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Linguistic Approaches to Interval Complex Neutrosophic Sets in Decision Making

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ABSTRACT One of the most efficient tools for modeling uncertainty in decision-making problems is the neutrosophic set (NS) and its extensions, such as complex NS (CNS), interval NS (INS), and interval complex NS (ICNS). Linguistic variables have been long recognized as a useful tool in decision-making problems for solving the problem of crisp neutrosophic membership degree. In this paper, we aim to introduce new concepts: single-valued linguistic complex neutrosophic set (SVLCNS-2) and interval linguistic complex neutrosophic set (ILCNS-2) that are more applicable and adjustable to real-world implementation than those of their previous counterparts. Some set-theoretic operations and the operational rules of SVLCNS-2 and ILCNS-2 are designed. Then, gather classifications of the candidate versus criteria, gather the significance weights, gather the weighted rankings of candidates versus criteria and a score function to arrange the candidates are determined. New TOPSIS decision-making procedures in SVLCNS-2 and ICNS-2 are presented and applied to lecturer selection in the case study of the University of Economics and Business, Vietnam National University. The applications demonstrate the usefulness and efficiency of the proposal.

INDEX TERMS Lecturer selection, linguistic interval complex neutrosophic set, multi-criteria decision-making, neutrosophic set.

I. INTRODUCTION

One of the most efficient tools for demonstrating uncertainty and vagueness in decision making is the NS [1] which is the more generality of classical set, fuzzy set and intuitionistic fuzzy set (IFS) by adding three grades of truth, falsehood, and indeterminacy of a confirmed statement. It has been employed in various decision making processes such as in [2]–[8]. Yet, in order to adapt NS with more real complex cases, CNS and INS have been proposed accordingly. Wang *et al.* [9] suggested the notion of INS which is described by the degree of truth, falsehood and indeterminacy whose values and standards are intervals rather than real numbers. Ali and Smarandache [10] suggested the idiom CNS which is an expansion form of complex fuzzy set and complex IFS to handle the unnecessary nature of ambiguity, incompleteness, indefiniteness and changeability in periodic data. These extensions have been applied to decision making problems successfully [7].

As an expansion to this trend, Ali *et al.* [11] have recently proposed the notion of ICNS by fusing CNS and INS in a homogeneous way. Therein, the authors defined some set notional procedures of ICNS such as intersection, union and complement, and afterwards the operational principles. A decision-making transaction in ICNS was presented and applied to green supplier selection [11]. It has been realized from this research that ICNS with suitable ranking methods generated from the score, accuracy and certainty functions can handle the real decision cases that have not been solved by the relevant works such as of Ye [12]. However, this

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research remains a problem: It is not simple to discover a crisp neutrosophic membership degree (as in the Single-Valued Neutrosophic Set (SVN)). In many real applications, we have to deal with undecided and imprecise information in our everyday life that could be represented by linguistic variables instead of the crisp neutrosophic membership degree [13].

The idea of **linguistic variables** in decision making problems has been long recognized as a useful approach. Li, Zhang and Wang [13] advanced two multi-criteria decision-making (MCDM) techniques in which the interrelationships among individual data are considered under linguistic neutrosophic environments. Fang and Ye [14] gave the connotation of a linguistic neutrosophic number which is categorized independently by the truth, indeterminacy, and falsity linguistic variables for multiple attribute group decision-making. Interval neutrosophic linguistic numbers (INLNs) has also been defined by Ma, Wang, Wang & Wu [15] for an application of practical treatment selection using interval neutrosophic linguistic multi-criteria group decision-making. SVN linguistic trapezoid linguistic aggregation operators were developed for decision making problems [22]. Ye [24] studied some aggregation operators of INLNs for multiple attribute decision making (MADM). Some more literature can be seen in [4], [16]–[27].

TOPSIS is popular decision making technique for interval neutrosophic unclear semantic variables [23]. Pouresmaeil *et al.* [35] utilized TOPSIS for defining the weights of decision makers with single valued neutrosophic information. Otay and Kahraman [36] employed interval neutrosophic TOPSIS method to evaluate Six Sigma projects, which aimed at providing almost defect-free products and/or services to customers. Pramanik *et al.* [37] planned TOPSIS method for MADM under neutrosophic cubic, which is the generalized form of cubic set and interval neutrosophic set. Liang, Zhao and Wu [38] designed a new term called linguistic neutrosophic numbers and integrated it into TOPSIS for investment and development of mineral resources. A multi-criteria group decision-making methodology incorporating power combination factors, TOPSIS-based QUALIFLEX and life cycle assessment technique was proposed in [21] to find the key to green product design selection using neutrosophic linguistic information. Altinirmak *et al.* [39] used single valued Neutrosophic Set based entropy to rank the banks for analyzing m-banking quality factors. Eraslan and Çağman [40] combined TOPSIS and Grey Relational Analysis under fuzzy soft sets for drug selection. It has been shown that TOPSIS is a well-known method for decision making under uncertain environments of neutrosophic and linguistic [2], [11], [18], [23], [33], [41], [42]. However, the current research on TOPSIS model do not mention the period of time when describing observation data in their model.

Meanwhile, many complex real-world problems about decision support system in which data contains some characters such as: uncertain, heterogeneous, inconsistent and have concerned with the period of time. To consider a financial corporation or company this chooses to set up novel software to process and analyses company data. For this, the company goes into a huddle some experts who give the information concerning: various choices of software which data process and analysis in financial fields, corresponding software version and other information. Surveying and observing the software is done within a period of time. After that, the company desires to select the most favorable alternative of software with its newest version concurrently. Here, we need to pay attention two things (a) to choose the best candidate of software (b) its newest version. This cannot be simplified accurately using classical concept of Fuzzy Set or NS. So the preferable way to show all of the information in this problem is using the theory of Linguistic Variables and ICNS.

In this paper, we aim to introduce new concepts namely **Single-Valued Linguistic Interval Complex Neutrosophic Set** (SVLCNS-2) and **Interval Linguistic Interval Complex Neutrosophic Set** (ILCNS-2) that are more pliable and adjustable to real-world implementations than those of their previous counter parts motivated from the mentioned analysis. Specifically, we define the SVLCNS-2 and ILCNS-2. Next, we describe some set notional operations such as the intersection, union and complement. Moreover, we set the functioning basics of SVLCNS-2 and ILCNS-2. Then, we develop gather classifications of candidate versus criteria, gather the significance weights, gather the weighted classifications of candidates versus criteria and determine a score function to rank the candidates. Lastly, new TOPSIS decision making procedures in SVLCNS-2 and ICNS-2 are presented.

Personnel selection plays a crucial role in human resource administration since the inappropriate personnel might reason various problems affecting productivity, accuracy, pliability and goodness of the products adversely [28]. It is a complicated process in the meaning that several factors should be estimated concurrently in order to find the right people for the appropriate jobs [28]. Personnel selection is a decision making problem where quality of decision affects the success of a person in an organization [29]. In the context of university selection, the consideration for reasonable and realistic selection measures of adequate candidates and effective prediction of possible success at university, therefore, becomes more and more important [30]. It has been long recognized that measuring of intelligence is no longer enough as a medium for a person's skills and success estimation [31]. It is indeed adopted by various factors to judge the suitability and adaptability of a candidate in a university context. Hence, developing effective selection or decision making techniques is critical indeed [32].

The proposed TOPSIS methods are applied to lecturer selection in the case study of University of Economics and Business - Vietnam National University (UEB-VNU), which is one of the leading universities in Hanoi, Vietnam. A committee of four decision makers (DMs) and six selection criteria are presented in the application. The applications demonstrate the usefulness and efficiency of the proposal.

The rest of this paper is prepared as follows. The formulation of SVLCNS-2 and its operations are presented in

Sections 2 and 3 while ILCNS-2 and operations are given in Sections 4 and 5. The TOPSIS decision making procedures on SVLCNS-2 and ILCNS-2 are explained in Section 6. Lastly, an application of the procedures for lecturer selection on a real case study is illustrated in Section 7. Section 8 compares the suggested method with another decision making method. Conclusions and further studies allocate in Section 9.

II. SINGLE-VALUED LINGUISTIC COMPLEX NEUTROSOPHIC SET (SVLCNS-2)

Definition 1 (Type-1 Single VALUED Linguistic Complex Neutrosophic Set (SVLCNS-1)): Let $\coprod$ be a universe of discourse and a complex neutrosophic set A included in $\coprod$. Let $\c{S} = \{\c{S}_1, \c{S}_2, \ldots, \c{S}_n$, for $2 \leq \eta < \infty$, be a set of totally ordered labels (therefore the classical min/max operators work on S), with $\c{S}_i < \c{S}_j$ for $i < j$, where $i, j \in \{1, 2, 3, \ldots, \eta\}$. Let $\bar{R} = \{[\c{S}_i, \c{S}_j], \c{S}_i, \c{S}_j \in \c{S}, i < j\}$ be a set of label intervals. A **single-valued type-1 complex neutrosophic set** (SVLCNS-1) is a set $A \subset \coprod$ such that each element x in A has linguistic degree of complex truth membership $T_A(x) \in S \times S$, a linguistic degree of complex indeterminate membership $I_A(x) \in S \times S$, and a linguistic degree of complex falsity membership $F_A(x) \in S \times S$ and $s_{\theta(x)} \in S$. A SVLCNS set A can be written as,

$$A = \{\langle X, [\c{S}_{\theta(X)}, (T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))]\rangle\}$$

where

$$\left.\begin{aligned} T_A(X) &= T_{1A}(X)\cdot e^{j.T_{2A}(X)} \\ \widehat{I}_A(X) &= \widehat{I}_{1A}(X)\cdot e^{j.\widehat{I}_{2A}(X)} \\ \mathcal{F}_A(X) &= \mathcal{F}_{1A}(X)\cdot e^{j.\mathcal{F}_{2A}(X)} \end{aligned}\right\}$$

where $T_{1A}(x)$ is representing linguistic amplitude truth membership and $e^{j.T_{2A}(x)}$ is denoting the linguistic phase truth membership function. Moreover, $I_{1A}(x)$ refers to linguistic amplitude indeterminate membership while $e^{j.I_{2A}(x)}$ indicates linguistic phase indeterminate membership. Further, $F_{1A}(x)$ is called the linguistic amplitude falsity membership and $e^{j.F_{2A}(x)}$ is said to be the linguistic phase falsehood membership function:

$$\begin{gathered} 3 * s_1 \leq \min\{T_{1A}(x)\} + \min\{I_{1A}(x)\} + \min\{F_{1A}(x)\}, \\ \max\{T_{1A}(x)\} + \max\{I_{1A}(x)\} + \max\{F_{1A}(x)\} \leq 3 * s_n, \\ 3 * s_1 \leq \min\{T_{2A}(x)\} + \min\{I_{2A}(x)\} + \min\{F_{2A}(x)\}, \\ \max\{T_{2A}(x)\} + \max\{I_{2A}(x)\} + \max\{F_{2A}(x)\} \leq 3 * s_n. \end{gathered}$$

Definition 2 (Type-2 Single Valued Linguistic Complex Neutrosophic Set (SVLCNS-2)): Let $\coprod$ be a universe of discourse and a complex NS A included in $\coprod$. Let $\c{S} = \{\c{S}_1, \c{S}_2, \ldots, \c{S}_n$, for $\eta >= 2$, be a set of ordered labels with $s_i < s_j$ with $i, j \in \{1, 2, 3, \ldots n\}$. Let $R = \{[s_i, s_j], s_i, s_j \in S, i < j\}$ be a collection of label intervals. A single-valued type-2 linguistic complex neutrosophic set (SVLCNS-2) is a set $A \subset \coprod$ such that each element x in A has linguistic degree of complex truth membership $T_A(x) \in R$, a linguistic degree of complex indeterminate membership $I_A(x) \in R$, and a linguistic degree of complex falsity membership $\mathcal{F}_A(X) \in \bar{R}$ and $\Theta_{\theta(X)} \in \c{S}$. A SVLCNS set A can be written as,

$$A = \left\{\left\langle X, \left[\Theta_{\theta(X)}, \left(T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X)\right)\right]\right\rangle | X \in \Pi\right\}$$

where

$$\left.\begin{aligned} T_A(X) &= T_{1A}(X)\cdot e^{j.T_{2A}(X)} \\ \widehat{I}_A(X) &= \widehat{I}_{1A}(X)\cdot e^{j.\widehat{I}_{2A}(X)} \\ \mathcal{F}_A(X) &= \mathcal{F}_{1A}(X)\cdot e^{j.\mathcal{F}_{2A}(X)} \end{aligned}\right\}$$

where $T_{1A}(x)$ represents the amplitude truth membership and $e^{j.T_{2A}(x)}$ denotes the phase truth membership function. Moreover, $I_{1A}(x)$ refers to the amplitude indeterminate membership while $e^{j.I_{2A}(x)}$ indicates the phase indeterminate membership function. Further, $F_{1A}(x)$ is called the amplitude falsity membership and $e^{j.F_{2A}(x)}$ is said to be the phase falsehood membership function while $0 \leq T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X) \leq 3$.

Due to complexity of higher computation involved in SVLCNS-1, in this paper, we will use SVLCNS-2 for developing the TOPSIS method.

Definition 3: Let A and B be two SVLCNSs-2 over $\coprod$ which are defined by $\langle \Theta_{\theta_A(X)}, (T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))\rangle$, and $\langle \Theta_{\theta_B(X)}, (T_B(X), \widehat{I}_B(X), \mathcal{F}_B(X))\rangle$, respectively. Their union signified as $A \cup B$ and is defined as:

$$\begin{aligned} \Theta_{\theta_{A \cup B}(X)} &= \Theta_{\theta_{1A \cup B}(X)}, \\ T_{A \cup B}(X) &= T_{1A \cup B}(X)\cdot e^{j.T_{2A \cup B}(X)}, \\ \widehat{I}_{A \cup B}(X) &= \widehat{I}_{1A \cup B}(X)\cdot e^{j.\widehat{I}_{2A \cup B}(X)}, \\ \mathcal{F}_{A \cup B}(X) &= \mathcal{F}_{1A \cup B}(X)\cdot e^{j.F_{2A \cup B}(X)}, \end{aligned}$$

where

$$\begin{aligned} \Theta_{\theta_{1A \cup B}(X)} &= \vee\left(\Theta_{\theta_A(X)}, \Theta_{\theta_B(X)}\right), \\ T_{1A \cup B}(X) &= \vee\left(T_A(X), T_B(X)\right), \\ T_{2A \cup B}(X) &= \vee\left(T_A(X), T_B(X)\right), \\ \widehat{I}_{1A \cup B}(X) &= \wedge\left(\widehat{I}_A(X), \widehat{I}_B(X)\right), \\ T_{2A \cup B}(X) &= \wedge\left(\inf T_A(X), \inf T_B(X)\right), \\ \mathcal{F}_{1A \cup B}(X) &= \wedge\left(\mathcal{F}_A(X), \mathcal{F}_B(X)\right), \\ \mathcal{F}_{2A \cup B}(X) &= \wedge\left(\mathcal{F}_A(X), \mathcal{F}_B(X)\right). \end{aligned}$$

for all $x \in X$. The symbols $\vee, \wedge$ represents maximize and minimize operators.

Definition 4: Let A and B be two SVLCNSs-2 over $\coprod$ which are defined by $\langle \Theta_{\theta_A(X)}, (T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))\rangle$, and $\langle \Theta_{\theta_B(X)}, (T_B(X), \widehat{I}_B(X), \mathcal{F}_B(X))\rangle$, respectively. Their **intersection** signified as $A \cup B$ and is defined as:

$$\Theta_{\theta_{A \cap B}(X)} = \Theta_{\theta_{1A \cap B}(X)},$$

$$\mathcal{T}_{A\cap B}(X) = \mathcal{T}_{1A\cap B}(X) \cdot e^{j.T_{2A\cap B}(X)},$$
$$\widehat{I}_{A\cap B}(X) = \widehat{I}_{1A\cap B}(X) \cdot e^{j.\widehat{I}_{2A\cap B}(X)},$$
$$\mathcal{F}_{A\cap B}(X) = \mathcal{F}_{1A\cap B}(X) \cdot e^{j.F_{2A\cap B}(X)},$$

where

$$\Theta_{\theta_{1A\cap B}(X)} = \wedge\left(\Theta_{\theta_A(X)}, \Theta_{\theta_B(X)}\right),$$
$$\mathcal{T}_{1A\wedge B}(X) = \wedge\left(\mathcal{T}_A(X), \mathcal{T}_B(X)\right),$$
$$\mathcal{T}_{2A\wedge B}(X) = \wedge\left(\mathcal{T}_A(X), \mathcal{T}_B(X)\right),$$
$$\widehat{I}_{1A\cap B}(X) = \vee\left(\widehat{I}_A(X), \widehat{I}_B(X)\right),$$
$$\mathcal{T}_{2A\cup B}(X) = \vee\left(\inf\mathcal{T}_A(X), \inf\mathcal{T}_B(X)\right),$$
$$\mathcal{F}_{1A\cap B}(X) = \vee\left(\mathcal{F}_A(X), \mathcal{F}_B(X)\right),$$
$$\mathcal{F}_{2A\cap B}(X) = \vee\left(\mathcal{F}_A(X), \mathcal{F}_B(X)\right).$$

for all $x \in X$. The symbols $\vee, \wedge$ represents max and min operators.

Proposition 2: Let A and B be two SVLCNS-2 over $\coprod$. Then

a) $A \cup B = B \cup A$,
b) $A \cap B = B \cap A$,
c) $A \cup A = A$,
d) $A \cap A = A$,

Proof: Straightforward.

Proposition 6: Let A, B and C be three SVLCNS-2 over $\coprod$. Then

a) $A\cup(B\cup C) = (A\cup B) \quad \cup C$,
b) $A\cap(B \cap C) = A \cap C$,
c) $A\cup(B \cap C) = (A\cup B) \quad \cap (A\cup C)$
d) $A\cap(B\cup C) = (A \cap B) \quad \cup (A \cap C)$
e) $A\cup(B \cap C) = A$,
f) $A\cup(A \cap B) = A$.

Theorem 7: The SVLCNS-2$A\cup B$ is the minimum set comprising together A and B.

Proof: Straightforward.

Theorem 8: The SVLCNS-2$A\cup B$ is the leading one comprised in together A and B.

Proof: Straightforward.

Theorem 9: Let P be the power set of all SVLCNSs-2. Then $(P, \cup, \cap)$ forms a distributive lattice.

Proof: Straightforward.

III. OPERATIONAL RULES OF SVLCNS-2

Let A and B be two SVLCNSs-2 over $\coprod$ which are defined by $\langle\Theta_{\theta_A(X)}, (\mathcal{T}_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))\rangle$, and $\langle\Theta_{\theta_B(X)}, (\mathcal{T}_B(X), \widehat{I}_B(X), \mathcal{F}_B(X))\rangle$, correspondingly. the operational rules of SVLCNS-2 are definite as:

a) The **product** of A and B signified as

$A \otimes B = \langle\Theta_{\theta_{A\otimes B}}, (\mathcal{T}_{A\otimes B}(X), \widehat{I}_{A\otimes B}(X), \mathcal{F}_{A\otimes B}(X))\rangle$, is defined as:

$$\Theta_{\theta_{A\otimes B}(x)} = \Theta_{\theta_A(x)}.\Theta_{\theta_B(x)},$$
$$[\Theta_j, \Theta_k]^v = [\Theta_{j^v}, \Theta_{k^v}], v > 0.$$
$$T_{A\otimes B}(x) = (T_{1A}(x).T_{1B}(x)) \cdot e^{j(T_{2A}(x).T_{2B}(x))},$$
$$I_{A\otimes B}(X) = (I_{1A}(X) + I_{1B}(X) - I_{1A}(X)\, I_{1B}(X)) \cdot e^{j(I_{2A}(X).I_{2B}(X))},$$
$$F_{A\otimes B}(X) = (F_{1A}(X) + F_{1B}(X) - F_{1A}(X)\, F_{1B}(X)) \cdot e^{j(F_{2A}(X).F_{2B}(X))},$$

b) The **addition** of A and B indicated as $A \oplus B = \langle\Theta_{\theta_{A\oplus B}}, (\mathcal{T}_{A\oplus B}(X), \widehat{I}_{A\oplus B}(X), \mathcal{F}_{A\oplus B}(X))\rangle$, is well-defined as:

$$\Theta_{\theta_{A\oplus B}(x)} = \Theta_{\theta_A(x)} + \Theta_{\theta_B(x)},$$
$$T_{A\oplus B}(x) = ((T_{1A}(x) + T_{1B}(x)) - (T_{1A}(x).T_{1B}(x))) \cdot e^{j(T_{2A}(x)+T_{2B}(x))},$$
$$I_{A\oplus B}(x) = (I_{1A}(x).I_{1B}(x)) \cdot e^{j(I_{2A}(x)+I_{2B}(x))},$$
$$F_{A\oplus B}(x) = (F_{1A}(x).F_{1B}(x)) \cdot e^{j(F_{2A}(x)+F_{2B}(x))}.$$

c) The **scalar multiplication** of A is a SVLCNS-2 denoted as $C = kA$ defined as:

$$k\Theta_{\theta_A(x)} = \Theta_{k\theta_A(x)}$$
$$T_C(x) = \left(1 - (1 - T_{1A}(x))^k\right) \cdot e^{j(T_{2A}(x))^k},$$
$$I_C(x) = \left((T_{1A}(x))^k\right) \cdot e^{j(I_{2A}(x))^k},$$
$$F_C(x) = \left((F_{1A}(x))^k\right) \cdot e^{j(F_{2A}(x))^k}.$$

Proposition 10: Let A and B be two SVLCNSs-2 over $\coprod$ which are defined by $\langle\Theta_{\theta_A(X)}, (\mathcal{T}_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))\rangle$ and $\langle\Theta_{\theta_B(X)}, (\mathcal{T}_B(X), \widehat{I}_B(X), \mathcal{F}_B(X))\rangle$, respectively. Then

a) $A \otimes B = B \otimes A$,
b) $A \oplus B = B \oplus A$,
c) $k(A \otimes B) = k(B \otimes A)$,
d) $(k_1 \otimes k_2)A = k_1 \otimes A k_2 A$.

IV. INTERVAL LINGUISTIC COMPLEX NEUTROSOPHIC SET (ILCNS-2)

Definition 11: Let $\coprod$ be a universe of discourse and let $S = \{s_1, s_2, \ldots, s_n\}$, for $\infty > n \geq 2$, be a collection of single value, linguistic markers, where $s_1 < s_2 < \ldots < s_n$ and they are the qualitative values of a linguistic variable. The linguistic relation of order $s_i < s_j$, means that label s_i is less important than label s_j An interval linguistic type-2 complex neutrosophic set (ILCNS-2) is a set $A \subset \coprod$ such that each element x in A has linguistic degree of complex interval-membership $T_A(x) \subseteq R \times R$, a linguistic degree of complex interval-indeterminate membership $I_A(x) \subseteq R \times R$, and a linguistic degree of complex interval-falsity membership $F_A(x) \subseteq R \times R$, $\Theta_{\theta(x)} \in S$. An ILCNS-2 set A can be written as,

$$A = \left\{\left\langle X, \left[\Theta_{\theta(X)}, \left(\mathcal{T}_A(X), \widehat{I}_A(X), \mathcal{F}_A(X)\right)\right]\right\rangle | X \in \Pi\right\},$$

where

$$\left.\begin{aligned} T_A(x) &= \left[\inf T_{1A}(x), \sup T_{1A}(x)\right] \cdot e^{j\left[\inf T_{2A}(x), \sup T_{2A}(x)\right]} \\ I_A(x) &= \left[\inf I_{1A}(x), \sup I_{1A}(x)\right] \cdot e^{j\left[\inf I_{2A}(x), \sup I_{2A}(x)\right]} \\ F_A(x) &= \left[\inf F_{1A}(x), \sup F_{1A}(x)\right] \cdot e^{j\left[\inf F_{2A}(x), \sup F_{2A}(x)\right]} \end{aligned}\right\}$$

where $\left[\inf T_{1A}(x), \sup T_{1A}(x)\right]$ represents the interval amplitude truth membership and $e^{j\left[\inf T_{2A}(x), \sup T_{2A}(x)\right]}$ denotes the interval phase truth membership function. Moreover, $\left[\inf I_{1A}(x), \sup I_{1A}(x)\right]$ refers to the interval amplitude indeterminate membership while $e^{j\left[\inf I_{2A}(x), \sup I_{2A}(x)\right]}$ indicates the interval phase indeterminate membership function. Further, $\left[\inf F_{1A}(x), \sup F_{1A}(x)\right]$ is called the interval amplitude falsity membership and $e^{j\left[\inf F_{2A}(x), \sup F_{2A}(x)\right]}$ is said to be the interval phase falsehood membership function.

Definition 12: Let A and B be two ILCNSs-2 over $\coprod$ which are defined by $\langle X, [\Theta_{\theta(X)}(T_A(X), \widehat{I}_A(X), \mathcal{F}_A(X))]\rangle$, and $\langle X, [\Theta_{\theta(X)}(T_B(X), \widehat{I}_B(X), \mathcal{F}_B(X))]\rangle$, respectively . Their union:

$$A \cup B = \{\langle X, [S_{\Theta A \cup B(X)}, (T_{A\cup B}(X), \widehat{I}_{A\cup B}(X), \mathcal{F}_{A\cup B}(X))]\rangle | X \in \Pi\},$$

is defined as:

$$\Theta_{\theta_{A\cup B}}(X) = \Theta_{\theta_{1A\cup B}}(X),$$
$$T_{A\cup B}(X) = \left[\inf T_{1A\cup B}(X), \sup T_{1A\cup B}(X),\right] \cdot e^{j\left[\inf T_{2A\cup B}(X), \sup T_{2A\cup B}(X)\right]},$$
$$I_{A\cup B}(X) = \left[\inf I_{1A\cup B}(X), \sup I_{1A\cup B}(X),\right] \cdot e^{j\left[\inf I_{2A\cup B}(X), \sup I_{2A\cup B}(X)\right]},$$
$$\mathcal{F}_{A\cup B}(X) = \left[\inf \mathcal{F}_{1A\cup B}(X), \sup \mathcal{F}_{1A\cup B}(X),\right] \cdot e^{j\left[\inf F_{2A\cup B}(X), \sup F_{2A\cup B}(X)\right]},$$

where

$$\Theta_{\theta 1A\cup B(x)} = \vee\left(\theta_{\theta A(x)}, \Theta_{\theta B(x)}\right),$$
$$\inf T_{1A\cup B}(x) = \vee(\inf T_{1A}(x), \inf T_{1B}(x)),$$
$$\sup T_{1A\cup B}(x) = \vee(\sup T_{1A}(x), \sup T_{1B}(x)),$$
$$\inf I_{1A\cup B}(x) = \wedge(\inf I_{1A}(x), \inf I_{1B}(x)),$$
$$\sup I_{1A\cup B}(x) = \wedge(\sup I_{1A}(x), \sup I_{1B}(x)),$$
$$\inf F_{1A\cup B}(x) = \wedge(\inf F_{1A}(x), \inf F_{1B}(x)),$$
$$\sup F_{1A\cup B}(x) = \wedge(\sup F_{1A}(x), \sup F_{1B}(x)),$$

for all $x \in X$. The symbols $\vee, \wedge$ represents max and min operators, respectively.

Definition 13: Let A and B be two ILCNSs-2 over $\coprod$ which are defined by $\langle X, [\Theta_{\theta(X)}(T_A(X), \widehat{I}_A(X), F_A(X))]\rangle$, and $\langle X, [\Theta_{\theta(X)}(T_B(X), I_B(X), \mathcal{F}_B(X))]\rangle$, respectively. Their intersection denoted as, $A \cap B = \{\langle X, [\Theta_{\theta A\cap B(X)}, (T_{A\cap B}(X), \widehat{I}_{A\cap B}(X), \mathcal{F}_{A\cap B}(X))]\rangle | X \in \Pi\}$, is defined as:

$$\Theta_{\theta_{A\cap B}}(X) = \Theta_{\theta_{1A\cap B}}(X)$$
$$T_{A\cap B}(X) = \left[\inf T_{1A\cap B}(X), \sup T_{1A\cap B}(X),\right] \cdot e^{j\left[\inf T_{2A\cap B}(X), \sup T_{2A\cap B}(X)\right]},$$
$$I_{A\cap B}(X) = \left[\inf I_{1A\cap B}(X), \sup I_{1A\cap B}(X),\right] \cdot e^{j\left[\inf I_{2A\cap B}(X), \sup I_{2A\cap B}(X)\right]},$$
$$\mathcal{F}_{A\cap B}(X) = \left[\inf \mathcal{F}_{1A\cap B}(X), \sup \mathcal{F}_{1A\cap B}(X),\right] \cdot e^{j\left[\inf F_{2A\cap B}(X), \sup F_{2A\cap B}(X)\right]},$$

where

$$\Theta_{\theta 1A\cap B(x)} = \wedge\left(\Theta_{\theta A(x)}, \Theta_{\theta B(x)}\right),$$
$$\inf T_{1A\cap B}(x) = \wedge(\inf T_{1A}(x), \inf T_{1B}(x)),$$
$$\sup T_{1A\cap B}(x) = \wedge(\sup T_{1A}(x), \sup T_{1B}(x)),$$
$$\inf I_{1A\cap B}(x) = \vee(\inf I_{1A}(x), \inf I_{1B}(x)),$$
$$\sup I_{1A\cap B}(x) = \vee(\sup I_{1A}(x), \sup I_{1B}(x)),$$
$$\inf F_{1A\cap B}(x) = \vee(\inf F_{1A}(x), \inf F_{1B}(x)),$$
$$\sup F_{1A\cap B}(x) = \vee(\sup F_{1A}(x), \sup F_{1B}(x)),$$

for all $x \in X$. The symbols $\vee, \wedge$ represents max and min operators, respectively.

Proposition 14: Let A and B be two ILCNS-2 over $\coprod$. Then

a) $A \cup B = B \cup A$,
b) $A \cap B = B \cap A$,
c) $A \cup A = A$,
d) $A \cap A = A$.

Proof: Straightforward.

Proposition 15: Let A, B and C be three ILCNS over $\coprod$. Then

a) $A \cup (B \cup C) = (A \cup B) \quad \cup C$,
b) $A \cap (B \cap C) = A \cap C$,
c) $A \cup (B \cap C) = (A \cup B) \quad \cap \left(A \cup C\right)$
d) $A \cap (B \cup C) = (A \cap B) \quad \cup \left(A \cap C\right)$
e) $A \cup (B \cap C) = A$,
f) $A \cup (A \cap B) = A$.

Proof: Straightforward.

Theorem 16: The ILCNS $A \cup B$ is the minimum set comprising together A and B.

Proof: Straightforward.

Theorem 17: The ILCNS $A \cap B$ is the leading one enclosed in A and B.

Proof: Straightforward.

Theorem 18: Let P be the power set of all ILCNSs. Then,$(P, \cup, \cap)$ forms a distributive lattice.

Proof: Straightforward.

Definition 19: Let A and B be two ILCNSs over $\coprod$ which are defined by Eq. (1, 2), as shown at the top of the next page.

The **Hamming and Euclidian distances** between two ILCNS A and B for **phase** terms are defined as follows by Eqs. (3, 4), as shown at the top of the next page

$$A = \langle x, [\Theta_{\theta A(x)}, ([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)])]\rangle$$

and

$$B = \langle x, [\Theta_{\theta B(x)}, ([T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)])]\rangle,$$

respectively; where $[T_A^L(x), T_A^U(x)] = [t_A^L(x), t_A^U(x)] e^{j[\omega_A^L(x), \omega_A^U(x)]}$, $[I_A^L(x), I_A^U(x)] = [i_A^L(x), i_A^U(x)] e^{j[\psi_A^L(x), \psi_A^U(x)]}$,

$$d_H^a(A,B)=\frac{1}{6(n-1)}(|\theta_A\times t_A^L-\theta_B\times t_B^L|+|\theta_A\times t_A^R-\theta_B\times t_B^R| \\ +|\theta_A\times i_A^L-\theta_B\times i_B^L|+|\theta_A\times i_A^R-\theta_B\times i_B^R|+|\theta_A\times f_A^L-\theta_B\times f_B^L|+|\theta_A\times f_A^R-\theta_B\times f_B^R| \tag{1}$$

$$d_E^a(A,B) \\ =\sqrt{\frac{1}{6(n-1)}((\theta_A\times t_A^L-\theta_B\times t_B^L)^2+(\theta_A\times t_A^R-\theta_B\times t_B^R)^2+(\theta_A\times i_A^L-\theta_B\times i_B^L)^2+(\theta_A\times i_A^R-\theta_B\times i_B^R)^2+(\theta_A\times f_A^L-\theta_B\times f_B^L)^2+(\theta_A\times f_A^R-\theta_B\times f_B^R)^2)} \tag{2}$$

$$d_H^p(A,B) \\ =|\omega_A^L(x)-\omega_B^L(x)|+|\omega_A^R(x)-\omega_B^R(x)|+|\psi_A^L(x)-\psi_B^L(x)|+|\psi_A^R(x)-\psi_B^R(x)|+|\phi_A^L(x)-\phi_B^L(x)|+|\phi_A^R(x)-\phi_B^R(x)| \tag{3}$$

$$d_E^p(A,B) \\ =\sqrt{(\omega_A^L(x)-\omega_B^L(x))^2+(\omega_A^R(x)-\omega_B^R(x))^2+(\psi_A^L(x)-\psi_B^L(x))^2+(\psi_A^R(x)-\psi_B^R(x))^2+(\phi_A^L(x)-\phi_B^L(x))^2+(\phi_A^R(x)-\phi_B^R(x))^2} \tag{4}$$

$[F_A^L(x), F_A^U(x)] = [f_A^L(x), f_A^U(x)]e^{j[\phi_A^L(x),\phi_A^U(x)]}$, $[T_B^L(x), T_B^U(x)] = [t_B^L(x), t_B^U(x)]e^{j[\omega_B^L(x),\omega_B^U(x)]}[I_B^L(x), I_B^U(x)] = [i_B^L(x), i_B^U(x)]e^{j[\psi_B^L(x),\psi_B^U(x)]}[F_A^L(x), F_A^U(x)] = [f_A^L(x), f_A^U(x)]$ $e^{j[\phi_A^L(x),\phi_A^U(x)]}$.

The Hamming and Euclidian distances between two ILCNS A and B for amplitude terms are well-defined as:

V. OPERATIONAL RULES OF ILCNS

Let A and B be two ILCNSs over $\coprod$ which are illustrated by $\langle x, [\Theta_{\theta_A(x)}, (T_A(x), I_A(x), F_A(x))]\rangle$ and $\langle x, [\Theta_{\theta_B(x)}, (T_B(x), I_B(x), F_B(x))]\rangle$ respectively. Then, the operational rules of ILCNS-2 are illustrated as:

a) The product of A and B indicated as

$$A\otimes B=\langle x, [\Theta_{\theta_{A\otimes B}(x)}, (T_{A\otimes B}(x), I_{A\otimes B}(x), F_{A\otimes B}(x))]\rangle$$

is defined as:

$$\Theta_{\theta_{A\otimes B}(x)} = \Theta_{\theta_A(x)}.\Theta_{\theta_B(x)}$$
$$T_{A\otimes B}(x) = (\inf T_{1A}(x).\inf T_{1B}(x))\cdot e^{j(\inf T_{2A}(x).\inf T_{2B}(x))}$$
$$T_{A\otimes B}(x) = (\sup T_{1A}(x).\sup T_{1B}(x))\cdot e^{j(\sup T_{2A}(x).\sup T_{2B}(x))}$$
$$I_{A\otimes B}(x) = (\inf I_{1A}(x).\inf I_{1B}(x))\cdot e^{j(\inf I_{2A}(x).\inf I_{2B}(x))}$$
$$I_{A\otimes B}(x) = (\sup I_{1A}(x).\sup I_{1B}(x))\cdot e^{j(\sup I_{2A}(x).\sup I_{2B}(x))}$$
$$F_{A\otimes B}(x) = (\inf F_{1A}(x).\inf F_{1B}(x))\cdot e^{j(\inf F_{2A}(x).\inf F_{2B}(x))}$$
$$F_{A\otimes B}(x) = (\sup F_{1A}(x).\sup F_{1B}(x))\cdot e^{j(\sup F_{2A}(x).\sup F_{2B}(x))}$$

b) The addition of A and B denoted as

$$A\oplus B=\langle x, [\Theta_{\theta_{A\oplus B}(x)}, (T_{A\oplus B}(x), I_{A\oplus B}(x), F_{A\oplus B}(x))]\rangle$$

is defined as:

$$\Theta_{\theta_{A\oplus B}(x)} = \Theta_{\theta_A(x)} + \Theta_{\theta_B(x)},$$
$$T_{A\oplus B}(x) = \begin{pmatrix}(\inf T_{1A}(x)+\inf T_{1B}(x))\\ -(\inf T_{1A}(x).\inf T_{1B}(x))\end{pmatrix} \cdot e^{j(\inf T_{2A}(x)+\inf T_{2B}(x))}$$
$$T_{A\oplus B}(x) = \begin{pmatrix}(\sup T_{1A}(x)+\sup T_{1B}(x))\\ -(\sup T_{1A}(x).\sup T_{1B}(x))\end{pmatrix} \cdot e^{j(\sup T_{2A}(x)+\sup T_{2B}(x))},$$
$$I_{A\oplus B}(x) = (\inf I_{1A}(x).\inf I_{1B}(x))\cdot e^{j(\inf I_{2A}(x)+\inf I_{2B}(x))},$$
$$I_{A\oplus B}(x) = (\sup I_{1A}(x).\sup I_{1B}(x))\cdot e^{j(\sup I_{2A}(x)+\sup I_{2B}(x))},$$
$$F_{A\oplus B}(x) = (\inf F_{1A}(x).\inf F_{1B}(x))\cdot e^{j(\inf F_{2A}(x)+\inf F_{2B}(x))},$$
$$F_{A\oplus B}(x) = (\sup F_{1A}(x).\sup F_{1B}(x))\cdot e^{j(\sup F_{2A}(x)+\sup F_{2B}(x))}.$$

c) The scalar multiplication of A is an ILCNS-2 denoted as $C = kA$ is defined as:

$$k\Theta_{\theta_A(x)} = \Theta_{k\theta_A(x)},$$
$$\inf T_C(x) = \left(1-(1-\inf T_{1A}(x))^k\right)\cdot e^{jk\inf T_{2A}(x)},$$
$$\sup T_C(x) = \left(1-(1-\sup T_{1A}(x))^k\right)\cdot e^{jk\sup T_{2A}(x)},$$
$$\inf I_C(x) = \left((\inf T_{1A}(x))^k\right)\cdot e^{jk\inf T_{2A}(x)},$$
$$\sup I_C(x) = \left((\sup T_{1A}(x))^k\right)\cdot e^{jk\sup T_{2A}(x)},$$
$$\inf F_C(x) = \left((\inf F_{1A}(x))^k\right)\cdot e^{jk\inf F_{2A}(x)},$$
$$\sup F_C(x) = \left((\sup F_{1A}(x))^k\right)\cdot e^{jk\sup F_{2A}(x)}.$$

Proposition 20: Let A and B be two SVLCNSs-2 over $\coprod$ which are defined by $\langle\Theta_{\theta_A(x)}, (T_A(x), I_A(x), F_A(x))\rangle$, and $\langle\Theta_{\theta_B(x)}, (T_B(x), I_B(x), F_B(x))\rangle$ respectively. We have

a) $A\otimes B = B\otimes A$,
b) $A\oplus B = B\oplus A$,
c) $k(A\otimes B) = k(B\otimes A)$, $(k_1\otimes k_2)A = k_1A\otimes k_2A$.

VI. A TOPSIS MODEL FOR SVLCNS-2 AND ILCNS-2

For simplicity, we only describe the model for ILCNS-2. The model for SVLCNS-2 can be deduced similarly. Let us suppose that a team of h DMs ($D_q, q = 1,\ldots,h$) is accountable for assessing m alternatives ($A_m, m = 1,\ldots,t$) under p selection criteria ($C_p, p = 1,\ldots,n$), the stages of the proposed TOPSIS technique are as:

A. AGGREGATE RATINGS OF ALTERNATIVES VERSUS CRITERIA

Let

$$x_{mpq} = \left\langle x, \left\{\Theta_{\theta_{mpq}}(x)\begin{pmatrix}[T_{mpq}^L(x), T_{mpq}^U(x)],\\ [I_{mpq}^L(x), I_{mpq}^U(x)],\\ [F_{mpq}^L(x), F_{mpq}^U(x)]\end{pmatrix}\right\}\right\rangle \tag{5}$$

$$T_{mp}(x) = \left[1-\left(1-\sum_{q=1}^{h} T^{L}_{pmq}(x)\right)^{\frac{1}{h}}, 1-\left(1-\sum_{q=1}^{h} T^{R}_{pmq}(x)\right)^{\frac{1}{h}}\right] e^{j\left[\frac{1}{h}\sum_{q=1}^{h} w^{L}_{mq}(x), \frac{1}{h}\sum_{q=1}^{h} w^{U}_{mq}(x)\right]}$$

$$I_{mp}(x) = \left[\left(\sum_{q=1}^{h} I^{L}_{pmq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h} I^{R}_{pmq}\right)^{\frac{1}{h}}\right] e^{j\left[\frac{1}{h}\sum_{q=1}^{h} \psi^{L}_{mq}(x), \frac{1}{h}\sum_{q=1}^{h} \psi^{U}_{mq}(x)\right]}$$

$$F_{mp}(x) = \left[\left(\sum_{q=1}^{h} F^{L}_{pmq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h} F^{R}_{pmq}\right)^{\frac{1}{h}}\right] e^{j\left[\frac{1}{h}\sum_{q=1}^{h} \phi^{L}_{mq}(x), \frac{1}{h}\sum_{q=1}^{h} \phi^{U}_{mq}(x)\right]}$$

be the suitability assessment allocated to alternative A_m by DM D_q for criterion C_p, where: $[T^{L}_{mpq}, T^{U}_{mpq}] = [t^{L}_{mpq}, t^{U}_{mpq}] \cdot e^{j[\omega^{L}_{mpq}(x), \omega^{U}_{mpq}(x)]}$, $[I^{L}_{mpq}, I^{U}_{mpq}] = [i^{L}_{mpq}, i^{U}_{mpq}] \cdot e^{j[\psi^{L}_{mpq}(x), \psi^{U}_{mpq}(x)]}$, $[F^{L}_{mpq}, F^{U}_{mpq}] = [f^{L}_{mpq}, f^{U}_{mpq}] \cdot e^{j[\phi^{L}_{mpq}(x), \phi^{U}_{mpq}(x)]}$, $m = 1, \ldots, t;$ Ᵽ $= 1, \ldots,$ ɳɋ $= 1, \ldots,$ ɧ Using the operational rules of the ILCNS, the averaged suitability rating $x_{mp} = \left\langle x, \left\{ \Theta_{\theta_{mp}}(x) \begin{pmatrix} [T^{L}_{mp}(x), T^{U}_{mp}(x)], \\ [I^{L}_{mp}(x), I^{U}_{mp}(x)], \\ [F^{L}_{mp}(x), F^{U}_{mp}(x)] \end{pmatrix} \right\} \right\rangle$ can be evaluated $T_{mp}(x)$, $I_{mp}(x)$, $F_{mp}(x)$, as shown at the top of the this page.

B. AGGREGATE THE IMPORTANCE WEIGHTS

Let

$$w_{pq} = \left\langle x, \left\{ \Theta_{\rho_{pq}}(x) \begin{pmatrix} [T^{L}_{pq}(x), T^{U}_{pq}(x)], \\ [I^{L}_{pq}(x), I^{U}_{pq}(x)], \\ [F^{L}_{pq}(x), F^{U}_{pq}(x)] \end{pmatrix} \right\} \right\rangle$$

be the weight allocated by DM D_q to criterion C_p, where $[T^{L}_{pq}, T^{U}_{pq}] = [t^{L}_{pq}, t^{U}_{pq}] \cdot e^{j[\omega^{L}_{pq}(x), \omega^{U}_{pq}(x)]}$, $[I^{L}_{pq}, I^{U}_{pq}] = [i^{L}_{pq}, i^{U}_{pq}] \cdot e^{j[\psi^{L}_{pq}(x), \psi^{U}_{pq}(x)]}$, $[F^{L}_{pq}, F^{U}_{pq}] = [f^{L}_{pq}, f^{U}_{pq}] \cdot e^{j[\phi^{L}_{pq}(x), \phi^{U}_{pq}(x)]}$, $F^{U}_{pq} = f^{U}_{pq} \cdot e^{j[\phi^{L}_{pq}(x), \phi^{U}_{pq}(x)]}$, Ᵽ $= 1, \ldots,$ ɳɋ $= 1, \ldots,$ ɧ Using the operational rules of the ILCNS, the average weight $w_p = \left\langle x, \left\{ \Theta_{\rho_p}(x) \begin{pmatrix} [T^{L}_{p}(x), T^{U}_{p}(x)], \\ [I^{L}_{p}(x), I^{U}_{p}(x)], \\ [F^{L}_{p}(x), F^{U}_{p}(x)] \end{pmatrix} \right\} \right\rangle$ can be evaluated as:

$$w_p = \left(\frac{1}{h}\right) \otimes (w_{p1} \oplus w_{p2} \oplus \ldots \oplus w_{ph}), \tag{6}$$

where

$$T_p(x) = \begin{bmatrix} 1-\left(1-\sum_{q=1}^{h} T^{L}_{pq}(x)\right)^{\frac{1}{h}}, \\ 1-\left(1-\sum_{q=1}^{h} T^{R}_{pq}(x)\right)^{\frac{1}{h}} \end{bmatrix} e^{j\left[\frac{1}{h}\sum_{q=1}^{h} w^{L}_{q}(x), \frac{1}{h}\sum_{q=1}^{h} w^{U}_{q}(x)\right]}$$

$$I_p(x) = \left[\left(\sum_{q=1}^{h} I^{L}_{pq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h} I^{R}_{pq}\right)^{\frac{1}{h}}\right] e^{j\left[\frac{1}{h}\sum_{q=1}^{h} \psi^{L}_{q}(x), \frac{1}{h}\sum_{q=1}^{h} \psi^{U}_{q}(x)\right]}$$

$$F_p(x) = \left[\left(\sum_{q=1}^{h} F^{L}_{pq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h} F^{R}_{pq}\right)^{\frac{1}{h}}\right] e^{j\left[\frac{1}{h}\sum_{q=1}^{h} \phi^{L}_{q}(x), \frac{1}{h}\sum_{q=1}^{h} \phi^{U}_{q}(x)\right]}$$

C. AGGREGATE THE WEIGHTED RATINGS OF ALTERNATIVES VERSUS CRITERIA

The weighted ratings of alternatives can be advanced via the operations of ILCNS as follows:

$$G_m = \frac{1}{n}\sum_{p=1}^{n} x_{mp} * w_p, \quad m = 1, \ldots, t; \; p = 1, \ldots, n. \tag{7}$$

D. CALCULATION OF A^{+}, A^{-}, d_i^{+} AND d_i^{-}

The positive-ideal solution (FPIS, A^{+}) and fuzzy negative ideal solution (FNIS, A^{-}) are obtained as Eq. (8, 9), as shown at the top of the next page. The distances of each alternative A_m, $m = 1, \ldots, t$ from A^{+} and A^{-} for the amplitude terms and the phase terms are calculated as:

$$d^{a+}_{m} = \sqrt{(G^{a}_{m} - A^{a+})^2} \tag{10}$$

$$d^{a-}_{m} = \sqrt{(G^{a}_{m} - A^{a-})^2} \tag{11}$$

$$d^{p+}_{m} = \sqrt{(G^{p}_{m} - A^{p+})^2} \tag{12}$$

$$d^{p-}_{m} = \sqrt{(G^{p}_{m} - A^{p-})^2} \tag{13}$$

where d^{a+}_{m}, d^{p+}_{m} characterizes the shortest distances of candidate A_m, and d^{a-}_{m}, d^{p-}_{m}, characterizes the farthest distance of candidate A_m.

E. OBTAIN THE CLOSENESS COEFFICIENT

The closeness coefficients for the amplitude terms and the phase terms of every candidate, which are cleared to define

$$A^{+} = \left\langle x, \left\{ \Theta_{\max(\theta_{mpq}, \rho_{pq})}(x)([1, 1]e^{j\max([\omega^{L}_{mpq}(x).\omega^{L}_{pq}(x), \omega^{U}_{mpq}(x).\omega^{U}_{pq}(x)])}, [0, 0], [0, 0] \right\} \right\rangle \tag{8}$$

$$A^{-} = \left\langle x, \left\{ \Theta_{\min_{(\theta_{mpq}, \rho_{pq})}}(x)([0, 0], [1, 1]e^{j\max([\psi^{L}_{mpq}(x).\psi^{L}_{pq}(x), \psi^{U}_{mpq}(x).\psi^{U}_{pq}(x)])}, [1, 1]e^{j\max([\phi^{L}_{mpq}(x).\phi^{L}_{pq}(x), \phi^{U}_{mpq}(x).\phi^{U}_{pq}(x)])} \right\} \right\rangle \tag{9}$$

the classification order of all candidates, are calculated as:

$$CC^{a}_{i} = \frac{d^{a-}_{i}}{d^{a+}_{i} + d^{a-}_{i}} \tag{14}$$

$$CC^{p}_{i} = \frac{d^{p-}_{i}}{d^{p+}_{i} + d^{p-}_{i}} \tag{15}$$

A higher value of the closeness coefficient designates that an candidate is closer to PIS and farther from NIS concurrently. Let A_1 and A_2 be any two ILCNS-2. Then, the classification method can be cleared as follows:

If $CC^{a}_{A_1} > CC^{a}_{A_2}$ then $A_1 > A_2$

If $CC^{a}_{A_1} = CC^{a}_{A_2}$ and $CC^{p}_{A_1} > CC^{p}_{A_2}$ then $A_1 > A_2$

If $CC^{a}_{A_1} = CC^{a}_{A_2}$ and $CC^{p}_{A_1} = CC^{p}_{A_2}$ then $A_1 = A_2$.

VII. AN APPLICATION OF THE PROPOSED TOPSIS METHOD

This section applies the proposed TOPSIS method for lecturer selection in the case of University of Economics and Business - Vietnam National University (UEB-VNU), which is one of the leading universities in Hanoi, Vietnam. Assume that UEB-VNU need to choose an alternative for the teaching position. Data were gathered by conducting semi-structured discussions with UEB-VNU's Board of management, Office of Human resources and department head. A commission of four DMs, i.e. $D_1, \ldots, D_3$, and D_4, were requested to distinctly proceed to their own evaluation for the significance weights of selection criteria and the ratings of four potential alternatives. Based on the discussion with the commission members, six selection criteria are considered including number of publications (C_1), quality of publications (C_2), personality factors (C_3), activity in professional society (C_4), classroom teaching experience (C_5), and fluency in a foreign language (C_6). The computational proceeding is concised as follows.

A. AGGREGATION OF THE RATINGS OF CANDIDATES VERSUS CRITERIA

Four DMs decide the suitability rankings of four potential alternatives versus the criteria using the ILCNS $\Theta = \{\Theta_1 = VP, \Theta_2 = P, \Theta_3 = M, \Theta_4 = G, \Theta_5 = VG\}$ where VP = Very Poor $=<$ $(\Theta_1, ([0.1, 0.2]e^{j[0.5,0.6]}, [0.6, 0.7]e^{j[0.4,0.5]}, [0.6, 0.7]e^{j[0.3,0.4]}))$ $>$, P = Poor $=<$ $(\Theta_2, ([0.2, 0.3]e^{j[0.6,0.7]}, [0.5, 0.6]e^{j[0.5,0.6]}, [0.6, 0.7]e^{j[0.4,0.5]}))$ $>$, M = Medium $=<$ $(\Theta_3, ([0.3, 0.5]e^{j[0.7,0.8]}, [0.4, 0.6]e^{j[0.6,0.7]}, [0.4, 0.5]e^{j[0.5,0.6]}))$ $>$, G = Good $=<$ $(\Theta_4, ([0.5, 0.6]e^{j[0.8,0.9]}, [0.4, 0.5]e^{j[0.7,0.8]}, [0.3, 0.4]e^{j[0.6,0.7]}))$ $>$, and VG = Very Good $=<$ $(\Theta_5, ([0.6, 0.7]e^{j[0.9,1.0]}, [0.2, 0.3]e^{j[0.8,0.9]}, [0.2, 0.3]e^{j[0.7,0.8]}))$, to evaluate the appropriateness of the candidates under six criteria.

Table 1 presents the suitability rankings of four alternatives (A_1, A_2, A_3, A_4) versus six criteria ($C_1, .., C_6$) from four DMs ($\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4$) using the ILCNS. Using Eq. (5), the aggregated ratings of the candidates versus the criteria from the DMs are shown at the last column of Table 1.

B. AGGREGATE THE IMPORTANCE WEIGHTS

After defining the lecturer assortment criteria, the commission members are asked to define the level of significance of every criterion using the ILCNS, $V = \{v_1 = \text{UI}, v_2 = \text{OI}, v_3 = \text{I}, v_4 = \text{VI}, v_5 = \text{AI}\}$, where UI = Unimportant $=<$ $(v_1, ([0.1, 0.2]e^{j[0.4,0.5]}, [0.4, 0.5]e^{j[0.3,0.4]}, [0.6, 0.7]e^{j[0.2,0.3]}))$ $>$, OI = Ordinary Important $=<$ $(v_2, ([0.2, 0.4]e^{j[0.5,0.6]}, [0.5, 0.6]e^{j[0.4,0.5]}, [0.4, 0.5]e^{j[0.3,0.4]}))$ $>$, I = Important $=<$ $(v_3, ([0.4, 0.6]e^{j[0.6,0.7]}, [0.4, 0.5]e^{j[0.5,0.6]}, [0.3, 0.4]e^{j[0.4,0.5]}))$ $>$, VI = Very Important $=<$ $(v_4, ([0.6, 0.8]e^{j[0.7,0.8]}, [0.3, 0.4]e^{j[0.6,0.7]}, [0.2, 0.3]e^{j[0.5,0.6]}))$ $>$, and AI = Absolutely Important $=<$ $(v_5, ([0.7, 0.9]e^{j[0.8,0.9]}, [0.2, 0.3]e^{j[0.7,0.8]}, [0.1, 0.2]e^{j[0.6,0.7]}))$ $>$.

Table 2 shows the significance weights of the six criteria from the four DMs. The gathered weights of criteria attained by Eq. (6) are displayed in the last column of Table 2.

C. AGGREGATE THE WEIGHTED RATINGS OF ALTERNATIVES VERSUS CRITERIA

Table 3 presents the weighted ratings of alternatives of each candidate using Eq. (7).

D. CALCULATION OF A^+, A^-, d_i^+ AND d_i^-

As presented in Table 4, the distance of each candidate from A^+ and A^- for the amplitude term and the phase term can be calculated using Eqs.(8-13).

E. OBTAIN THE CLOSENESS COEFFICIENT

The closeness coefficients of each alternative can be computed by Equations (14)-(15), as shown in Table 5. Therefore, the ranking order of the four candidate is $A_1 \succ A_4 \succ A_3 \succ A_2$. Consequently, the best candidate is A_1.

The ILCNS is the generalization of ILNS and ICNS. Obviously, the extended decision making methods in [10], [12], [23], [25] are the special cases of the proposal in this paper.

F. SENSITIVITY ANALYSIS

A sensitivity analysis was performed to investigate the impact of criteria weights on the ranking of the candidates (lecturers). The detail of scenarios are shown in Table 6. The results show

TABLE 1. Aggregated ratings of lecturers versus the criteria.

Criteria	Candidates	Decision makers				Aggregated ratings
		$Đ_1$	$Đ_2$	$Đ_3$	$Đ_4$	
C_1	A_1	M	G	G	M	<($\Theta_{3.5}$, ([0.408, 0.553]$e^{j[0.75,0.85]}$, [0.4, 0.548]$e^{j[0.65,0.75]}$], [0.346, 0.447]$e^{j[0.55,0.65]}$])>
	A_2	G	G	VG	G	<($\Theta_{4.25}$, ([0.527, 0.628]$e^{j[0.825,0.925]}$, [0.336, 0.44]$e^{j[0.725,0.825]}$], [0.271, 0.372]$e^{j[0.625,0.725]}$])>
	A_3	M	G	G	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_4	G	G	G	M	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
C_2	A_1	G	VG	G	G	<($\Theta_{4.25}$, ([0.527, 0.628]$e^{j[0.825,0.925]}$, [0.336, 0.44]$e^{j[0.725,0.825]}$], [0.271, 0.372]$e^{j[0.625,0.725]}$])>
	A_2	M	G	G	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_3	VG	G	G	VG	<($\Theta_{4.5}$, ([0.553, 0.654]$e^{j[0.85,0.95]}$, [0.283, 0.387]$e^{j[0.75,0.85]}$], [0.245, 0.346]$e^{j[0.65,0.75]}$])>
	A_4	G	G	G	G	<($\Theta_{4.0}$, ([0.5, 0.6]$e^{j[0.8,0.9]}$, [0.4, 0.5]$e^{j[0.7,0.8]}$, [0.3, 0.4]$e^{j[0.6,0.7]}$))>
C_3	A_1	VG	VG	G	VG	<($\Theta_{4.75}$, ([0.577, 0.678]$e^{j[0.875,0.975]}$, [0.238, 0.341]$e^{j[0.775,0.875]}$], [0.221, 0.322]$e^{j[0.675,0.775]}$])>
	A_2	G	VG	G	G	<($\Theta_{4.25}$, ([0.527, 0.628]$e^{j[0.825,0.925]}$, [0.336, 0.44]$e^{j[0.725,0.825]}$], [0.271, 0.372]$e^{j[0.625,0.725]}$])>
	A_3	G	G	VG	G	<($\Theta_{4.25}$, ([0.527, 0.628]$e^{j[0.825,0.925]}$, [0.336, 0.44]$e^{j[0.725,0.825]}$], [0.271, 0.372]$e^{j[0.625,0.725]}$])>
	A_4	G	VG	G	VG	<($\Theta_{4.5}$, ([0.553, 0.654]$e^{j[0.85,0.95]}$, [0.283, 0.387]$e^{j[0.75,0.85]}$], [0.245, 0.346]$e^{j[0.65,0.75]}$])>
C_4	A_1	M	P	M	M	<($\Theta_{1.75}$, ([0.276, 0.456]$e^{j[0.675,0.775]}$, [0.423, 0.6]$e^{j[0.575,0.675]}$], [0.443, 0.544]$e^{j[0.475,0.575]}$])>
	A_2	M	G	G	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_3	M	M	G	M	<($\Theta_{2.25}$, ([0.356, 0.527]$e^{j[0.725,0.825]}$, [0.4, 0.573]$e^{j[0.625,0.725]}$], [0.372, 0.473]$e^{j[0.525,0.625]}$])>
	A_4	G	G	M	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
C_5	A_1	G	M	G	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_2	G	G	G	G	<(Θ_{4} , ([0.5, 0.6]$e^{j[0.8,0.9]}$, [0.4, 0.5]$e^{j[0.7,0.8]}$, [0.3, 0.4]$e^{j[0.6,0.7]}$))>
	A_3	G	G	M	G	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_4	VG	G	G	VG	<($\Theta_{4.5}$, ([0.553, 0.654]$e^{j[0.85,0.95]}$, [0.283, 0.387]$e^{j[0.75,0.85]}$], [0.245, 0.346]$e^{j[0.65,0.75]}$])>
C_6	A_1	G	G	G	G	<(Θ_{4} , ([0.5, 0.6]$e^{j[0.8,0.9]}$, [0.4, 0.5]$e^{j[0.7,0.8]}$, [0.3, 0.4]$e^{j[0.6,0.7]}$))>
	A_2	G	G	G	M	<($\Theta_{3.75}$, ([0.456, 0.577]$e^{j[0.775,0.875]}$, [0.4, 0.523]$e^{j[0.675,0.775]}$], [0.322, 0.423]$e^{j[0.575,0.675]}$])>
	A_3	VG	G	VG	G	<($\Theta_{4.5}$, ([0.553, 0.654]$e^{j[0.85,0.95]}$, [0.283, 0.387]$e^{j[0.75,0.85]}$], [0.245, 0.346]$e^{j[0.65,0.75]}$])>
	A_4	G	VG	G	G	<($\Theta_{4.25}$, ([0.527, 0.628]$e^{j[0.825,0.925]}$, [0.336, 0.44]$e^{j[0.725,0.825]}$], [0.271, 0.372]$e^{j[0.625,0.725]}$])>

TABLE 2. The importance and aggregated weights of the criteria.

Criteria	DMs				Aggregated weights
	D_1	D_2	D_3	D_4	
$Ç_1$	I	OI	I	I	<($v_{2.75}$, ([0.355, 0.557]$e^{j[0.575,0.675]}$, [0.423, 0.523]$e^{j[0.475,0.575]}$], [0.322, 0.423]$e^{j[0.375,0.475]}$)>
$Ç_2$	I	I	OI	OI	<($v_{2.5}$, ([0.307, 0.51]$e^{j[0.55,0.65]}$, [0.447, 0.548]$e^{j[0.45,0.55]}$], [0.346, 0.447]$e^{j[0.35,0.45]}$)>
$Ç_3$	I	I	I	VI	<($v_{3.25}$, ([0.458, 0.664]$e^{j[0.625,0.725]}$, [0.372, 0.473]$e^{j[0.525,0.625]}$], [0.271, 0.372]$e^{j[0.425,0.525]}$)>
$Ç_4$	AI	VI	AI	VI	<($v_{4.5}$, ([0.654, 0.859]$e^{j[0.75,0.85]}$, [0.245, 0.346]$e^{j[0.65,0.75]}$], [0.141, 0.245]$e^{j[0.55,0.65]}$)>
$Ç_5$	VI	VI	I	VI	<($v_{3.75}$, ([0.557, 0.762]$e^{j[0.675,0.775]}$, [0.322, 0.423]$e^{j[0.575,0.675]}$, [0.221, 0.322]$e^{j[0.475,0.575]}$))>
$Ç_6$	VI	I	VI	I	<($v_{3.5}$, ([0.51, 0.717]$e^{j[0.65,0.75]}$, [0.346, 0.447]$e^{j[0.55,0.65]}$], [0.245, 0.346]$e^{j[0.45,0.55]}$)>

TABLE 3. Weighted assessments of each candidate.

Candidates	Aggregated weights
A_1	<($\Theta v_{12.688}$, ([0.212, 0.372]$e^{j[0.497,0.634]}$, [0.594, 0.682]$e^{j[0.365,0.484]}$], [0.495, 0.57]$e^{j[0.369,0.433]}$)>
A_2	<($\Theta v_{13.313}$, ([0.232, 0.39]$e^{j[0.507,0.646]}$, [0.602, 0.684]$e^{j[0.374,0.494]}$], [0.48, 0.556]$e^{j[0.379,0.443]}$)>
A_3	<($\Theta v_{13.320}$, ([0.225, 0.385]$e^{j[0.508,0.646]}$, [0.584, 0.671]$e^{j[0.374,0.494]}$], [0.479, 0.554]$e^{j[0.381,0.444]}$)>
A_4	<($\Theta v_{13.927}$, ([0.243, 0.402]$e^{j[0.518,0.658]}$, [0.58, 0.659]$e^{j[0.383,0.504]}$], [0.466, 0.542]$e^{j[0.391,0.454]}$)>

TABLE 4. The distance of every alternative from A^+ and A^-.

Candidates	Amplitude terms		Phase term	
	d^+	d^-	d^+	d^-
A_1	4.255	2.443	0.832	0.807
A_2	4.265	2.404	0.851	0.821
A_3	4.242	2.431	0.852	0.822
A_4	4.228	2.425	0.872	0.837

TABLE 5. Closeness coefficients of candidates.

Candidates	Closeness coefficient		Ranking
	Amplitude terms	Phase term	
A_1	0.3647	0.4923	1
A_2	0.3605	0.4911	4
A_3	0.3643	0.4910	3
A_4	0.3645	0.4899	2

that eight out of eleven scenarios, the candidate is ranked either as the first or the second candidate. This confirms domination of the candidate A_1 compared to other alternatives. Therefore, the candidate selection decision is relatively insensitive to criteria weights.

VIII. COMPARISON OF THE SUGGESTED METHOD WITH ANOTHER DECISION MAKING METHOD

This section compares the proposed TOPSIS decision making procedure in ICNS with a different MCDM methodology to illustrate applicability and its advantages. We recall an example explored by Sahin and Yigider [33] in which a production industry wishes to choose and assess their suppliers. In this model, four DMs ($D_1, \ldots, D_4$) have been selected to valuate five suppliers ($S_1, \ldots, S_5$) with respect to five performance criteria including delivery (C_1), quality (C_2), flexibility (C_3), service (C_4) and price (C_5). The information of weights provided to the five criteria by the four DMs are offered in Table 7. The gathered weights of criteria gained by Eq. (4) are displayed in the last column of Table 7.

TABLE 6. Scenarios for sensitivity analysis.

Scenarios	Weights of criteria	Closeness coefficient (CC_i)								Ranking
		A_1		A_2		A_3		A_4		
		CC_1^a	CC_1^p	CC_2^a	CC_2^p	CC_3^a	CC_3^p	CC_4^a	CC_4^a	
1	$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 =$ UI	0,235	0,5088	0,220	0,5080	0,222	0,5077	0,210	0,5069	$A_1 > A_3 > A_2 > A_4$
2	$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 =$ OI	0,276	0,4998	0,266	0,4990	0,267	0,4986	0,259	0,4978	$A_1 > A_3 > A_2 > A_4$
3	$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 =$ I	0,3433	0,4936	0,3375	0,4928	0,3406	0,4925	0,3378	0,4917	$A_1 > A_3 > A_4 > A_2$
4	$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 =$ VI	0,4141	0,4892	0,4123	0,4884	0,4176	0,4880	0,4194	0,4873	$A_4 > A_3 > A_1 > A_2$
5	$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 =$ AI	0,4693	0,4859	0,4696	0,4851	0,4769	0,4847	0,4817	0,4840	$A_4 > A_3 > A_2 > A_1$
6	$w_1 =$ AI, $w_2 = w_3 = w_4 = w_5 = w_6 =$ UI	0,2715	0,5029	0,2412	0,5011	0,2588	0,5016	0,2540	0,5010	$A_1 > A_3 > A_4 > A_2$
7	$w_2 =$ AI, $w_1 = w_3 = w_4 = w_5 =$ UI	0,2704	0,5017	0,2765	0,5019	0,2614	0,5004	0,2679	0,5006	$A_2 > A_1 > A_4 > A_3$
8	$w_3 =$ AI, $w_1 = w_2 = w_4 = w_5 = w_6 =$ UI	0,2836	0,5009	0,2875	0,5011	0,2912	0,5008	0,2817	0,4999	$A_3 > A_2 > A_1 > A_4$
9	$w_4 =$ AI, $w_1 = w_2 = w_3 = w_5 = w_6 =$ UI	0,2558	0,5043	0,2127	0,5019	0,2331	0,5024	0,2119	0,5010	$A_1 > A_3 > A_2 > A_4$
10	$w_5 =$ AI, $w_1 = w_2 = w_3 = w_4 = w_6 =$ UI	0,2719	0,5026	0,2565	0,5015	0,2677	0,5016	0,2452	0,4999	$A_1 > A_3 > A_2 > A_4$
11	$w_6 =$ AI, $w_1 = w_2 = w_3 = w_4 = w_5 =$ UI	0,2736	0,5021	0,2740	0,5019	0,2572	0,5004	0,2588	0,5002	$A_2 > A_1 > A_4 > A_3$

TABLE 7. The significance and aggregated weights of the criteria.

Criteria	DMs				Aggregated weights
	D_1	D_2	D_3	D_4	
C_1	AI	AI	AI	VI	$<(v_{4.75}, ([0.678, 0.881]e^{j[0.775,0.875]}, [0.221, 0.322]e^{j[0.675,0.775]}], [0.119, 0.221]e^{j[0.575,0.675]})>$
C_2	VI	I	I	VI	$<(v_{3.5}, ([0.51, 0.717]e^{j[0.65,0.75]}, [0.346, 0.447]e^{j[0.55,0.65]}], [0.245, 0.346]e^{j[0.45,0.55]})>$
C_3	AI	AI	VI	AI	$<(v_{4.75}, ([0.678, 0.881]e^{j[0.775,0.875]}, [0.221, 0.322]e^{j[0.675,0.775]}], [0.119, 0.221]e^{j[0.575,0.675]})>$
C_4	VI	VI	I	OI	$<(v_{3.25}, ([0.474, 0.687]e^{j[0.625,0.725]}, [0.366, 0.468]e^{j[0.525,0.625]}], [0.263, 0.366]e^{j[0.425,0.525]})>$
C_5	I	I	AI	AI	$<(v_{4.0}, ([0.576, 0.8]e^{j[0.7,0.8]}, [0.283, 0.387]e^{j[0.6,0.7]}], [0.173, 0.283]e^{j[0.5,0.6]})>$

The averaged ratings of suppliers versus the criteria are shown in Table 8.

Table 9 shows the last fuzzy valuation values of every supplier using Eq. (7).

The distance of each supplier from A^+ and A^- for the amplitude term and the phase term can be calculated using Eqs. (8-13) as shown in Table 10.

The closeness coefficients of each supplier can be calculated by Eqs. (14-15), as shown in Table 11. Therefore, the ranking order of the five suppliers is $A_5 \succ A_2 \succ A_3 \succ A_4 \succ A_1$.

The result indicates that there is a slightly different among the rating order of suppliers using the suggested method and Sahin and Yigider [33]. This is due to the proposed technique

TABLE 8. Aggregated evaluations of suppliers versus the criteria.

Criteria	Suppliers	DMs				Aggregated ratings
		D_1	D_2	D_3	D_4	
C_1	A_1	G	M	G	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}, [0.322, 0.423]e^{i[0.575,0.675]})>$
	A_2	G	G	M	M	$<(\Theta_{3.5}, ([0.408, 0.553]e^{i[0.75,0.85]}, [0.4, 0.548]e^{i[0.65,0.75]}], [0.346, 0.447]e^{i[0.55,0.65]})>$
	A_3	P	G	M	P	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.675,0.775]}, [0.447, 0.573]e^{i[0.575,0.675]}, [0.456, 0.560]e^{i[0.475,0.575]}))>$
	A_4	G	M	G	M	$<(\Theta_{3.5}, ([0.408, 0.553]e^{i[0.75,0.85]}, [0.4, 0.548]e^{i[0.65,0.75]}], [0.346, 0.447]e^{i[0.55,0.65]})>$
	A_5	M	G	G	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}], [0.322, 0.423]e^{i[0.575,0.675]})>$
C_2	A_1	G	G	M	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}], [0.322, 0.423]e^{i[0.575,0.675]})>$
	A_2	G	M	P	M	$<(\Theta_{3.0}, ([0.335, 0.486]e^{i[0.7,0.8]}, [0.423, 0.573]e^{i[0.6,0.7]}, [0.412, 0.514]e^{i[0.5,0.6]}))>$
	A_3	P	G	G	G	$<(\Theta_{3.5}, ([0.438, 0.54]e^{i[0.75,0.85]}, [0.423, 0.523]e^{i[0.65,0.75]}, [0.357, 0.46]e^{i[0.55,0.65]}))>$
	A_4	M	P	G	P	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.675,0.775]}, [0.447, 0.573]e^{i[0.575,0.675]}, [0.456, 0.560]e^{i[0.475,0.575]}))>$
	A_5	G	G	M	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}], [0.322, 0.423]e^{i[0.575,0.675]})>$
C_3	A_1	M	M	P	P	$<(\Theta_{2.5}, ([0.252, 0.408]e^{i[0.65,0.75]}, [0.447, 0.6]e^{i[0.55,0.65]}, [0.49, 0.592]e^{i[0.45,0.55]}))>$
	A_2	G	G	G	G	$<(\Theta_{4.0}, ([0.5, 0.6]e^{i[0.8,0.9]}, [0.4, 0.5]e^{i[0.7,0.8]}, [0.3, 0.4]e^{i[0.6,0.7]}))>$
	A_3	P	G	M	M	$<(\Theta_{3.0}, ([0.335, 0.486]e^{i[0.7,0.8]}, [0.423, 0.573]e^{i[0.6,0.7]}, [0.412, 0.514]e^{i[0.5,0.6]}))>$
	A_4	G	M	G	M	$<(\Theta_{3.5}, ([0.408, 0.553]e^{i[0.75,0.85]}, [0.4, 0.548]e^{i[0.65,0.75]}], [0.346, 0.447]e^{i[0.55,0.65]})>$
	A_5	M	G	G	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}], [0.322, 0.423]e^{i[0.575,0.675]})>$
C_4	A_1	G	P	M	P	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.675,0.775]}, [0.447, 0.573]e^{i[0.575,0.675]}, [0.456, 0.560]e^{i[0.475,0.575]}))>$
	A_2	G	G	P	G	$<(\Theta_{3.5}, ([0.438, 0.54]e^{i[0.75,0.85]}, [0.423, 0.523]e^{i[0.65,0.75]}, [0.357, 0.46]e^{i[0.55,0.65]}))>$
	A_3	M	M	M	M	$<(\Theta_{3.0}, ([0.3, 0.5]e^{i[0.7,0.8]}, [0.4, 0.6]e^{i[0.6,0.7]},[0.4, 0.5]e^{i[0.5,0.6]}))>$
	A_4	P	P	M	G	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.675,0.775]}, [0.447, 0.573]e^{i[0.575,0.675]}, [0.456, 0.560]e^{i[0.475,0.575]}))>$
	A_5	M	G	G	G	$<(\Theta_{3.75}, ([0.456, 0.577]e^{i[0.775,0.875]}, [0.4, 0.523]e^{i[0.675,0.775]}], [0.322, 0.423]e^{i[0.575,0.675]})>$
C_5	A_1	P	M	G	P	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.675,0.775]}, [0.447, 0.573]e^{i[0.575,0.675]}, [0.456, 0.560]e^{i[0.475,0.575]}))>$
	A_2	G	P	G	G	$<(\Theta_{3.5}, ([0.438, 0.54]e^{i[0.75,0.85]}, [0.423, 0.523]e^{i[0.65,0.75]}, [0.357, 0.46]e^{i[0.55,0.65]}))>$
	A_3	G	G	P	M	$<(\Theta_{3.25}, ([0.388, 0.514]e^{i[0.725,0.825]}, [0.423, 0.548]e^{i[0.925,1.025]}, [0.383, 0.486]e^{i[0.775,0.875]}))>$
	A_4	P	P	M	G	$<(\Theta_{2.75}, ([0.312, 0.44]e^{i[0.725,0.825]}, [0.447, 0.573]e^{i[0.625,0.725]}, [0.456, 0.560]e^{i[0.525,0.625]}))>$
	A_5	G	G	G	G	$<(\Theta_{4.0}, ([0.5, 0.6]e^{i[0.8,0.9]}, [0.4, 0.5]e^{i[0.7,0.8]}, [0.3, 0.4]e^{i[0.6,0.7]}))>$

TABLE 9. The last fuzzy valuation values of every supplier.

Suppliers	Aggregated weights
A_1	$<(\Theta_{12.263}, ([0.208, 0.388]e^{j[0.497,0.648]}, [0.594, 0.698]e^{j[0.366,0.497]}, [0.517, 0.601]e^{j[0.357,0.427]}))>$
A_2	$<(\Theta_{14.625}, ([0.249, 0.435]e^{j[0.526,0.681]}, [0.583, 0.677]e^{j[0.391,0.526]}, [0.47, 0.554]e^{j[0.386,0.456]}))>$
A_3	$<(\Theta_{12.225}, ([0.205, 0.390]e^{j[0.496,0.647]}, [0.592, 0.699]e^{j[0.365,0.496]}, [0.515, 0.598]e^{j[0.356,0.426]}))>$
A_4	$<(\Theta_{12.338}, ([0.208, 0.392]e^{j[0.495,0.646]}, [0.592, 0.699]e^{j[0.365,0.495]}, [0.515, 0.599]e^{j[0.355,0.425]}))>$
A_5	$<(\Theta_{15.2}, ([0.27, 0.461]e^{j[0.546,0.704]}, [0.574, 0.668]e^{j[0.408,0.546]}, [0.445, 0.527]e^{j[0.406,0.476]}))>$

TABLE 10. The distance of each supplier from A^+ and A^-.

Suppliers	Amplitude terms		Phase term	
	d^+	d^-	d^+	d^-
A_1	4.528	2.303	0.834	0.822
A_2	4.487	2.300	0.888	0.862
A_3	4.522	2.317	0.833	0.821
A_4	4.523	2.309	0.832	0.820
A_5	4.470	2.355	0.925	0.891

TABLE 11. Closeness coefficients of suppliers.

Suppliers	Closeness coefficient		Rank
	Amplitude terms	Phase term	
A_1	0.33708	0.4964	5
A_2	0.33884	0.4925	2
A_3	0.33878	0.4964	3
A_4	0.33794	0.4964	4
A_5	0.34502	0.4906	1

applying the ILCNS, which is the generalization of ILNS, ICNS and INS.

IX. CONCLUSIONS

Linguistic based strategies are very useful tool in decision making problems for solving the problem of crisp values. In this paper, we proposed the Single-Valued Linguistic Interval Complex Neutrosophic Set (SVLCNS) and Interval Linguistic Interval Complex Neutrosophic Set (ILCNS) for decision making under uncertainty situations. Some basic set notional operations such as the intersection, union and complement as well as the functioning rules of SVLCNS and ILCNS were also defined of the proposed framework. Moreover, we also developed a new TOPSIS decision making method in SVLCNS and ICNS that was applied to lecturer selection problem for the case study of (UEB-VNU) with four DMs and six selection criteria. It has been explained throughout the elaborated computation in the application that the suggested decision making methods are efficient.

Further works of this research involve deriving variants of the TOPSIS methods in terms of multi-attribute decision making [11], [43]–[48]. Strategies for decision support in real-time and dynamic decision-making tasks are also our next target. In the follow up study, this work can be extended to the triangular and trapezoidal linguistic numbers of SVLCNS and ILCNS. Several types of similarity measures can be utilized to extend the proposed framework in the near future. The different types of correlation coefficients can also be studied in this regard. Linguistic complex interval neutrosophic prioritized aggregation operators can be designed for decision making issues based on the proposed work. Some other types of aggregation operators such as Hammy mean operators, weighted aggregation operators, arithmetic and harmonic aggregation operators, power aggregation operators etc. can be developed in the follow up works. Moreover, linguistic hesitant complex interval neutrosophic set can be another possible study in this regard. The proposed framework can be embedded in soft set to develop linguistic complex interval neutrosophic set.

APPENDIX

This section reviews some basic notions and definitions of neutrosophic set, single-value neutrosophic set, interval-valued complex neutrosophic set and single-valued neutrosophic linguistic variable as follows [1], [9], [10], [13]:

Let U be a universe of discourse and a set $N \subset U$, such that

$$N = \{x(T_A(x), I_A(x), F_A(x)), x \in U\},$$

where $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$ are real subsets, for all $x \in U$, is called a neutrosophic set (NS)

If $T_A(x), I_A(x), F_A(x) \in [0,1]$ are real (crisp) numbers, for all $x \in U$, then N is called a *single-valued neutrosophic set* (SVNS).

If $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$ are real intervals, for all $x \in U$, then N is called a *interval-valued neutrosophic set* (IVNS).

If $CN = \{x(T_{1A}(x)e\hat{}\ (jT_{2A}(x)), I_{1A}(x)e\hat{}\ (jI_{2A}(x)), F_{1A}(x)e\hat{}\ (jF_{2A}(x)), x \in U\}$, where $T_{1A}(x), T_{2A}(x), I_{1A}(x), I_{2A}(x), F_{1A}(x), F_{2A} \subseteq [0, 1]$ are real subsets, for all $x \in U$, then CN is called a *complex neutrosophic set* (CNS).

If $T_{1A}(x), T_{2A}(x), I_{1A}(x), I_{2A}(x), F_{1A}(x), F_{2A} \in [0, 1]$ are real (crisp) numbers, for all $x \in U$, then CN is called a *single-valued complex neutrosophic set* (SVCNS).

If $T_{1A}(x), T_{2A}(x), I_{1A}(x), I_{2A}(x), F_{1A}(x), F_{2A} \subset [0, 1]$ are real intervals, for all $x \in U$, then CN is called a *interval-valued complex neutrosophic set* (IVCNS).

Let U be a universe of discourse and $S = \{s_1, s_2, \ldots, s_n\}$ be a set of labels. A *single-valuedlinguistic variable* (L) with respect to the attribute A is defined as:

$$L: U \rightarrow S, L(x) = s_x \in \{s_1, s_2, \ldots, s_n\}.$$

A *single-valuedneutrosophic linguistic variable* (NL) with respect to the attribute A is defined as:

$$NL: U \rightarrow S^3, NL\ (x) = (t_x, i_x, f_x),$$
$$\text{where } t_x, i_x, f_x \in \{s_1, s_2, \ldots, s_n\},$$

and t_x represents the positive degree of the element x with respect to the attribute A, i_x represents the indeterminate degree of the element x with respect to the attribute A, while f_x represents the false degree of the element x with respect to the attribute A.

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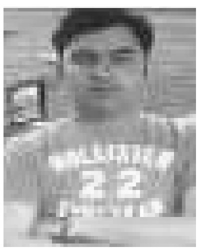

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