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Selecting an outsourcing provider based on the combined MABAC–ELECTRE method using single-valued neutrosophic linguistic sets

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ABSTRACT

Outsourcing has become a common and important link for enterprises. When selecting an outsourcing provider, single-valued neutrosophic linguistic sets can successfully express qualitative and fuzzy information. Furthermore, the selection of an outsourcing provider is a multi-criteria decision-making problem that can be tackled by the multi-attribute border approximation area comparison (MABAC) method. In MABAC, criteria are assumed to be compensatory. However, criteria may be non-compensatory in outsourcing provider selection. Thus, this paper introduces the main idea of the elimination and choice translating reality (ELECTRE) method. An MABAC–ELECTRE method is established under single-valued neutrosophic linguistic environments. This method uses the mean-squared deviation weight method to obtain the weights of criteria. Moreover, an illustrative example is conducted to explain the procedure of the MABAC–ELECTRE method. A comparative analysis verifies its feasibility in solving problems with non-compensatory criteria.

1. Introduction

Outsourcing has become a significant link in the production chain for many enterprises. It can reduce manpower and financial investment and achieve maximum efficiency. Enterprises may outsource a part of or all work to professional organisations as needed. Recently, an increasing number of enterprises have been outsourcing to seek assistance for their production and operation. In 2015, the amount of money involved in outsourcing service contracts signed by Chinese enterprises reached 130.93 billion dollars, and the amount of money involved in execution reached 96.69 billion dollars. It is of great economic value to investigate the decision-making method for outsourcing provider selection.

Many scholars have focused on finding a suitable one from a multitude of outsourcing providers. Some of these scholars mentioned that the outsourcing provider selection is a multi-criteria decision-making problem (Hsu, Liou, & Chuang, 2013; Liou & Chuang, 2010; Liou, Wang, Hsu, & Yin, 2011; Tavana, Zareinejad, Caprio, & Kaviani, 2016). These scholars introduced multi-criteria decision-making (MCDM) methods to establish the outsourcing provider selection methods. The characterisation of decision information is a significant issue in constructing these MCDM selection methods. A few researchers considered that uncertain information may exist in the selection process due to the complexity of human cognition (Li & Wan, 2014a; Uygun, Kaçamak, & Kahraman, 2015; Wan, Wang, Lin, & Dong, 2015). Fuzzy sets (FSs) can depict uncertain information (Krupka & Lastovicka, 2017; Li & Wang, 2017) and have been introduced in many fields, such as decision making (Geng, Liu, Teng, & Liu, 2017; Wang, Yang, & Li, 2016; Wang, Zhang, & Wang, 2018; Yu, Wang, & Wang, 2018), treatment selection (Ji, Zhang, & Wang, 2017), game model (Erman, 2010) and green supplier evaluation (Wang, Liu, Liu, & Huang, 2017). Likewise, FSs have been applied to denote uncertain information in the outsourcing provider selection.

However, limitations exist in existing research on the fuzzy outsourcing provider selection.

(1) The outsourcing provider selection is a complicated problem, in which uncertain decision information may not be expressed quantitatively. Such information may be improper to be characterised by FSs that comprise quantitative values. Nevertheless, this fact has not been considered in existing fuzzy outsourcing provider selection methods. These methods applied FSs composed of quantitative values (see Section 2). Information loss and distortion may exist in current outsourcing provider selection methods.
(2) Existing studies on outsourcing provider selection are based on an impertinent assumption. They suppose that multiple criteria involved in outsourcing provider selection are complementary. However, criteria may be non-compensatory. For example, six criteria may be considered in information technology (IT) outsourcing provider selection. These criteria include research and development

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capability, product quality, technology level, flexibility, delivery time and cost. Low cost cannot compensate for poor research and development capability, inferior quality of product, low technology level, poor flexibility or long delivery time.

A novel fuzzy outsourcing provider selection method is constructed. It overcomes the aforementioned deficiencies. The proposed outsourcing provider selection method expresses decision information using single-valued neutrosophic linguistic sets (SVNLSs). Moreover, the new selection method is established by using multi-attribute border approximation area comparison (MABAC) method. This paper presents two weighted aggregation operators for SVNLSs that will be used in MABAC. These two operators include the single-valued neutrosophic linguistic weighted average (SVNLWA) and single-valued neutrosophic linguistic weighted geometric (SVNLWG) operators. As for the non-compensation problem, our outsourcing provider selection method introduces the main idea of the elimination and choice translating reality (ELECTRE) method into MABAC.

The use of SVNLSs is motivated by the following reason. An SVNLS (Ye, 2015) comprises a linguistic part and a fuzzy part. The fuzzy part is used to depict the degrees of confidence of linguistic value in the linguistic part. The fuzzy part of an SVNLS is in the form of single-valued neutrosophic numbers (SVNNs). SVNLS is a useful tool to denote decision information in the outsourcing provider selection. For example, when an expert is required to assess an outsourcing provider, he or she provides a qualitative evaluation (e.g. the outsourcing provider is good) instead of a numerical rating. Furthermore, the degree of which he or she thinks the assessment is true is not 100% but a decimal between zero and one, such as 0.8. The degree of which he or she thinks the assessment is false is not zero but a decimal between zero and one, such as 0.3. The degree of which he or she does not sure about the assessment is a decimal between zero and one, such as 0.2. A single specific value cannot reflect all information simultaneously. Moreover, these three degrees are mutually independent, and there is no restriction on their sum. SVNNs can depict information regarding the confidence of the assessment perfectly and SVNLSs can denote the assessments in the outsourcing provider selection successfully.

The application of the proposed outsourcing provider selection method is explained in an illustrative example. In addition, the proposed outsourcing provider selection method is compared with several existing methods in a comparative analysis. Results of comparative analysis indicate the good performance of the proposed outsourcing provider selection method.

We contribute to existing research on outsourcing provider selection. The contributions of this study are threefold: (1) Characterisation of decision information: SVNLSs are perfect for describing the qualitative and uncertain information in outsourcing provider selection. Unlike single-valued neutrosophic sets (SVNSs), SVNLSs can depict qualitative information. Different from linguistic values, SVNLSs characterise fuzziness in the selection process. (2) Consideration of non-compensation of criteria: Our outsourcing provider selection method introduces the main idea behind ELECTRE method into MABAC. Unlike the traditional MABAC, our method considers the non-complementary property of criteria. (3) Weight method: Our MABAC–ELECTRE method determines the weight vector of criteria by using the mean-squared deviation weight method.

The structure of this paper is organised as follows. Section 2 reviews the existing research on outsourcing provider selection. Section 3 introduces definitions that will be used in our outsourcing provider selection method. Furthermore, we define the SVNLWA and SVNLWG operators. Section 4 constructs the combined MABAC–ELECTRE method to address the outsourcing provider selection problem. Section 5 provides an illustrative example of IT outsourcing provider selection. Section 6 presents a comparative analysis. Finally, Section 7 concludes the paper and provides interesting directions for future research.

2. Literature review

Scholars have been investigating on how to find a proper one from multiple outsourcing providers and have obtained some achievements (Hsu et al., 2013; Liou & Chuang, 2010; Liou et al., 2011). For example, Lin, Lin, Yu, and Tzeng (2010) proposed an outsourcing provider selection method, which uses the interpretive structural modelling and the analytic network process. Tavana et al. (2016) developed a selection method for outsourcing reverse logistics. The proposed method uses a preference programming model to derive the weights of criteria. Li and Wan (2014a) and Qiang and Li (2015) constructed linear programming methods for outsourcing provider selection. Uygun et al. (2015) introduced an MCDM method to establish an outsourcing provider selection method. Moreover, Sivakumar, Kannan, and Murugesan (2015) combined the analytic hierarchy process and Taguchi loss functions to address outsourcing provider selection problems.

Some scholars emphasized the existence of uncertain information in outsourcing provider selection due to the complexity of human cognition (Li & Wan, 2014a; Uygun et al., 2015; Wan et al., 2015). These researchers investigated the expression of uncertain information in outsourcing provider selection and suggested the introduction of fuzzy logic. For instance, Wan et al. (2015) used intuitionistic FSs to depict uncertain information in outsourcing provider selection. In addition, Li and Wan (2014a) introduced trapezoidal fuzzy number to express uncertain information. Uygun et al. (2015) characterised uncertain information using triangular fuzzy number. Moreover, Wang, Wang, and Zhang (2016) applied multi-hesitant fuzzy linguistic term sets in denoting decision information in outsourcing provider selection. Diversely, Ji, Wang, and Zhang (2016) employed SVNSs to express uncertain information in outsourcing provider selection.

However, two deficiencies regarding characterising uncertain information exist in the aforementioned studies. (1) Firstly, FSs in (Li & Wan, 2014a; Uygun et al., 2015; Wan et al., 2015) can only depict quantitative information. Section 1 indicates that decision makers may fail to express their evaluations using quantitative values. They prefer to use qualitative values rather than precise values in evaluating outsourcing provider. Nevertheless, FSs in (Li & Wan, 2014a; Uygun et al., 2015; Wan et al., 2015) cannot depict qualitative information in outsourcing provider selection. (2) Secondly, FS in Wang, Wang, et al. (2016) can express qualitative evaluation information. However, it cannot reflect the degrees of truth-membership, falsity-membership and indeterminacy-membership of each qualitative evaluation. Section 1 explains that the degree of which a decision maker thinks his or her assessment is true may not be 100%; the degree of which a decision maker thinks his or her assessment is false may not be zero; the degree of which a decision maker is not sure about his or her assessment may not be zero. This is because of his or her limited knowledge and cognition.

The outsourcing provider selection is an MCDM problem. MCDM methods can be introduced to solve outsourcing provider selection problems. Many fuzzy MCDM methods have been proposed (Pouresmaeil, Shivanian, Khorram, & Fathabadi, 2017). For example, Stanujkic, Zavadskas, Smarandache, Brauers, and Karabasevic (2017) and Zavadskas, Bausys, Juodagalviene, and Garnyte-Sapranaviciene (2017) investigated the multi-objective optimisation by a ratio analysis and the full multiplicative form under neutrosophic environments. Moreover, a weighted aggregated sum product assessment with SVNSs was developed by Zavadskas, Baušys, Stanujkic, and Magdalinovic-Kalinovic (2017) and Zavadskas, Baušys, and Lazauskas (2015). In addition, Liang, Wang, and Zhang (2017) presented a single-valued trapezoidal neutrosophic decision making trial and evaluation laboratory method. Zavadskas, Bausys, Kaklauskas, et al. (2017) proposed a multi-attribute market value assessment method with neutrosophic sets. Furthermore, Ye (2014) constructed a neutrosophic MCDM method based on cross-entropy.

MABAC method is a simple and effective MCDM method with simple calculation, systematic process and sound logic (Xue, You, Lai, & Liu, 2016). It is a particularly pragmatic and reliable measure for MCDM problems. MABAC was recently proposed by Pamučar and Ćirović (2015). It divides alternatives under each criterion into three areas (Debnath, Roy, Kar, Zavadskas, & Antucheviciene, 2017; Shi, Liu, Li, & Xu, 2017). These areas comprise the border, upper and lower approximation areas. MABAC has been extended into fuzzy environments (Gigović, Pamučar, Božanić, & Ljubojević, 2017; Peng, Dai, & Yuan, 2016; Shi et al., 2017). For instance, Xue et al. (2016) and Liu, You, and Duan (2017) applied MABAC to address problems under interval-valued intuitionistic fuzzy environments. In addition, Peng and Yang (2016) established the Choquet integral-based MABAC under Pythagorean fuzzy environments. Yu, Wang, and Wang (2017) investigated the extension of MABAC under interval type-2 fuzzy environments. Pamučar, Petrović, and Ćirović (2017) and Pamučar, Mihajlović, Obradović, and Atanasković (2017) presented MABAC with interval-valued fuzzy rough numbers. They combined the best–worst method with MABAC. Moreover, MABAC has been extended into hesitant fuzzy (Peng & Dai, 2017b) and interval-valued neutrosophic environments (Peng & Dai, 2017a). MABAC has not been investigated under single-valued neutrosophic linguistic environments. It is an interesting research topic to apply MABAC in outsourcing provider selection with SVNLSs.

MABAC assumes that criteria are compensatory. However, as narrated in Section 1, criteria may be non-compensatory in outsourcing provider selection. ELECTRE is a significant method to address the non-compensation problem of criteria (Del Vasto-Terrientes, Valls, Slowinski, & Zielniewicz, 2015). ELECTRE has been investigated and extended by many researchers. A family of ELECTRE methods, including ELECTRE I, ELECTRE II and ELECTRE III, has been formed (Zhang, Peng, & Wang, 2017). In addition, many researchers devote themselves to the application of ELECTRE methods in fuzzy decision making (Liu, You, Chen, & Chen, 2016; Wang, Wang, Chen, Zhang, & Chen, 2014; Zhou, Wang, & Zhang, 2016). For example, Vahdani and Hadipour (2011) applied ELECTRE to settle decision-making problems with interval-valued FSs. In addition, ELECTRE has been extended into intuitionistic fuzzy (Wu & Chen, 2011) and neutrosophic environments (Zhang, Wang, & Chen, 2016). Moreover, Peng, Wang, Wang, Yang, and Chen (2015) defined the outranking relations of multi-hesitant FSs and extended ELECTRE III into multi-hesitant fuzzy environments. ELECTRE III has also been investigated under hesitant interval-valued fuzzy environments (Wang, Peng, Zhang, & Chen, 2017).

In this study, decision information in outsourcing provider selection is characterised by single-valued neutrosophic linguistic numbers (SVNLNs). The SVNLWA and SVNLWG operators are presented. Then, an outsourcing provider selection method is constructed by using MABAC. The proposed outsourcing provider selection method tackles the non-compensation problem of criteria by introducing the main idea of ELECTRE III.

3. Preliminary definitions

In this section, some concepts of SVNLSs and SVNLNs are reviewed. These concepts will be involved in our proposed outsourcing provider selection method. Moreover, the SVNLWA and the SVNLWG operators are developed and will be used in the construction of the proposed outsourcing provider selection method.

Definition 1 (*Ye, 2015*). Let $X = \{x_1,x_2,...,x_n\}$ be a non-empty fixed set. Let $S = \{s_0,s_1,s_2,...,s_l\}$ be a finite and totally ordered discrete linguistic term set. An SVNLS can be defined as:

$$A = \{\langle x_i,[s_{\theta(x_i)},(T_A(x_i),I_A(x_i),F_A(x_i))]\rangle\},$$

where $x_i \in X$, $s_{\theta(x_i)} \in S$, $T_A(x_i) \in [0,1]$, $I_A(x_i) \in [0,1]$ and $F_A(x_i) \in [0,1]$. $T_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ are the degrees of truth-membership, indeterminate-membership and false-membership of x_i in X to $s_{\theta(x_i)}$, respectively. Moreover, $0 \leqslant T_A(x_i) + I_A(x_i) + F_A(x_i) \leqslant 3$ exists for any $x_i \in X$.

In addition, $a = \langle s_{\theta(x_i)},(T_A(x_i),I_A(x_i),F_A(x_i))\rangle$ is called an SVNLN. For convenience, hereafter, an SVNLN is denoted as $a = \langle s_{\theta(x_i)},(T_a,I_a,F_a)\rangle$. An SVNLS can be considered as a collection of SVNLNs and can be represented as $A = \{\langle s_{\theta(x_i)},(T_A(x_i),I_A(x_i),F_A(x_i))\rangle\}$.

Tian, Wang, Wang, and Zhang (2017) introduced the linguistic scale function to define the operations of SVNLNs.

Definition 2 (*Tian et al., 2017*). Let $a = \langle s_{\theta(a)},(T_a,I_a,F_a)\rangle$ and $b = \langle s_{\theta(b)},(T_b,I_b,F_b)\rangle$ be two SVNLNs. Let f^* be a linguistic scale function, f^{*-1} be the anti-function of f^* and $\lambda > 0$. Then, the operations of SVNLNs can be defined as follows:

(1) $a + b = \left\langle f^{*-1}(f^*(s_{\theta(a)}) + f^*(s_{\theta(b)})),\left(\frac{f^*(s_{\theta(a)})T_a+f^*(s_{\theta(b)})T_b}{f^*(s_{\theta(a)})+f^*(s_{\theta(b)})},\frac{f^*(s_{\theta(a)})I_a+f^*(s_{\theta(b)})I_b}{f^*(s_{\theta(a)})+f^*(s_{\theta(b)})},\frac{f^*(s_{\theta(a)})F_a+f^*(s_{\theta(b)})F_b}{f^*(s_{\theta(a)})+f^*(s_{\theta(b)})}\right)\right\rangle$;
(2) $a \times b = \langle f^{*-1}(f^*(s_{\theta(a)})f^*(s_{\theta(b)})),(T_aT_b,I_a + I_b-I_aI_b,F_a + F_b-F_aF_b)\rangle$;
(3) $\lambda a = \langle f^{*-1}(\lambda f^*(s_{\theta(a)})),(T_a,I_a,F_a)\rangle$;
(4) $a^\lambda = \langle f^{*-1}((f^*(s_{\theta(a)}))^\lambda),((T_a)^\lambda,1-(1-I_a)^\lambda,1-(1-F_a)^\lambda)\rangle$; and
(5) $neg(a) = \langle f^{*-1}(f^*(s_{\theta(l)})-f^*(s_{\theta(a)})),(F_a,1-I_a,T_a)\rangle$.

Three linguistic scale functions are listed by Tian et al. (2017). The first scale function is $f_1(s_{\theta(j)}) = \frac{\theta(j)}{l}$, $\theta(j) \in [0,l]$. Its linguistic information is divided on average. The second and third linguistic scale functions are piecewise functions. They are presented as follows:

$$f_2(s_{\theta(j)}) = \begin{cases} \frac{r^{l/2}-r^{l/2-\theta(j)}}{2r^{l/2}-2} & \theta(j) \in [0,l/2], \\ \frac{r^{l/2}+r^{\theta(j)-l/2}-2}{2r^{l/2}-2} & \theta(j) \in (l/2,l] \end{cases} \text{,and}$$

$$f_3(s_{\theta(j)}) = \begin{cases} \frac{(l/2)^\beta-(l/2-\theta(j))^\beta}{2(l/2)^\beta} & \theta(j) \in [0,l/2], \\ \frac{(l/2)^\gamma+(\theta(j)-l/2)^\gamma}{2(l/2)^\gamma} & \theta(j) \in (l/2,l] \end{cases}.$$

All three functions are strictly monotonically increasing and continuous functions.

Tian et al. (2017) developed the comparison method of SVNLNs based on the proposed score, accuracy and certainty functions.

Definition 3 (*Tian et al., 2017*). Let $a = \{\langle s_{\theta(a)},(T_a,I_a,F_a)\rangle\}$ be an SVNLN. Let f^* be a linguistic scale function. The score function of a can be defined as follows:

$$S(a) = f^*(s_{\theta(a)})(T_a + 1-I_a + 1-F_a).$$

The accuracy function of a can be defined as follows:

$$A(a) = f^*(s_{\theta(a)})(T_a-F_a).$$

The certainty function of a can be defined as follows:

$$C(a) = f^*(s_{\theta(a)})T_a.$$

Definition 4 (*Tian et al., 2017*). Let $a = \{\langle s_{\theta(a)},(T_a,I_a,F_a)\rangle\}$ and $b = \{\langle s_{\theta(b)},(T_b,I_b,F_b)\rangle\}$ be two SVNLNs. The comparison method of SVNLNs can be defined as follows:

(1) If $S(a) > S(b)$, then $a \succ b$.
(2) If $S(a) = S(b)$ and $A(a) > A(b)$, then $a \succ b$.
(3) If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) > C(b)$, then $a \succ b$.
(4) If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) = C(b)$, then $a = b$.

Definition 5 (*Ye, 2015*). Let $a = \langle s_{\theta(a)},(T_a,I_a,F_a)\rangle$ and $b = \langle s_{\theta(b)},(T_b,I_b,F_b)\rangle$ be two SVNLNs. The generalised distance measure between a and b can be defined as follows:

$$d(a,b) = |\theta(a)T_a-\theta(b)T_b|^p + |\theta(a)I_a-\theta(b)I_b|^p + |\theta(a)F_a-\theta(b)F_b|^p, \quad (1)$$

when $p = 1$, Eq. (1) reduces to the Hamming distance; and when $p = 2$, Eq. (1) reduces to the Euclidean distance.

Two aggregation operators of SVNLNs, namely, SVNLWA and the SVNLWG, are defined on the basis of existing studies.

Definition 6. Let $x_i = \langle s_{\theta(x_i)},(T_{x_i},I_{x_i},F_{x_i})\rangle$ $(i = 1,2,...,n)$ be a collection of SVNLNs. $w = (w_1,w_2,...,w_n)^T$ is the weight vector of x_i, where $\sum_{i=1}^{n} w_i = 1$ and $w_i \geqslant 0$ $(i = 1,2,...,n)$. The SVNLWA operator can be defined as follows:

$$SVNLWA(x_1,x_2,...,x_n) = \sum_{i=1}^{n} (w_i x_i).$$

If the weight vector $w = \left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then the SVNLWA operator reduces to the single valued neutrosophic linguistic arithmetic average (SVNLAA) operator.

Theorem 1. *Let $x_i = \langle s_{\theta(x_i)},(T_{x_i},I_{x_i},F_{x_i})\rangle$ $(i = 1,2,...,n)$ be a collection of SVNLNs. $w = (w_1,w_2,...,w_n)^T$ is the weight vector of x_i, where $\sum_{i=1}^{n} w_i = 1$ and $w_i \geqslant 0$ $(i = 1,2,...,n)$. The aggregated result by the SVNLWA operator is also an SVNLN, and*

$$\begin{aligned} &SVNLWA(x_1,x_2,...,x_n) \\ &= \left\langle f^{*-1}\left(\sum_{i=1}^{n} w_i f^*(s_{\theta(x_i)})\right),\left(\frac{\sum_{i=1}^{n} w_i T_{x_i} f^*(s_{\theta(x_i)})}{\sum_{i=1}^{n} w_i f^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{n} w_i I_{x_i} f^*(s_{\theta(x_i)})}{\sum_{i=1}^{n} w_i f^*(s_{\theta(x_i)})}\right.\right. \\ &\left.\left.,\frac{\sum_{i=1}^{n} w_i F_{x_i} f^*(s_{\theta(x_i)})}{\sum_{i=1}^{n} w_i f^*(s_{\theta(x_i)})}\right)\right\rangle. \end{aligned} \tag{2}$$

The proof of Theorem 1 is presented in Appendix A.

Definition 7. Let $x_i = \langle s_{\theta(x_i)},(T_{x_i},I_{x_i},F_{x_i})\rangle$ $(i = 1,2,...,n)$ be a collection of SVNLNs. $w = (w_1,w_2,...,w_n)^T$ is the weight vector of x_i, where $\sum_{i=1}^{n} w_i = 1$ and $w_i \geqslant 0$ $(i = 1,2,...,n)$. The SVNLWG operator can be defined as follows:

$$SVNLWG(x_1,x_2,...,x_n) = \prod_{i=1}^{n} (x_i)^{w_i}.$$

If the weight vector $w = \left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then the SVNLWG operator reduces to the single valued neutrosophic linguistic geometric average (SVNLGA) operator.

Theorem 2. *Let $x_i = \langle s_{\theta(x_i)},(T_{x_i},I_{x_i},F_{x_i})\rangle$ $(i = 1,2,...,n)$ be a collection of SVNLNs. $w = (w_1,w_2,...,w_n)^T$ is the weight vector of x_i, where $\sum_{i=1}^{n} w_i = 1$ and $w_i \geqslant 0$ $(i = 1,2,...,n)$. The aggregated result by the SVNLWG operator is also an SVNLN, and*

$$\begin{aligned} SVNLWG(x_1,x_2,...,x_n) = &\left\langle f^{*-1}\left(\prod_{i=1}^{n} (f^*(\theta(x_i)))^{w_i}\right),\left(\prod_{i=1}^{n} T_{x_i}^{w_i},1\right.\right. \\ &\left.\left. -\prod_{i=1}^{n} (1-I_{x_i})^{w_i},1-\prod_{i=1}^{n} (1-F_{x_i})^{w_i}\right)\right\rangle. \end{aligned} \tag{3}$$

The proof of Theorem 2 is provided in Appendix A.

4. Combined MABAC–ELECTRE selection method

This section presents a combined MABAC–ELECTRE method for outsourcing provider selection. The combined MABAC–ELECTRE method comprises three stages. The first stage aims to acquire the weight vector of criteria. The second stage obtains the differences between outsourcing providers and corresponding border approximation area. The outsourcing providers are ranked in the last stage. Fig. 1 depicts the structure of the combined MABAC–ELECTRE method. Its details are illustrated in the rest of this section.

An outsourcing provider selection problem involves m providers $(A_1,A_2,...,A_m)$ and n criteria $(C_1,C_2,...,C_n)$. Each outsourcing provider is evaluated by e $(e \geqslant 1)$ decision makers $(DM_1,DM_2,...,DM_e)$ against each criterion. The weighted vector of decision makers is given as $\omega = (\omega_1,\omega_2,...,\omega_e)^T$. The evaluations provided by decision maker DM_g $(g = 1,2,...,e)$ can be characterised by SVNLNs. The details of the characterisation have been explained in Section 1. The decision matrix U^g of DM_g can be denoted as follows:

$$U^g = \begin{pmatrix} U_{11}^g & U_{12}^g & \cdots & U_{1n}^g \\ U_{21}^g & U_{22}^g & \cdots & U_{2n}^g \\ \vdots & \vdots & \ddots & \vdots \\ U_{m1}^g & U_{m2}^g & \cdots & U_{mn}^g \end{pmatrix},$$

where $U_{rj}^g = \langle \tilde{s}_{rj}^g,(\tilde{T}_{rj}^g,\tilde{I}_{rj}^g,\tilde{F}_{rj}^g)\rangle$ is the SVNLN of A_r $(r = 1,2,\cdots,m)$ against C_j $(j = 1,2,\cdots,n)$ provided by DM_g $(g = 1,2,...,e)$.

4.1. Stage 1: Obtain the weight vector of criteria

In this stage, the weight vector of criteria is obtained. The weights of criteria are determined by a mean-squared deviation weight method. The details of this stage are explained as follows.

Step 1: Normalise decision matrices.

An outsourcing provider problem may have cost and benefit criteria. Thus, the decision matrix of each decision maker should be normalised. If C_j is a cost criterion, then U_{rj}^g should be normalised; otherwise, normalising U_{rj}^g is unnecessary. For ease of description, CC is used to denote the set of all cost criteria. The formula of normalisation is defined as follows:

$$N_{rj}^g = \langle s_{rj}^g,(T_{rj}^g,I_{rj}^g,F_{rj}^g)\rangle = \begin{cases} neg(U_{rj}^g) & C_j \in CC, \\ U_{rj}^g & \text{otherwise.} \end{cases} \tag{4}$$

where $neg(U_{rj}^g)$ is the compensatory set of U_{rj}^g. The compensatory set of an SVNLN is defined in Definition 2. The normalised decision matrices $N^g = (N_{rj}^g)_{m\times n}$ $(g = 1,2,...,e)$ can be determined by using Eq. (4).

Step 2: Aggregate normalised decision matrices.

The most suitable outsourcing provider should be obtained using the evaluations of all decision makers. In this step, the normalised decision matrices of e decision makers are aggregated. This purpose can be achieved by using the SVNLWA operator in Eq. (2):

$$N_{rj} = \langle s_{rj},(T_{rj},I_{rj},F_{rj})\rangle = SVNLWA(N_{rj}^1,N_{rj}^2,...,N_{rj}^e), \tag{5}$$

where N_{rj} is the aggregated decision information of A_r against C_j. The aggregated decision matrix N comprises N_{rj} $(r = 1,2,...m;j = 1,2,...,n)$ and can be denoted as $N = (N_{rj})_{m\times n}$.

Step 3: Obtain mean values.

A mean-squared deviation weight method is developed in the subsequent steps. This method uses the mean value of all outsourcing providers against each criterion. In this step, the mean value $E(N_j)$ of all outsourcing providers against C_j is obtained. $E(N_j)$ is obtained by using the SVNLAA operator:

$$E(N_j) = SVNLAA(N_{1j},N_{2j},...,N_{mj}). \tag{6}$$

The SVNLAA operator is an SVNLWA operator (i.e. Eq. (2)) with equal weights of aggregated elements.

Step 4: Obtain mean-squared deviation values.

This step intends to calculate the mean-squared deviation values. The mean-squared deviation value $\sigma(N_j)$ against C_j can be obtained by

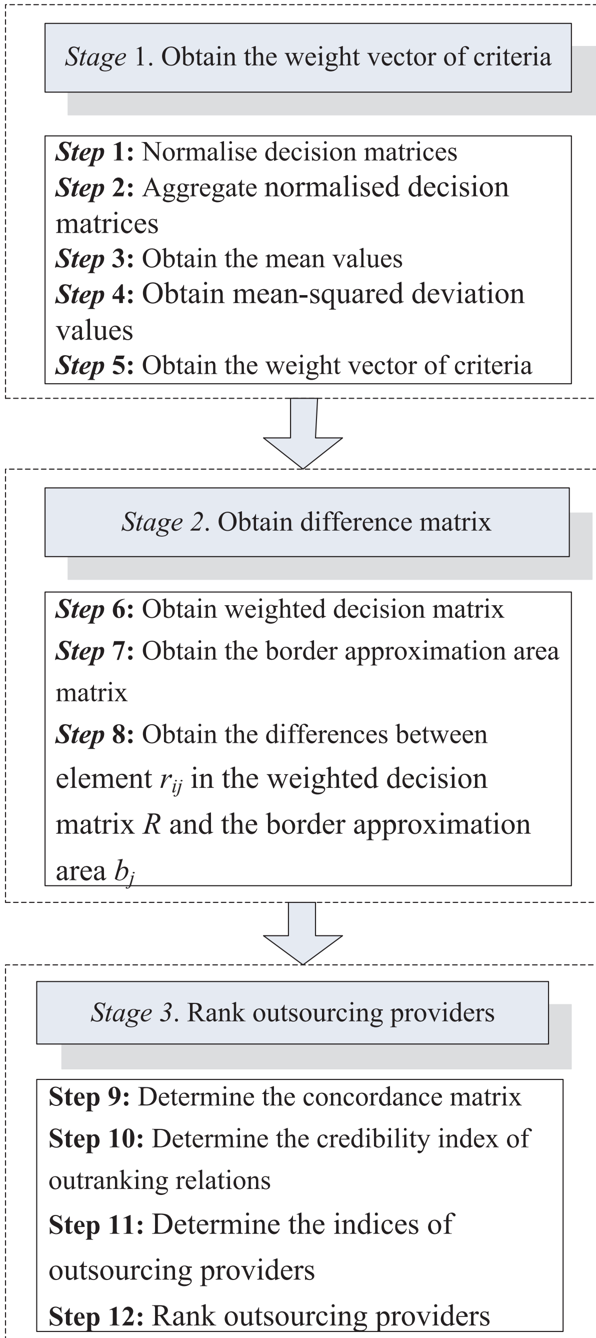

Fig. 1. Structure of the combined MABAC-ELECTRE method.

using Eq. (7):

$$\sigma(N_j) = \sqrt{\sum_{i=1}^{m} (d(N_{ij}-E(N_j)))^2}. \tag{7}$$

Step 5: Obtain the weight vector of criteria.

The weight w_j of C_j can be obtained by using Eq. (8):

$$w_j = \frac{\sigma_j}{\sum_{j=1}^{n} \sigma_j}. \tag{8}$$

The weight vector of criteria comprises their weight, and it is represented by $w = (w_1,w_2,...,w_n)^T$ in the two next stages.

4.2. Stage 2: Obtain difference matrix

Step 6: Obtain weighted decision matrix.

Stage 1 obtains the aggregated decision matrix N in Step 2. Stage 1 also determines the weight vector of criteria $\boldsymbol{w}$ in Step 5. This step obtains the weighted decision matrix by multiplying N by $\boldsymbol{w}$. Elements in the weighted decision matrix $R = (r_{ij})_{m\times n}$ can be obtained by Eq. (9):

$$r_{ij} = w_j N_{ij}. \tag{9}$$

Step 7: Obtain the border approximation area matrix.

MABAC involves the notion of border approximation area. The border approximation area b_j can be calculated by using the SVNLGA operator in Eq. (3) as:

$$b_j = SVNLGA(N_{1j},N_{2j},...,N_{mj}). \tag{10}$$

Then, the border approximation area matrix $B = (b_j)_{n\times 1}$ can be determined.

Step 8: Obtain the difference between element r_{ij} in the weighted decision matrix R and the border approximation area b_j.

The difference matrix $D = (d_{ij})_{m\times n}$ can be obtained by using Eq. (1) in Definition 3 as:

$$d_{ij} = \begin{cases} d(N_{ij},b_j) & \text{if} \quad N_{ij} \succ b_j \\ -d(N_{ij},b_j) & \text{otherwise} \end{cases}. \tag{11}$$

Whether $N_{ij} \succ b_j$ or not is determined by the comparison method in Definition 4.

4.3. Stage 3: Rank outsourcing providers

MABAC ranks alternatives according to the closeness coefficient to the border approximation area. This closeness coefficient is the arithmetic sum of the row elements of D (i.e., $\sum_{j=1}^{n} d_{ij}$). That is, MABAC irrationally assumes that criteria are complementary. The main idea of ELECTRE is to manage the integration of the row elements of D. The details of this stage are explained as follows.

Step 9: Determine the concordance matrix.

The concordance index $C(x_i,x_k)$ between A_i and A_k can be calculated by Eq. (12):

$$C(A_i,A_k) = \sum_{j=1}^{n} \frac{1}{n} c(d_{ij},d_{kj}), \tag{12}$$

where $c(d_{ij},d_{kj})$ is the degree of concordance between d_{ij} and d_{kj}.

$$c(d_{ij},d_{kj}) = \begin{cases} 0 & d_{kj}-d_{ij} \geqslant p_j \\ \frac{p_j+d_{ij}-d_{kj}}{p_j-q_j} & q_j \leqslant d_{kj}-d_{ij} \leqslant p_j, \\ 1 & d_{kj}-d_{ij} \leqslant q_j \end{cases} \tag{13}$$

where q_j is the preference threshold under c_j, p_j is the indifference threshold under c_j and $0 \leqslant q_j \leqslant p_j$.

Therefore, the concordance matrix C is as follows:

$$C = \begin{pmatrix} C(A_1,A_1) & C(A_1,A_2) & \cdots & C(A_1,A_n) \\ C(A_2,A_1) & C(A_2,A_2) & \cdots & C(A_2,A_n) \\ \cdots & \cdots & \cdots & \cdots \\ C(A_n,A_1) & C(A_n,A_2) & \cdots & C(A_n,A_n) \end{pmatrix}.$$

Step 10: Determine the credibility index of outranking relations.

$$\sigma(A_i,A_k) = C(A_i,A_k)\cdot \prod_{j=1}^{n} \delta_j(A_i,A_k). \tag{14}$$

Here,

$$\delta_j(A_i,A_k) = \begin{cases} \frac{1-disc(d_{ij},d_{kj})}{1-C(d_i,d_k)} & \text{if } disc(d_{ij},d_{kj}) > C(A_i,A_k) \\ 1 & \text{otherwise} \end{cases},$$

where $disc(d_{ij},d_{kj})$ is the degree of discordance between d_{ij} and d_{kj}, and $disc(d_{ij},d_{kj})$ can be obtained by:

$$disc(d_{ij},d_{kj}) = \begin{cases} 0 & d_{kj}-d_{ij} \leqslant p_j \\ \frac{d_{kj}-d_{ij}-p_j}{v_j-p_j} & p_j \leqslant d_{kj}-d_{ij} \leqslant v_j, \\ 1 & d_{kj}-d_{ij} \geqslant v_j \end{cases}$$

where p_j is the indifference threshold under c_j, v_j is the veto threshold under c_j and $0 \leqslant p_j \leqslant v_j$.

Step 11: Determine the indices of outsourcing providers.

The index of an outsourcing provider can be defined as follows:

$$index(A_i) = \sum_{k=1}^{m} \sigma(A_i,A_k) - \sum_{k=1}^{m} \sigma(A_k,A_i), \tag{15}$$

where $i = 1,2,...,m$.

Step 12: Rank outsourcing providers.

Outsourcing providers can be ranked according to $index(A_i)$ ($i = 1,2,...,m$). A greater value of $index(A_i)$ yields a better outsourcing provider A_i.

5. Sample of outsourcing provider selection

This section applies the combined MABAC–ELECTRE method to tackle an outsourcing provider selection problem. This section mainly aims to explain the application of the combined MABAC–ELECTRE method.

The following case, which stems from (Li & Wan, 2014b), is about IT outsourcing provider selection. San'an Optoelectronics Company Limited (hereafter termed San'an), which was founded in November 2000, is China's earliest-founded and largest production base to manufacture full-colour, ultra-high bright LED epitaxial products and chips with best quality. San'an principally engages in research & development, production and marketing of products, such as full-colour, ultra-high brightness LED epitaxial products, chips, compound solar cells and high-power concentrating solar products. The general headquarter of San'an is located in Xiamen and its industrialisation bases distribute in multiple regions, including Xiamen, Tianjin, Wuhu and Huainan. In 2015, the revenue of San'an reached RMB 4.858 billion with a growth rate of 6% from 2014, and the profit of San'an reached RMB 1.695 billion with a growth rate of 16% from 2014.

San'an contributes the great majority of manpower and financial resources to its core competition instead of IT. The outsourcing of IT becomes an optimal option for San'an because of its lack of professional competence. San'an selects five potential outsourcing providers, including x_1 Shanghai Ingens IT Company Limited (hereafter termed Ingens), x_2 Beijing Teamsun Technology Company Limited (hereafter termed Teamsun), x_3 Shenzhen Sinoway IT Outsourcing Company Limited (hereafter termed Sinoway), x_4 Lenovo and x_5 Taiji Company Limited (hereafter termed Taiji), for further evaluation after forward selection. To find the most suitable IT outsourcing provider, San'an invites three experts, denoted by e_1, e_2 and e_3, to evaluate each outsourcing provider against six criteria, namely, c_1 research and development capability, c_2 product quality, c_3 technology level, c_4 flexibility, c_5 delivery time and c_6 cost. The weight vector of these three experts is $(1/3,1/3,1/3)^T$.

Section 1 stated that evaluations can be characterised by SVNLNs. Each expert assesses every provider against each criterion. The linguistic values in $S = \{s_0,s_1,s_2,s_3,s_4\}$ are used. For example, e_1 evaluated x_1 against C_1 using the linguistic value s_3. Moreover, each manager is requested to provide the following information: (1) The degree of which he or she thinks the assessment is true. (2) The degree of which he or she thinks the assessment is false. (3) The degree of which he is not sure about the assessment. These three kinds of information can be depicted by a SVNN. For instance, e_1 provided these three kinds of information about his evaluation (s_3) on x_1 against C_1. The degree that s_3 is true is 0.4; the degree that s_3 is false is 0.1; and the degree that he is unsure is 0.6. These kinds of information can be characterised by a SVNN $\langle 0.4,0.6,0.1\rangle$. All the aforementioned information on x_1 against C_1

Table 1
Decision matrix of e_1 with SVNLNs.

	c_1	c_2	c_3
x_1	$\langle s_3,(0.4,0.6,0.1)\rangle$	$\langle s_4,(0.9,0.2,0.6)\rangle$	$\langle s_4,(0.9,0.7,0.1)\rangle$
x_2	$\langle s_1,(0.9,0.7,0.1)\rangle$	$\langle s_2,(0.8,0.4,0.2)\rangle$	$\langle s_1,(0.5,0.4,0.9)\rangle$
x_3	$\langle s_2,(0.5,0.1,0.3)\rangle$	$\langle s_3,(0.6,0.5,0.1)\rangle$	$\langle s_4,(0.9,0.2,0.1)\rangle$
x_4	$\langle s_1,(0.5,0.3,0.5)\rangle$	$\langle s_2,(0.4,0.1,0.1)\rangle$	$\langle s_2,(0.2,0.8,0.4)\rangle$
x_5	$\langle s_4,(0.5,0.3,0.5)\rangle$	$\langle s_4,(0.7,0.5,0.1)\rangle$	$\langle s_2,(0.5,0.6,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_3,(0.1,0.8,0.6)\rangle$	$\langle s_0,(0.2,0.9,0.4)\rangle$	$\langle s_4,(0.8,0.1,0.1)\rangle$
x_2	$\langle s_0,(0.3,0.8,0.6)\rangle$	$\langle s_3,(0.1,0.9,0.4)\rangle$	$\langle s_2,(0.2,0.1,0.1)\rangle$
x_3	$\langle s_3,(0.1,0.3,0.5)\rangle$	$\langle s_0,(0.2,0.9,0.3)\rangle$	$\langle s_1,(0.5,0.1,0.2)\rangle$
x_4	$\langle s_0,(0.1,0.7,0.9)\rangle$	$\langle s_3,(0.1,0.8,0.5)\rangle$	$\langle s_4,(0.6,0.1,0.3)\rangle$
x_5	$\langle s_1,(0.2,0.3,0.8)\rangle$	$\langle s_0,(0.1,0.9,0.8)\rangle$	$\langle s_1,(0.6,0.3,0.1)\rangle$

Table 2
Decision matrix of e_2 with SVNLNs.

	c_1	c_2	c_3
x_1	$\langle s_4,(0.8,0.6,0.1)\rangle$	$\langle s_4,(0.5,0.2,0.2)\rangle$	$\langle s_4,(0.7,0.4,0.1)\rangle$
x_2	$\langle s_1,(0.8,0.4,0.1)\rangle$	$\langle s_3,(0.6,0.4,0.2)\rangle$	$\langle s_3,(0.5,0.2,0.3)\rangle$
x_3	$\langle s_1,(0.2,0.1,0.3)\rangle$	$\langle s_2,(0.8,0.5,0.2)\rangle$	$\langle s_4,(0.6,0.5,0.1)\rangle$
x_4	$\langle s_1,(0.6,0.4,0.4)\rangle$	$\langle s_2,(0.7,0.3,0.3)\rangle$	$\langle s_3,(0.6,0.8,0.2)\rangle$
x_5	$\langle s_4,(0.9,0.2,0.6)\rangle$	$\langle s_4,(0.9,0.2,0.1)\rangle$	$\langle s_2,(0.7,0.8,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_3,(0.1,0.6,0.9)\rangle$	$\langle s_1,(0.3,0.6,0.6)\rangle$	$\langle s_4,(0.5,0.1,0.3)\rangle$
x_2	$\langle s_1,(0.1,0.6,0.7)\rangle$	$\langle s_3,(0.4,0.9,0.7)\rangle$	$\langle s_4,(0.8,0.1,0.1)\rangle$
x_3	$\langle s_3,(0.2,0.5,0.8)\rangle$	$\langle s_0,(0.1,0.9,0.4)\rangle$	$\langle s_1,(0.6,0.1,0.1)\rangle$
x_4	$\langle s_0,(0.1,0.6,0.9)\rangle$	$\langle s_3,(0.1,0.5,0.4)\rangle$	$\langle s_4,(0.6,0.1,0.1)\rangle$
x_5	$\langle s_0,(0.2,0.6,0.4)\rangle$	$\langle s_0,(0.1,0.9,0.7)\rangle$	$\langle s_1,(0.2,0.1,0.1)\rangle$

Table 3
Decision matrix of e_3 with SVNLNs.

	c_1	c_2	c_3
x_1	$\langle s_2,(0.5,0.6,0.1)\rangle$	$\langle s_4,(0.4,0.8,0.1)\rangle$	$\langle s_3,(0.8,0.5,0.1)\rangle$
x_2	$\langle s_1,(0.4,0.4,0.1)\rangle$	$\langle s_4,(0.5,0.4,0.2)\rangle$	$\langle s_2,(0.8,0.1,0.3)\rangle$
x_3	$\langle s_3,(0.6,0.1,0.1)\rangle$	$\langle s_4,(0.5,0.5,0.5)\rangle$	$\langle s_4,(0.6,0.8,0.1)\rangle$
x_4	$\langle s_1,(0.1,0.8,0.2)\rangle$	$\langle s_2,(0.4,0.5,0.8)\rangle$	$\langle s_4,(0.8,0.8,0.1)\rangle$
x_5	$\langle s_4,(0.4,0.7,0.1)\rangle$	$\langle s_4,(0.8,0.2,0.1)\rangle$	$\langle s_2,(0.9,0.1,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_3,(0.1,0.7,0.9)\rangle$	$\langle s_2,(0.5,0.9,0.1)\rangle$	$\langle s_4,(0.2,0.1,0.2)\rangle$
x_2	$\langle s_2,(0.6,0.2,0.9)\rangle$	$\langle s_3,(0.1,0.9,0.4)\rangle$	$\langle s_3,(0.6,0.1,0.4)\rangle$
x_3	$\langle s_3,(0.3,0.7,0.2)\rangle$	$\langle s_0,(0.6,0.9,0.8)\rangle$	$\langle s_1,(0.4,0.1,0.6)\rangle$
x_4	$\langle s_0,(0.1,0.8,0.9)\rangle$	$\langle s_3,(0.1,0.8,0.6)\rangle$	$\langle s_4,(0.3,0.1,0.2)\rangle$
x_5	$\langle s_2,(0.2,0.6,0.7)\rangle$	$\langle s_0,(0.1,0.9,0.3)\rangle$	$\langle s_1,(0.4,0.2,0.1)\rangle$

provided by e_1 can be denoted by an SVNLN $\langle s_3,(0.4,0.6,0.1)\rangle$. Table 1 lists the SVNLNs for all outsourcing providers against each criterion provided by e_1. Likewise, Tables 2 and 3 show the SVNLNs for all outsourcing providers against each criterion provided by e_2 and e_3, respectively. SVNLN for x_i against C_j provided by e_1 (e_2 or e_3) is presented in the $(i + 1)$-th row and $(j + 1)$-th column of Table 1 (Tables 2 or 3).

5.1. Stage 1: Obtain the weight vector of criteria

Step 1: Normalise decision matrix.

c_4 and c_5 are cost criteria whereas the others are benefit criteria. Let the scale function be $f_1(s_{\theta(j)}) = \frac{\theta(j)}{l}$. In this example, $l = 4$ because $S = \{s_0,s_1,s_2,s_3,s_4\}$. Decision matrices can be normalised by using Eq. (4). The normalised decision matrices are listed in Tables 4–6.

Table 4
Normalised decision matrix N^1 of e_1.

	c_1	c_2	c_3
x_1	$\langle s_3,(0.4,0.6,0.1)\rangle$	$\langle s_4,(0.9,0.2,0.6)\rangle$	$\langle s_4,(0.9,0.7,0.1)\rangle$
x_2	$\langle s_1,(0.9,0.7,0.1)\rangle$	$\langle s_2,(0.8,0.4,0.2)\rangle$	$\langle s_1,(0.5,0.4,0.9)\rangle$
x_3	$\langle s_2,(0.5,0.1,0.3)\rangle$	$\langle s_3,(0.6,0.5,0.1)\rangle$	$\langle s_4,(0.9,0.2,0.1)\rangle$
x_4	$\langle s_1,(0.5,0.3,0.5)\rangle$	$\langle s_2,(0.4,0.1,0.1)\rangle$	$\langle s_2,(0.2,0.8,0.4)\rangle$
x_5	$\langle s_4,(0.5,0.3,0.5)\rangle$	$\langle s_4,(0.7,0.5,0.1)\rangle$	$\langle s_2,(0.5,0.6,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_1,(0.6,0.2,0.1)\rangle$	$\langle s_4,(0.4,0.1,0.2)\rangle$	$\langle s_4,(0.8,0.1,0.1)\rangle$
x_2	$\langle s_4,(0.6,0.2,0.3)\rangle$	$\langle s_1,(0.4,0.1,0.1)\rangle$	$\langle s_2,(0.2,0.1,0.1)\rangle$
x_3	$\langle s_1,(0.5,0.7,0.1)\rangle$	$\langle s_4,(0.3,0.1,0.2)\rangle$	$\langle s_1,(0.5,0.1,0.2)\rangle$
x_4	$\langle s_4,(0.9,0.3,0.1)\rangle$	$\langle s_1,(0.5,0.2,0.1)\rangle$	$\langle s_4,(0.6,0.1,0.3)\rangle$
x_5	$\langle s_3,(0.8,0.7,0.2)\rangle$	$\langle s_4,(0.8,0.1,0.1)\rangle$	$\langle s_1,(0.6,0.3,0.1)\rangle$

Table 5
Normalised decision matrix N^2 of e_2.

	c_1	c_2	c_3
x_1	$\langle s_4,(0.8,0.6,0.1)\rangle$	$\langle s_4,(0.5,0.2,0.2)\rangle$	$\langle s_4,(0.7,0.4,0.1)\rangle$
x_2	$\langle s_1,(0.8,0.4,0.1)\rangle$	$\langle s_3,(0.6,0.4,0.2)\rangle$	$\langle s_3,(0.5,0.2,0.3)\rangle$
x_3	$\langle s_1,(0.2,0.1,0.3)\rangle$	$\langle s_2,(0.8,0.5,0.2)\rangle$	$\langle s_4,(0.6,0.5,0.1)\rangle$
x_4	$\langle s_1,(0.6,0.4,0.4)\rangle$	$\langle s_2,(0.7,0.3,0.3)\rangle$	$\langle s_3,(0.6,0.8,0.2)\rangle$
x_5	$\langle s_4,(0.9,0.2,0.6)\rangle$	$\langle s_4,(0.9,0.2,0.1)\rangle$	$\langle s_2,(0.7,0.8,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_1,(0.9,0.4,0.1)\rangle$	$\langle s_3,(0.6,0.4,0.3)\rangle$	$\langle s_4,(0.5,0.1,0.3)\rangle$
x_2	$\langle s_3,(0.7,0.4,0.1)\rangle$	$\langle s_1,(0.7,0.1,0.4)\rangle$	$\langle s_4,(0.8,0.1,0.1)\rangle$
x_3	$\langle s_1,(0.8,0.5,0.2)\rangle$	$\langle s_4,(0.4,0.1,0.1)\rangle$	$\langle s_1,(0.6,0.1,0.1)\rangle$
x_4	$\langle s_4,(0.9,0.4,0.1)\rangle$	$\langle s_1,(0.4,0.5,0.1)\rangle$	$\langle s_4,(0.6,0.1,0.1)\rangle$
x_5	$\langle s_4,(0.4,0.4,0.2)\rangle$	$\langle s_4,(0.7,0.1,0.1)\rangle$	$\langle s_1,(0.2,0.1,0.1)\rangle$

Table 6
Normalised decision matrix N^3 of e_3.

	c_1	c_2	c_3
x_1	$\langle s_2,(0.5,0.6,0.1)\rangle$	$\langle s_4,(0.4,0.8,0.1)\rangle$	$\langle s_3,(0.8,0.5,0.1)\rangle$
x_2	$\langle s_1,(0.4,0.4,0.1)\rangle$	$\langle s_4,(0.5,0.4,0.2)\rangle$	$\langle s_2,(0.8,0.1,0.3)\rangle$
x_3	$\langle s_3,(0.6,0.1,0.1)\rangle$	$\langle s_4,(0.5,0.5,0.5)\rangle$	$\langle s_4,(0.6,0.8,0.1)\rangle$
x_4	$\langle s_1,(0.1,0.8,0.2)\rangle$	$\langle s_2,(0.4,0.5,0.8)\rangle$	$\langle s_4,(0.8,0.8,0.1)\rangle$
x_5	$\langle s_4,(0.4,0.7,0.1)\rangle$	$\langle s_4,(0.8,0.2,0.1)\rangle$	$\langle s_2,(0.9,0.1,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_1,(0.9,0.3,0.1)\rangle$	$\langle s_2,(0.1,0.1,0.5)\rangle$	$\langle s_4,(0.2,0.1,0.2)\rangle$
x_2	$\langle s_2,(0.9,0.8,0.6)\rangle$	$\langle s_1,(0.4,0.1,0.1)\rangle$	$\langle s_3,(0.6,0.1,0.4)\rangle$
x_3	$\langle s_1,(0.2,0.3,0.3)\rangle$	$\langle s_4,(0.8,0.1,0.6)\rangle$	$\langle s_1,(0.4,0.1,0.6)\rangle$
x_4	$\langle s_4,(0.9,0.2,0.1)\rangle$	$\langle s_1,(0.6,0.2,0.1)\rangle$	$\langle s_4,(0.3,0.1,0.2)\rangle$
x_5	$\langle s_2,(0.7,0.4,0.2)\rangle$	$\langle s_4,(0.3,0.1,0.1)\rangle$	$\langle s_1,(0.4,0.2,0.1)\rangle$

Table 7
Integrated decision matrix.

	c_1	c_2	c_3
x_1	$\langle s_3,(0.6,0.6,0.1)\rangle$	$\langle s_4,(0.6,0.4,0.3)\rangle$	$\langle s_3,(0.8,0.5,0.1)\rangle$
x_2	$\langle s_1,(0.7,0.5,0.1)\rangle$	$\langle s_3,(0.6,0.4,0.2)\rangle$	$\langle s_2,(0.6,0.2,0.4)\rangle$
x_3	$\langle s_2,(0.5,0.1,0.2)\rangle$	$\langle s_3,(0.6,0.5,0.3)\rangle$	$\langle s_4,(0.7,0.5,0.1)\rangle$
x_4	$\langle s_1,(0.4,0.5,0.3)\rangle$	$\langle s_2,(0.5,0.3,0.4)\rangle$	$\langle s_3,(0.6,0.8,0.2)\rangle$
x_5	$\langle s_4,(0.6,0.4,0.4)\rangle$	$\langle s_4,(0.8,0.3,0.1)\rangle$	$\langle s_2,(0.7,0.5,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_1,(0.8,0.3,0.1)\rangle$	$\langle s_3,(0.4,0.2,0.3)\rangle$	$\langle s_4,(0.5,0.1,0.2)\rangle$
x_2	$\langle s_3,(0.7,0.4,0.3)\rangle$	$\langle s_1,(0.5,0.1,0.2)\rangle$	$\langle s_3,(0.6,0.1,0.2)\rangle$
x_3	$\langle s_1,(0.5,0.5,0.2)\rangle$	$\langle s_4,(0.5,0.1,0.3)\rangle$	$\langle s_1,(0.5,0.1,0.3)\rangle$
x_4	$\langle s_4,(0.9,0.3,0.1)\rangle$	$\langle s_1,(0.5,0.3,0.1)\rangle$	$\langle s_4,(0.5,0.1,0.2)\rangle$
x_5	$\langle s_3,(0.6,0.5,0.2)\rangle$	$\langle s_4,(0.6,0.1,0.1)\rangle$	$\langle s_1,(0.4,0.2,0.1)\rangle$

Step 2: Obtain the comprehensive evaluation information.

The integrated decision matrix can be obtained by using Eq. (5). Table 7 lists the integrated decision matrix.

Step 3: Obtain the mean value of five outsourcing providers against each criterion.

The mean value of outsourcing providers against each criterion can be obtained by using Eq. (6). The results are presented as follows: $E(N_1) = \langle s_{2.2},(0.5727,0.4182,0.2455)\rangle$, $E(N_2) = \langle s_{3.2},(0.6375,0.3813,0.2437)\rangle$, $E(N_3) = \langle s_{2.8},(0.6857,0.5214,0.1643)\rangle$, $E(N_4) = \langle s_{2.4},(0.7333,0.3917,0.1833)\rangle$, $E(N_5) = \langle s_{2.6},(0.5077,0.1385,0.2154)\rangle$ and $E(N_6) = \langle s_{2.6},(0.5154,0.1077,0.2)\rangle$.

Step 4: Obtain the mean square deviation value of outsourcing providers against each criterion.

The mean squared deviation value $\sigma(N_j)$ of outsourcing providers under criterion C_j can be obtained by using Eq. (7). Results are shown in the second column of Table 8. The last row of the second column of Table 8 is the sum of mean squared deviation values of all criteria.

Step 5: Obtain the weight vector of criteria.

The Euclidean distance is taken as an example, that is, $p = 2$. The weights of criteria can be obtained by using Eq. (8). The obtained weights are presented in the third column of Table 8. The last row of the third column of Table 8 is the sum of weights of all criteria.

5.2. Stage 2: Obtain difference matrix

Step 6: Obtain the weighted decision matrix R.

The weighted decision matrix can be determined by using Eq. (9). Table 9 shows the weighted decision matrix.

Step 7: Obtain the border approximation area matrix.

The border approximation area matrix can be obtained by using Eq. (10). The results are listed as follows: $b_1 = \langle s_{0.4099},(0.5502,0.4422,0.2292)\rangle$, $b_2 = \langle s_{0.4288},(0.6128,0.3847,0.2669)\rangle$, $b_3 = \langle s_{0.4058},(0.676,0.5427,0.1894)\rangle$, $b_4 = \langle s_{0.5543},(0.6853,0.4067,0.1835)\rangle$, $b_5 = \langle s_{0.271},(0.4959,0.164,0.205)\rangle$ and $b_6 = \langle s_{0.2146},(0.4959,0.121,0.2025)\rangle$.

Step 8: Obtain the difference between element r_{ij} in the weighted decision matrix R and the border approximation area b_j.

d_{ij} between r_{ij} in R and b_j can be obtained by using Eq. (11) ($p = 2$). Table 10 lists the values of d_{ij}.

Table 8
Mean squared deviation value under each criterion.

	Mean squared deviation value	Weight
c_1	3.3721	0.217
c_2	2.1460	0.138
c_3	2.3332	0.150
c_4	4.2050	0.271
c_5	1.9409	0.125
c_6	1.5368	0.099
Sum	15.5340	1

Table 9
Weighted decision matrix.

	c_1	c_2	c_3
x_1	$\langle s_{0.651},(0.6,0.6,0.1)\rangle$	$\langle s_{0.553},(0.6,0.4,0.3)\rangle$	$\langle s_{0.451},(0.8,0.5,0.1)\rangle$
x_2	$\langle s_{0.217},(0.7,0.5,0.1)\rangle$	$\langle s_{0.414},(0.6,0.4,0.2)\rangle$	$\langle s_{0.3},(0.6,0.2,0.4)\rangle$
x_3	$\langle s_{0.434},(0.5,0.1,0.2)\rangle$	$\langle s_{414},(0.6,0.5,0.3)\rangle$	$\langle s_{0.601},(0.7,0.5,0.1)\rangle$
x_4	$\langle s_{0.217},(0.4,0.5,0.3)\rangle$	$\langle s_{0.276},(0.5,0.3,0.4)\rangle$	$\langle s_{0.451},(0.6,0.8,0.2)\rangle$
x_5	$\langle s_{0.868},(0.6,0.4,0.4)\rangle$	$\langle s_{0.553},(0.8,0.3,0.1)\rangle$	$\langle s_{0.3},(0.7,0.5,0.1)\rangle$
	c_4	c_5	c_6
x_1	$\langle s_{0.271},(0.8,0.3,0.1)\rangle$	$\langle s_{0.375},(0.4,0.2,0.3)\rangle$	$\langle s_{0.396},(0.5,0.1,0.2)\rangle$
x_2	$\langle s_{0.812},(0.7,0.4,0.3)\rangle$	$\langle s_{0.125},(0.5,0.1,0.2)\rangle$	$\langle s_{0.297},(0.6,0.1,0.2)\rangle$
x_3	$\langle s_{0.271},(0.5,0.5,0.2)\rangle$	$\langle s_{0.5},(0.5,0.1,0.3)\rangle$	$\langle s_{0.099},(0.5,0.1,0.3)\rangle$
x_4	$\langle s_{1.083},(0.9,0.3,0.1)\rangle$	$\langle s_{0.125},(0.5,0.3,0.1)\rangle$	$\langle s_{0.396},(0.5,0.1,0.2)\rangle$
x_5	$\langle s_{0.812},(0.6,0.5,0.2)\rangle$	$\langle s_{0.5},(0.6,0.1,0.1)\rangle$	$\langle s_{0.099},(0.4,0.2,0.1)\rangle$

Table 10
Difference d_{ij} between r_{ij} and b_j.

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	0.072	0.0105	0.0085	−0.0531	0.0044	0.0098
x_2	−0.0159	−0.0012	−0.0364	0.0656	−0.0071	0.0054
x_3	0.0191	−0.0021	0.0281	−0.0702	0.0223	−0.0037
x_4	−0.0253	−0.0223	−0.0199	0.3635	−0.0071	0.0098
x_5	0.1791	0.0357	−0.0112	0.0478	0.0274	−0.0056

5.3. Stage 3: Rank outsourcing provider

Step 9: Determine the concordance matrix.

Let $q_j = 0.01$ and $p_j = 0.02$ be the preference and indifference thresholds for all criteria c_j ($j = 1,2,3,4,5,6$), respectively. The concordance matrix can be obtained by using Eqs. (12) and (13). Table 11 lists the degrees of concordance between x_i and x_k ($i,k = 1,2,3,4,5$).

Step 10: Determine the credibility index of outranking relations.

Let $v = 0.06$ be the veto threshold for all criteria c_j ($j = 1,2,3,4,5,6$). The credibility index of outranking relations can be obtained by using Eq. (14). The indices are presented in Table 12.

Step 11: Determine the indices of outsourcing providers.

The indices of outsourcing providers can be obtained by using Eq. (15). The results are presented as follows: $index(x_1) = 0.7084 - 1.0288 = -0.3204$, $index(x_2) = 0 - 1.6869 = -1.6869$, $index(x_3) = 0.2807 - 1.6295 = -1.3488$, $index(x_4) = 0.9211$ and $index(x_5) = 2.435$.

Step 12: Rank outsourcing providers.

Table 13 shows that the rankings of outsourcing providers are $x_5 \succ x_4 \succ x_1 \succ x_3 \succ x_2$ because $index(x_5) > index(x_4) > index(x_1) > index(x_3) > index(x_2)$. The best IT outsourcing provider is Taiji (x_5).

Table 11
Degrees of concordance between x_i and x_k.

	x_1	x_2	x_3	x_4	x_5
x_1		0.8333	0.7084	0.8333	0.3333
x_2	0.6125		0.5	0.7247	0.3333
x_3	0.6127	0.8333		0.7748	0.5
x_4	0.4752	0.8333	0.3333		0.5
x_5	0.7481	0.8536	0.8333	0.7424	

Table 12
Credibility index of outranking relations.

	x_1	x_2	x_3	x_4	x_5	Sum
x_1		0	0.7084	0	0	0.7084
x_2	0		0	0	0	0
x_3	0.2807	0		0	0	0.2807
x_4	0	0.8333	0.0878		0	0.9211
x_5	0.7481	0.8536	0.8333	0		2.435
Sum	1.0288	1.6869	1.6295	0	0	

Table 13
Rankings of IT outsourcing providers.

Provider	Index	Rank
Ingens (x_1)	−0.3204	3
Teamsun (x_2)	−1.6869	5
Sinoway (x_3)	−1.3488	4
Lenovo (x_4)	0.9211	2
Taiji (x_5)	2.4350	1

6. Comparative analysis

In this section, the MABAC–ELECTRE method is compared with several decision-making methods to verify its feasibility. The details of the comparative analysis are introduced in the remainder of this section.

Three decision-making methods are used to solve the IT outsourcing provider selection presented in Section 5.

(1) Method 1: This method is an extended TOPSIS method developed by Ye (2015). Ye (2015) defined the distance measurement of SVNLNs, based on which a TOPSIS-based decision-making method was established.
(2) Method 2: This method is established by Tian et al. (2017). They proposed a weighted Bonferrroni mean operator for SVNLNs. Subsequently, they constructed Method 2 using this operator.
(3) Method 3: This method is a traditional fuzzy MABAC method. The input data of Method 3 is SVNLNs. Only one difference, which lies in the attainment of the ranking indices of outsourcing providers, exists between Method 3 and the MABAC–ELECTRE method. In Method 3, the ranking order of outsourcing providers is determined by $\sum_{j=1}^{n} d_{ij}$.

The ranking results obtained by these three methods are compared with the ranking result of the MABAC–ELECTRE method. Table 14 depicts the ranking results of Methods 1–3 and the MABAC–ELECTR method. To offer clear visual representation, Fig. 2 describes the ranking orders of these four methods based on the results in Table 14. Table 14 and Fig. 2 show that the different ranking orders of outsourcing providers can be obtained with distinct methods. Furthermore, the best outsourcing providers of these four methods may be different. The best outsourcing provider of the MABAC–ELECTRE method is the same as that of Method 2. Taiji (x_5) is the best outsourcing provider in the rankings obtained by these two methods. The best outsourcing provider obtained by Method 1 is Teamsun (x_2), whereas that of Method 3 is Lenovo (x_4).

Table 14
Ranking orders of four different methods.

Method	Ranking order	Best provider	Worst provider
Method 1	$x_2 \succ x_4 \succ x_3 \succ x_1 \succ x_5$	x_2	x_5
Method 2	$x_5 \succ x_1 \succ x_4 \succ x_3 \succ x_2$	x_5	x_2
Method 3	$x_4 \succ x_5 \succ x_1 \succ x_2 \succ x_3$	x_4	x_3
MABAC–ELECTRE	$x_5 \succ x_4 \succ x_1 \succ x_3 \succ x_2$	x_5	x_2

In this comparative analysis, the method in (Jahan, Ismail, Shuib, Norfazidah, & Edwards, 2011), which is an aggregation method, is used to obtain the most suitable one from the four decision-making methods. Please refer to (Jahan et al., 2011) for the details of the aggregation method. This method finds the optimal rankings from the rankings of multiple decision-making methods. The most suitable decision-making method obtains the most consistent ranking result with the optimal rankings. Table 15 shows the number of times the outsourcing provider is assigned to different ranks. Table 16 lists the values of E_{ik}, which is obtained on the basis of the values in Table 15. The computation of E_{ik} prepares for the construction of model (16):

$$Max\ Z = \sum_{i=1}^{5}\sum_{k=1}^{5} E_{ik} * \frac{5^2}{k} * N_{ik}$$
$$s.\ t. \begin{cases} \sum_{k=1}^{5} N_{ik} = 1 & i = 1,2,...,5, \\ \sum_{i=1}^{5} N_{ik} = 1 & i = 1,2,...,5, \\ N_{ik} = \begin{cases} 0 \\ 1 \end{cases} & \text{for all } i \text{ and } k \end{cases} \tag{16}$$

This linear programming model helps find optimal rankings.

MATLAB software is used to solve this linear programming problem. The results of the method in (Jahan et al., 2011) and the ranking results of Methods 1–3 and the MABAC–ELECTRE method are illustrated in Fig. 3. The MABAC–ELECTRE method determines the same ranking order with the aggregation method. That is, the MABAC–ELECTRE method is more suitable than Methods 1–3 in tackling the outsourcing provider selection problem.

The difference between the ranking result of each decision-making method and optimal rankings is obtained. The difference is calculated by using the formula in (Zhang, Ji, Wang, & Chen, 2016), as follows:

$$sumDiv = \sum_{i=1}^{n} |(r_i - B_i)/B_i|,$$

where r_i represents the ranking order of x_i obtained by a decision-making method, B_i denotes the optimal ranking of x_i. Table 17 presents the differences between the results of each method and optimal rankings. The results in Table 17 agree with those of Fig. 3. Method 1 achieves the highest deviation, followed by Method 2. The deviation of Method 3 is lower than that of Method 2. Moreover, the MABAC–ELECTRE method exhibits the lowest deviation. Results reveal that the MABAC–ELECTRE method is the most suitable among the four decision-making methods.

The possible explanations for these results are provided as follows.

(1) The operations used in Method 1 are different from those in the other three methods. In Method 1, the operations regarding the linguistic terms are directly based on their subscripts. The correlation between the linguistic and fuzzy parts is ignored. On the contrary, the other three methods, including the MABAC–ELECTRE method, use the linguistic scale function to define the operations. Moreover, the correlation between the linguistic and fuzzy parts in SVNLNs is considered. Hence, the result of Method 1 may be different from those of the other three methods.
(2) The MABAC–ELECTRE method acknowledges that criteria may be non-compensatory in some actual situations and introduces the main idea of ELECTRE. Nevertheless, Methods 1 and 3 assume that criteria are compensatory and independent. Method 3 also assumes that criteria are compensatory. In addition, this method considers the interrelationships among criteria by using the Bonferrroni mean. Therefore, the ranking result of the MABAC–ELECTRE method may not be identical to the results of the other three methods.

In general, results in this comparative analysis prove the effectiveness of the combined MABAC–ELECTRE method. The combined

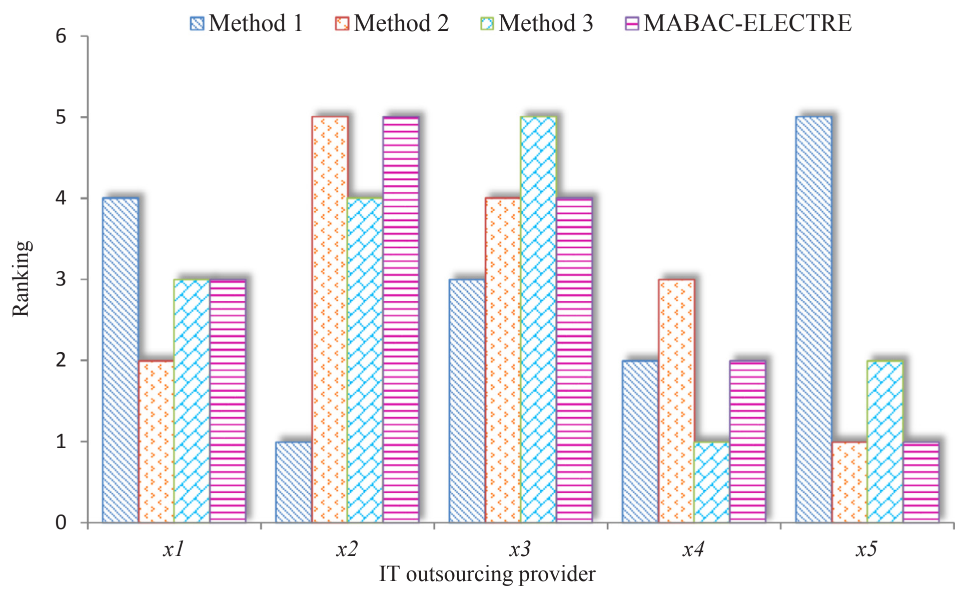

Fig. 2. Rankings of each outsourcing provider obtained by four different methods.

Table 15
Number of times an outsourcing provider is assigned to different ranks.

Outsourcing providers	Rank				
	1	2	3	4	5
x_1		1	2	1	
x_2	1			1	2
x_3			1	2	1
x_4	1	2	1		
x_5	2	1			1

Table 16
Smoothing of outsourcing provider assignment over ranks (E_{ik}).

Outsourcing providers	Rank				
	1	2	3	4	5
x_1	0	1	3	4	4
x_2	1	1	1	2	4
x_3	0	0	1	3	4
x_4	1	3	4	4	4
x_5	2	3	3	3	4

Table 17
Deviation of the four methods.

Method	Method 1	Method 2	Method 3	MABAC–ELECTRE
sumDiv	4.77	3.75	1.28	**0**

Note: The minimum value is in **boldface**.

MABAC–ELECTRE method introduces the main idea of ELECTRE to obtain the ranking order of outsourcing providers. In other words, the proposed outsourcing provider selection method is better in addressing actual situations with non-compensatory criteria than other SVNLN methods. As presented in Section 5, the combined MABAC–ELECTRE method may require large computation. However, the situation can be improved with the assistance of computer software, such as MATLAB.

7. Conclusion

This paper has established a fuzzy outsourcing provider selection method. The outsourcing provider selection method applies SVNLNs to depict qualitative and fuzzy information. MABAC is extended to handle SVNLNs. Moreover, the outsourcing provider selection method introduces the main idea of ELECTRE into MABAC and considers the non-

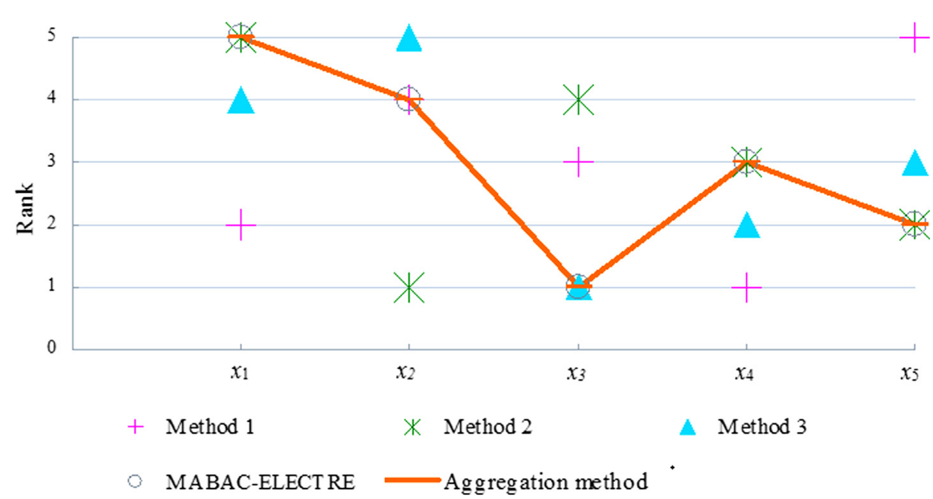

Fig. 3. Ranking results of different methods.

compensation of criteria. Moreover, the mean-squared deviation weight method with SVNLNs is developed to obtain criteria weights.

The outsourcing provider selection method has been applied to an illustrative example of IT outsourcing provider selection. Results show its feasibility in solving outsourcing provider selection problems. A comparative analysis has been conducted in this study. Several MCDM methods with SVNLNs are compared with the proposed outsourcing provider selection method in the comparative analysis. Results indicate the advantages of the proposed outsourcing provider selection method. From a managerial point of view, our method can assist enterprises in selecting proper outsourcing providers. This can further improve the performance of enterprises.

Several interesting directions may be investigated in our future research. Firstly, the current study proposed a fuzzy outsourcing provider selection method. This method can also be applied to address problems in various other fields, such as purchasing decision. The use of our findings to solve problems in these fields still needs to be investigated. Secondly, the combined MABAC–ELECTRE method was constructed on the basis of a perfect rationality assumption. However, decision makers may be bounded rational in actual situations. The performance of our outsourcing provider selection method can be improved by overcoming this deficiency, such as considering human psychological characteristics. To do that, the prospect theory or regret theory can be introduced into the outsourcing provider selection method.

Conflict of interest

No potential conflict of interest was reported by the authors.

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Appendix A

Proof of Theorem 1. By the mathematical induction, Theorem 1 can be proved to be true.

α. When $n=2$, $SVNLWA(x_1,x_2)=w_1x_1+w_2x_2$. By the operation (3) in Definition 2, $w_1x_1=\langle f^{*-1}(w_1f^*(s_{\theta(x_1)})),(T_{x_1},I_{x_1},F_{x_1})\rangle$ and $w_2x_2=\langle f^{*-1}(w_2f^*(s_{\theta(x_2)})),(T_{x_2},I_{x_2},F_{x_2})\rangle$. By the operation (1) in Definition 2,

$$w_1x_1+w_2x_2=\left\langle f^{*-1}(w_1f^*(s_{\theta(x_1)})+w_2f^*(s_{\theta(x_2)})),\left(\frac{w_1f^*(s_{\theta(x_1)})T_{x_1}+w_2f^*(s_{\theta(x_2)})T_{x_2}}{w_1f^*(s_{\theta(x_1)})+w_2f^*(s_{\theta(x_2)})},\frac{w_1f^*(s_{\theta(x_1)})I_{x_1}+w_2f^*(s_{\theta(x_2)})I_{x_2}}{w_1f^*(s_{\theta(x_1)})+w_2f^*(s_{\theta(x_2)})},\frac{w_1f^*(s_{\theta(x_1)})F_{x_1}+w_2f^*(s_{\theta(x_2)})F_{x_2}}{w_1f^*(s_{\theta(x_1)})+w_2f^*(s_{\theta(x_2)})}\right)\right\rangle$$

.

That is to say, Eq. (2) holds for $n=2$.

β. Suppose that Eq. (2) is true when $n=k$. That is,

$$SVNLWA(x_1,x_2,...,x_k)=\sum_{i=1}^{k}(w_ix_i)=\left\langle f^{*-1}\left(\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})\right),\left(\frac{\sum_{i=1}^{k}w_iT_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k}w_iI_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k}w_iF_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})}\right)\right\rangle.$$

When $n=k+1$, $SVNLWA(x_1,x_2,...,x_k,x_{k+1})=\sum_{i=1}^{k}(w_ix_i)+w_{k+1}x_{k+1}$. By the operation (3) in Definition 2, $w_{k+1}x_{k+1}=\langle f^{*-1}(w_{k+1}f^*(s_{\theta(x_{k+1})})),(T_{x_{k+1}},I_{x_{k+1}},F_{x_{k+1}})\rangle$. By the operation (1) in Definition 2, the following equation can be obtained.

$$\begin{aligned}SVNLWA(x_1,x_2,...,x_k,x_{k+1})&=\left\langle f^{*-1}\left(\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})\right),\left(\frac{\sum_{i=1}^{k}w_iT_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k}w_iI_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k}w_iF_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k}w_if^*(s_{\theta(x_i)})}\right)\right\rangle\\&\quad+\langle f^{*-1}(w_{k+1}f^*(s_{\theta(x_{k+1})})),(T_{x_{k+1}},I_{x_{k+1}},F_{x_{k+1}})\rangle\\&=\left\langle f^{*-1}\left(\sum_{i=1}^{k+1}w_if^*(s_{\theta(x_i)})\right),\left(\frac{\sum_{i=1}^{k+1}w_iT_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k+1}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k+1}w_iI_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k+1}w_if^*(s_{\theta(x_i)})},\frac{\sum_{i=1}^{k+1}w_iF_{x_i}f^*(s_{\theta(x_i)})}{\sum_{i=1}^{k+1}w_if^*(s_{\theta(x_i)})}\right)\right\rangle.\end{aligned}$$

Therefore, Eq. (2) is right when $n=k+1$. Thus, Eq. (2) is proved to be correct for all n.

Hence, Theorem 1 holds.

Proof of Theorem 2. By the mathematical induction, Theorem 2 can be proved to be true.

α. When $n=2$, $SVNLWG(x_1,x_2)=(x_1)^{w_1}\times(x_2)^{w_2}$. By the operation (4) in Definition 2, $x_1^{w_1}=\langle f^{*-1}((f^*(s_{\theta(x_1)}))^{w_1}),(T_{x_1}^{w_1},1-(1-I_{x_1})^{w_1},1-(1-F_{x_1})^{w_1})\rangle$ and $x_2^{w_2}=\langle f^{*-1}((f^*(s_{\theta(x_2)}))^{w_2}),(T_{x_2}^{w_2},1-(1-I_{x_2})^{w_2},1-(1-F_{x_2})^{w_2})\rangle$. By the operation (2) in Definition 2, $(x_1)^{w_1}\times(x_2)^{w_2}=\langle f^{*-1}((f^*(s_{\theta(x_1)}))^{w_1}(f^*(s_{\theta(x_2)}))^{w_2}),(T_{x_1}^{w_1}T_{x_2}^{w_2},1-(1-I_{x_1})^{w_1}(1-I_{x_2})^{w_2},1-(1-F_{x_1})^{w_1}(1-F_{x_2})^{w_2})\rangle$. That is to say, Eq. (3) holds for $n=2$.

β. Suppose that Eq. (3) is true when $n=k$. That is,

$$SVNLWG(x_1,x_2,...,x_k)=\prod_{i=1}^{k}(x_i)^{w_i}=\left\langle f^{*-1}\left(\prod_{i=1}^{k}(f^*(\theta(x_i)))^{w_i}\right),\left(\prod_{i=1}^{k}T_{x_i}^{w_i},1-\prod_{i=1}^{k}\left(1-I_{x_i}\right)^{w_i},1-\prod_{i=1}^{k}\left(1-F_{x_i}\right)^{w_i}\right)\right\rangle.$$

When $n=k+1$, $SVNLWG(x_1,x_2,...,x_k,x_{k+1})=\prod_{i=1}^{k}(x_i)^{w_i}\times(x_{k+1})^{w_{k+1}}$. By the operation (4) in Definition 2, $x_{k+1}^{w_{k+1}}=\langle f^{*-1}((f^*(s_{\theta(x_{k+1})}))^{w_{k+1}}),(T_{x_{k+1}}^{w_{k+1}},1-(1-I_{x_{k+1}})^{w_{k+1}},1-(1-F_{x_{k+1}})^{w_{k+1}})\rangle$. By the operation (2) in Definition 2, the following equation can be obtained.

$$SVNLWG(x_1,x_2,\ldots,x_k,x_{k+1}) = \left\langle f^{*-1}\left(\prod_{i=1}^{k}(f^*(\theta(x_i)))^{w_i}\right),\left(\prod_{i=1}^{k}T_{x_i}^{w_i},1-\prod_{i=1}^{k}(1-I_{x_i})^{w_i},1-\prod_{i=1}^{k}(1-F_{x_i})^{w_i}\right)\right\rangle$$
$$\times\langle f^{*-1}((f^*(s_{\theta(x_{k+1})}))^{w_{k+1}}),(T_{x_{k+1}}^{w_{k+1}},1-(1-I_{x_{k+1}})^{w_{k+1}},1-(1-F_{x_{k+1}})^{w_{k+1}})\rangle$$
$$= \left\langle f^{*-1}\left(\prod_{i=1}^{k+1}(f^*(\theta(x_i)))^{w_i}\right),\left(\prod_{i=1}^{k+1}T_{x_i}^{w_i},1-\prod_{i=1}^{k+1}(1-I_{x_i})^{w_i},1-\prod_{i=1}^{k+1}(1-F_{x_i})^{w_i}\right)\right\rangle.$$

Therefore, Eq. (3) is right when $n = k + 1$. Thus, Eq. (3) is proved to be correct for all n.

Hence, Theorem 2 holds.

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