International J.Math. Combin. Vol.1(2019), 62-72

On Hemi-Slant Submanifold of Kenmotsu Manifold

Chhanda Patra

(Balichak Girls' High School(H.S.), Balichak, Paschim Medinipur 721124, India)

Barnali Laha

(Department of Mathematics, Shri Shikshayatan College, Kolkata-700071, India)

Arindam Bhattacharyya

(Department of Mathematics Jadavpur University, Kolkata-700032, India)

E-mail: patra.chhanda@gmail.com, barnali.laha87@gmail.com, bhattachar1968@yahoo.co.in

Abstract: We present here a brief analysis on some properties of hemi-slant submanifold of Kenmotsu manifold. After the introduction some preliminaries about this manifold have been discussed. Necessary and sufficient condition for distributions to be integrable are worked out. Some important results have been obtained in this direction. The last section emphasizes the geometry of leaves of hemi-slant submanifold of Kenmotsu manifold.

Key Words: Kenmotsu manifold, hemi-slant submanifold, integrability, leaves of distribution.

AMS(2010): 53C05, 53C15.

§1. Introduction

The notion of Kenmotsu manifold was defined by K. Kenmotsu in 1972 [9]. Then several works have been done on Kenmotsu manifold by G.Pitis [20] in 1988; J.B.Jun, U.C.De and G.Pathak [8] in 2005; C.S. Bagewadi and Venkatesha in 2007.

An interesting topic in the differential geometry is the theory of submanifolds in space endowed with additional structures [4], [5]. B.Y.Chen in 1990 initiated the study of slant manifold of an almost Hermitian manifold as a natural generalization of both holomorphic and totally real submanifolds. N.Papaghiuc have studied semi-invariant submanifolds in a Kenmotsu manifold [17], [18]. He also studied the geometry of leaves on a semi-invariant $\xi^{\perp}$-submanifolds in a Kenmotsu manifolds [18]. Afterwords in 1994, N.Papaghiuc introduced a class of submanifolds in an almost Hermitian manifold, called the semi-slant submanifolds, which includes the class of proper CR-submanifolds and slant submanifolds. Then in 1996, A. Lotta extended the notion of slant immersions in the setting of almost contact metric manifold. Later slant submanifolds of K-contact and Sasakian manifolds have been characterized by Cabrerizo, Carriazo and Fernandez in some papers (1999-2002).

[1]Received April 21, 2018, Accepted February 26, 2019.

The idea of hemi-slant submanifold was given by Carriazo as a particular class of bislant submanifolds and he called them anti slant submanifolds. After him B.Sahin in 2009 mentioned anti-slant submanifolds as hemi-slant submanifolds.

§2. **Preliminaries**

Let $\tilde{M}^{(2n+1)}(\phi, \xi, \eta, \tilde{g})$ be an almost contact Riemannian manifold where ϕ is a tensor field of type $(1,1)$, ξ is a vector field and η is a 1-form and $\tilde{g}$ is the induced Riemannian metric on $\tilde{M}$ satisfying

$$\eta \circ \phi = 0, \quad \phi\xi = 0, \quad \eta(\xi) = 1, \tag{2.1}$$

$$\phi^2 X = -X + \eta(X)\xi, \tag{2.2}$$

$$\tilde{g}(X, \xi) = \eta(X), \tag{2.3}$$

$$\tilde{g}(\phi X, Y) = -\tilde{g}(X, \phi Y), \tag{2.4}$$

$$\tilde{g}(\phi X, \phi Y) = \tilde{g}(X, Y) - \eta(X)\eta(Y), \tag{2.5}$$

for all vector fields X, Y on $\tilde{M}$. Now if

$$(\tilde{\nabla}_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi, \tag{2.6}$$

$$\tilde{\nabla}_X \xi = X - \eta(X)\xi, \tag{2.7}$$

$\tilde{\nabla}$ is the Riemannian connection of $\tilde{g}$, then$(\tilde{M}, \phi, \xi, \eta, \tilde{g})$ is called a Kenmotsu manifold.

In Kenmotsu manifold the following relations hold [9]:

$$(\tilde{\nabla}_X \eta)Y = \tilde{g}(\phi X, \phi Y), \tag{2.8}$$

$$\eta(R(X,Y)Z) = -\tilde{g}(Y, Z)\eta(X) + \tilde{g}(X, Z)\eta(Y), \tag{2.9}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.10}$$

$$R(\xi, X)Y = \eta(Y)X - \tilde{g}(X, Y)\xi, \tag{2.11}$$

$$S(X, \xi) = -2n\eta(X), \tag{2.12}$$

$$(\tilde{\nabla}_Z R)(X,Y)\xi = \tilde{g}(Z, X)Y - \tilde{g}(Z, Y)X - R(X,Y)Z, \tag{2.13}$$

where R is the Riemannian curvature tensor and S is the Ricci tensor.

Let M be a submanifold of $\tilde{M}$ with Kenmotsu structure $(\phi, \xi, \eta, \tilde{g})$ with induced metric g and let ∇ is the induced connection on the tangent bundle TM and $\nabla^{\perp}$ is the induced connection on the normal bundle $T^{\perp}M$ of M.

The Gauss and Weingarten formulae are characterized by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.14}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{2.15}$$

for any $X, Y \in TM$, $N \in T^{\perp}M$, h is the second fundamental form and A_N is the Weingarten map associated with N via

$$g(A_N X, Y) = g(h(X, Y), N). \tag{2.16}$$

The mean curvature H is denoted by

$$H = \frac{1}{k}\sum_{i=1}^{k} h(e_i, e_i), \tag{2.17}$$

where k is the dimension of M and $\{e_1, e_2, e_3, \cdots, e_k\}$ is the local orthonormal frame on M. For any $X \in \Gamma(TM)$ we can write,

$$\phi X = TX + FX, \tag{2.18}$$

where TX is the tangential component and FX is the normal component of ϕX. Similarly for any $V \in \Gamma(T^{\perp}M)$ we can put

$$\phi V = tV + fV, \tag{2.19}$$

where tV denote the tangential component and fV denote the normal component of ϕV. The covariant derivatives of the tensor fields T, F, t and f are defined as

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{2.20}$$

$$(\nabla_X F)Y = \nabla^{\perp}_X FY - F\nabla_X Y, \tag{2.21}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla^{\perp}_X V, \tag{2.22}$$

$$(\nabla_X f)V = \nabla^{\perp}_X fV - f\nabla^{\perp}_X V, \tag{2.23}$$

for all $X, Y \in TM$and for all $V \in T^{\perp}M$. A submanifold M is said to be invariant if F is identically zero, i.e., $\phi X \in \Gamma(TM)$ for any $X \in \Gamma(TM)$. On the other hand, M is said to be anti-invariant if T is identically zero, i.e., $\phi X \in \Gamma(T^{\perp}M)$ for any $X \in \Gamma(TM)$.

A submanifold M of $\tilde{M}$ is called totally umbilical if

$$h(X, Y) = g(X, Y)H, \tag{2.24}$$

for any $X, Y \in \Gamma(TM)$, where H is the mean curvature. A submanifold M is said to be totally geodesic if $h(X, Y) = 0$ for each $X, Y \in \Gamma(TM)$ and is minimal if $H = 0$ on M.

Now to study slant submanifolds let M be a Riemannian manifold,isometrically immersed in an almost contact metric manifold $(\tilde{M}, \phi, \xi, \eta, g)$ and ξ be tangent to M.Then the tangent bundle TM decomposes as $TM = D \oplus < \xi >$ where D is the orthogonal distribution to ξ.Now for each nonzero vector X tangent to M at x,such that X is not proportional to ξ_x,we denote the angle between ϕX and D_x by $\theta(X)$.M is said to be slant submanifold if the angle $\theta(X)$ is constant,which is independent of the choice of $x \in M$ and $X \in T_x M - < \xi_x >$. The constant angle $\theta \in [0, \pi/2]$ is then called slant angle of M in $\tilde{M}$. If $\theta = 0$ the submanifold is invariant submanifold, if $\theta = \pi/2$ then it is anti-invariant submanifold and if $\theta \neq 0, \pi/2$ then it is proper slant submanifold.

According to A. Lotta [16], when M is a proper slant submanifold of $\tilde{M}$ with slant angle θ then

$$T^2X = -cos^2\theta(X - \eta(X)\xi), \tag{2.25}$$

for all $X \in \Gamma(TM)$.

Cabrerizo et. al. [2] extended the above result into a characterization for a slant submanifold in a contact metric manifold.

Theorem 2.1 *Let M be a slant submanifold of an almost contact metric manifold $\tilde{M}$ such that $\xi \in \Gamma(TM)$. Then M is slant submanifold if and only if there exists a constant $\lambda \in [0,1]$ such that $T^2 = -\lambda(I - \eta \otimes \xi)$. Furthermore, in such case, if θ is the slant angle of M, then $\lambda = cos^2\theta$.*

This theorem has the following consequences:

$$g(TX, TX) = cos^2\theta(g(X,Y) - \eta(X)\eta(Y)), \tag{2.26}$$

$$g(FX, FY) = sin^2\theta(g(X,Y) - \eta(X)\eta(Y)) \tag{2.27}$$

for all $X, Y \in \Gamma(TM)$.

§3. Hemi-slant Submanifolds of Kenmotsu Manifold

A.Carriazo [3] introduced hemi-slant submanifolds as a special case of bislant submanifolds and he called them pseudo-slant submanifolds.

Definition 3.1([10]) *A submanifold M of a Kenmotsu manifold $\tilde{M}$ is said to be a hemi-slant submanifold if there exist two orthogonal complementary distributions D^θ and $D^\perp$ satisfying the following properties from:*

(1) $TM = D^\theta \oplus D^\perp \oplus < \xi >$;

(2) D^θ *is a slant distribution with slant angle* $\theta \neq \pi/2$;

(3) $D^\perp$ *is totally real i.e.,* $\phi D^\perp \subseteq T^\perp M$.

A hemi-slant submanifold is called proper hemi-slant submanifold if $\theta \neq 0, \frac{\pi}{2}$.

It is clear from above that CR-submanifolds and slant submanifolds are hemi-slant submanifolds with slant angle $\theta = \frac{\pi}{2}$ and $D^\theta = 0$, respectively.

In the rest of the paper, we use M as hemi-slant submanifold of a Kenmotsu manifold $\tilde{M}$. If we denote the dimensions of the distribution $D^\perp$ and D^θ by m_1 and m_2 respectively, then we have the following cases:

(i) If $m_2 = 0$ then M is anti-invariant submanifold;

(ii) If $m_1 = 0$ and $\theta = 0$, then M is an invariant submanifold;

(iii) If $m_1 = 0$ and $\theta \neq 0$, then M is a proper slant submanifold with slant angle θ;

(iv) If $m_1m_2 \neq 0$ and $\theta \in (0, \frac{\pi}{2})$, then M is a proper hemi-slant submanifold.

Suppose M to be a hemi-slant submanifold of a Kenmotsu manifold manifold $\tilde{M}$, then for any $X \in TM$, we put

$$X = P_1X + P_2X + \eta(X)\xi \tag{3.1}$$

where P_1 and P_2 are projection maps on the distribution $D^\perp$ and D^θ. Now operating ϕ on both sides of above equation, we arrive at

$$\phi X = \phi P_1X + \phi P_2X + \eta(X)\phi\xi.$$

Using (2.1) and (2.18) we have

$$TX + FX = FP_1X + TP_2X + FP_2X.$$

Comparing we get

$$TX = TP_2X,\ FX = FP_1X + FP_2X.$$

If we denote the orthogonal complement of ϕTM in $T^\perp M$ by μ, then the normal bundle $T^\perp M$ can be decomposed as

$$T^\perp M = F(D^\perp)\ \oplus\ F(D^\theta)\ \oplus\ <\mu>, \tag{3.2}$$

as $F(D^\perp)$ and $F(D^\theta)$ are orthogonal distributions. Now $g(Z, W) = 0$ for each $Z \in D^\perp$ and $W \in D^\theta$. Thus by (2.5) and (2.18) we obtain

$$g(FZ, FX) = g(\phi Z, \phi X) = g(Z, X) = 0,$$

which shows that the distributions $F(D^\perp)$ and $F(D^\theta)$ are mutually perpendicular. In fact, the decomposition (3.2) is an orthogonal direct decomposition.

In this section we will derive some results on involved distributions of a hemi-slant submanifold, which play a crucial role from a geometrical point of view.

Theorem 3.1 *Let M be a hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$ then*

$$A_{\phi W}Z = A_{\phi Z}W + \eta(Z)\phi W - \eta(W)\phi Z$$

for all $Z, W \in D^\perp$.

Proof On using (2.16) we get

$$\begin{aligned} g(A_{\phi W}Z, X) &= g(h(Z, X), \phi W) = -g(\phi h(Z, X), W) \\ &= -g(\phi\tilde{\nabla}_X Z, W) + g(\phi\nabla_X Z, W) = -g(\phi\tilde{\nabla}_X Z, W) \\ &= -g(\tilde{\nabla}_X \phi Z - (\tilde{\nabla}_X\phi)Z, W) \\ &= -g(\tilde{\nabla}_X\phi Z, W) + g((\tilde{\nabla}_X\phi)Z, W). \end{aligned}$$

Again using (2.6) and (2.15) we obtain

$$g(A_{\phi W}Z, X) = -g(-A_{\phi Z}X + \nabla^{\perp}_{X}\phi Z, W) + g(-\eta(Z)\phi X - g(X, \phi Z)\xi, W)$$

After some steps of calculations we get

$$g(A_{\phi W}Z, X) = g(h(W, X), \phi Z) + \eta(Z)g(\phi W, X) - g(\phi Z, X)\eta(W)$$

$$= g(A_{\phi Z}W + \eta(Z)\phi W - \eta(W)\phi Z, X).$$

Hence the theorem. □

Theorem 3.2 *Let M be a hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$. Then the distribution $D^{\theta} \oplus D^{\perp}$ is integrable if and only if $g([X, Y], \xi) = 0$ for all $X, Y \in D^{\theta} \oplus D^{\perp}$.*

Proof For $X, Y \in D^{\theta} \oplus D^{\perp}$ we have

$$g([X, Y], \xi) = g(\tilde{\nabla}_X Y, \xi) - g(\tilde{\nabla}_Y X, \xi) = -g(\tilde{\nabla}_X \xi, Y) + g(\tilde{\nabla}_Y \xi, X)$$

$$= -g(X - \eta(X)\xi, Y) + g(Y - \eta(Y)\xi, X) = 0.$$

Since $TM = D^{\theta} \oplus D^{\perp} \oplus < \xi >$, therefore $[X, Y] \in D^{\theta} \oplus D^{\perp}$. So, $D^{\theta} \oplus D^{\perp}$ is integrable.

Conversely, let $D^{\theta} \oplus D^{\perp}$ is integrable. Then for all $X, Y \in D^{\theta} \oplus D^{\perp}$, $[X, Y] \in D^{\theta} \oplus D^{\perp}$. As $TM = D^{\theta} \oplus D^{\perp} \oplus < \xi >$,therefore $g([X, Y], \xi) = 0$. □

Theorem 3.3 *Let M be a hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$. Then the anti-invariant distribution $D^{\perp}$ is integrable if and only if $\eta(Z)FW = \eta(W)FZ$ for any $Z, W \in D^{\perp}$.*

Proof For $Z, W \in D^{\perp}$ we have from (2.6),

$$(\tilde{\nabla}_Z \phi)W = -\eta(W)\phi Z - g(Z, \phi W)\xi. \tag{3.3}$$

After some steps of calculations and using Gauss and Weingarten formula we can obtain

$$-A_{FW}Z + \nabla^{\perp}_{Z}FW - T\nabla_Z W - F\nabla_Z W - th(Z, W) - fh(W, Z) \tag{3.4}$$

$$= -\eta(W)TZ - \eta(W)FZ - g(Z, TW + FW)\xi.$$

Comparing the tangential components, we have

$$-A_{FW}Z - T\nabla_Z W - th(Z, W) = -\eta(W)TZ - g(Z, TW)\xi. \tag{3.5}$$

Interchanging Z and W, we get

$$-A_{FZ}W - T\nabla_W Z - th(W, Z) = -\eta(Z)TW - g(W, TZ)\xi. \tag{3.6}$$

Subtracting equation (3.6) from (3.5) and using the fact that h is symmetric we get

$$A_{FW}Z - A_{FZ}W + T(\nabla_Z W - \nabla_W Z) = \eta(W)TZ + g(Z,TW)\xi - \eta(Z)TW - g(W,TZ)\xi. \quad (3.7)$$

Notice that $D^{\perp}$ is integrable iff $[Z,W] \in D^{\perp}$. Now $D^{\perp}$ is anti-invariant, i.e. $\phi D^{\perp} \subseteq T^{\perp}M$. Hence $T(Z)=0,\ T(W)=0,\ T[Z,W]=0$.

Again from (4.7)

$$A_{FW}Z - A_{FZ}W + T[Z,W] = 0. \quad (3.8)$$

So $D^{\perp}$ is integrable $\Longleftrightarrow A_{FW}Z - A_{FZ}W = 0$. By Theorem 3.1 we get the result. □

Theorem 3.4 *Let M be a hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$. Then the slant distribution D^{θ} is integrable if and only if*

$$P_1(\nabla_X TY - \nabla_Y TX + R(\xi,TX)Y - R(\xi,TY)X) = 0 \quad (3.9)$$

for any $X,Y \in D^{\theta}$.

Proof We denote by P_1 and P_2 the projections on $D^{\perp}$ and D^{θ} respectively. For any vector fields $X,Y \in D^{\theta}$, we have from (2.6),

$$(\tilde{\nabla}_X\phi)Y = -\eta(Y)\phi X - g(X,\phi Y)\xi. \quad (3.10)$$

On applying (2.14), (2.15), (2.18) and (2.19) we get

$$(\tilde{\nabla}_X\phi)Y = \nabla_X TY + h(X,TY) - A_{FY}X + \nabla^{\perp}_X FY - (T\nabla_X Y + F\nabla_X Y) - (th(X,Y) + fh(X,Y)). \quad (3.11)$$

Therefore from (3.10) and (3.11)

$$\begin{aligned}\nabla_X TY \quad &+ \quad h(X,TY) - A_{FY}X + \nabla^{\perp}_X FY - (T\nabla_X Y + F\nabla_X Y) \\ &- \quad (th(X,Y) + fh(X,Y)) = -\eta(Y)(T+F)X - g(X,(T+F)Y)\xi. \quad (3.12)\end{aligned}$$

Comparing the tangential components

$$\nabla_X TY - A_{FY}X - T\nabla_X Y - th(X,Y) = -\eta(Y)TX - g(X,TY)\xi. \quad (3.13)$$

Interchanging X and Y and subtracting from above equation we obtain

$$\nabla_X TY - \nabla_Y TX - A_{FY}X + A_{FX}Y - T\nabla_X Y + T\nabla_Y X \quad (3.14)$$

$$= -\eta(Y)TX + \eta(X)TY - g(X,TY)\xi + g(Y,TX)\xi.$$

From (2.11) we get

$$\nabla_X TY - \nabla_Y TX - A_{FY}X + A_{FX}Y - T\nabla_X Y + T\nabla_Y X = -R(\xi,TX)Y + R(\xi,TY)X. \quad (3.15)$$

Since $X, Y \in D^\theta$, $FX = 0$ and $FY = 0$. Hence applying P_1 to both sides of above equation we conclude our theorem. □

Theorem 3.5 *Let M be a hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$. If the leaves of $D^\perp$ are totally geodesic in M, then*

$$g(h(Z,X),FW) + g(th(Z,W),X) = 0 \tag{3.16}$$

for all $X \in D^\theta$ and $Z, W \in D^\perp$.

Proof From (2.6), (2.14) and (2.15) we get

$$\nabla_Z \phi W + h(Z,\phi W) - A_{FW} Z + \nabla^\perp_Z FW - \phi\nabla_z W - \phi h(Z,W) = -\eta(W)\phi Z - g(Z,\phi W)\xi. \tag{3.17}$$

Comparing the tangential components and on taking inner product with $X \in D^\theta$, we obtain

$$-g(A_{FW}Z,X) - g(th(Z,W),X) - g(T\nabla_Z W,X) = 0. \tag{3.18}$$

The leaves of $D^\perp$ are totally geodesic in M if for $Z, W \in D^\perp$,$\nabla_Z W \in D^\perp$.So $T\nabla_Z W = 0$. Hence we have

$$-g(A_{FW}Z,X) - g(th(Z,W),X) = 0. \tag{3.19}$$

This completes the proof. □

Theorem 3.6 *Let M be a totally umbilical hemi-slant submanifold of Kenmotsu manifold $\tilde{M}$. Then at least one of the following holds:*

(*i*) *dim* $D^\perp = 1$;

(*ii*) $H \in \mu$;

(*iii*) *M is proper hemi-slant submanifold.*

Proof In a Kenmotsu manifold for any $z \in D^\perp$ we have from (2.6),

$$(\tilde{\nabla}_Z \phi)Z = -\eta(Z)\phi Z - g(Z,\phi Z)\xi. \tag{3.20}$$

Using (2.14) and (2.18) we obtain

$$\tilde{\nabla}_Z FZ - \phi(\nabla_Z Z + h(Z,Z)) = -\eta(Z)FZ - g(Z,FZ)\xi. \tag{3.21}$$

Since $Z \in D^\perp, TZ = 0$. Now from (2.15),(2.18) and (2.19)

$$-A_{FZ}Z + \nabla^\perp_Z FZ - F\nabla_Z Z - th(Z,Z) - fh(Z,Z) = -\eta(Z)FZ - g(Z,FZ)\xi. \tag{3.22}$$

Comparing the tangential components

$$-A_{FZ}Z - th(Z,Z) = 0. \tag{3.23}$$

Taking inner product with $W \in D^{\perp}$,we get on using the fact that M is totally umbilical submanifold

$$g(g(Z,W)H, FZ) + g(tg(Z,Z)H, W) = 0. \tag{3.24}$$

After some brief calculations we get

$$g(Z,W)g(H, FZ) = 0. \tag{3.25}$$

Hence either $g(Z,W) = 0$ or $g(H, FZ) = 0$. If $g(Z,W) = 0$ then either $Z = 0$ or $Z = W$. As Z is arbitrary taken from $D^{\perp}$, so if $Z = 0$ then $D^{\perp} = 0$. And if $Z = W$ then dim $D^{\perp} = 1$. Now, if $g(H, FZ) = 0$, then $H \in \mu$. $\square$

§4. An Example of Hemi-slant Submanifold of a Kenmotsu Manifold

Let us consider a 9-dimensional submanifold M of $\Re^9$ defined by [7]

$$(u_1, -\sqrt{2}u_2, u_2 sin\theta_1, u_2 cos\theta_1, scos\theta_2, -cos\theta_2, ssin\theta_2, -sin\theta_2, z).$$

The independent vector fields

$$e_1 = \frac{\partial}{\partial x_1},$$

$$e_2 = -\sqrt{2}\frac{\partial}{\partial y_1} \ + \ sin\theta_1\frac{\partial}{\partial x_2} \ + \ cos\theta_1\frac{\partial}{\partial y_2},$$

$$e_3 = cos\theta_2\frac{\partial}{\partial x_3} \ + \ sin\theta_2\frac{\partial}{\partial x_4},$$

$$e_4 = -ssin\theta_2\frac{\partial}{\partial x_3} \ + \ sin\theta_2\frac{\partial}{\partial x_3} + scos\theta_2\frac{\partial}{\partial x_4} \ - \ cos\theta_2\frac{\partial}{\partial y_4},$$

$$e_5 = \xi = \frac{\partial}{\partial z}$$

span the tangent bundle of M. Let ϕ be the $(1,1)$ tensor field defined by

$$\phi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i}, \phi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial x_j}, \phi(\frac{\partial}{\partial z}) = 0 \quad 1 \le i,j \le 4.$$

For any vector field

$$X = \lambda_i\frac{\partial}{\partial x_i} + \mu_j\frac{\partial}{\partial y_j} + \gamma\frac{\partial}{\partial z},$$

$$Y = \lambda_i'\frac{\partial}{\partial x_i} + \mu_j'\frac{\partial}{\partial y_j} + \gamma'\frac{\partial}{\partial z} \in \Gamma(T\Re^9)$$

where $i,j \in \{1,2,3,4\}$.

After calculations we have

$$\phi^2 X = -\lambda_i \frac{\partial}{\partial x_i} - \mu_j \frac{\partial}{\partial y_j},$$

$$-X + \eta(X)\xi = -\lambda_i \frac{\partial}{\partial x_i} - \mu_j \frac{\partial}{\partial y_j}$$

$$g(\phi X, \phi Y) = \lambda_i \lambda_i' + \mu_j \mu_j'$$

Again

$$\phi(X,Y) = \lambda_i \lambda_i' + \mu_j \mu_j' + \gamma\gamma'$$

$$\eta(X)\eta(Y) = \gamma\gamma'.$$

Therefore we can see that $\phi^2 X = -X + \eta(X)\xi$ .Moreover equation (2.1) and (2.5) are also satisfied. Hence (ϕ, η, ξ, g) is an almost contact structure.

By direct calculation we can infer $D^\theta = span\{e_1, e_2\}$ ia a slant distribution with slant angle $\theta = cos^{-1}(\frac{\sqrt{6}}{3})$. Since ϕe_3 and ϕe_4 are orthogonal to M, $D^\perp = span\{e_3, e_4\}$ is an anti-invariant distribution. Thus M is a $5-$ dimensional proper semi-slant submanifold of $\Re^9$ with (ϕ, η, ξ, g).

Let $\tilde{\nabla}$ be the Levi-Civita connection on $\Re^9$. $[e_1, e_2]f = 0$. By similar calculation we get $[e_i, e_j] = 0,\ i, j \in \{1, 2, 3, 4, 5\}$. We can also calculate that

$$g(e_1, e_1) = g(e_3, e_3) = 1, g(e_2, e_2) = 3,$$

$$g(e_4, e_4) = s^2 + 1, g(e_5, e_5) = 1,$$

$$g(e_i, e_j) = 0 for\ i \neq j.$$

By using Koszul formula for g we can find the values of $\nabla_{e_i} e_j$ and verify (2.6) and (2.7). Therefore (ϕ, η, ξ, g) is a Kenmotsu manifold.

Let $z', w' \in D^\perp$ so $z' = \lambda_3 e_3 + \lambda_4 e_4, w' = \mu_3 e_3 + \mu_4 e_4$ for some $\lambda_3, \lambda_4, \mu_3, \mu_4$.

$\eta(z')\phi(w') = g(\lambda_3 e_3 + \lambda_4 e_4, e_5) \times \{-\mu_4 sin\theta_2 \frac{\partial}{\partial x_3} + \mu_4 cos\theta_2 \frac{\partial}{\partial x_4} + \frac{\partial}{\partial x_3}(\mu_3 cos\theta_2 - s\mu_4 sin\theta_2) + \frac{\partial}{\partial y_4}(\mu_3 sin\theta_2 + s cos\theta_2 \mu_4)\} = 0$. Similarly we compute $\eta(w')\phi(z') = 0$ which indicates $\eta(z')\phi(w') = \eta(w')\phi(z')$.

Now $g([e_3, e_4], e_5) = g([e_3, e_4], e_1) = g([e_3, e_4], e_2) = 0$. Therefore $[e_3, e_4] \in D^\perp$. Hence $D^\perp$ is integrable.

References

[1] D.E.Blair, Contact manifolds in Riemannian geometry, *Lecture notes in Mathematics*, Springer -Verlag, Berlin (1976).

[2] J.L.Cabrerizo, A. Carriazo, L.M. Fernandez, M. Fernandez, Slant submanifolds in Sasakian manifolds, *Glasglow Mathematical Journal*, 42(2000), 125-138.

[3] A.Carriazo, *New Developments in Slant Submanifold Theory*, New Delhi:Narosa Publishing House (2002).

[4] B.Y.Chen, *Geometry of Slant Submanifolds*, Katholieke Universiteit Leuven, Leuven, Belgium (1990).

[5] B.Y.Chen, Slant immersions, *Bulletin of the Australian Mathematical Society*, Vol.41, No.1(1990), 135-147.

[6] A. De, On Kenmotsu manifold, *Bulletin of Mathematical Analysis And Applications* , Vol.2, No.3(2010), 1-6.

[7] S. Dirik, M. Ateken, Pseoudo-slant submanifolds in Cosymplectic space forms, *Acta Univ. Sapientiae,Mathematica*, 8,1(2016), 53-74.

[8] J.B. Jun, U.C. De, and G. Pathak, On Kenmotsu manifolds, *J.Korean Math.Soc.*, 42(2005), 435-445.

[9] K. Kenmotsu, A class of almost contact Riemannian manifolds, *Tohoku Mathematical Journal*, Vol.24(1972), 93-103.

[10] M.A. Khan, K.Singh and V.A. Khan, Slant submanifold of LP-contact manifolds, *Differential Geometry-Dynamical System, Balkan Society of Geometers*, Vol.12(2010), 102-108.

[11] B.Laha, A.Bhattacharyya, On generalized quasi-Kenmotsu manifold, *International J.Math. Combin.*, Vol.4(2014), 39-46.

[12] B.Laha, A.Bhattacharyya, Totally umbilical hemislant submanifolds of Lorentzian (α)-Sasakian manifold, *International J.Math.Combin.*, Vol.1(2015), 49-56.

[13] B.Laha, A.Bhattacharyya, Totally umbilical hemislant submanifolds of LP-Sasakian manifold, *Lobachevskii Journal of Mathematics*, Vol.36, No.2(2015), 127-131.

[14] B.Laha, Slant and hemislant submanifolds of 3-dimensional trans-sasakian manifold, *The Bulletin of Society for Mathematical Services and Standards*, Vol.15(2015), 10-21.

[15] M.A. Lone, M.S. Lone, and M.H. Shahid, Hemi-slant Submanifolds of cosymplectic manifolds, *Cogent Mathematics*, 3:1204143(2016).

[16] A.Lotta, Slant submanifolds in contact geometry, *Bulletin Mathematiques de la Societe des Sciences Mathematiques de Roumanie*, 39(1996), 183-198.

[17] N.Papaghiuc, Semi-invariant submanifolds in a Kenmotsu manifolds, *Rend.Mat.*(7), 3(4) (1983), 607-622.

[18] N.Papaghiuc, On the geometry of leaves on a semi-invariant $\xi^{\perp}$-submanifold in a Kenmotsu manifold, *An.Stiint.Univ, Al.I.Cuza Iasi*, 38(1)(1992), 111-119.

[19] N.Papaghuic, Semi-slant submanifolds of a Kaehlarian manifold, *Analele Stiintifice ale Universitatii Al.I.Cuza Iasi*, Page 55-61(1994).

[20] G.Pitis, A remark on Kenmotsu manifolds, *Buletinal Universitatii Din Brasov.*, 30(1988).

[21] B.Sahin, Warped product submanifolds of a Kaehler manifold with a slant factor, *Annales Polonici Mathematici*,95(2009), 107-226.

[22] K.Yano, and M.Kon, *Structures on Manifolds*, In Pure Mathematics, Vol.3, World Scientific, Singapore (1984).