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An outline of cellular automaton universe via cosmological KdV equation

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Abstract. It has been known for long time that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abound. However, such a sound wave model of cosmology is rarely developed fully into a complete framework. This paper can be considered as our *second* attempt towards such a complete description of the Universe based on soliton wave solution of cosmological KdV equation. Then we advance further this KdV equation by virtue of Cellular Automaton method to solve the PDEs. We submit wholeheartedly Robert Kuruczs hypothesis that *Big Bang should be replaced with a finite cellular automaton universe with no expansion* [4][5]. Nonetheless, we are fully aware that our model is far from being complete, but it appears the proposed cellular automaton model of the Universe is very close in spirit to what Konrad Zuse envisaged long time ago. It is our hope that the new proposed method can be verified with observation data. But we admit that our model is still in its infancy, more researches are needed to fill all the missing details.

1. Introduction

Konrad Zuse is probably the first scholar who imagine a *Computing Universe*. In recent years, there are a few researchers who suggest similar vision in terms of cellular automata, for example Stephen Wolfram, Gerardus t Hooft, and Robert Kurucz from Harvard Smithsonian of Astrophysics. Nonetheless, it seems that there is no existing model which can be connected with a nonlinear PDE of the Universe. In this paper, we try to offer some working CA models based on the KdV equation, which can be modelled and solved using computer algebra packages such as Mathematica.

Meanwhile, Korteweg-de Vries (KdV) equation is a non-linear wave equation plays a fundamental role in diverse branches of mathematical and theoretical physics. Its significance to cosmology has been discussed by a number of authors, such as Rosu and recently Lidsey [3][7]. It is suggested that the KdV equation arises in a number of important scenarios, including inflationary cosmology etc. Analogies can be drawn between cosmic dynamics and the propagation of the solitonic wave solution to the equation, whereby quantities such as the speed and amplitude profile of the wave can be identified with cosmological parameters such as the spectral index of the density perturbation spectrum and the energy density of the universe.

2. Cosmological KdV equation

The Korteweg-de Vries (KdV) equation is the completely integrable, third-order, non-linear partial differential equation (PDE) [3]:

$$\partial_t u + \partial_x^3 u + \frac{3}{u_0} u \partial_x u = 0, \tag{1}$$

where $u = u(x,t), \partial_t = \frac{\partial}{\partial t}, \partial_x^3 = \frac{\partial^3}{\partial x^3}$, etc., u_0 is a constant and (x,t) represent space and time coordinates, respectively. This equation was originally derived within the context of small-amplitude, non-linear water wave theory and it is well known that it admits a solitonic wave solution of the form

$$u = u_0 \lambda^2 \sec h^2 \left[\frac{\lambda(x - \lambda^2 t)}{2} \right], \tag{2}$$

where the constant $\frac{\lambda}{2}$ represents the wavenumber of the soliton. The KdV soliton is characterized by the property that its speed and amplitude are proportional to the square of the wavenumber.

Rosu [7] and also Lidsey [3] both have considered some cosmological applications of KdV equation. We will consider here one application in inflationary universe model. It can be shown that Friedmann equation after some steps which have been discussed in [3], yields to an equation which takes the form of (2), as follows:

$$H^2(\phi) = H_0^2 \lambda^2 \sec h^2 \left[\frac{\lambda A}{2} \right], \tag{3}$$

where

$$A = \frac{\sqrt{8\pi}}{m_p} \phi. \tag{4}$$

Therefore, it appears quite reasonable to consider this equation as originated from certain cosmological KdV physics.

3. Cellular automata model of KdV equation: Towards cellular automaton universe

There are several methods to consider discretization of KdV equation into cellular automata models. Here we briefly discuss only few methods:

(i) Based on paper by [11], KdV equation can be written as a conservation law:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left(-\frac{u^2}{2} - \frac{\partial^2 u}{\partial x^2} \right) = 0. \tag{5}$$

It follows that, after the simplest discretization, we obtain the cellular automata:

$$u_j(t+1) = u_j(t)[u_{j+1}(t) - u_j(t)] + u_{j+2}(t) - u_{j+1}(t) - u_{j-1}(t). \tag{6}$$

Thus $\Sigma_{j=0}^{N-1} u_j(t)$ is not an invariant.

(ii) The discrete analogue of the KdV equation is known thanks to the pioneering work of Hirota. It has the form [15]:

$$\frac{1}{u_{l+1}^{t+1}} - \frac{1}{u_l^t} = \delta(u_{l+1}^t - u_l^{t+1}) \tag{7}$$

(iii) Another model was proposed by Tokihiro et al around twenty years ago. They suggested that an integrable discretization (differential-difference equation) of the KdV equation is the Lotka-Volterra equation [14]:

$$\frac{d}{dt}b_j t = b_j(t)[b_{j+t}(t) - b_{j-1}(t)] \tag{8}$$

In other words, it appears possible at least in theory to consider a Cellular Automaton-KdV Universe, based on discretization of the original KdV equation. Nonetheless, further analysis is required to study its potential applications.

4. Discussion

We advance further KdV equation by virtue to Cellular Automaton method to solve the PDEs. Here, we consider some mathematical methods to discretize the original KdV equation in order to be transformed into cellular automata cosmology models. As far as our knowledge, this approach is different from existing cellular cosmology, such as by Conrad Ranzan (www.cellularuniverse.com).

References

[1] Pain H J 2005 *The Physics of Vibrations and Waves* (New York: John Wiley and Sons)
[2] Hu W and White M 2004 *The Cosmic Symphony*, Scientific American 2004. URL: https://www.scientificamerican.com/author/wayne-hu-and-martin-white/
[3] Lidsey J E Cosmology and the Korteweg-de Vries equation, Phys. Rev. D. vol. 86, URL: https://journals.aps.org/prd/pdf/10.1103/PhysRevD.86.123523.
[4] Kurucz R L Radiatively-driven cosmology in the cellular automaton universe. Harvard-Smithsonian Center for Astrophysics, 60 Garden St, Cambridge, MA 02138. URL: http://kurucz.harvard.edu/papers/radcellular/radiativelycell.ps
[5] Kurucz R L Elementary physics in the cellular automaton universe. Harvard-Smithsonian Center for Astrophysics, 60 Garden St, Cambridge. URL: http://kurucz.harvard.edu/papers/elementary/elementary.ps
[6] Adiwidjaja H 1995 On the Korteweg-de Vries equation. CDS 141 Project, Caltech Univ. URL: www.cds.caltech.edu/ marsden/wiki/uploads/projects/geomech/Adiwidjaja1995.pdf
[7] Rosu H C 2002 *Mod. Phys. Lett.* A **17** 667
[8] Scott A C 2007 *The Nonlinear Universe* (Berlin: Springer-Verlag)
[9] Wolfram S 1994 *Cellular Automata and Complexity* (Addison-Wesley)
[10] Wolfram S. Cellular Automaton fluids I: basic theory. URL: www.stephenwolfram.com/publications/academic/cellular-automaton-fluids-theory.pdf
[11] Steeb W H and Hardy Y 2007 *Naturforsch* **62a** 567
[12] Yang X S and Young Y 1983 Cellular automata, PDEs, and pattern formation. Chapter in Handbook of Bioinspired Algorithms and Applications, edited by Stephan Olariu, Albert Y. Zomaya. (New York: Chapman and Hall/CRC Computer and Information Science Series, 2010)
[13] de Oliveira P P B and Verardo M 2014 *Math. J.* **16** url: http://www.mathematica-journal.com/2014/08/representing-families-of-cellular-automata-rules/
[14] Tokihiro T 1996 *Phys. Rev. Lett.* **76** 18
[15] Willox R 2010 *J. Phys. A: Math. Theor.* **43** 482003
[16] Kurucz R L 2000 The formation of life. Harvard-Smithsonian Center for Astrophysics, 60 Garden St, Cambridge, MA 02138
[17] Walker S I 2017 Origins of life: A problem for physics. *Reports on Progress in Physics, IOP* url: http://iopscience.iop.org/article/10.1088/1361-6633/aa7804/pdf, https://doi.org/10.1088/1361-6633/aa7804