Multi-valued Neutrosophic Linguistic Power Operators and their Applications

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***Abstract*—Motivated by the advantages of multi-valued neutrosophic sets (MVNSs) and linguistic variables (LVs), we introduce the concept of multi-valued neutrosophic linguistic sets (MVNLSs) and define the operational laws of multi-valued neutrosophic linguistic numbers (MVNLNs) based on Algebraic operations and the distance measure for MVNLNs. Then, the multi-valued neutrosophic linguistic power weighted average (MVNLPWA) operator and the multi-valued neutrosophic linguistic power weighted geometric (MVNLPWG) operator are proposed to aggregate the multi-valued neutrosophic linguistic information, and some desirable properties of two operators are analyzed. Furthermore, a multi-criteria decision-making (MCDM) method based on the power aggregation operator is developed, where the criterion values corresponding to alternatives are the form of MVNLNs. Finally, an illustrative example is provided to demonstrate the effectiveness and practicality of the proposed method.**

***Index Terms*—multi-criteria decision-making, Algebraic, power operator, multi-valued neutrosophic linguistic**

I. INTRODUCTION

Smarandache [1] originally proposed the concept of neutrosophic set, which can be better to deal with incomplete, indeterminate and inconsistent information. Therefore, some researchers have developed a series of neutrosophic set to solve multi-criteria decision-making problem. Wang [2, 3] defined the notions of single-valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs). Ye [4] proposed simplified neutrosophic sets (SNSs). Ye [5] introduced the concept of the single-valued neutrosophic hesitant fuzzy sets (SVNHFSs). Wang [6] developed the definition of multi-valued neutrosophic sets (MVNSs). In fact, the MVNSs and SVNHFSs are the same notion.

Generally, people refer to apply linguistic terms to express evaluation information. Hence, Zadeh [7] introduced the concept of linguistic variables. Then, some achievements on linguistic variables have been developed. Intuitionistic linguistic sets (ILSs) [8] are proposed to handle MCDM problem. Hesitant fuzzy linguistic sets (HFLSs) [9] are also introduced, which combine hesitant fuzzy sets and linguistic term sets.

However, linguistic variables commonly can express the truth-membership degree of a linguistic term is 1 but cannot imply the indeterminacy-membership degree and the falsity-membership degree. To overcome this shortcoming, ye [10, 11] defined the concepts of single-valued neutrosophic linguistic sets (SVNLSs) and interval neutrosophic linguistic sets (INLSs), and extended weighted average (WA) operator and weighted geometric (WG) operator to the interval neutrosophic linguistic environment. In some cases, the degrees of truth, indeterminacy and falsity regarding the linguistic term cannot be expressed exactly with a crisp value or interval values. For example, when an expert is asked for their opinion about an investment, he or she gives the statement is good. And he or she may say the possibility of the statement being true is 0.7 or 0.8, the possibility of it being indeterminacy is 0.2 or 0.3, and the one of it being false is 0.1 or 0.2. This issue is beyond the scope of SVNLSs and INLSs. Considering this situation, Li [12] defined the concept of multi-valued neutrosophic linguistic sets (MVNLSs), and developed the Bonferroni mean (BM) operator based on Hamacher operations.

Aggregation operator plays an important role in MCDM problem, because it can fuse multiple values into a single comprehensive value. Many practical aggregation operators have been proposed, such as the weighted arithmetic average (WAA) operator, the weighted geometric average (WGA) operator [13, 14], ordered weighted aggregation (OWA) operator, Bonferroni Mean (BM) operator, Maclaurin symmetric mean (MSM) operator.

To consider the relationship between the arguments being aggregated, Yager [15] firstly defined the power average (PA) operator, which makes the arguments being fused to support and reinforce each other. Xu and Yager [16] introduced the power geometric (PG) operator. Zhou [17] proposed a generalized power average operator. Yang [18] introduced power aggregation operator for single-valued neutrosophic sets. Liu [19] proposed some power generalized operators and

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studied some properties of them under interval neutrosophic environment. Peng [20] applied power weighted average (PWA) operator and power weighted geometric (PWG) operator to multi-valued neutrosophic sets. Li [21] proposed a novel generalized simplified neutrosophic number Einstein aggregation operator.

However, to the best of our knowledge, the existing PA operator has not been applied to deal with MCDM problem in which the input arguments are multi-valued neutrosophic linguisitic numbers (MVNLNs). To accommodate these situations, the purposes of this paper are: (1) to define operational laws based on Algebraic operations and a distance measure between MVNLNs, (2) to extend the traditional PA operator to multi-valued neutrosophic linguisitic environments, MVNLPWA and MVNLPWG operators are proposed, and discuss their desirable properties.

Therefore, the rest of the paper is organized as follows. Section Ⅱ introduces some basic concepts and originally defines the operational rules, and distance measure between MVNLNs. Section Ⅲ proposes MVNLPWA and MVNLPWG operators and investigates the properties. Section Ⅳ presents an example for MCDM to demonstrate the feasibility and application of the proposed method, and a comparison analysis is also conducted in this section. Section Ⅴ contains a conclusion and future work.

II. PRELIMINARIES

A. MVNLSs and algebraic operations

This section introduces the concept and operations for MVNLSs, which will be useful in the next analysis.

Definition 1 Let X be a set of points, an MVNLS A in X is defined as follows [12]:

$$A=\left\{\left\langle x,\left[s_{\theta(x)},\left(\tilde{T}_A(x),\tilde{I}_A(x),\tilde{F}_A(x)\right)\right]\right\rangle \middle| x\in X\right\},$$

Where $s_{\theta(x)}\in S$, $S=\{s_1,s_2,\cdots,s_t\}$ is an ordered and finite linguistic set, in which s_j denotes a linguistic variable value and t is an odd value. $\tilde{T}_A(x)=\{\gamma \mid \gamma\in\tilde{T}_A(x)\}$, $\tilde{I}_A(x)=\{\delta \mid \delta\in\tilde{I}_A(x)\}$, $\tilde{F}_A(x)=\{\eta \mid \eta\in\tilde{F}_A(x)\}$, $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, and $\tilde{F}_A(x)$ are three sets of crisp values in $[0,1]$, denoting three degrees of x in X belonging to $s_{\theta(x)}$, that are true, indeterminacy and falsity, satisfying these conditions $0\le\gamma,\delta,\eta\le1$, and $0\le\sup\tilde{T}_A(x)+\sup\tilde{I}_A(x)+\sup\tilde{F}_A(x)\le3$.

Suppose there is only one element in X, then tuple $\left\langle s_{\theta(x)},\left(\tilde{T}_A(x),\tilde{I}_A(x),\tilde{F}_A(x)\right)\right\rangle$ is depicted as a multi-valued neutrosophic linguistic number (MVNLN). For simplicity, the MVNLN can also be represented as $A=\left\langle s_{\theta(x)},\left(\tilde{T}_A(x),\tilde{I}_A(x),\tilde{F}_A(x)\right)\right\rangle$.

The t-norms and t-conorms play an important role in the building process of operation laws because different aggregation operators are all depended on different t-norms and t-conorms. Algebraic t-norm and t-conorm consist of the following equations, $a\otimes b=ab$, $a\oplus b=a+b-ab$.

Then, the operational laws of MVNLNs based on algebraic operations are given as follows.

Definition 2

Let $a_1=\left\langle s_{\theta(a_1)},\left(\tilde{T}(a_1),\tilde{I}(a_1),\tilde{F}(a_1)\right)\right\rangle$ and $a_2=\left\langle s_{\theta(a_2)},\left(\tilde{T}(a_2),\tilde{I}(a_2),\tilde{F}(a_2)\right)\right\rangle$ be two MVNLNs, and $\lambda>0$, then the operations of MVNLNs can be defined based on algebraic operations.

(1) $a_1\oplus a_2$

$$=\Big\langle s_{\theta(a_1)+\theta(a_2)},$$
$$\Big(\bigcup_{\gamma_1\in\tilde{T}(a_1),\gamma_2\in\tilde{T}(a_2)}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\},$$
$$\bigcup_{\delta_1\in\tilde{I}(a_1),\delta_2\in\tilde{I}(a_2)}\{\delta_1\delta_2\},$$
$$\bigcup_{\eta_1\in\tilde{F}(a_1),\eta_2\in\tilde{F}(a_2)}\{\eta_1\eta_2\}\Big)\Big\rangle;$$

(2) $a_1\otimes a_2$

$$=\Big\langle s_{\theta(a_1)\times\theta(a_2)},$$
$$\Big(\bigcup_{\gamma_1\in\tilde{T}(a_1),\gamma_2\in\tilde{T}(a_2)}\{\gamma_1\gamma_2\},$$
$$\bigcup_{\delta_1\in\tilde{I}(a_1),\delta_2\in\tilde{I}(a_2)}\{\delta_1+\delta_2-\delta_1\delta_2\},$$
$$\bigcup_{\eta_1\in\tilde{F}(a_1),\eta_2\in\tilde{F}(a_2)}\{\eta_1+\eta_2-\eta_1\eta_2\}\Big)\Big\rangle;$$

(3) $\lambda a_1=\Big\langle s_{\lambda\theta(a_1)},$

$$\Big(\bigcup_{\gamma_1\in\tilde{T}(a_1)}\{1-(1-\gamma_1)^\lambda\},$$
$$\bigcup_{\delta_1\in\tilde{I}(a_1)}\{\delta_1^{\lambda}\},$$
$$\bigcup_{\eta_1\in\tilde{F}(a_1)}\{\eta_1^{\lambda}\}\Big)\Big\rangle;$$

(4) $a_1^{\lambda}=\Big\langle s_{\theta^{\lambda}(a_1)},$

$$\Big(\bigcup_{\gamma_1\in\tilde{T}(a_1)}\{\gamma_1^{\lambda}\},$$
$$\bigcup_{\delta_1\in\tilde{I}(a_1)}\{1-(1-\delta_1)^\lambda\},$$
$$\bigcup_{\eta_1\in\tilde{F}(a_1)}\{1-(1-\eta_1)^\lambda\}\Big)\Big\rangle.$$

B. Distance measure

The Hamming distance is commonly applied in real field, which is one typical measure. Then, we give the definition for the Hamming distance for two MVNLNs

Definition 3

Let $a_1=\left\langle s_{\theta(a_1)},\left(\tilde{T}(a_1),\tilde{I}(a_1),\tilde{F}(a_1)\right)\right\rangle$ and $a_2=\left\langle s_{\theta(a_2)},\left(\tilde{T}(a_2),\tilde{I}(a_2),\tilde{F}(a_2)\right)\right\rangle$ be two MVNLNs, then the Hamming distance between a_1 and a_2 can be defined as follows:

$$d(a_1,a_2)=\frac{1}{\max(\theta(x))+2\min(\theta(x))}\cdot\left(\begin{aligned}&\left|\theta(a_1)\frac{1}{l_{\tilde{T}(a_1)}}\sum_{\gamma_1\in\tilde{T}(a_1)}\gamma_1-\theta(a_2)\frac{1}{l_{\tilde{T}(a_2)}}\sum_{\gamma_2\in\tilde{T}(a_2)}\gamma_2\right|\\&+\left|\theta(a_1)\frac{1}{l_{\tilde{I}(a_1)}}\sum_{\delta_1\in\tilde{I}(a_2)}\delta_1-\theta(a_2)\frac{1}{l_{\tilde{I}(a_2)}}\sum_{\delta_2\in\tilde{I}(a_2)}\delta_2\right|\\&+\left|\theta(a_1)\frac{1}{l_{\tilde{F}(a_1)}}\sum_{\eta_1\in\tilde{F}(a_1)}\eta_1-\theta(a_2)\frac{1}{l_{\tilde{F}(a_2)}}\sum_{\eta_2\in\tilde{F}(a_2)}\eta_2\right|\end{aligned}\right) \quad (1)$$

Where $l_{\tilde{T}(a_1)}$, $l_{\tilde{I}(a_1)}$, and $l_{\tilde{F}(a_1)}$ are the numbers of the values in $\tilde{T}(a_1)$, $\tilde{I}(a_1)$ and $\tilde{F}(a_1)$ respectively, $l_{\tilde{T}(a_2)}$, $l_{\tilde{I}(a_2)}$, and $l_{\tilde{F}(a_2)}$ are the numbers of the values in $\tilde{T}(a_2)$, $\tilde{I}(a_2)$ and $\tilde{F}(a_2)$ respectively. Where $s_{\theta(x)}$ denotes a linguistic variable value.

For any three MVNLNs a_1, a_2, and a_3, it is easy to prove the distance defined above satisfies the following properties.

(1) $d(a_1,a_1)=0$,

(2) $d(a_1,a_2)=d(a_2,a_1)$, $d(a_1,a_2)\in[0,1]$,

(3) $d(a_1,a_2)+d(a_2,a_3)\ge d(a_1,a_3)$.

III. NOVEL OPERATORS

In this section, the power weighted average operator and the power weighted geometric operator are developed, and some properties of them are also discussed.

A. PA operator

Definition 4 Let $a_i(i=1,2,\cdots,n)$ be a collection of data, the PA operator is defined as [15]

$$PA(a_1,\cdots,a_n)=\frac{\sum_{i=1}^{n}(1+S(a_i))a_i}{\sum_{i=1}^{n}(1+S(a_j))}$$

Where $S(a_i)=\sum_{j=1,j\neq i}^{n}Supp(a_i,a_j)$ and $Supp(a_i,a_j)$ is the support for a_i and a_j, which meets the following properties:

(1) $Supp(a_i,a_j)\in[0,1]$

(2) $Supp(a_i,a_j)=Supp(a_j,a_i)$

(3) $Supp(a_i,a_j)\ge Supp(a_p,a_q) iff d(a_i,a_j)<d(a_p,a_q)$

Where $d(a_i,a_j)$ is the distance between a_i and a_j. The smaller distance is, the more they support each other.

B. MVNLPWA operator

Definition 5 Let $a_i(i=1,2,\cdots,n)$ be a collection of MVNLNs, $a_i=\left\langle s_{\theta(a_i)},\left(\tilde{T}(a_i),\tilde{I}(a_i),\tilde{F}(a_i)\right)\right\rangle$, and $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ be the weighted vector for a_i, $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$. Then the operator of MVNLPWA is given as below, and the aggregation result is still an MVNLN.

$$MVNLPWA(a_1,a_2,\cdots a_n)=\frac{\oplus_{i=1}^{n}\omega_i(1+S(a_i))a_i}{\sum_{i=1}^{n}\omega_i(1+S(a_i))} \quad (2)$$

Where $S(a_i)=\sum_{j=1,j\neq i}^{n}\omega_j Supp(a_i,a_j)$, satisfying the following conditions.

(1) $Supp(a_i,a_j)\in[0,1]$

(2) $Supp(a_i,a_j)=Supp(a_j,a_i)$

(3) $Supp(a_i,a_j)\ge Supp(a_p,a_q)$. If $d(a_i,a_j)<d(a_p,a_q)$,

Here $d(a_i,a_j)$ is the Hamming distance between a_i and a_j defined in Definition 3.

Based on the operations in Definition 2 and Eq. (2), we can derive the following Theorem 1.

Theorem 1 Let $a_i(i=1,2,\cdots,n)$ be a collection of MVNLNs, $a_i=\left\langle s_{\theta(a_i)},\left(\tilde{T}(a_i),\tilde{I}(a_i),\tilde{F}(a_i)\right)\right\rangle$, and $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ be the weighted vector for a_i, $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$. Then the aggregated result of MVNLPWA is also an MVNLN.

$$MVNLPWA(a_1,a_2,\cdots a_n)=\left\langle s_{\sum_{i=1}^{n}\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}\cdot\theta(a_i)},\left(\begin{aligned}&\bigcup_{\gamma_i\in\tilde{T}(a_i)}\left\{1-\prod_{i=1}^{n}(1-\gamma_i)^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}}\right\},\\&\bigcup_{\delta_i\in\tilde{I}(a_i)}\left\{\prod_{i=1}^{n}\delta_i^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}}\right\},\\&\bigcup_{\eta_i\in\tilde{F}(a_i)}\left\{\prod_{i=1}^{n}\eta_i^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}}\right\}\end{aligned}\right)\right\rangle \quad (3)$$

Proof. The proof can be done by using the mathematical induction. For simplicity, let $\xi_i=\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}$ in the process of proof.

(1) If n=2, based on the operations (1) and (3) in Definition 2.

$$\xi_1 a_1=\left\langle s_{\xi_1\cdot\theta(a_1)},\left(\bigcup_{\gamma_1\in\tilde{T}(a_1)}\left\{1-(1-\gamma_1)^{\xi_1}\right\},\bigcup_{\delta_1\in\tilde{I}(a_1)}\left\{\delta_1^{\xi_1}\right\},\bigcup_{\eta_1\in\tilde{F}(a_1)}\left\{\eta_1^{\xi_1}\right\}\right)\right\rangle$$

$$\xi_2 a_2 = \Big\langle s_{\xi_2 \cdot \theta(a_2)},$$
$$\Big(\bigcup_{\gamma_2 \in \tilde{T}(a_2)} \{1-(1-\gamma_2)^{\xi_2}\},$$
$$\bigcup_{\delta_2 \in \tilde{I}(a_2)} \{\delta_2^{\xi_2}\},$$
$$\bigcup_{\eta_2 \in \tilde{F}(a_2)} \{\eta_2^{\xi_2}\}\Big)\Big\rangle$$

Thus,

$$MVNLPWA(a_1, a_2) = \xi_1 a_1 \oplus \xi_2 a_2$$
$$= \Big\langle s_{\xi \cdot \theta(a_1) + \xi_2 \cdot \theta(a_2)},$$
$$\Big(\bigcup_{\gamma_1 \in \tilde{T}(a_1), \gamma_2 \in \tilde{T}(a_2)} \left\{\begin{matrix}(1-(1-\gamma_1)^{\xi_1}) + (1-(1-\gamma_2)^{\xi_2}) \\ -(1-(1-\gamma_1)^{\xi_1})(1-(1-\gamma_2)^{\xi_2})\end{matrix}\right\},$$
$$\bigcup_{\delta_1 \in \tilde{I}(a_1), \delta_2 \in \tilde{I}(a_2)} \{\delta_1^{\xi_1} \cdot \delta_2^{\xi_2}\},$$
$$\bigcup_{\eta_1 \in \tilde{F}(a_1), \eta_2 \in \tilde{F}(a_2)} \{\eta_1^{\xi_1} \cdot \eta_2^{\xi_2}\}\Big)\Big\rangle$$
$$= \Big\langle s_{\sum_{i=1}^{2} \xi_i \cdot \theta(a_i)},$$
$$\Big(\bigcup_{\gamma_1 \in \tilde{T}(a_1), \gamma_2 \in \tilde{T}(a_2)} \{1-(1-\gamma_1)^{\xi_1}(1-\gamma_2)^{\xi_2}\},$$
$$\bigcup_{\delta_1 \in \tilde{I}(a_1), \delta_2 \in \tilde{I}(a_2)} \Big\{\prod_{i=1}^{2} \delta_i^{\xi_i}\Big\},$$
$$\bigcup_{\eta_1 \in \tilde{F}(a_1), \eta_2 \in \tilde{F}(a_2)} \Big\{\prod_{i=1}^{2} \eta_i^{\xi_i}\Big\}\Big)\Big\rangle$$

(2) If Eq. (3) holds for n=k, then

$$MVNLPWA(a_1, a_2, \cdots a_k) = \Big\langle s_{\sum_{i=1}^{k} \xi_i \cdot \theta(a_i)},$$
$$\Big(\bigcup_{\gamma_i \in \tilde{T}(a_i)} \Big\{1-\prod_{i=1}^{k}(1-\gamma_i)^{\xi_i}\Big\},$$
$$\bigcup_{\delta_i \in \tilde{I}(a_i)} \Big\{\prod_{i=1}^{k} \delta_i^{\xi_i}\Big\},$$
$$\bigcup_{\eta_i \in \tilde{F}(a_i)} \Big\{\prod_{i=1}^{k} \eta_i^{\xi_i}\Big\}\Big)\Big\rangle$$

(3) If n=k+1, by the operations (1) and (3) in Definition 2.

$$MVNLPWA(a_1, a_2, \cdots, a_k, a_{k+1}) =$$
$$\Big\langle s_{\sum_{i=1}^{k} \xi_i \cdot \theta(a_i) \xi_i \cdot \theta(a_i) + \xi_{k+1} \cdot \theta(a_{k+1})},$$
$$\Big(\bigcup_{\gamma_i \in \tilde{T}(a_i), \gamma_{k+1} \in \tilde{T}(a_{k+1})} \left\{\begin{matrix}(1-\prod_{i=1}^{k}(1-\gamma_i)^{\xi_i}) \\ +(1-(1-\gamma_{k+1})^{\xi_{k+1}}) \\ -(1-\prod_{i=1}^{k}(1-\gamma_i)^{\xi_i}) \\ \cdot(1-(1-\gamma_{k+1})^{\xi_{k+1}})\end{matrix}\right\},$$
$$\bigcup_{\delta_i \in \tilde{I}(a_i), \delta_{k+1} \in \tilde{I}(a_{k+1})} \Big\{\prod_{i=1}^{k} \delta_i^{\xi_i} \cdot \delta_{k+1}^{\xi_{k+1}}\Big\},$$
$$\bigcup_{\eta_i \in \tilde{F}(a_i), \eta_{k+1} \in \tilde{F}(a_{k+1})} \Big\{\prod_{i=1}^{k} \eta_i^{\xi_i} \cdot \eta_{k+1}^{\xi_{k+1}}\Big\}\Big)\Big\rangle$$
$$= \Big\langle s_{\sum_{i=1}^{k+1} \xi_i \cdot \theta(a_i)},$$
$$\Big(\bigcup_{\gamma_i \in \tilde{T}(a_i)} \Big\{1-\prod_{i=1}^{k+1}(1-\gamma_i)^{\xi_i}\Big\},$$
$$\bigcup_{\delta_i \in \tilde{I}(a_i)} \Big\{\prod_{i=1}^{k+1} \delta_i^{\xi_i}\Big\},$$
$$\bigcup_{\eta_i \in \tilde{F}(a_i)} \Big\{\prod_{i=1}^{k+1} \eta_i^{\xi_i}\Big\}\Big)\Big\rangle$$

Eq. (3) holds for n=k+1. Thus, Eq. (3) holds for all n.

The MVNLPWA operator has the following properties.

(1) Commutativity: Let $a_i = \langle s_{\theta(a_i)}, (\tilde{T}(a_i), \tilde{I}(a_i), \tilde{F}(a_i))\rangle$ be a collection of MVNLNs, if $a_i^*(i=1,2,\cdots,n)$ is any permutation of $a_i\,(i=1,2,\cdots,n)$, then

$MVNLPWA(a_1, a_2, \cdots, a_n) = MVNLPWA(a_1^*, a_2^*, \cdots, a_n^*)$.

(2) Idempotency: Let $a_i = \langle s_{\theta(a_i)}, (\tilde{T}(a_i), \tilde{I}(a_i), \tilde{F}(a_i))\rangle$, $(i=1,2,\cdots,n)$ be a collection of MVNLNs, and $a = \langle s_{\theta(a)}, (\tilde{T}(a), \tilde{I}(a), \tilde{F}(a))\rangle$ be a MVNLN, if $a_i = a\,(i = 1, 2, \cdots, n)$, then

$MVNLPWA(a_1, a_2, \cdots, a_n) = a.$

(3) Boundness: Let $a_i = \langle s_{\theta(a_i)}, (\tilde{T}(a_i), \tilde{I}(a_i), \tilde{F}(a_i))\rangle$, $(i=1,2,\cdots,n)$ and $a_i^* = \langle s_{\theta(a_i^*)}, (\tilde{T}(a_{i^*}), \tilde{I}(a_{i^*}), \tilde{F}(a_{i^*}))\rangle$ $(i=1,2,\cdots,n)$ be two collections of MVNLNs. If $\theta(a_i) \le \theta(a_i^*), \gamma_i \le \gamma_i^*, \delta_i \ge \delta_i^*, \eta_i \ge \eta_i^*$ for all i, then $MVNLPWA(a_1, a_2, \cdots, a_n) \le MVNLPWA(a_1^*, a_2^*, \cdots, a_n^*)$.

Where γ_i, δ_i and η_i are elements of $\tilde{T}(a_i), \tilde{I}(a_i)$ and $\tilde{F}(a_i)$ respectively, $\gamma_i^* \delta_i^* and \eta_i^*$ are elements of $\tilde{T}(a_{i^*}), \tilde{I}(a_{i^*}) and \tilde{F}(a_{i^*})$ respectively.

C. MVNLPWG operator

Definition 6 Let $a_i (i = 1, 2, \cdots, n)$ be a collection of MVNLNs, $a_i = \langle s_{\theta(a_i)}, (\tilde{T}(a_i), \tilde{I}(a_i), \tilde{F}(a_i))\rangle$, and

$\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ be the weighted vector for $a_i, \omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Then the operator of MVNLPWG is achieved as below, and the aggregation result is still an MVNLN.

$$MVNLPWG(a_1, a_2, \cdots a_n) = \bigotimes_{i=1}^{n} (a_i)^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}} \quad (4)$$

Where $S(a_i) = \sum_{j=1, j\neq i}^{n} \omega_j Supp(a_i, a_j)$, satisfying the following conditions.

(1) $Supp(a_i, a_j) \in [0,1]$

(2) $Supp(a_i, a_j) = Supp(a_j, a_i)$

(3) $Supp(a_i, a_j) \geq Supp(a_p, a_q)$. If $d(a_i, a_j) < d(a_p, a_q)$,

Here $d(a_i, a_j)$ is the Hamming distance between a_i and a_j defined in Definition 3.

Based on the operations in Definition 2 and Eq. (4), we can derive the following Theorem 2.

Theorem 2 Let $a_i (i = 1, 2, \cdots, n)$ be a collection of MVNLNs, $a_i = \langle s_{\theta(a_i)}, (\tilde{T}(a_i), \tilde{I}(a_i), \tilde{F}(a_i)) \rangle$, and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ be the weighted vector for $a_i, \omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Then the aggregated result of MVNLPWG is also an MVNLN.

$$MVNLPWG(a_1, a_2, \cdots a_n) = \left\langle s_{\prod_{i=1}^{n} \theta(a_i)^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}}}, \left(\bigcup_{\gamma_i \in \tilde{T}(a_i)} \left\{ \prod_{i=1}^{n} \gamma_i^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}} \right\}, \bigcup_{\delta_i \in \tilde{I}(a_i)} \left\{ 1 - \prod_{i=1}^{n} (1-\delta_i)^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}} \right\}, \bigcup_{\eta_i \in \tilde{F}(a_i)} \left\{ 1 - \prod_{i=1}^{n} (1-\eta_i)^{\frac{\omega_i(1+S(a_i))}{\sum_{i=1}^{n}\omega_i(1+S(a_i))}} \right\} \right) \right\rangle \quad (5)$$

Where $S(a_i) = \sum_{j=1, j\neq i}^{n} \omega_j Supp(a_i, a_j)$, satisfying the conditions in Definition 6.

Similarly, the MVNLPWG operator Eq. (5) can be proved using the mathematical induction, and the MVNLPWG operator also has the properties of ommutativity, dempotency and boundness.

IV. ILLUSTRATIVE EXAMPLE

In this section, we will use the novel operators to deal with the multi-criteria decision-making problems under the multi-valued neutrosophic linguistic environment, where the alternative values are in the form of MVNLN s and the criteria weights are in the form of crisp values.

A. Example

Next, we will consider the same decision-making problem adapted from Li [12].

An investment company wants to expand its business. Four alternatives will be chosen, A_1 represents auto corporation, A_2 represents food corporation, A_3 represents computer company corporation), A_4 represents weapon corporation. Each alternative is evaluated under three criteria, C_1 denotes risk, C_2 denotes growth, C_3 denotes the impact of environment, where C_3 is the minimizing criteria. The corresponding weighted vector is $\omega = \{0.35, 0.25, 0.4\}$. In real situation, the decision maker may hesitant and give several possible value for the satisfaction, indeterminacy and dissatisfaction regarding the alternative A_i corresponding to the criteria C_j under the linguistic term set S. Therefore, the assessment value is given in the form of MVNLNs, and the linguistic term set is employed as $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ $= \{\text{extremely poor,very poor,poor,medium,good,very good,extremely good}\}$.

The multi-valued neutrosophic linguistic decision matrix $R = [a_{ij}]_{4\times3}$ is shown as follows.

$$R = [a_{ij}]_{4\times3} = \begin{bmatrix} \langle s_5, (\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\}) \rangle & \langle s_6, (\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3, 0.4\}) \rangle & \langle s_5, (\{0.2, 0.3\}, \{0.1, 0.2\}, \{0.5, 0.6\}) \rangle \\ \langle s_6, (\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.2, 0.3\}) \rangle & \langle s_5, (\{0.6, 0.7\}, \{0.1\}, \{0.3\}) \rangle & \langle s_5, (\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.1, 0.2\}) \rangle \\ \langle s_6, (\{0.5, 0.6\}, \{0.4\}, \{0.2, 0.3\}) \rangle & \langle s_5, (\{0.6\}, \{0.3\}, \{0.4\}) \rangle & \langle s_4, (\{0.5, 0.6\}, \{0.1\}, \{0.3\}) \rangle \\ \langle s_4, (\{0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\}) \rangle & \langle s_4, (\{0.6, 0.7\}, \{0.1\}, \{0.2\}) \rangle & \langle s_6, (\{0.3, 0.5\}, \{0.2\}, \{0.1, 0.2, 0.3\}) \rangle \end{bmatrix}$$

Step1. Normalize the decision matrix.

Suppose that $R = [a_{ij}]_{m\times n}$ is the original decision matrix, which can be normalized as follows:

$$b_{ij} = \begin{cases} a_{ij}, & \text{for maximizing criteria} \\ \langle s_{t-\theta(a_{ij})}, (\tilde{T}(a_{ij}), \tilde{I}(a_{ij}), \tilde{F}(a_{ij})) \rangle, & \text{for minimizing criteria} \end{cases}$$

Thus, the normalized matrix $B = [b_{ij}]_{m\times n}$ is gained.

Because C_3 is the minimizing criteria, which should be converted to the maximizing criteria, then the normalized decision matrix $B = [b_{ij}]_{m\times n}$ can be obtained as follows:

$$B = [b_{ij}]_{4\times 3} = \begin{bmatrix} \langle s_5, (\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\})\rangle & \langle s_6, (\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3, 0.4\})\rangle & \langle s_2, (\{0.2, 0.3\}, \{0.1, 0.2\}, \{0.5, 0.6\})\rangle \\ \langle s_6, (\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.2, 0.3\})\rangle & \langle s_5, (\{0.6, 0.7\}, \{0.1\}, \{0.3\})\rangle & \langle s_2, (\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.1, 0.2\})\rangle \\ \langle s_6, (\{0.5, 0.6\}, \{0.4\}, \{0.2, 0.3\})\rangle & \langle s_5, (\{0.6\}, \{0.3\}, \{0.4\})\rangle & \langle s_3, (\{0.5, 0.6\}, \{0.1\}, \{0.3\})\rangle \\ \langle s_4, (\{0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\})\rangle & \langle s_4, (\{0.6, 0.7\}, \{0.1\}, \{0.2\})\rangle & \langle s_1, (\{0.3, 0.5\}, \{0.2\}, \{0.1, 0.2, 0.3\})\rangle \end{bmatrix}$$

Step2. Calculate the supports $Supp(b_{ij}, b_{ip})$.

As an example, $Supp(b_{11}, b_{12})$ can be obtained as follows:

$$Supp(b_{11}, b_{12}) = 1 - d(b_{11}, b_{12})$$
$$= 1 - d(\langle s_5, (\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\})\rangle, \langle s_6, (\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3, 0.4\})\rangle)$$
$$= 0.7056$$

Where $d(b_{11}, b_{12})$ is the Hamming distance defined in Eq. (1).

Then, $Supp(b_{ij}, b_{ip})$ $(i = 1,2,3,4;\ j, p = 1,2,3;\ j \neq p)$ can be calculated.

$Supp(b_{11}, b_{12}) = Supp(b_{12}, b_{11}) = 0.7056;$
$Supp(b_{11}, b_{13}) = Supp(b_{13}, b_{11}) = 0.6278;$
$Supp(b_{12}, b_{13}) = Supp(b_{13}, b_{12}) = 0.4444;$
$Supp(b_{21}, b_{22}) = Supp(b_{22}, b_{21}) = 0.8833;$
$Supp(b_{21}, b_{23}) = Supp(b_{23}, b_{21}) = 0.5111;$
$Supp(b_{22}, b_{23}) = Supp(b_{23}, b_{22}) = 0.6278;$
$Supp(b_{31}, b_{32}) = Supp(b_{32}, b_{31}) = 0.8111;$
$Supp(b_{31}, b_{33}) = Supp(b_{33}, b_{31}) = 0.5167;$
$Supp(b_{32}, b_{33}) = Supp(b_{33}, b_{32}) = 0.5944;$
$Supp(b_{41}, b_{42}) = Supp(b_{42}, b_{41}) = 0.9333;$
$Supp(b_{41}, b_{43}) = Supp(b_{43}, b_{41}) = 0.6444;$
$Supp(b_{42}, b_{43}) = Supp(b_{43}, b_{42}) = 0.6667.$

Step3. Calculate the weights ξ_{ij}.

The weighted support $S(b_{ij})$ can be obtained using the weights ω_j $(j = 1,2,3)$ of the criteria C_j $(j = 1,2,3)$

$$S(b_{ij}) = \sum_{p=1, p\neq j}^{3} \omega_p Supp(b_{ij}, b_{ip}) \ (i = 1,2,3,4;\ p = 1,2,3)$$

Then, the weights ξ_{ij} $(i = 1,2,3,4;\ j = 1,2,3)$ associated with the MVNLN b_{ij} can be calculated by the following formula:

$$\xi_{ij} = \frac{\omega_j(1+S(b_{ij}))}{\sum_{j=1}^{3}\omega_j(1+S(b_{ij}))}$$

As an example, $S(b_{11})$ can be calculated as follows:

$$S(b_{11}) = \omega_2 \cdot Supp(b_{11}, b_{12}) + \omega_3 \cdot Supp(b_{11}, b_{13})$$
$$= 0.25 \cdot 0.7056 + 0.4 \cdot 0.6278$$
$$= 0.4275;$$

Then,

$$(S(b))_{4\times 3} = \begin{bmatrix} 0.4275 & 0.4247 & 0.3308 \\ 0.4253 & 0.5603 & 0.3358 \\ 0.4095 & 0.5216 & 0.3294 \\ 0.4911 & 0.5933 & 0.3922 \end{bmatrix}$$

Therefore, as an example, ξ_{11} can be calculated as follows:

$$\xi_{11} = \frac{\omega_1(1+S(b_{11}))}{\sum_{j=1}^{3}\omega_j(1+S(b_{1j}))} = \frac{0.4996}{1.3881} = 0.3599$$

Then,

$$\xi_{4\times 3} = \begin{bmatrix} 0.3599 & 0.2566 & 0.3835 \\ 0.3505 & 0.2741 & 0.3754 \\ 0.3510 & 0.2707 & 0.3783 \\ 0.3533 & 0.2696 & 0.3771 \end{bmatrix}$$

Step4. Calculate the comprehensive evaluate value of each alternative.

Utilize the MVNLPWA operator in Eq. (3) to aggregate all the values of each alternative. Then, the comprehensive value b_1 of alternative A_1 can be obtained as follows:

$$b_1 = MVNLPWA(b_{11}, b_{12}, b_{13})$$
$$= \Big\langle s_{\xi_{11}\theta(b_{11})+\xi_{12}\theta(b_{12})+\xi_{13}\theta(b_{13})},$$
$$\Big(\bigcup_{\gamma_{1j}\in \tilde{T}(b_{1j})}\left\{1-(1-\gamma_{11})^{\xi_{11}}\cdot(1-\gamma_{12})^{\xi_{12}}\cdot(1-\gamma_{13})^{\xi_{13}}\right\},$$
$$\bigcup_{\delta_{1j}\in \tilde{I}(b_{1j})}\left\{\delta_{11}^{\xi_{11}}\cdot\delta_{12}^{\xi_{12}}\cdot\delta_{13}^{\xi_{13}}\right\},$$
$$\bigcup_{\eta_{1j}\in \tilde{F}(b_{1j})}\left\{\eta_{11}^{\xi_{11}}\cdot\eta_{12}^{\xi_{12}}\cdot\eta_{13}^{\xi_{13}}\right\}\Big)\Big\rangle$$
$$= \Big\langle s_{4.1061},$$
$$\Big(\left\{\begin{matrix}0.3242, 0.3579, 0.3618, 0.3936, 0.3606, 0.3926,\\ 0.3962, 0.4264, 0.4012, 0.4311, 0.4346, 0.4628\end{matrix}\right\},$$
$$\{0.1195, 0.1558, 0.1326, 0.1729\},$$
$$\left\{\begin{matrix}0.3649, 0.3914, 0.3929, 0.4213, 0.4047, 0.4340,\\ 0.4357, 0.4673\end{matrix}\right\}\Big)\Big\rangle$$

$$b_2 = MVNLPWA(b_{21}, b_{22}, b_{23})$$
$$= \Big\langle s_{4.2243},$$
$$\Big(\left\{\begin{matrix}0.6, 0.6409, 0.6303, 0.6682, 0.6384, 0.6754,\\ 0.6658, 0.7\end{matrix}\right\},$$
$$\{0.1, 0.1297, 0.1275, 0.1654\},$$
$$\{0.1723, 0.2235, 0.1986, 0.2576\}\Big)\Big\rangle$$

$$b_3 = MVNLPWA(b_{31}, b_{32}, b_{33})$$
$$= \Big\langle s_{4.5944},$$
$$\Big(\{0.5293, 0.5674, 0.5648, 0.6\},$$
$$\{0.2190\},$$
$$\{0.2813, 0.3243\}\Big)\Big\rangle$$

$$b_4 = MVNLPWA(b_{41}, b_{42}, b_{43})$$
$$= \Big\langle s_{2.8687},$$
$$\left(\begin{Bmatrix} 0.5538, 0.6069, 0.5871, 0.6363, 0.6133, 0.6594, \\ 0.6422, 0.6848 \end{Bmatrix},\right.$$
$$\{0.1299\},$$
$$\left.\{0.1205, 0.1566, 0.1824, 0.1540, 0.2, 0.2330\}\right)\Big\rangle$$

Step5. Calculate the Hamming distance between an alternative A_j and the ideal solution/negative ideal solution.

The ideal solution is given as $y^+ = \langle s_{\max\theta(x)}, (1,0,0)\rangle$, and the negative ideal solution is given as $y^- = \langle s_{\min\theta(x)}, (0,1,1)\rangle$. The distance measure is given in the following.

$$d_j^+ = d(b_j, y^+),\ d_j^- = d(b_j, y^-).$$
$$d_1^+ = 0.8526, d_1^- = 0.2338;$$
$$d_2^+ = 0.6329, d_2^- = 0.3671;$$
$$d_3^+ = 0.7555, d_3^- = 0.3328;$$
$$d_4^+ = 0.6762, d_4^- = 0.3238.$$

Step6. Get the relative closeness coefficient.

$$R_j = \frac{d_j^+}{d_j^+ + d_j^-}\ j = 1, 2, 3, 4.$$

Thus,

$$R_1 = 0.7848, R_2 = 0.6329,$$
$$R_3 = 0.6942, R_4 = 0.6762.$$

Step7. Rank the alternatives.

According to the relative closeness coefficient, the final ranking order of the alternatives is $A_2 \succ A_4 \succ A_3 \succ A_1$. The smaller R_j is, the better the alternative A_j is. Apparently, the best alternative is A_2 while the worst alternative is A_1.

B. Comparative analysis

To verify the effectiveness of the proposed method in this paper, a comparison analysis with the relevant papers [2-6, 10-12] is conducted. On the one hand, the decision information in [2-6] is SVNSs, INSs, and MVNSs, respectively, which apply real values or interval values to express evaluation information with the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Whereas the decision information used in this paper is MVNLSs, which apply the linguistic variable and multiple real values to express evaluation information. The MVNLSs is an extension of the existing methods, which is more reasonable and useful in handling complex decision-making problems. On the other hand, the proposed method based on the MVNLNs power aggregation operators in this paper is compared with some methods in Ye [10, 11] and Li [12]. Firstly, the method proposed by Ye [10] was based on an extended Topsis method for SVNLNs, which cannot realize the information aggregation. Secondly, the method proposed by Ye [11] was based on INLWAA operator and INLWGA operator to handle INLNs, which cannot take all the decision arguments and their relationship into account, and the proposed method is more scientific to make decision. Thirdly, the method proposed in this paper is compared with method in Li [12], which extends traditional power operators to MVNLNs environment and is more generation.

V. Conclusion

Linguistic variables can express qualitative information, and MVNSs can describe hesitant and uncertainties information. The MVNLSs is a combination of LVs and MVNSs, and it has both the advantages of LVs and MVNSs. Thus, it is meaningful to solve MCDM problems with MVNLSs.

Based on the related research achievements, this paper firstly introduced the concepts of MVNLSs and MVNLNs. The main contributions of this paper are: Firstly, the operations of MVNLNs based on Algebraic operations were developed and the Hamming distance measure between the MVNLNs was originally defined. Secondly, the conventional PWA operator and PWG operator fail in handling MVNLSs. Thus, the conventional PWA operator and PWG operator are extended to the multi-valued neutrosophic linguistic environment, and their properties are also discussed. Finally, in order to demonstrate the practicality and effectiveness for MCDM problem, an illustrative example based on the MVNLPWA operator is given.

In this paper, the proposed operators can not only own the advantages of multi-valued neutrosophic sets (MVNSs) and linguistic variables (LVs), but also extend conventional power operators to MVNLNs environment, which consider all the arguments and their relationship. Meanwhile, the comparison analysis shows that the proposed method is more scientific and flexible in solving complex MCDM problems with MVNL environment, in which the evaluation values take the form of MVNLNs and criteria weights are known real value.

In the future, we will develop more aggregation operators for MVNLNs and apply them to different fields, such as medical diagnosis, pattern recognition and group decision making.

References

[1] F. Smarandache, A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[2] H.Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multispace and Multistructure*, vol. 4, pp. 410－413, 2010.

[3] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, Interval Neutrosophic Sets and Logic:Theory and Applications in Computing, Hexis, Phoenix, Ariz, USA, 2005.

[4] J.Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets," *Journal of Intelligent & Fuzzy Systems*, vol.26, no. 5, pp. 2459-2466, 2014.

[5] J.Ye, "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment," J Intell Syst , vol.24, no. 1, pp. 23–36, 2015.

[6] J.Q. Wang and X.E. Li, "An application of the TODIM method with multi-valued neutrosophic set," Control Decis , vol.30, no. 6, pp. 1139–1142, 2015.

[7] L.A.Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," Inf Sci,vol. 8, no. 3, pp. 199–249, 1975.

[8] J.Q. Wang and J.J.Li, "The multi-criteria group decision making method based on multi-granularity intuitionistic two semantics," Science &Technology Inforamtion, vol.33, pp. 8–9, 2009.
[9] F.Y.Meng, X.H. Chen, Q.Zhang, "Multi-attribute decision analysis under a linguistic hesitant fuzzy environment," Inf Sci, vol. 267, pp. 287-305, 2015.
[10] J .Ye, "An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers," J Intell Fuzzy Syst, vol. 28, no.1, pp. 247–255, 2015.
[11] J.Ye, "Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making," J Intell Fuzzy Syst , vol. 27, no.5, pp. 2231–2241,2014.
[12] B.L.Li, J.R.Wang, L.H.Yang, and X.T.Li, "Multiple criteria decision making approach with multi-valued neutrosophic linguistic normalized weighted Bonferroni mean Hamacher operator," Mathematical problems in Engineering, 2018.Doi:10.1155/2018/2432167.
[13] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decision making," *IEEE Trans. on Systems, Man and Cybernetics Cybernetics*, vol. 18, no. 1, pp. 183-190, 1988.
[14] Z. S. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems*, vol. 35,no. 4, pp. 417-433, 2006.
[15] R. R. Yager, "The power average operator," IEEE Trans. on Systems, Man, Cybernetics-Part A:Systems and Humans, vol. 31, (2001),pp. 724-731.
[16] Z. S. Xu and R. R. Yager, "Power-geometric operators and their use in group decision making,"IEEE Trans. on Fuzzy Systems, vol. 18,(2010), pp. 94-105.
[17] L. G. Zhou, H. Y. Chen, and J. P. Liu, "Generalized power aggregation operators and their applications in group decision making," Computers & IndustrialEngineering, vol. 62, (2012),pp. 989-999.
[18] L.H.Yang and B.L.Li, "A Multi-Criteria Decision-Making Method Using Power Aggregation Operators for Single-valued Neutrosophic Sets," *International Journal of Database Theory and Application*. vol. 9, no. 2, pp. 23-32, 2016.
[19] P. D. Liu and Y. Liu, "An approach to multiple attribute group decision making based on intuitionistic trapezoidal fuzzy power generalized aggregation operator," International Journal of Computational Intelligence Systems, vol. 7, no. 2, (2014),pp. 291-304.
[20] J.J. Peng, J.Q. Wang and X.H. Wu, "Multi-valued Neutrosophic Sets and Power Aggregation Operators with Their Applciation in Multi-criteria Group Decision-making Problems", International Journal of Computational Intelligence Systems, vol.8, no.2, pp.345-363, 2015.
[21] B.L.Li, J.R.Wang, L.H.Yang, and X.T.Li, "A novel generalized simplified neutrosophic number Einstein aggregation operator," *IAENG International Journal of Applied Mathematics*, vol. 48, no. 1, pp. 67-72, 2018.