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Sample allocation decisions for program evaluation: a case of multiplicative effect on units due to the developmental initiatives carried out in a phased manner

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Abstract

Evaluation is primarily a measure of the effects and elaboration on the current performance of a program. It facilitates identification and choice of the ways to achieve the intended program objectives. The evaluation includes selection of samples, as well as collection and analysis of data.

This paper discusses the issue of the allocation of samples for the population for which programs have been implemented in a phased manner. This needs special attention due to the temporal impact of allocations on the successive units. A method of proportional allocation has been proposed for the estimation of the sample size in different phases under the assumption that the impact of the latter phase is a multiplicative product of constant factor and the former phase.

Key words

stratified sampling, temporal impact, sample allocation, weight

Introduction

Evaluation is the process of determining the present existing state of units under the influence of an implemented program or developmental activities. It determines the positive and negative impacts and associated losses and gains that directly result from the program. The results can be used as policy instruments for future planning and can also provide some tools for retrospectively determining the performance of individuals for implementing programs.

Generally the developmental program such as poverty alleviation, literacy drive, health scheme *etc*. is being implemented on units across a large geographical area. Therefore, evaluation of all units is generally not feasible, thus sample evaluation is recommended. Therefore, all the issues pertaining to sampling should be taken into consideration for precise evaluation. It includes nature of population units, efficient sample size, and estimation procedures depending on the applied sampling scheme. The program which has been implemented under different phases leads to the time-dependent impact, therefore for evaluation; temporal impacts cannot be ignored. Therefore, for estimating the parameter of interest, the sample size and its allocation should consider the temporal effects of the popula-

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tion units. Sampling theory provides a solution for the allocation of sample size (Cochran 1977).

The impact distribution of such populations is usually highly skewed with long tails in one direction due to time dependent impacts. This makes population units heterogeneous and stratified sampling may be suitable. The different phases may be considered strata, as program implementation attracted all units of a phase at a time. Therefore, the essence of stratified sampling designs *i.e.* the number of the strata formed; and the construction of the stratum boundaries will be addressed automatically. However, sample allocation within a strata is tactical keeping in view the time-dependent impact on the population units. It means, the initial units may have different impacts and, hence, the response than the latter stage units. Therefore, for uniformity, a weight may be assigned depending on the realised impacts on units. This will nullify the impacts of the implementation phase and minimise the bias on the resulting estimates of impact evaluation.

The large sample will yield for waste of recourse, and the small sample may provide no efficient result. Therefore to get a precise estimate of the parameter for impact evaluation, the impact has to be account for through assigning temporal weights to these units, as per the actual time or phase of implementation of the program for those units. Lacking these accounting will lead to improper estimation of impact evaluation. This assigning of weight is possible, if the influence pattern of the impact is known. The lack of this information affects the rationale of practical decisions that will be derived from the estimation.

The issue of finding the stratum boundaries to minimize the variance of the stratified sample estimator was discussed by several researchers based on the population characteristics (Dalenius 1950; Dalenius and Hodges 1957; Lavallée and Hidiroglou 1988; Horgan 2006). This heterogeneity in the population may be intrinsic in nature or may be arisen due to some artificial or extrinsic causes, e.g. the varied response in population units due to the operation of the developmental program in a phased manner. The variation in response to a specified unit is related to the implementation phase of the program. That is, the units, which fall in the first phase, will be having more time to realize the actual effect of the program, therefore will have a different impact, logically. This kind of problem of sample allocation in different strata for impact evaluation of any program/projects implemented in a phased manner was addressed for the additive impacts in respective units by Pandey and Verma 2008.

For efficient estimation of impact evaluation in stratified sampling, the pertinent point is allocation of the sample size in different strata. Conventionally, it can be decided by considering three factors, *viz*; total number of units in the stratum, variability within the stratum, and cost of taking observations per sampling unit in the stratum. However, it becomes more critical, as the phased rational operation influences the study variable. The conventional allocation of the sample size by assigning weight in different strata depending on the characteristics of strata will not account for this variability. Therefore, it is logical to consider the time-dependent response for weight, so that the unit's response should be free from phase implementation. This paper attempts to address the issue of allocation of samples in different phases through the integration of this information for weight estimation with the assumption that the response in successive phases is governed by the multiplicative effect.

Sample size determination in different strata

The sample size in different strata is *a priori* fixed scientifically. The allocation of the sample to different strata is based on the principle of optimum allocation (Sukhatme et. al. 1984). The methods for sample allocation in different strata are equal allocation, proportional allocation and Neyman allocation (Cochran 1977).

Symbolically, let there are N units/villages/sectors, where the program has been implemented in h (say) phases. The phase is generally based on an administrative regions, therefore may be treated as a stratification criterion. Then

$$N = \sum_{i=1}^{h} N_i \tag{1}$$

A total of 'n' sample will be selected randomly with pre-decided sampling scheme for impact evaluation. The sample size in different strata will be

$$n = \sum_{i=1}^{h} n_i \tag{2}$$

The proportional allocation is considered for determination of the sample size. It will be

$$n_i = n\frac{N_i}{N} \quad (3)$$

Here, n_i is not true for those populations which attract some external impact with a lagged response. These allocations used available information about the population with consideration that the study variable is without influence of external factors. Therefore, these methods are not suitable for those situations where the study variables are under influence due to some external interventions such as the present case, in which the impacts are time-dependent.

Therefore, theoretically, if this is used, the impact evaluation will be erroneous. This may also happen due to the fact that the realized impact under different phases will be different due to the time lag in implementation (Pandey and Verma, 2008). It means that, because of the dynamic nature of the population under the influence of temporal impacts from phase to phase, the general weight $W_i = N_i / N$ cannot be applied for the estimation of sample mean, sample variance and allocation of sample size.

This can be addressed by selecting units in such a way that the impact on all units should be treated identically by applying some artificial measure. This can be achieved through assigning weight to each stratum depending on the impact of the program being implemented. These weights can be used to obtain the hypothetical population stratum size by making all the units of the population similar for evaluation.

To achieve the weight, it may be considered that the impact of the program is uniformly distributed within each phases and that it is constant and multiplicative in nature with respect to different phases. In other words, if there are h phases, and if the impact of the last phase (the most recent one) is M, and the impact of the immediately preceding phase is Q then $Q = MU$, where U is the constant multiplicative factor *i.e.* improvement factor due to the time lag (realized impact) between the preceding and succeeding ones. In other words, we can say that the units, which received the programs in the preceding to last phases, will have U times more impact than the units, which receive the last phase of the program with the above assumptions. This multiplicative factor may assume values greater than one ($U > 1$), with the assumptions that the developmental program will be implemented with the objectives of improvement over time.

Mathematically, let us consider a population of size N containing the population units $N_1, N_2, \ldots, N_h$ for the implementation of the h phase of the developmental program. Suppose that the development program is implemented in the h phase with the impact of the last phase being M, then the impact of the immediately preceding one will be MU, where U is defined above. Therefore, for the first phase, if there are N_1 units, then the total actual impact will be $MU(h-1)$ times more than the last beneficiaries. Hence, for comparison with the last units, the hypothetical stratum size may be $(MU^{(h-1)})N_1$. In the second phase, the development program considers N_2 units. Hence, the total hypothetical population for that phase will be $(MU^{(h-2)})N_2$. In general, there will be $(MU^{(h-i)})N_i$ beneficiaries for the i-th stratum. There will be $(M)N_h$ units – beneficiaries in the last phase of implementation of the development program. For more clarity, this has been presented in Tab. 1. A similar problem with additive impacts of the programme in different phases was considered by Pandey and Verma, 2008. The weight for such impacts was defined as $(h-i+1)$ N_i, where h is the additive impact factor between two successive phases.

Based on the above logic, the weight will be based on the impact and implementation phase. Therefore, the general formula for the weight is:

$$w_i = \frac{(MU^{(h-i)})N_i}{N_{Hy}} \quad (4)$$

where:

- h – number of the development program phase,
- i – stratum number, $i = 1, 2, 3, \ldots, h$,
- N_i – actual numbers of beneficiaries in the i-th stratum,
- N_{Hy} – total numbers of hypothetical units in the population adjusted by the impact or phase factor with the product of the actual population *i.e.* the sum of the units adjusted by the impact factor,
- U – constant multiplicative factor of the preceding phase with the succeeding one,
- M – realised impact of the last phase.

Estimation of the mean and variance for the proposed method

The population mean for a program implemented in h phases i.e. h strata will be

$$\bar{Y} = \sum_{i=1}^{h} W_i \bar{Y}_i$$

for the study variable Y, where $\bar{Y}_i$ is the mean of the i-th phase.

The unbiased estimator of the population mean $\bar{Y}$ is:

$$\bar{y}_{st} = \sum_{i=1}^{h} W_i \bar{y}_i \qquad (5)$$

where $\bar{y}_i$ is the unbiased estimate of $\bar{Y}_i$ for the i-th stratum or phase. The variance of $\bar{y}_{st}$ is given as follows (Sukhatme, *et al.*, 1984):

$$V(\bar{y}_{st}) = \sum_{i=1}^{h} \frac{W_i^2 S_i^2}{n_i} - \sum_{i=1}^{h} \frac{W_i S_i^2}{N_i} \qquad (6)$$

The formula shows that the variance of $\bar{y}_{st}$ depends only on the variances of the estimates S_i^2 of the individual stratum means $\bar{Y}_i$ with N_i number of units and the sample size of n_i.

However, for estimating the mean of the studied variable for the development program implemented in different phases, one cannot use the conventional weight W_i for the stratum due to non accounting of the temporal impact. Therefore, the modified weight ω_i defined in equation (4) may be more realistic at least theoretically. Therefore, on a logical basis, the unbiased estimator of the population mean is:

$$\bar{y}_{stm} = \sum_{i=1}^{h} w_i \bar{y}_i \qquad (7)$$

where $\bar{y}_{stm}$ is the mean of the character under stratified sampling with modified weights.

Similarly, the variance of $\bar{y}_{stm}$ will be estimated as

$$V(\bar{y}_{stm}) = \sum_{i=1}^{h} \frac{w_i^2 S_i^2}{n_i} - \sum_{i=1}^{h} \frac{w_i S_i^2}{N_i} \qquad (8)$$

New Proportional Sample Allocation Method

This section addresses the allocation of samples to different strata for the present case. The conventional proportional allocation will not be applied for allocating samples to strata for impact evaluation of the development program due to non-accounting of the temporal variation. Hence the modified method is proposed in the paper. In this allocation, the number of units selected from each stratum depends directly on the number of units in the stratum and the temporal effect of the developmental program. Mathematically, it can be derived as:

– from the i-th strata of the stratified population

$$n_i \propto N_i \qquad (9)$$

– weight assigning due to program effect

$$n_i \propto (MU^{(h-i)}) \qquad (10)$$

– based on (9) and (10) we get

$$n_i \propto (MU^{(h-i)}) N_i \qquad (11)$$

– the equation can be rewritten as with constant k.

$$n_i = k(MU^{(h-i)}) N_i \qquad (12)$$

– taking the summation on both sides in (12) will result into the total sample size,

$$\sum_{i=1}^{L} n_i = n = k \sum_{i=1}^{L} (MU^{(h-i)}) N_i \qquad (13)$$

– however, for proportional allocation with a hypothetical population

$$n = kN_{Hy} \qquad (14)$$

– now putting the value of k in equation (12), the sample size from the i-th strata will be

$$n_i = \left[\frac{MU^{(h-i)} N_i}{N_{Hy}} \right] n \qquad (15)$$

– since $w_i = \left[\frac{MU^{(h-i)} N_i}{N_{Hy}} \right]$, hence we can write that

$$n_i = \omega_i n \qquad (16)$$

This equation (16) can be used to allocate sample sizes in different strata for impact evaluation of a development program, which has been implemented in phases and have a multiplicative impact in respective units.

Empirical Study for Sample Allocation

Let us consider that the development program is being implemented in five phases in different villages. In the first phase the development program is started in 20 villages. In the second phase, it is implemented in another 30 villages. Similarly, the development program is being implemented in different 40, 50 and 60 villages in the third, fourth and fifth phases, respectively. It was assumed that the development program had multiplicative effects in succeeding villages. In this case, conventional allocation may lead to improper selection, keeping in view the high impact in initial villages. The allocation of samples in different phases using conventional proportional allocation and new proportional allocation is illustrated in Tab. 2 for comparative analysis. As seen in the Table, there are differences in the allocation of the sample size in different phases with more representation of the initial phases in the proposed method.

Tab. 1. Formula for the allocation of the sample size for impact evaluation

Stratum number	Stratum population size	Stratum sample size, say	Impact of program (Impact Factor)	Stratum population size due to calibration of impact (Hypothetical Size)	Estimated Stratum Weight with calibration of impact	Stratum sample size
1	N_1	n_1	$MU^{(h-1)}$	$(MU^{(h-1)})\,N_1$	$\frac{(MU^{(h-1)})\,N_1}{N_{Hy}} = \omega_1$, Say	$n\omega_1$
2	N_2	n_2	$MU^{(h-2)}$	$(MU^{(h-2)})\,N_2$	$\frac{(MU^{(h-2)})\,N_2}{N_{Hy}} = \omega_2$, Say	$n\omega_2$
3	N_3	n_3	$MU^{(h-3)}$	$(MU^{(h-3)})\,N_3$	$\frac{(MU^{(h-3)})\,N_3}{N_{Hy}} = \omega_3$, Say	$n\omega_3$
4	N_4	n_4	$MU^{(h-4)}$	$(MU^{(h-4)})\,N_4$	$\frac{(MU^{(h-4)})\,N_4}{N_{Hy}} = \omega_4$, Say	$n\omega_4$
…	…	…	…	…	…	…
$h-1$	N_{h-1}	n_{h-1}	MU	MUN_{h-1}	$\frac{MUN_{h-1}}{N_{Hy}} = \omega_{h-1}$, Say	$n\omega_{h-1}$
h	N_h	n_h	M	MN_h	$\frac{MN_h}{N_{Hy}} = \omega_h$,	$n\omega_h$
Total	N	n	$\frac{M(1-U^h)}{1-M}$	N_{Hy} (Say)	$\omega_1 + \omega_2 + ... + \omega_n = 1$	n

Tab. 2. Numerical example for a five-year implementation program

Stratum number	Population size N	Stratum sample size (proportional allocation, $n = 20$)	Impact factor ($M = 2$, $U = 2$)	Hypothetical stratum population N_{Hy}	Weight ω_i	Sample size for program evaluation with Proposed Methods n_i
1	20	2	32	640	640/1760 ≈ 0.36	7.20 ≈ 7
2	30	3	16	480	480/1760 ≈ 0.27	5.40 ≈ 6
3	40	4	8	320	320/1760 ≈ 0.18	3.60 ≈ 4
4	50	5	4	200	200/1760 ≈ 0.11	2.20 ≈ 2
5	60	6	2	120	120/1760 ≈ 0.07	1.40 ≈ 1
Total	200	20	62	1760	1	20

Conclusions

Mid-term impact evaluation of any program/projects is presently in vogue in the developing countries. The allocation of samples in different strata based on phased implementations needs special attention due to its temporal impact on the units. The accounting of temporal impact for allocation of samples may provide a better mid-term evaluation of the program at least based on the theoretical logic than the conventional approach *i.e.* inclusion of impacts. Pandey and Verma, 2008 have reported the case of sample allocation for the additive impacts in successive strata. This study considers the issue of implementation of the developmental program in phased manner with multiplicative effects and reported an empirical study of sample allocation only. This also gives different estimates of the sample size for each stratum *i.e.* by considering the impact of each phase.

Overall, it appears that: compared to the classical sampling analysis for the pre-specified margin of error approach, as well as the conventional approach, the proposed method may result in a better and theoretically efficient estimate keeping in view the real coverage incorporating the significance of impacts. The theoretical efficiency may seem to be logical due to the incorporation of the influential components in the form of weight for sample allocation in different strata.

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