The lateral area of revolution solids

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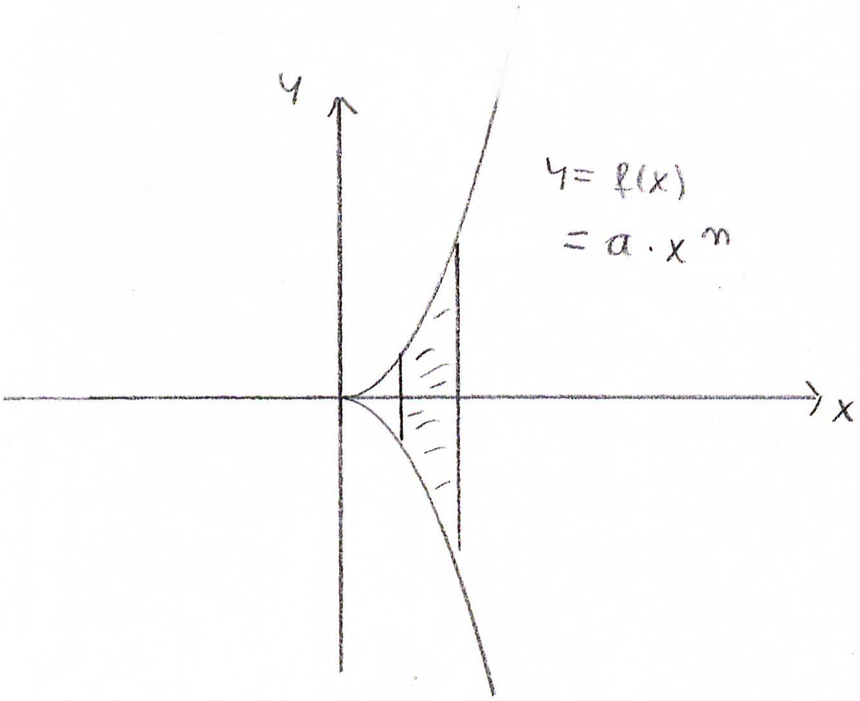
We view a revolution solid around the x-axis as in the following figure:



It is valid $y = f(x)$ and for the lateral area we have:

$$M = 2\pi \cdot \int_{x_1}^{x_2} y \cdot \sqrt{1 + y'^2}\, dx$$

y' is the differentiation from f with respect to x. This formula is proved at Forster [1] chapter 14 p.141,142. This formula is a special case of a more general formula in Forster [1]. As function we define $f(x)=ax^n$ with a and n as positive real numbers. If we insert this function into the lateral area formula and we choose $x_1=0$ and $x_2=h$, we get:

$$M = 2\pi a \cdot \int_0^h x^n \cdot \sqrt{1+n^2x^{2n-2}}\,dx$$

Now we view examples for $n=3$ respectively 4 and 5. h is a natural number from 1 to 10 and $a=1$ is chosen. This integral cannot be calculated exactly. We use the Gauss-Kronrod-method to numerical integration. The result is the following table with the values of lateral area:

h \ n	3	4	5
1	3.5631	3.4365	3.3616
2	203.0436	805.7179	3218.1523
3	2294.819	20615.423	185510.637
4	12876.226	205893.599	3294203.589
5	49100.36	1227194.35	30679623.52
6	146592.9	5276683.3	189960105.5
7	369630.8	18110675.6	887422182.4
8	823583	52707204	3373259446
9	1669613	135235294	$1.095405628 \cdot 10^{10}$
10	3141645	314159305	$3.141592657 \cdot 10^{10}$

References

[1] "Analysis 3" Otto Forster Vieweg Verlag 2. edition Brunswick 1983