Discovering a Practical Impossibility: The Internal Configuration of a Problem in Mathematical Reasoning

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Introduction

Some years ago[1], in what was a discussion of other matters, we wondered about the analytic potentiality of ignorance (Sharrock and Anderson, 1980). What had caught our eye was the possibility, no more, that the initial fieldwork experience, involving as it does the overcoming of alienness and separation, could provide rich resources for the analysis of culture. In coming to know a culture, in finding one's feet, the fieldworker finds an organisation to activities, knowledge and practices. That organisation constitutes the culture for him. In focussing on this "discovery", it might be possible to draw out some of the ways that bodies of knowledge and associated activities make themselves accessible, comprehensible and utilisable by anyone who comes to them. Such a suggestion is, of course, little more than an extension of Schutz' (1962) insights with regard to the social role and position of "the stranger", and, perhaps Wittgenstein's remarks (1958) on the nature of logical compulsion. Others, for example Staten (1984) would argue that it can also be seen in Derrida's "deconstruction".

In the present paper, we return to this analytic potentiality and will try to make some preliminary observations (forays might be better) concerning its character. We will try to bring out how a body of knowledge and the courses of reasoning associated with it, can be viewed as organised to be found, to be used, to be understood, and how the use and understanding of such knowledge and reasoning is the display of its organisation.

As our title suggests, our primary interest is in the topography of mathematics as an encountered phenomenon; encountered, that is, with stocks of knowledge and relevances to hand and tasks to be pursued. Our interest is not, as it is in the work of Eric Livingston (1983), in what is essential to the activity of mathematical reasoning, but in mathematical knowledge and reasoning as an environment of action with an in-built, discoverable organisation. We do not want to say what mathematics is, nor what essentially it must be, what defines its character. We would rather bring out how it is organised to be whatever anyone finds in it.

[1] *Editor's note:* This paper was originally drafted by W.W. Sharrock and R.J. Anderson and circulated in the mid-eighties under the title "The Internal Configuration of a Problem in Practical Mathematical Reasoning" (n.d.). The draft didn't find its way into print, although it inspired a later paper co-authored by Wes Sharrock and Nozomi Ikeya, titled 'Instructional matter. Readable properties of an introductory text in matrix algebra' (in S. Hester, D. Francis (Eds.) (2000), *Local Educational Order*. Amsterdam: John Benjamins). Although the initial paper is not that recent, it offers a remarkably elegant and highly pertinent contribution to the topic of this special issue. This is why we decided to have it included. Special acknowledgements are due to Wes Sharrock for allowing us to publish the paper, even though under a slightly changed title and with interspersed subtitles (all indicated with an asterisk*).

Our survey of what might best be thought of as "encountered maths"[2] will focus primarily on two aspects. First, we will try to describe from within, so to speak, the character of reasoning in encountered maths. Second, we will sketch how in and through our encounter with it, we were able to grasp and follow a course of reasoning, and hence to sense something of its trajectory. The ostensible topic of our discussion is, then, our recognition and overcoming of ignorance of a field of mathematics. We will speak of how we found our way around, how things which were mystifying became sensible. But it is not that we learned some maths which is of interest, just as it is not that the fieldworker eventually got to know his surroundings which draws out his reports. For, given time and patience we could all do that. Neither is it the mathematics we learned which we wish to talk about. Were it so, there would be little to choose between sociology and traveller's tales, or between topographical surveys and tourist guides. Both would be taken up with recounting lists of the things to be found "out there" in the world. What we (and, we venture, the fieldworker) wish to talk about is culture as display; as an organisation made available in and for reporting.

'Encountered math', a practical impossibility, and sociological scrutiny*

To the layman and the mathematician alike, there is something almost tactile about mathematical competence. Either one has a "feel" for maths or one does not. No amount of slavish devotion and drill can compensate for "having the feel for it". Yet such a feel is not mathematical omniscience. Gifted mathematicians can still be puzzled, stumped, unable to see what is going on. But in not seeing an argument, a proof, what is invisible to them is how to make the steps necessary, why they follow one from another. Our objective in this paper is to make some preliminary moves towards seeing what encountered mathematics as a course of reasoning in action consists in. We shall do so by examining the discovery from within of an object which as far as we are aware has received little or no attention in the sociological literature. This social object is "the practical impossibility". The instance of a practical impossibility which we have discovered for ourselves is to be found in a branch of mathematics known as Matrix Algebra. We use the term 'discovered for ourselves' here in a particular way, for we are not, of course, claiming to have made an original mathematical discovery nor a contribution to mathematics as a body of knowledge. Rather, what we have done, and what we will be reporting on, is seeing for ourselves the practical impossibility of fulfilling a particular mathematical task. It follows from this that our topic is, once again, how the peculiar and specific character of a course of technical reasoning is made available. Our interest in Matrix Algebra is not in the mathematics *per se* but in what it displays to us about mathematics as a course of reasoning, and thus about the social organisation of bodies of reasoning such as mathematics.

In subjecting mathematics to sociological scrutiny, we have to be careful to avoid two pitfalls. The first is the overhasty and overzealous generalisation of what are thought of as sociological "discoveries" about mathematics 'premises'. These discoveries concern the social bases of mathematical practice, who "owns" mathematical knowledge[3], who controls and disseminates it, and so on. Treating mathematics as just another collection of knowledge which someone controls, as just another instance of professional ideology and the domination of vested interests, tells us very little about the working business of

[2] A phrase that is shorthand for "the body of mathematical reasoning and knowledge made available in and through our encounters with it".

[3] The idea of "owning knowledge" was explored in W.W Sharrock (1974)

mathematics. At the same time, focussing too closely on the mathematics, on the way that mathematicians think and speak about their work can lead us either to reify 'mathematical objects' or to overstress their socially constructed, conventional character. We can end up talking mathematics and not about mathematics as a sociological phenomenon. Rather than take up the "ontological status" of mathematical objects, we would prefer to look at the "cans and cann'ts, do's and don'ts" of mathematics. We offer these observations as indications of what we take as our own boundary markers and not as allusive criticism of other people's work. The interests they pursue are, of course, theirs to define.

Means and Ends*

We hope to be able to fulfil our objectives by concentrating on the limitations of mathematical reasoning as we encountered them. Again, notice the personal pronoun. We do not say that we have drawn a line around mathematics and have defined what it can say and what it cannot. Rather, we have followed a course of mathematical reasoning, taking one step after another until *we* were pulled up short. The incremental cumulative character of the reasoning ceased and a quantum transformation was required. Either you could see why the next step followed, or you could not. In seeing why the next step was not just permissible but obvious, one reasoned mathematically for oneself. One gained a "feel" for the mathematics involved. What had pulled us up short was not the assertion that this or that move was a practical impossibility, or a demonstration of why it should be so viewed, but its treatment as such. The disregard for its obviousness rendered it impossible for us to see[4].

Seeing the practical impossibility of the task in hand, and thus the necessity for taking the steps that were taken, counted as acquiring some (albeit not very much) competence in the form of mathematical reasoning explored by Matrix Algebra.

Before we go on to recount the details of our re-discovery, let us say something concerning the sociological importance, as we see it, of the phenomenon of practical impossibility. Any concern with the character of human action as action, as praxeology as Mises (1963) originally defined it[5], must take up the co-ordination of effort and effectiveness, and the correlation of means and ends. This will involve, if we might echo a previous formulation of our own (Anderson and Sharrock, 1979), an interest in the economics of activities, with the budgeting of time, resources and, of course, effort. Here lies the interest of the practical impossibility.

Practical impossibilities lie off the indifference curves. Existing budget lines cannot provide for their attainment. To do so will require the re-scaling of ends and means and the re-organisation of preferences and priorities. In this sense, practical impossibilities set the boundaries of daily life as daily life. It is in recognising and respecting the unacknowledged obviousness of such things as the practical impossibility of some line of action that cultural competence is to be found and displayed.

We will now turn to the specific character of the impossibility which we discovered and the nature of the course of reasoning in which it is to be found.

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[4] The notion of a rendering practice is taken up by Lynch (1985). The same practices are illustrated under a different title in our earlier paper, Anderson and Sharrock (1982).

[5] Mises defines action as reasoning and thus avoids the separation of doing and thinking.

Discovering a practical impossibility 'from within'*

The story is a relatively simple one. While developing some knowledge of programming, one of us was asked to help write a program to compute a Principal Components Analysis (PCA) for a set of data. This program was to run on a desk top micro. It had become necessary to write the program because no easily available and utilisable "off the shelf" ones were to be had. The general outline of the procedure of PCA, a kind of Factor Analysis, and what results it generated, were already to hand. It was known to be a procedure for condensing and simplifying complex data arrays. It allowed the extraction of 'underlying' or 'latent' structures which were "producing" or could "explain" the given arrays of data. This extraction is achieved by a process of transformation. The steps can be set out as follows.

A primary data matrix **D** is reduced to a matrix of correlation co-efficients **R**. This matrix is then progressively transformed by finding those factors of the matrix which maximise its variance. The maximal variance for r is $r^2 = 1$. Once these factors have been 'extracted', the investigator's task is to find a causal story which will attribute some real world, objective status to them. Statistical objects are taken to stand for real processes.

The data set which was to be subjected to analysis consisted of measurements of peat erosion from a number of selected sites together with data on climatic and other relevant variables. Thus the array of primary data might look like this:

MEASUREMENT NO 1.

peat loss	rainfall	wind speed	days of frost	no. of burns
x_{11}	x_{12}	x_{13}	x_{14}	x_{15}

with the measurements being repeated for different sites on different occasions. This complex data set would then be reduced the following array:

	x_1	x_2	x_3	x_4	x_5
x_1	1.0	r_{12}	r_{13}	r_{14}	r_{15}
x_2	r_{21}	1.0	r_{23}	r_{24}	r_{25}
x_3	r_{31}	r_{32}	1.0	r_{34}	r_{35}
x_4	r_{41}	r_{42}	r_{43}	1.0	r_{45}
x_5	r_{51}	r_{52}	r_{53}	r_{54}	1.0

The application of PCA to this array would enable the extraction of 5 principal components which decrease in order of influence upon the variance of the matrix. These would be set out to display this influence like this.

PRINCIPAL COMPONENTS

	Variance	Cumulative Variance	%Cum. Var.
P.C.1			
P.C.2			
P.C.3			
P.C.4			
P.C.5			

FACTOR SCORES

	P.C.1	P.C.2	P.C.3	P.C.4	P.C.5
x_1					
x_2					
x_3					
x_4					
x_5					

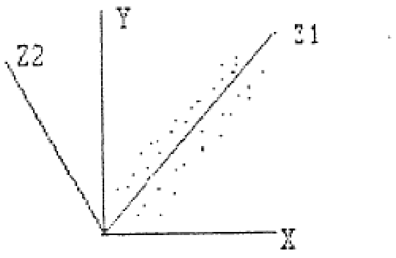
The computation problem as it was first encountered, then, consisted simply in finding a set of procedures which would reduce **D** to **R** and then transform **R** so that the principal components could be extracted and arrayed. To see how to produce such a set of procedures, recourse was made to texts (Cooper and Weekes, 1982; Kim and Mueller, 1978; Van De Geer 1971) which set out to explain PCA as an analytic technique. But consulting such texts merely gave us our problem back. In looking to the explanatory texts for guidance on how to compute factors for a matrix such as ours, we exposed what we have come to think of as a "gap in the texts". This gap was the recognisability of and documentation for the practical impossibility of achieving what we had set out to do. The gap in the texts marked the limits of our mathematical reasoning.

As we came upon it, it appeared as a presumption of massive proportions. But its grounds were invisible to us. What was, for us, the first and most pressing problem was, for the writers of our texts, no concern at all.

The gap in the text

The explanations we were offered of PCA were all pitched at the same two levels.

Elementary introductions to the technique would concentrate on visual presentations using two dimensional graphs of the following sort:



scatter plot of two dimensional array

If it were possible to transform the co-ordinates $\{X,Y\}$ to $\{Z_1,Z_2\}$ where Z_1 is the line of greatest elongation (i.e. where the variance is maximised) then Z_1 will be the "principal component" which explains the greater proportion of the variance. The transformation keeps Z_2 orthogonal to Z_1 so that the value of the variance is unchanged. The distribution between the variables $\{Z_1,Z_2\}$ has been transformed, that is all. Z_1 and Z_2 and the transformational factors are derived variables of the distribution. A presentation such as this sets those using it a sort of "eyeball" test of comprehension[6]. The procedure is made visible on the page. It is in being seen that it is understood. Thus a first presumption here is of the interchangeability of algebraic and geometrical formulations. If the reader could not see this much then nothing further could be said.

What is of general interest here is the use that is made of what, to borrow a formulation of Lynch (1985:41), might be called the naturalistically visible space rendered by graphical disclosure. The set of duplets $\{X,Y\}$ become Platonic objects in the real world by being summarised in the scattergram. Similarly the pairing Z_1Z_2, which are statistical artefacts, take on a visible, manipulable, workable character. The steps from $\{X,Y\}$ to $\{Z_1,Z_2\}$ involve *obvious transformation*. The rotation of the axes does no more than bring out what was there all along, namely the correlation of $\{X,Y\}$ revealed in the clustering of the points. The mathematical procedure rests, therefore, on what anyone can see.

The visual presentation makes it quite apparent what the technique is aiming to achieve. However, no-one uses the geometric technique in actual analyses. Visual presentations beyond three dimensions are impossible but the principle of the procedure is the same. Hence the direct move to multidimensional arrays is presumed to offer no intrinsic problems. The multi-dimensional presentations are all of the format which we set out above, produced by the use of a statistical package such as SPSS running on a large main frame computer. Between the levels there is nothing. So, all the explanations concerning computed PCA for multi-dimensional data made reference to the use of a large computer and therefore a higher order language such as Fortran. But none

[6] This term is used by Hendry (1980) with regard to correlation displays.

of the texts say that this must be so. Hence the gap. The practical impossibility of achieving a workable solution for PCA without these resources is, for the writers of these texts, so obvious that they do not refer to it. Anyone who understood the procedure would have to see that this was so. But we could not.

We were up against the limits of our mathematical reasoning and of the reasoning given in the texts. To see why no explanations were given, and thus to extend the scope of our reasoning, it was necessary to take an interest in Matrix Algebra. What had started out as a problem in computing had turned into a problem in mathematics, namely how to grasp the reasoning underlying Matrix Algebra. We could not bridge the gap in the texts without at least the beginnings of an understanding of the principles of Matrix Algebra. That understanding could only be gained by an exercise in “reasoning together”, that is in this instance by reading and using texts. Such “reasoning together” would provide a view of Matrix Algebra as a course of reasoning from within. We could not ‘invent’ the procedures nor could we ‘discover’ them of ourselves. The character of our encounter with it is, then, as Matrix Algebra introduced and set out for beginners.

Again, we went to Matrix Algebra with a problem. Whatever we found there would have to stand as the solution for that problem. Such a solution may not be the best possible solution, from a mathematician’s or computer scientist’s point of view say, but it would be the only one we could come up with here and now, given our mathematical competences and the texts we had to hand. That is its praxeological character.

Reasoning Together

We should all be familiar with the argument that texts are not determinate objects whose meanings are stored in them and merely await release. The finding of meaning, of comprehension and understanding is social and collaborative. In our case such comprehension would involve the acquisition of elementary Matrix Algebra as a course of reasoning. Here what we have is the following through of a series of pre-given steps to an envisaged conclusion as one person guides others through a course of reasoning. The “followability” of this reasoning is one of its primary features. In making these steps, reasoners assist one another to see why steps follow on and which of alternatives should now be selected. Such collaboration allows us to say that novices discover for themselves what the course of reasoning can come to. They discover for themselves what is already known to others. This re-discovery is achieved through the use of prepared exercises.

The reasoners in our case are the readers and writers of texts. A great deal has already been said about the problematic nature of this relational pairing. Much of this discussion, though, is not germane to our interests for it concerns the design procedures by which writers provide for given readings and the achievement of a community of reader and writer. It is obvious that texts may project different readers and may offer the reconciliation of a diversity of interests, say those of the beginner and those in need of a reference manual, as a central feature of their own organisation. Our problem is not to discover how the text comes to say what it says, how it fixes meaning, but how it could provide for whatever readings we made of it in seeking to remedy our gap in the texts. As we shall see in a moment, the very phenomenon of a gap in the texts is testimony to the collaborative character of reasoning in this form. Such collaboration is none the less real for being at long distance. The writer is present at the reading of the text in much the same way that the deceased is present at the reading of a will. No matter how clear they thought their instructions were, and how precisely they formulated their intentions, the text is no longer their property, theirs to control. It is now what others make of it, what they know the deceased would have wanted, had in mind, must have meant, and so forth.

The existence of a gap in the texts is the outcome of the fact that texts project an intended reader, a social type motivated by given sets of interests. In the case of our original PCA texts, this intended reader was presumed to want to know what PCA was and how it was applied. The intended reader was also presumed to bring to the reasoning transaction sets of deployable resources, namely the availability of the programs for computation and a distinctive set of problems for which the texts furnished solutions. In the case in hand, the intended reader and his presumed interests and resources were asymmetrical with the readings we were trying to give.

The structures of relevance embodied in the texts set aside the requirement to know how to compute the factors. The prime consideration was the preparation of the original data matrix and with the interpretation of the results. The rest was mere computation. In the case of the Matrix Algebra texts (e.g., Namboodiri, 1984; Lipschutz, 1982; Hohn, 1973), the intended reader was presumed to have different sorts of interest so that all they needed to know was what the computation would show if it were to be done.

Hence, the computation did not feature at all. Here, too, the presumed resources were different. The reader did not need to have a main frame computer to solve the examples set since none were more than three dimensional in form. (The reasons for this will become apparent in a moment). What was being conveyed here were the properties and procedures of the algebra not the facility to apply it in given situations. The texts were "application free" in a way that the PCA manuals were "application fixed". One of the clear presumptions in these texts was that some linear algebra was already to hand and some familiarity with simple matrices could be acquired quite quickly, thus it was presumed right at the start that the equivalence of the pairing

$$2x + 3y = 7$$

$$3x - y = 5$$

and $$\begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

would be fairly obvious once some basic properties of matrices and their manipulation had been run through. The texts, then, are written for those with some knowledge of algebra and some ability to see algebraic relations. Without this, a start cannot be made. This time our "eyeball test" is facilitated with the use of a prompt through the rules of matrix algebra.

We said one has to be able to start, but not all starts need be in the same place nor are all starts made by the intended reader. The text projects an intended reader, but actual readers may not match the intended one. The text's organisation facilitates anyone who has enough matrix algebra, to start anywhere and to move backwards and forwards around the text until they find what they want. It is designed for one reader but can be employed by many others. Thus the intended reader may be envisaged as reading from beginning to end, but not every imagined reader is an intended one. The text makes itself available for such readings by making the resources offered to the intended reader, references to previous chapters and examples, promises of where topics will be taken up again, available for anyone to use. So although there are first things to do and things to move onto, not everyone has to cover the first things first, nor take things in the order projected. The intended reader follows an unfolding, step by step, course of reasoning. Others may recover the sense of the reasoning by leaping from place to place. This gives the encountered topography of texts their distinctive character. They are designed to be usable by anyone who comes to them. "Sense" is not recoverable in just one way.

This openness of the text to readings is an expression of the unbounded character of possible structures of relevance. Using the "intended reader" as an orienting device, we can see that the text can be taken up by the novice, the semi-trained or the fluent. Each of these finds the text to have a different character. Those parts which are, for now, just signs whose meaning is not yet known to some, are to others familiar forms. For the novice and the semi-trained, the meaningless signs will only take on meaning when they are worked to. The text, with its conception of the intended reader, has a designed order of reading sequence. But while designed to be used in this careful, stepwise manner nonetheless it is capable of being used differently. It presumes that a start will be made with simple things and stepwise progression will be followed from there. Similarly it presumes that, in this instance, if you know that the factors of $x^2+5x+6 = 0$ are $(x+2)$ and $(x+3)$ then you should have no problem grasping the notion of factors for matrices, even if, at the moment you cannot see what sort of 'Platonic objects' they might be nor how they could possibly be computed. The notion of a designed order of reading provides a unity of presentation which goes beyond the coherence of the text as a produced object, a collection of parts merely pasted together between two covers and roughly 'about' the same topic.

The designed sequencing co-ordinates the textual division of labour. Its organisation provides what can be thought of as a set of delineated expectations for collaboration. In this sense, the text's organisation displays the grounds for motivated compliance in its collaborative reading. That is, its reasoning together. In respect of text-books in general, the finding of a unity in this way, can be the assigned to the reader requiring the collecting up of allusions, exemplars and the like. It can also be laid down in the structure of the text itself.

Thus far, we have been talking of the distribution of knowledge and the allocation of work tasks within the community of reader and writer. We will now turn to the nature of the reasoning itself. One way of thinking about text writing and the reproduction of courses of reasoning is to see it as translation. The writer relates for novices, non-speakers, something of the achievement of fluency. But the achievement of fluency is not merely a textual phenomenon. The novice has to learn to reason for him- or herself. The translation is, so to speak, in and out of pidgin. Both the native and the writer have to learn and communicate in a second language[7]. Here we find in the notion of mathematics as a "logical language" rather more than just a well worn metaphor that brings out the conventionalised character of its axiomatic character and procedural rules.

From the point of view of the intended reader, what is communicated in this pidgin is the seamless construction of the course of reasoning. Roy Turner used this idea when speaking of the stories which, typically, lawyers provide as accounts of their client's activities. No portions of time are left unaccounted for; every action is contextualised and explained. Such seamlessness is, of course, a collaborative

[7] Here the connection to our (1980) paper becomes obvious. To begin with, no matter how linguistically fluent, the fieldworker works with a pidgin version of the culture he studies.

achievement. Those with asymmetric interests (such as ours) or professional doubters, such as the Police, may find gaps in the text and notice where the whole has been stitched together. Seamlessness here, then, is a collaborative achievement of reading and writing. Such collaboration involves essentially a documentary reading from within, the bringing to mind what has been already covered, the setting aside of what is deemed to be irrelevant just now. In this way, the seamless construction is one achieved by inching forward from one point to another, one example to the next, one form to the next more complex with the effort required to make the move from step to step being minimised. Integral to this inching forward is the use of examples. These are expositions and demonstrations of what is now being talked about, how it relates to what has been covered before, and so on.

What is accomplished through reasoning together in this way is the internalisation of the trajectory of a course of reasoning. Its various parts are all aligned and the route taken emerges as both straightforward and the most direct. By way of our own exemplification of the procedures we have been discussing we will offer a demonstration in reasoning together, a demonstration which will bring out the essentially mathematical character of the practical impossibility we spoke of earlier.

Reasoning together, an example

The matrix **R** which we used as an illustration earlier is a square matrix. Square matrices have two useful properties. The determinant of the matrix is defined as the difference in the sums of the cross products. The trace is the sum of the right hand diagonal elements. These two can be illustrated using the simple 2 × 2 matrix **A**

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$$

The determinant of **A**,

$$|A|. = (3 \times 4) - (2 \times 1) = 10$$

The trace of **A**,

$$\mathrm{tr}\, A. = 3 + 4 = 7$$

Every square matrix has a corresponding identity matrix **I** made up of zeros and unities. For a 2 × 2 matrix this would be

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In its simplest form, the problem is to find a value, or scalar, **b** which would satisfy the determinental equation

$$|\mathbf{A}| = [\,\mathbf{A} - \mathbf{bI}\,] = 0$$

which for **A** would be the expression

$$|A| = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} = 0$$

The value **b** which satisfies this equation would be the eigenvalue of the matrix **A** and would be decomposable into eigenvectors of **A**. The elements of the eigenvectors are the factors which we seek.

We know from our earlier definition of **|A|** and tr **A** that since |**A**| = [**A - bI**] = 0 |**A**| = $b_1 \times b_2$ and tr **A** = $b_1 + b_2$.

Thus we have a pair of simultaneous equations

$$b_1 \times b_2 = 10$$

$$b_1 + b_2 = 7$$

By inspection we can see that $b_1 = 5$ and $b_2 = 2$. Thus **b** is a vector whose elements are {5,2}.

Taking each of the elements in turn, we can specify the factors as follows. Pre-multiplying the vector $\begin{bmatrix} x \\ y \end{bmatrix}$ by b_1 is the same as

post multiplying **A** by $\begin{bmatrix} x \\ y \end{bmatrix}$

that is

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix}$$

This is the matrix formulation of the simultaneous equations

$$3x + 2y = 5x$$

$$x + 4y = 5y$$

There is no formulaic solution for the extraction of factors for such equations. They are achieved by setting one value to unity. If in $3x + 2y = 5x$, x is set to 1 then $y = 1$. Thus the elements or factors of b_1 are {1,1}.

Reiterating with

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix}$$

We obtain $b_2 = \{1, - 0.5\}$.

Thus far the reasoning can be followed both on paper and in the head with little difficulty, providing some basic matrix algebra has been mastered. It is amenable to setting out in our inching forward manner, and has something of the seamlessness we mentioned. However, the phrase "by inspection" directs attention to the outcome of the procedure and not to its specific character. It disregards the number of trials, guesses and approximations which were necessary before we hit upon the right combination. The size and the number of permutations which might be run through multiply astronomically as the number of dimensions is increased. Since we do not know in advance the number of iterations required to discover the values, we have here a practical impossibility. The internal configuration of our problem in mathematical reasoning has turned out to be a practical impossibility. As we see it now, and as the competent mathematician saw it before, now one would want to attempt to do it this way. That is why PCA resorts to computers and programs to extract them.

But what of our original computing problem? How is that to be left? Two difficulties arise here. The maximal value of any variance is 1.0. The factors will contribute a proportion of this. The amount of storage space for decimalisation on micro-computers is limited. As each pass is made through the sorting procedure, the discriminations which will have to be made will have to be finer and finer to be accurate. Any low level language which does allow decimalised functions may still be too inflexible for use. There are algorithms in the literature[8] which will determine eigenvalues. But even with the simple examples used for exposition, say of 5 variables, it may take up to 20 passes through the array to determine the first component. The next could take even longer, and so on. Routines written in low level languages are usually extremely cumbersome and slow in operation since normal replication routines or loops require specification of the number of times the loop is to be iterated. We cannot specify this. Thus even if we could solve the programming problems, to make them

[8] See for instance Van Der Geer (1971).

viable for any actual case we might want to apply the PCA to, the routines would be likely to tie the processor up for days on end. It too is a practical impossibility, as those whom we consulted knew. For them to be able to explain why, it would have been necessary for us to be where we are now. Thus were the limits of our reasoning displayed. If we could see that it was a practical impossibility we understood the problem in hand.

To conclude*

Our discovery of our practical impossibility was made from within a course of reasoning. It was, on this occasion, our discovery. Such a discovery shows the social character of reasoning in overcoming its own limitations. The texts could not tell us unless we could already see it for ourselves. Thus the gap in the text was testimony to the essential character of the limitations of mathematical reasoning. There was no way that the understanding of the procedures could be given "from the outside". It had to be discovered "from within". The practical impossibility of factoring multidimensional matrices with pencil and paper, calculator and patience was that problem's internal configuration.

Let us finish as we started with some general considerations. In the paper to which we have already referred more than once, Lynch poses the problem faced by the sociology of the natural and mathematical sciences, like this. On the one hand, there is the wish to see science and scientific objects as wholly *constructed*, and hence as artefacts of theory and method. On the other hand, there is the conviction that science does describe what is "there", and that its descriptions are *more or less determined*. It seems to us this dichotomizing encourages a strategy of synthesis with the pressure to reconcile the views by saying that scientific objects are both constructed and determined. In one way, it could then be argued the disputes in the sociology of scientific knowledge are really squabbles about how to constitute this synthesis. However, from the point of view of a descriptive sociology, scientific objects are *neither* constructed nor determined in quite the ways that this view suggests. Scientific and mathematical reasoning are members' methods for the discovery of the objectivity of scientific objects. Their discoverable properties are just those which the methods reveal. There seems then little point in debating the ontological status of quarks, leptons, eigenvalues and the like when the frameworks determining their "real-world" character are part and parcel of the procedures used to discover them. The task is, surely, not to decide where such objects reside, "in the world" or "in the theory", but how in the world constituted according to the theory their objective status is achieved? It is our hope that in a preliminary way, our discussion of reasoning together through texts indicates some of the ways in which this can be achieved.

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