neighbourly edge irregular bipolar fuzzy graphs [11]. Pal and Hossein introduced irregular interval-valued fuzzy graphs [17]. Sunitha and Mathew discussed about growth of fuzzy graph theory [15]. N.R.S. Maheswari and C. Sekar introduced pseudo degree and total pseudo degree in fuzzy graphs and pseudo regular fuzzy graphs and discussed some of its properties [12]. These motivate us to introduce neighbourly pseudo irregular fuzzy graphs, and neighbourly pseudo totally irregular fuzzy graphs discussed some of its properties.

§3. Preliminaries

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted by $V(G)$ and $E(G)$ respectively [2].

Definition 3.1([5]) *A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non-empty set V and $\mu : V \times V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\mu(uv) \leq \sigma(u)\Lambda\sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(uv) = \sigma(u)\Lambda\sigma(v)$ is satisfied.*

Definition 3.2([4]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. The degree of a vertex u in G is denoted by $d(u)$ and is defined as $d(u) = \sum \mu(uv)$, for all $uv \in E$.*

Definition 3.3([6]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. The total degree of a vertex u in G is denoted by $td(uv)$ and is defined as $td(uv) = d(u) + \sigma(u)$ for all $u \in V$.*

Definition 3.4([1]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Then G is said to be an irregular fuzzy graph, if there is a vertex which is adjacent to vertices with distinct degrees.*

Definition 3.5 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$.Then G is said to be a totally irregular fuzzy graph if there is vertex which is adjacent to vertices with distinct degrees.*

Definition 3.6 *let $G : (\sigma, \mu)$be a fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct degree.*

Definition 3.7 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly total irregular fuzzy graph if every two adjacent vertices have distinct total degrees.*

Definition 3.8 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. The 2-degree of a vertex v is defined as the sum of degrees of vertices incident at v and it is denoted by $t(v)$.*

Definition 3.9 *A pseudo degree of a vertex v is denoted by $d_a(v)$ and defined as $\frac{t(v)}{d^*_G(v)}$, where $d^*_G(v)$ is the number of edges incident at v.*

Definition 3.10 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. The pseudo total degree of a vertex v in G is denoted by $td_a(v)$ and is defined as $td_a(v) = d_a(v) + \sigma(v)$ for all $v \in V$.*

§4. Neighbourly Pseudo Irregular Fuzzy Graphs

Definition 4.1 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly pseudo irregular fuzzy graph if every two adjacent vertices of G have distinct pseudo degree.*

Example 4.2 Consider a graph on $G^*(V, E)$.

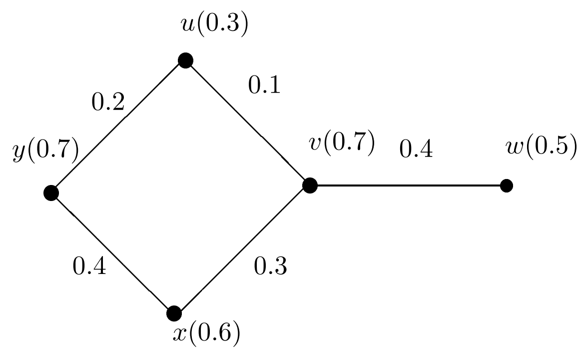

Figure 1

From Figure 1, $d_G(u) = 0.3$, $d_G(v) = 0.8$, $d_G(w) = 0.4$, $d_G(x) = 0.7$, $d_G(y) = 0.6$. Also, $d_a(u) = 0.7$, $d_a(v) = 0.46$, $d_a(w) = 0.8$, $d_a(x) = 0.7$, $d_a(y) = 0.5$. Here, pseudo degrees of all pair of adjacent vertices are distinct. Hence G is neighbourly pseudo irregular fuzzy graph.

Definition 4.3 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly pseudo totally irregular fuzzy graph if every two adjacent vertices of G have distinct total pseudo degree.*

Example 4.4 Consider a graph on $G^*(V, E)$.

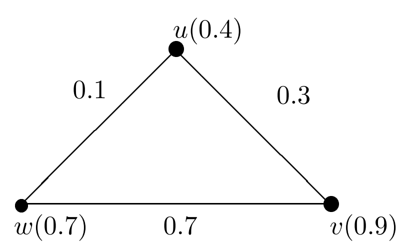

Figure 2

From Figure 2, $d_G(u) = 0.4$, $d_G(v) = 1.0$, $d_G(w) = 0.8$. Here, $d_G*(u) = 2$ for all u in G.

Also, $d_a(u) = 0.9$, $d_a(v) = 0.6$, $d_a(w) = 0.7$, $td_a(u) = 1.3$, $td_a(v) = 1.5$, $td_a(w) = 1.4$. Here, total pseudo degrees of all pair of adjacent vertices are distinct. Hence G is neighbourly pseudo totally irregular fuzzy graph.

Remark 4.5 A neighbourly pseudo irregular fuzzy graph need not be a neighbourly pseudo totally irregular fuzzy graph.

Example 4.6 Consider a graph on $G^*(V, E)$.

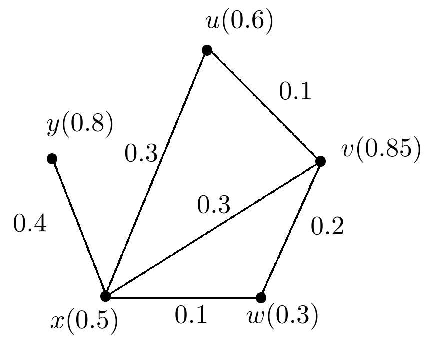

Figure 3

From the above figure, $d_G(u) = 0.4$, $d_G(v) = 0.6$, $d_G(w) = 0.3$, $d_G(x) = 1.1$, $d_G(y) = 0.4$. Also, $d_a(u) = 0.85$, $d_a(v) = 0.6$, $d_a(w) = 0.85$,$d_a(x) = 0.425$, $d_a(y) = 1.1$, $td_a(u) = 1.45$, $td_a(v) = 1.45$, $td_a(w) = 1.15$, $td_a(x) = 0.925$, $td_a(y) = 1.9$. Here, pseudo degrees of all pair of adjacent vertices are distinct. Hence G is neighbourly pseudo irregular fuzzy graph. But u and v are the adjacent vertices having same total pseudo degree. Hence G is not a neighbourly pseudo totally irregular fuzzy graph.

Remark 4.6 A neighbourly pseudo totally irregular fuzzy graph need not be a neighbourly pseudo irregular fuzzy graph.

Example 4.7 Consider a graph on $G^*(V, E)$.

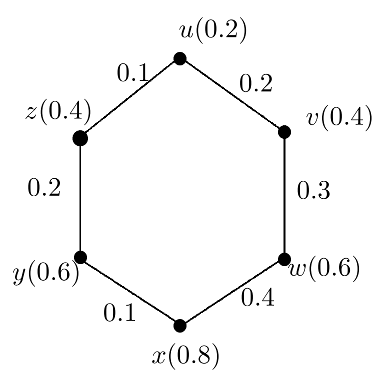

Figure 4

Here, $d_a(u) = 0.4$, $d_a(v) = 0.5$, $d_a(w) = 0.5$,$d_a(x) = 0.5$, $d_a(y) = 0.4$, $d_a(z) = 0.3$, $td_a(u) = 0.6$, $td_a(v) = 0.9$, $td_a(w) = 1.1$, $td_a(x) = 1.3$, $td_a(y) = 1.0$, $td_a(z) = 0.7$. Here, total pseudo degrees of all pair of adjacent vertices are distinct. Hence G is neighbourly pseudo totally irregular fuzzy graph. But the pairs v and w, w and x are the adjacent vertices having same pseudo degree. Hence G is not a neighbourly pseudo irregular fuzzy graph.

Theorem 4.9 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If σ is a constant function then the following are equivalent.*

(*i*) *G is neighbourly pseudo irregular fuzzy graph;*

(*ii*) *G is neighbourly pseudo totally irregular fuzzy graph.*

Proof Assume that σ is a constant function. Let $\sigma(u) = c$ for all $u \in V$. Suppose G is a neighbourly pseudo irregular fuzzy graph. Then every two pair of adjacent vertices have distinct pseudo degrees. Let u_1 and u_2 be two adjacent vertices with pseudo degrees k_1 and k_2 respectively. Then $k_1 \neq k_2$. Suppose G is not a neighbourly pseudo totally irregular fuzzy graph. Then at least two adjacent vertices have same total pseudo degree. Suppose $td_a(u_1) = td_a(u_2) \implies k_1 + c = k_2 + c \implies k_1 = k_2$, which is a contradiction. Hence G is a neighbourly pseudo totally irregular fuzzy graph. Then $(i) \implies (ii)$ proved

Now, Suppose G is a neighbourly pseudo totally irregular fuzzy graph. Then every pair of adjacent vertices have distinct total pseudo degrees. Let u_1 and u_2 be two adjacent vertices with pseudo degrees k_1 and k_2 respectively. Now, $td_a(u_1) \neq td_a(u_2) \implies k_1 + c \neq k_2 + c \implies k_1 \neq k_2$. Thus every pair of adjacent vertices have distinct average degrees. Hence G is a neighbourly pseudo irregular fuzzy graph. Thus $(ii) \implies (i)$ proved. □

Remark 4.10 The converse of the above theorem need not be true.

Example 4.11 Consider a graph on $G^*(V, E)$.

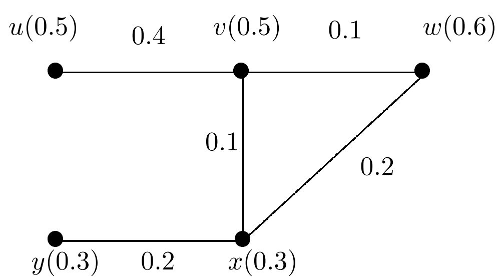

Figure 5

From the figure 5, $d_a(u) = 0.6$, $d_a(v) = 0.6$, $d_a(w) = 0.55$,$d_a(x) = 0.366$, $d_a(y) = 0.5$, $td_a(u) = 1.1$, $td_a(v) = 0.9$, $td_a(w) = 1.15$, $td_a(x) = 0.666$, $td_a(y) = 0.8$. Hence G is neighbourly pseudo irregular fuzzy graph and neighbourly pseudo totally irregular fuzzy graph. But σ is not a constant function.

Remark 4.12 Pseudo irregular fuzzy graph need not be a neighbourly pseudo irregular fuzzy graph.

Example 4.13 Consider a graph on $G^*(V, E)$.

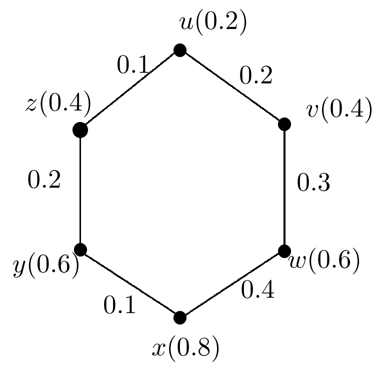

Figure 6

Here, $d_a(u) = 0.4$, $d_a(v) = 0.5$, $d_a(w) = 0.5$,$d_a(x) = 0.5$, $d_a(y) = 0.4$, $d_a(z) = 0.3$. Here But the pairs v & w and w & $x$are the adjacent vertices having same pseudo degree. Hence G is not a neighbourly pseudo irregular fuzzy graph. But G is pseudo irregular fuzzy graph, since the vertex u is adjacent to vertices v and z with distinct pseudo degrees

Theorem 4.14 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If the pseudo degrees of all vertices of G are distinct, then G is neighbourly pseudo irregular fuzzy graph.*

Proof Assume that the pseudo degrees of all vertices of G are distinct. Then every pair of adjacent vertices have distinct pseudo degree and hence G is neighbourly pseudo irregular fuzzy graph. □

Theorem 4.15 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If the pseudo degrees of all vertices of G are distinct and σ is constant, then G is neighbourly pseudo totally irregular fuzzy graph.*

Proof Assume that the pseudo degrees of all vertices of G are distinct. Then by theorem G is neighbourly pseudo irregular fuzzy graph. Since σ is constant, by theorem, G is neighbourly pseudo totally irregular fuzzy graph. □

Theorem 4.16 *If $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, a cycle of length n and μ is a constant function then G is not a neighbourly pseudo irregular fuzzy graph.*

Proof Assume that μ is a constant function, say $\mu(u_iu_j) = c$, $i \neq j$ for all $u_iu_j \in E$. Then $d_a(u_i) = 2c$ for all $u_i \in V$. Thus $d_a(u_i)$ is constant for all $u_i \in V$. Hence G is not a neighbourly pseudo irregular fuzzy graph. □

Theorem 4.17 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, a cycle of length n. If μ is a constant and σ is distinct, then G is neighbourly pseudo totally irregular fuzzy graph.*

Proof Assume that μ is a constant and σ is distinct. (i.e.) $\mu(u_iu_j) = c$, $i \neq j$ for all $u_iu_j \in E$ and $\sigma(u_i) = k_i$ for all $u_i \in V$. Thus $k_1 \neq k_2 \neq k_3 \neq \cdots \neq k_n$. Then $d_a(u_i) = 2c$ for all $u_i \in V$. Now $td_a(u_i) = d_a(u_i) + \sigma(u_i) = 2c + k_i$, for $i = 1, 2, 3, \cdots, n$. Hence G is a neighbourly pseudo totally irregular fuzzy graph. □

Theorem 4.18 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, an even cycle of length n and σ*

is distinct. If alternate edges have the same membership values, then G is neighbourly pseudo totally irregular fuzzy graph.

Proof Assume that alternate edges takes the same membership values and $\sigma(u_i) = k_i$, for $i = 1, 2, \cdots, n$ and $k_1 \neq k_2 \neq \cdots \neq k_n$. Let $e_1, e_2, \cdots, e_n$ be the edges of G. Since the alternate edges have the same membership values,

$$\mu(e_i) = \begin{cases} c_1 \text{if } i \text{ is odd,} \\ c_2 \text{ if } i \text{ iseven,} \end{cases}$$

$d_a(u_i) = c_1 + c_2$, $i = 1, 2, \cdots, n$,
$d_a(u_i) = \text{constant}$,
$td_a(u_i) = d_a(u_i) + \sigma(u_i)$,
$= d_a(u_i) + k_i$, $i = 1, 2, \cdots, n$ and $k_1 \neq k_2 \neq ... \neq k_n$.

So, every pair of adjacent vertices have distinct total pseudo degree. Hence G is neighbourly pseudo totally irregular fuzzy graph. □

Remarks 4.19 The above theorem does not hold for neighbourly pseudo irregular fuzzy graph.

Example 4.20 Consider a graph on $G^*(V, E)$.

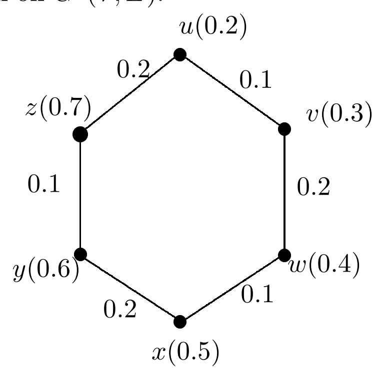

Figure 7

Here, $d_a(u) = 0.3$, $d_a(v) = 0.3$, $d_a(w) = 0.3$,$d_a(x) = 0.3$, $d_a(y) = 0.3$, $d_a(z) = 0.3$. Here $\sigma(u)$ is distinct. But G is not a neighbourly pseudo irregular fuzzy graph, since there is no pair of adjacent vertices having distinct pseudo degree.

Theorem 4.21 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, a cycle of length n and $n \geq 5$. If the membership values of the edges are $c_1, c_2, c_3, \cdots, c_n$ such that $c_1 < c_2 < c_3 < \cdots < c_n$. Then G is neighbourly pseudo irregular fuzzy graph.*

Proof Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, a cycle of length n and $n \geq 5$. Let $c_1, c_2, c_3, \cdots, c_n$ be the edges of the cycle C_n in that order. Let the membership values of the edges $e_1, e_2, e_3, \cdots, e_n$ be $c_1, c_2, c_3, \cdots, c_n$ such that $c_1 < c_2 < c_3 < \cdots < c_n$.

Now, $d(v_i) = \begin{cases} c_n + c_1 \text{ if } i = 1 \\ c_{i-1} + c_i \text{ if } i = 2, 3, 4, \cdots, n \end{cases}$

$$\implies d_a(v_i) = \begin{cases} \frac{d(v_2)+d(v_n)}{2} \text{ if } i = 1 \\ \frac{d(v_{i-1})+d(v_{i+1})}{2} \text{ if } i = 2, 3, \cdots, n-1 \\ \frac{d(v_{n-1})+d(v_1)}{2} \text{ if } i = n \end{cases}$$

$$\implies d_a(v_i) = \begin{cases} \frac{c_2+c_3+c_{n-1}+c_n}{2} \text{ if } i = 1 \\ \frac{c_n+c_1+c_2+c_3}{2} \text{ if } i = 2 \\ \frac{c_{i-2}+c_{i-1}+c_i+c_{i+1}}{2} \text{ if } i = 3, \cdots, n-1 \\ \frac{c_1+c_n+c_{n-1}+c_{n-2}}{2} \text{ if} i = n. \end{cases}$$

Also, since $c_1 < c_2 < c_3 < \cdots < c_n$, we have every pair of adjacent vertices have distinct pseudo degree. Hence the graph G is neighbourly pseudo irregular fuzzy graph. □

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