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A New Three Dimensional Solution for the Extrusions of Sections with Larger Dimensions than the Initial Billet

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Abstract

A new analysis for the three dimensional solution for the extrusion of sections with larger dimensions than the initial billet or container is presented in this paper. A generalized kinematically admissible velocity field was formulated using the upper bound theorem. The problem tackled in this paper is a practical one encountered in the extrusion industry where for the production of sections whose dimensions are larger than the container or billet diameter there exist some difficulties and sometime it is impossible to produce such products. The solution to this problem was suggested in a new design for the extrusion die which was done using the upper bound analysis. In this design unlike flat faced dies the material has to flow over two kinds of surfaces namely converging and diverging surfaces, the combination of which causes the material flow in a smooth manner and with the correct speed so that the required final shape would be achieved. For such geometries kinematically admissible velocity fields were obtained. Using this new formulation, extrusion of shapes such as square and rectangle were analyzed. Influence of the process parameters such as friction, extrusion ratio and aspect ratio on the extrusion load was investigated and the optimum die length was obtained. Finite element analysis for the same problem was also carried out and the comparison of the results showed good agreement. The finite element simulation was especially used to assist the theoretical analysis as regards the material flow and filling of the die cavity. Based on the analytical results, extrusion dies for the rectangular sections were designed and manufactured and experiments were carried out. The results of the tests showed that the dies performed very well and complete filling of the die cavity and a successful extruded profile was observed.

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Keywords: Extrusion, Shaped Sections, Spread Extrusion, Converging and Diverging Surfaces, Upper Bound, Material Flow, Velocity Field, FEM analysis.

Nomenclature

L	die length
R	radius of the initial billet
$\frac{L}{R}$	relative die length
a	smaller side of the rectangle (depth)
b	larger side of the rectangle (width)
B	the ratio of $\frac{b}{2R}$
AR	aspect ratio of the final extruded cross section
Ra	reduction of area
m	friction factor
n	an integer showing the number of divisions for the initial section
f, g and h	functions for the x, y and z components of the position vector
M	function for the axial velocity component
u, q and t	parameters changing between 0 and 1 in the x, y and z coordinates
$\bar{r}$	position vector for the moving particle
f_2	function defining the x component of the final cross section
g_2	function defining the y component of the final cross section
F_2, G_2	functions defining the x and y components of the perimeter for the final cross section
r_1	position vector defining the initial cross section
r_2	position vector defining the final cross section
s_e	surface velocity discontinuity at the entry
s_f	frictional velocity discontinuity at the die-material interface
s_x	surface velocity discontinuity at the exit
v_0	initial velocity of the billet material

v_x, v_y and v_z	component of the velocity in the x, y and z direction
Δv_e, Δv_x and Δv_f	velocity discontinuities due to entry and exit sections and material tool interface friction
S_e, S_x and S_f	surfaces of velocity discontinuities for the entry, exit and material tool interfaces
φ	the angle defining the entrance of the material
φ'	the angle defining the exit of the material
η	extrusion ratio (initial billet area to the exit area)
$\dot{\varepsilon}$	strain rate
σ	stress
σ_m	mean stress
W_i	power due to internal deformation
W_e	power due to entry surface of velocity discontinuities
W_x	power due to exit surface of velocity discontinuities
W_f	power due to die-material interface friction
J^*	upper bound on the extrusion power
P	extrusion pressure
P_{ave}	average extrusion pressure
P_i, P_s and P_f	components of extrusion pressure due to internal deformation, entrance and exit velocity discontinuities and die-material interface friction
subscripts u,q,t	indicate derivatives of functions with respect to these parameters

1-Introduction

The process of metal extrusion has been used in industry for a long time and many analytical and numerical solutions have been presented in the literature. However the problem of extrusion of shaped sections with larger dimensions than the initial billet or container has not been analyzed in the same fashion.

Chen and Ling [1] have given upper bound solutions to axisymmetric extrusion problems. They used three basic kinds of axisymmetric curved dies, namely, the cosine, elliptical and the hyperbolic types. Nagpal and Altan [2] showed that the mean pressure, necessary to extrude Al 1100 from a round stock to an elliptic shape, depended largely upon the reduction in area. They argued that the configuration of the extruded shape did not affect the extrusion pressure considerably, especially at high reductions. Yang and Lee [3] published an analysis of three dimensional extrusions of sections through curved dies using conformal transformation technique.

Hoshino and Gunasekera [4] published their work on upper bound solutions to the extrusion of square sections from round bars through converging dies. They proposed a theoretical solution in their paper for a new type of upper bound for the three dimensional metal flow through converging dies formed by an envelope of straight lines drawn from points on the perimeter of the circular type entry section to the corresponding points on the square type existing shape. An upper bound analysis of extrusion of square, rectangular, hexagonal and other asymmetric bars and wires was reported by Kiuichi et al [5].

An analysis of hydro film extrusion of three dimensional shapes from round bars was presented by Yang and Lang [6]. They proposed a new method of die construction which enabled the exact geometrical control of the die shape and ensured the initial sealing between the billet and the die. Yang et al [7] reported that in cold lubricated extrusion of round tubes the material properties and surface quality of the extruded products were influenced by the die profile. Streamlined dies induced less redundant work and rendered desirable distribution of strength. In this general analysis the shapes of the die and the mandrel were expressed by continuous functions.

A further investigation into the extrusion of toroidal gear sections considering three dimensional plastic flows was reported by Han and Yang [8]. The theoretical analysis used in this work was the same as that used in the previous work. The authors carried out experiments on two extrusion dies for a clover and a toroidal gear with eight teeth which were also designed and manufactured. Yang et al [9] gave details of a method of weighted residuals with the use of the finite difference method based on a coordinate transformation to non-orthogonal curvilinear coordinate system to fit the boundary of an arbitrary shape.

By employing a method of numerical analysis based on the method of weighted residuals, the stresses and strains in three-dimensional cold extrusion of arbitrarily shaped sections from round billets through continuous dies were calculated with sufficient accuracy reported in the work done by Lee et al [10]. They claimed that they had overcome the lack of generality observed in previous method using global flow functions for its application.

Kang and Yang [11] claimed that the variation of die bearing length was a primary way to control the metal flow in hot square die extrusion. Finite element computations were carried out to assess the influences of die bearing on metal flow and state variables. A finite element analysis was carried out for steady-state three-dimensional helical extrusion of noncircular twisted sections, such as clover and trocoidal gear sections, through curved dies by Park et al [12]. Curved die profiles were described by continuous functions by which smooth transitions of the die surface from the entrance to the exit were obtained.

Chitkara and Abrinia [13] gave a generalized upper bound solution for the forward extrusion of shaped sections in which cad-cam mathematical relations were incorporated. They presented an admissible velocity field which automatically satisfied the incompressibility condition regardless of the geometry of the die or the deformation zone. The design of three dimensional off-centric extrusions of arbitrarily shaped dies was applied to the off-centric extrusion of square sections from initially round billets with experimental verifications by Chitkara and Celik [14]. Abrinia and Bloorbar [15] reported a new generalized upper bound formulation for the three dimensional solution of the forward extrusion of shaped sections. They considered a velocity field in which curved surfaces for the entry and exit velocity discontinuities were formulated and hence the component of the velocity in the axial direction was not assumed constant as in all previous work. A new upper bound formulation for the extrusions of complicated shapes with no axis of symmetry was given by Abrinia and Zare [16]. In one of the very few works done on the extrusions of shaped sections with dimensions greater than the initial billet Imamura [17] et al used a ring in order to divert the material flow sideways. However a flat faced die was used in this work and new formulations were not presented.

In this paper a new generalized upper bound solution has been presented for the problem of spread extrusion for shaped sections such as a rectangle which have bigger dimensions than the initial billet. Finite element method has also been used to assist the solution and also to verify the analytical results. Experiments were also carried out to verify the dies designed in this work.

2-Theory

To formulate the problem first of all the deforming region must be defined and then a kinematically admissible velocity field will be derived based on which the upper bound solution will be presented.

2-1-Basic Formulation

For the extrusion considered in this paper which is a rectangular section profile with a side larger than the diameter of the initial billet, flat faced dies are not used and instead dies with converging and diverging surfaces are utilized. These surfaces are designed in such a way to lead the material flow in the desired direction so as to fill in the die cavity and form the final product (see figure (1)). Therefore the die surfaces together with the entry and exit material flow surfaces of discontinuities form the deforming region as shown in figures (2a) and (2b). In these figures a quarter of the geometry of the extrusion process has been shown. In figure (2a) a quarter of the initial billet is seen to be extruded through the die surface $MNLL'N'M'$ which consists of two surfaces namely a converging surface $NLL'N'$ and a diverging surface $MNN'M'$. After passing through the die, the material flows out and takes the shape of a rectangle which has a side ($L'N'$) greater than the initial billet radius (ON). Now consider the material flow on a general stream surface such as $OBB'O'$ (figure (2b)). For the formulation here the theory presented by [13] and [16] has been adopted and modified. A general material point such as P will move on a stream line such as CC'. Point P could be represented by a position vector:

$$\bar{r} = f(u,q,t)\vec{i} + g(u,q,t)\vec{j} + h(u,q,t)\vec{k} \tag{1}$$

where f, g and h are functions defining the x, y and z coordinates and u, q and t are parameters varying between 0 and 1 as defined below:

$$u = OC/R, \quad q = n\varphi/\pi, \quad t = L/Z \tag{2}$$

As u, q and t vary between 0 and 1 the radial, angular and axial position of point P is defined and different points in the deforming geometry are determined. Let us first define the entry section to the deforming geometry which has been assumed as a plane circular surface. Using the geometry in figure (2b) we have:

$$\vec{r}_1 = OC\sin\varphi\vec{i} + OC\cos\varphi\vec{j} \tag{3}$$

Now point C on line OB in figure (2b) could change from O to B and also by changing the angle φ between zero to 90 for this case all points on the entry section are defined. Considering equation (2) OC could be defined as uR and φ as $\pi q/n$ and hence equation (3) is defined as:

$$\vec{r}_1 = uR(\sin(\pi q/n)\vec{i} + \cos(\pi q/n)\vec{j}) \tag{4}$$

In equation (4) as u and q vary between 0 and 1 all points on the entry section are defined.

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Similarly the exit cross section which is a rectangular plane section ($O'L'N'M'$ -figure (2b)) could be defined by:

$$\vec{r}_2 = O'C'\sin\varphi'\vec{i} + O'C'\cos\varphi'\vec{j} + L\vec{k} \tag{5}$$

Here C' on line $O'B'$ defines a general point on the exit cross section which could change between O' and B' and also by varying the value of φ' between zero to 90 for this case all points on the exit section are defined. Line $O'C'$ in equation (5) is assumed to be given by(see figure (2b)):

$$O'C' = \frac{OC}{OB}O'B' = \frac{uR}{R}O'B' = uO'B' \tag{6}$$

Hence by substituting from equation (6) into (5) we have:

$$\vec{r}_2 = u(O'B'\sin\varphi'\vec{i} + O'B'\cos\varphi'\vec{j}) + L\vec{k} \tag{7}$$

Now considering the geometry of the exit section in figure (2b) and the definition of the extrusion ratio η the following relations could easily be derived:

$$\begin{aligned} O'B'\sin\varphi' &= F_2 = \frac{b}{2} \\ OB\cos\varphi' &= G_2 = \frac{\pi q\eta R^2}{b} \end{aligned} \qquad \text{for} \qquad 0\langle\varphi\langle\frac{\pi}{2} \tag{8-a}$$

$$\begin{aligned} O'B'\sin\varphi' &= F_2 = (\frac{1}{\eta}+1)\frac{b}{2} - \frac{\pi q R^2}{a} \\ O'B'\cos\varphi' &= G_2 = \frac{a}{2} \end{aligned} \qquad \text{for} \qquad \frac{\pi}{2}\langle\varphi\langle\pi \tag{8-b}$$

Here again by substituting from equations (8) into (7) (note that in equations (8) we have used $\varphi = \pi q / n$ from equations (2)) an equation in terms of u and q would be obtained as follows:

$$\vec{r}_2 = u(F_2\vec{i} + G_2\vec{j}) + L\vec{k} \tag{9}$$

Where F_2 and G_2 are given by equations (8) and by assigning values to u and q between 0 and 1 all points on the exit cross section are defined.

Now that all the points on the entry and exit cross sections are defined (equations (4) and (9)), by considering a bilinear surfaced die, that is a die with a surface defined by a linear relationship (interpolation) between the entry and exit cross section, the die surface is given by:

$$\vec{r} = [1 \quad t]\begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \vec{r}_1 \\ \vec{r}_2 \end{bmatrix} \tag{10}$$

(Note that in the interpolation the notation $Z = Lt$ has been used where t varies between 0 and 1). Equation (10) in fact defines the position vector of point P and by considering equation (1) the Cartesian components of the position of point P are given by:

$$\begin{aligned} X &= f(u,q,t) = u[R(\sin(\pi q/n)(1-t)+F_2 t] \\ Y &= g(u,q,t) = u[R(\cos(\pi q/n)(1-t)+G_2 t] \\ Z &= h(u,q,t) = Lt \end{aligned} \tag{11}$$

Therefore the above equations define the geometry of the deforming zone completely as u, q and t vary between 0 and 1. Note that for this case (a bilinear surfaced die) $h(u,q,t)$ is only a function of t.

2-2-A kinematically admissible velocity field

Having established the formulation for the die geometry and the deforming zone a kinematically admissible velocity field can now be obtained. Consider a point such as P in figure (2b) which is moving on a stream line CC'. The general vector equation for the velocity of this point is given by:

$$\vec{V} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} \tag{12}$$

Now since CC' is assumed to be a stream line then the unit tangent vectors of equation (1) for the position of point P and equation (12) for the velocity of point P coincide and the following relationships could easily be derived:

$$\begin{aligned} v_z &= M(u,q,t) \\ v_x &= \frac{f_t}{h_t}M \\ v_y &= \frac{g_t}{h_t}M \end{aligned} \tag{13}$$

where M is a general function which will be obtained from the incompressibility conditions and makes the velocity field automatically admissible. f_t, g_t and h_t are the derivatives of f, g and h with respect to t. Functions f, g and h are given by equations (11). We can now substitute the values for the components of velocity from equation (13) into the relationship for the incompressibility condition (equation (14)) and obtain M:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{14}$$

After some manipulation and integration, the following function is obtained for the function M:

$$M(u,q,t)=\frac{C.h_t}{h_u(f_q.g_t-f_t.g_q)+h_q(f_t.g_u-f_u.g_t)+h_t(f_u.g_q-f_q.g_u)} \tag{15}$$

Hence the equations (13) are solved and the velocity field is determined. Note that C is a constant in equation (15) and could be determined by substituting the boundary conditions.

Next the strain rate components are obtained using the following equations:

$$\dot{\varepsilon}_{xx}=\frac{\partial v_x}{\partial x},\ \dot{\varepsilon}_{xy}=\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial v_x},\ \dot{\varepsilon}_{yy}=\frac{\partial v_y}{\partial y},$$

$$\dot{\varepsilon}_{xz}=\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial v_x},\ \dot{\varepsilon}_{zz}=\frac{\partial v_z}{\partial z}\ \text{and}\ \dot{\varepsilon}_{zy}=\frac{\partial v_z}{\partial y}+\frac{\partial v_y}{\partial v_z}$$

(16)

2-3-Upper Bound on Power

The upper bound on power can now be calculated by the following relationship:

$$J^*=W_f+W_i+W_e+W_x$$

(17) The terms in equation (17) represent different components of the energy dissipated due to friction, internal deformation and velocity discontinuities and they are given as follows:

Energy dissipated due to friction between workpiece and die:

$$W_f=m\frac{\sigma_m}{\sqrt{3}}\int_{s_f}\Delta v_f ds_f \tag{18}$$

Energy dissipated due to internal deformation:

$$\dot{W}_i=\sigma_m\int_V\dot{\varepsilon}dV \tag{19}$$

Energy dissipated due to velocity discontinuities at the entrance section of the deforming zone:

$$W_e=\frac{\sigma_m}{\sqrt{3}}\int_{s_e}\Delta v_e ds_e \tag{20}$$

Energy dissipated due to velocity discontinuities at the exit section of the deforming zone:

$$W_x=\frac{\sigma_m}{\sqrt{3}}\int_{s_x}\Delta v_x ds_x \tag{21}$$

Summing up all the component of the energy in equation (17) the upper bound on power is obtained and using the following equation the average extrusion pressure is calculated:

$$P_{ave} = \frac{J^*}{\pi R^2 v_0} \qquad (22)$$

3-Finite element simulation

Using ABAQUS/ Explicit the same dies designed in the previous section were modeled and the extrusion process was simulated (figure (3)). The 3-D elements used for the workpiece were 8 nodes C3D8R and for the tools 4 nodes R3D4. All the dies considered in this paper were modeled and the simulation showed that the die cavity was filled and the complete product was extruded (figure (4)). This part of the simulation was vital in order for the analytical part of the work to become applicable. This is due to the fact that the velocity field in this paper is based on the completed deforming region geometry (i.e. unless the material fills the die cavity the analysis is not valid). However other results were also obtained from the finite element analysis to analyze the problem of the spread extrusion and to have data for comparison with the analytical work.

4-Experiments

In order to verify the theoretical work, dies for the extrusion of rectangular profiles with cross sections larger than the initial billets were designed and manufactured for the reduction of area of 40% and the ratios of the width of the rectangular profile to the diameter of the billet were taken as 1.2, 1.4 and 1.6. These dies were designed based on the results of the analysis carried out in this paper and optimum die length was taken to be $L/R=1$ for all the cases. The initial billet size was 20mm diameter and 30mm length. The billets were extruded using the experimental equipments designed and built for this purpose (figure (5)). An Instron 4028 experimental hydraulic press was used and tests were carried out. The material used for the billets was lead having a yield strength of 16MPa which was obtained using a standard compression test. The billets were extruded successfully for all die designs and the material filled the dies completely and the complete rectangular profiles were produced. The profiles extruded through the dies are seen in figure (6). In figure (7) the extruded product along with the initial billet and the die could be observed (note the size of the rectangle as compared to the initial billet). Figure (8) also illustrate the incomplete extruded billet in such a way that the flow of material on converging and diverging surfaces could be observed (the side of the extruded profile has been damaged during machining and splitting the die).

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5-Results and Discussion

The results obtained from the analytical and numerical (finite element method) solutions as well as experimental observations have been illustrated in figures (9) to (25).

Using the upper bound method presented above different dies for various process parameters were designed and the results given here are for the extrusion through these dies. In figure (9), relative extrusion pressure has been plotted against the die length. For this case the ratio of $b/2R$ (width of the rectangle to the diameter of the billet) has been varied as shown and the reduction of area and frictional conditions were kept constant. In general and as expected for lower values of $b/2R$ lower values of P/Y were obtained. Higher values of $b/2R$ mean that the flow of material along the converging surface of the die ($NLL'N'$ in figure (2a)) causes the material to spend more energy in passing over it. However the optimum die length for $b/2R=2$ is seen to be given by $L/R=1.5$ and for $b/2R=1$it is given by $L/R=1$. Hence the larger the width of the rectangle the larger should the die length be.

To investigate the above effects on each of the components of the upper bound on the extrusion pressure, plots of P_i/Y , P_s/Y and P_f/Y have been demonstrated in figures (10-1), (10-2) and (10-3). These components are due to internal deformation, entrance and exit velocity discontinuities and friction respectively. For the component of pressure due to internal deformation (figure (10-1)), it could be seen that $b/2R$has a pronounced influence on the relative extrusion pressure. For the optimum die length values, this effect is high. However the increase in die length does not have any effect on this component of pressure after a certain value of the die length ($L/R \approx 1$). It could be seen that in general the component of pressure due to velocity discontinuity (figure (10-2)) is not influenced very much by $b/2R$ for higher values of die length L/R . At the optimum value of the die length also the changes in this component are not appreciable. Note also that as the die length increases the pressure drops because the entrance and exit of the material to and from the deformation zone become smoother as the die length increases. The effect of $b/2R$ on the component of pressure due to friction (figure (10-3)) however is more pronounced and for optimum values of the die length there is an appreciable change in the value of pressure. Note that as the die length increases the pressure goes up rapidly. The influence of the friction on the extrusion pressure has been shown in figure (11). Clearly higher values of pressure are observed for higher friction factors. However for a given friction factor very high values of pressure are observed both for smaller values of the die length ($L/R\langle 0.5$) and for higher values of die length ($L/R\rangle 1$). The reason for this behavior could be explained by the fact that for the small values of die lengths the surfaces of the die make very steep angles with the direction of material flow and hence the normal pressure increases. For larger values of die length the die surfaces make smaller angles with the direction of flow but the interface between the material and die increases and hence the energy dissipated due to friction increases too. Variation of relative extrusion pressure for different values of the reduction of area is considered in figure (12). As expected higher reductions mean higher pressures. The optimum value of die length does not change with the reduction of area.

The analytical formulation given here is applicable only for the steady state part of the extrusion process. In other words at the beginning of the process when the billet is pushed into the die cavity, it can not be dealt with analytically. Hence finite element analysis was used for this part of the process and the simulation was carried out for the same die designed with the analytical theory. In figure (13) variation of the extrusion force with the advance of the ram is observed for different die lengths. In all cases the die is filled and force is increased until it reaches the stable stage of the extrusion. In figure (14) the effect of the larger dimension of the final extruded rectangular profile is shown on the extrusion force. For all values of $b/2R$, the filling of the die cavity is successful but for higher values of $b/2R$ higher extrusion force is required because the shape becomes more complex.

In figures (15) to (18) results of the finite element simulation and the upper bound analysis are compared. Effect of the die length on the extrusion pressure is shown in figure (15). It could be seen that the values of UBA and FEA are in good agreement considering that UBA values are upper bounds and must be higher than the real values anyway. Note that at lower die lengths FE simulation could not give realistic values due to high distortion of the elements while UBA does not suffer such limitations. In figure (16), comparison of results for FEA and UBA for the effect of $b/2R$ on the extrusion pressure is shown. Again relatively good agreement is observed. However at lower values of $b/2R$ FEA results are closer to UBA while as $b/2R$ values increase the difference becomes higher. This increase in the difference of results could also be explained by the fact that higher values of $b/2R$ mean more distortion of the elements in FEA and hence less accurate results. Influence of the reduction of area on the extrusion pressure as predicted by FEA and UBA is illustrated in figure (17). It could be said that both methods predict the same trend and the produced data are close to each other. The same sort of results is obtained in figure (18) where the influence of friction on the extrusion pressure is illustrated.

To investigate the magnitude of the share of power required to form different sub-sections of the final extruded shape (sub-sections $O'N'L'$ and $O'N'M'$ in figure (2a) showing the flow of material on the converging and diverging surfaces of the die), figures (19) to (22) have been presented. In figure (19) the influence of $b/2R$ on the extrusion pressure is shown separately for the two sub-sections. It could be said that the share of the subsection $O'N'L'$, (see figure (2a) for the sub-sections), in the extrusion pressure is much higher than the sub-section $O'N'M'$. This could be explained by the fact that the flow of material in the converging paths always requires more power than in the diverging paths (spread). However one interesting point to note in figure (19) is that as $b/2R$ increases, the difference of pressure required for the two surfaces actually decreases although the pressure in general increases as expected. A plot of the relative extrusion pressure for the two sub-sections $O'N'L'$ and $O'N'M'$ for different die lengths is shown in figure (20). Here again we see a bigger share of the extrusion pressure bared by sub-section $O'N'L'$, but as the die length changes there is no difference in the share of pressure for the two sub-sections. In figure (21) a very interesting result is observed. As the friction increases the share of the pressure needed to extrude the sub-section $O'N'L'$ increases much more than that required to extrude the sub-section $O'N'M'$. This may be explained by the fact that converging surfaces require more pressure than the diverging surfaces. The effect of the reduction of area on the share of the extrusion pressure for each sub-section could be observed in figure (22). It could be said that the increase in the reduction of area does not have any effect on share of pressure for each of the sub-sections.

To compare the spread extrusion (in which a section with larger dimension than the initial billet is extruded) and the ordinary extrusion (in which the final dimensions of the extruded product are smaller than the initial billet), plots of relative extrusion pressure versus relative die length have been presented in figures (23) and (24). In figure (23) a comparison is made for a constant aspect ratio. Clearly if one keeps the aspect ratio the same the reductions would then be different as shown in the graph. However it could be seen that in general the spread extrusion requires less pressure than the ordinary extrusion although for the spread extrusion the reduction of area is even higher. This may be explained by the fact that the material by flowing sideways or spreading, consumes less power. This is a very interesting conclusion and may have far reaching consequences which would be dealt with in the future work. However when reduction of area is kept constant and the aspect ratio is changed (figure (24)) it could be seen that the spread extrusion requires more energy than the ordinary extrusion. Comparing the results of the two recent graphs, it could be said that the influence of aspect ratio in increasing the pressure is very significant.

Finally the results obtained from experimental observations of the extrusion tests carried out for the verification of the theoretical work are shown in figure (25) along with the upper bound and finite element results. It could be seen that there are good agreements between the theoretical and experimental work. The successful extrusions of the rectangular billets (as shown in figures (6), (7) and (8)) proved that the dies designed in this way performed well and the material filled the die cavity completely. The final product did not suffer from any defects and was extruded completely. Also the pressure required to extrude the profiles was within reasonable bounds as indicated in figure (25).

6-Conclusions

-A theoretical method of analysis for the solution of extrusion of shaped sections with larger dimensions than the initial billet was presented.

-Comparison of the results of the finite element analysis and the upper bound solution showed good agreement.

-Using the theoretical analysis presented in this paper, dies were successfully designed for the extrusion of rectangular sections with larger dimensions than the initial billet.

-FE simulation was used in this paper to assist the analytical work and it was shown that the material successfully filled the die cavity.

-Results showed that the diverging surfaces of the die needed less power for the extrusion than the converging surfaces.

-The effect of friction on the extrusion pressure due to the converging surfaces was much more pronounced than that for the diverging surfaces.

-Comparing ordinary extrusion and spread extrusion it could be concluded that for a given aspect ratio, the spread extrusion requires less power than the ordinary extrusion.

-Using the theory presented, dies for the spread extrusion process could be designed for different shaped sections.

-Dies designed in this paper were manufactured and experiments carried out. Rectangular profiles with the ratio of side to billet diameters of 1.2, 1.4 and 1.6 were extruded successfully.

-Comparison of experimental and theoretical data showed good agreement.

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