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METHOD FOR SOLVING FUZZY ASSIGNMENT PROBLEM USING MAGNITUDE RANKING TECHNIQUE

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Abstract:

Assignment problems have various applications in the real world because of their wide applicability in industry, commerce, management science, etc. Traditional classical assignment problems cannot be successfully used for real life problem, hence the use of fuzzy assignment problems is more appropriate. In this paper, the fuzzy assignment problem is formulated to crisp assignment problem using Magnitude Ranking technique and Hungarian method has been applied to find an optimal solution. The Numerical examples show that the fuzzy Magnitude Ranking method offers an effective tool for handling the fuzzy assignment problem over Robust Ranking method.

1. Introduction:

An assignment problem is a special type of linear programming problem where the objective is to assign n number of persons to n jobs at a minimum cost (time). Zadeh (1965) introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years. Hungarian method proposed by Kuhn (1955) is widely used for the solution to APs. Chen (1985) proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Wang (1987) solved a similar model by graph theory. Lin and Wen (2004) investigated a fuzzy AP in which the cost depends on the quality of the job. Dubois and Fortemps (1999) proposed a flexible AP, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. Huang and Xu (2005) proposed a solution procedure for the APs with restriction of qualification. Mukherjee and Basu (2010) proposed a new method for solving fuzzy APs. Kumar et al (2009) proposed a method to solve the fuzzy APs, occurring in real life situations. Kumar and Gupta (2012) proposed two new methods for solving fuzzy APs and fuzzy travelling salesman problems. Kumar and Gupta (2011) proposed methods for solving fuzzy APs with different membership functions. K. Kalaiarasi et al (2014) proposed a fuzzy assignment model with TFN using Robust Ranking technique.Jatinder Pal Singh et al (2015)proposed method to solve fuzzy assignment problem using FHM and Operations for Subtraction and Division on TFN proposed by Gani and Assarudeen (2012). In this paperthe fuzzy assignment problem has been converted into crisp assignment problem using Method of Magnitude and Hungarian assignment has been applied to find an optimal solution.

2. Preliminaries:

2.1 Fuzzy Set:

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . A function $\mu_{\tilde{a}}$ such that the value assigned to the universal set X fall within a specified rangei.e. $\mu_{\tilde{a}} : X \to [0,1]$.The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{a}(x)}$ is called the membership function and the set $\hat{A} = \{(x, \mu_{\tilde{a}(x)}); x \in X\}$ defined by $\mu_{\tilde{a}(x)}$ for each $x \in X$ is called a fuzzy set.

2.2 Triangular Fuzzy Number (TFNs):

A fuzzy number $\hat{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\hat{a} : R \to [0,1]$ has the following characteristics

$$\mu_{\hat{a}(x)} = \begin{cases} (x-a_1)/(a_2-a_1) \ if \quad a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) \ if \quad a_1 \le x \le a_3 \\ 0 \quad otherwise \end{cases}$$

We denote this triangular fuzzy number by $\hat{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers. Also if $m = a_2$, represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\hat{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\hat{a}$ can be represented by the triple $\hat{a} = (\alpha, m, \beta)$ i.e. $\hat{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

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2.3 Defuzzification:

Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here Method of magnitude is used to defuzzify the TFNs because of its simplicity and accuracy.

2.3.1 Robust Ranking Technique:

For a convex fuzzy number $\tilde{a}$, the Robust's Ranking Index is defined by,

$$R(\tilde{a}) = \int_0^1 (0.5)(a^L{}_\alpha, a^U{}_\alpha)\, d\alpha$$

Where $(a^L{}_\alpha, a^U{}_\alpha) = \{(b-a)\alpha + a, c-(c-b)\alpha\}$ which is the α -level cut of the fuzzy number $\tilde{a}$.

2.3.2 Method of Magnitude:

A triangular fuzzy number $\breve{a} \in F(R)$ can also be represented as a pair $\breve{a} = (\underline{a}, \overline{a})$ of functions $(\underline{a}(r), \overline{a}(r))$ for $0 \le r \le 1$ which satisfies the following requirements:

- ✓ $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.
- ✓ $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.
- ✓ $\underline{a}(r) \le \overline{a}(r)$ for $0 \le r \le 1$

2.3.2.1 Definition:

For an arbitrary triangular fuzzy number $\breve{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1) + \overline{a}(1)}{2}\right)$ is said to be a location index number of $\breve{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$ and $a_* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index functions respectively. Hence every triangular fuzzy number $\breve{a} = (a_1, a_2, a_3)$ can also be represented by $\breve{a} = (a_0, a_*, a^*)$.

2.3.2.2 Ranking of Triangular Fuzzy Numbers:

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers. For an arbitrary triangular fuzzy number $\breve{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\breve{a} = (\underline{a}(r), \overline{a}(r))$ we define the magnitude of the triangular fuzzy number by $\breve{a}$ by

$$Mag(\breve{a}) = \frac{1}{2}\left(\int_0^1 (\overline{a} + \underline{a} + a_0) f(r)dr\right) = \frac{1}{2}\left(\int_0^1 (a^* + 3a_0 - a_*) f(r)dr\right)$$

[Since $\overline{a} + \underline{a} + a_0 = a^* + a_0 + a_0 - a_* + a_0 = a^* + 3a_0 - a_*$]. Where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0) = 0, f(1) = 1$ and $\int_0^1 f(r)dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$

Hence $Mag(\breve{a}) = \left(\frac{a^* + 3a_0 - a_*}{4}\right) = \left(\frac{\overline{a} + \underline{a} + a_0}{4}\right)$.

The magnitude of a triangular fuzzy number $\breve{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $Mag(\breve{a})$ is used to rank fuzzy numbers.

Theorem:

For any two fuzzy numbers, $A = \langle s_1, l_1, r_1 \rangle$ and $B = \langle s_2, l_2, r_2 \rangle$ and $A \le B$ if and only if $s_1 \le s_2$, $s_1 - l_1 \le s_2 - l_2$ and $s_1 + r_1 \le s_2 + r_2$.

Properties:

For any two triangular fuzzy number $\breve{a} = (a_0, a_*, a^*)$ and $\breve{b} = (b_0, b_*, b^*)$

- ✓ $Mag(\breve{a}) \ge Mag(\breve{b})$ if and only if $\breve{a} \ge \breve{b}$
- ✓ $Mag(\breve{a}) \le Mag(\breve{b})$ if and only if $\breve{a} \le \breve{b}$
- ✓ $Mag(\breve{a}) = Mag(\breve{b})$ if and only if $\breve{a} \approx \breve{b}$.

In this paper we use this Magnitude Ranking technique for ranking the objective values. $Mag(\breve{a})$ gives the representative value of the fuzzy number $\breve{a}$. It satisfies the linearity and additive property.

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3. Algorithm to Solve Fuzzy Assignment Problem:

Step 1: First test whether the given fuzzy cost matrix of a fuzzy assignment problem is a balanced one or not. If not change this unbalanced assignment problem into balanced one by adding the number of dummy row(s) / column(s) and the values for the entries are zero. If it is a balanced one (i.e. number of persons are equal to the number of works) then go to step 2.

Step 2: Defuzzify the fuzzy cost by using Magnitude ranking method.

Step 3: Apply Hungarian Algorithm to determine the best combination to produce the lowest total costs, where each machine should be assigned to only one job and each job requires only one machine.

4. Numerical Example:

Example 4.1: Here we are going to solve fuzzy Assignment problem using Magnitude Ranking Technique: To allocate 4 jobs to 4 different machines, the fuzzy assignment cost C_{ij} is given below:

$$\begin{bmatrix} (1,5,9) & (3,7,11) & (7,11,15) & (2,6,10) \\ (4,8,12) & (1,5,9) & (4,9,13) & (2,6,10) \\ (0,4,8) & (3,7,11) & (6,10,14) & (3,7,11) \\ (6,10,14) & (0,4,8) & (4,8,12) & (-1,3,7) \end{bmatrix}$$

Solution:

In conformation to model the fuzzy assignment problem can be formulation in the following

$$Min\, Mag(1,5,9)x_{11} + Mag(3,7,11)x_{12} + Mag(7,11,15)x_{13} + Mag(2,6,10)x_{14}$$
$$+Mag(4,8,12)x_{21} + Mag(1,5,9)x_{22} + Mag(4,9,13)x_{23} + Mag(2,6,10)x_{24}$$
$$+Mag(0,4,8)x_{31} + Mag(3,7,11)x_{32} + Mag(6,10,14)x_{33} + Mag(3,7,11)x_{34}$$
$$+Mag(6,10,14)x_{41} + Mag(0,4,8)x_{42} + Mag(4,8,12)x_{43} + Mag(-1,3,7)x_{44}$$

Subject to

$$x_{11}+x_{12}+x_{13}+x_{14}=1 \qquad x_{11}+x_{21}+x_{31}+x_{41}=1$$
$$x_{21}+x_{22}+x_{23}+x_{24}=1 \qquad x_{12}+x_{22}+x_{32}+x_{42}=1$$
$$x_{31}+x_{32}+x_{33}+x_{34}=1 \qquad x_{13}+x_{23}+x_{33}+x_{43}=1$$
$$x_{41}+x_{42}+x_{43}+x_{44}=1 \qquad x_{14}+x_{24}+x_{34}+x_{44}=1$$

Where $x_{ij} \in [0,1]$

Now we calculate $Mag(1,5,9)$ by applying method of magnitude. The membership function of the triangular fuzzy number $(1,5,9)$ is

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-1)}{4}, 1 \le x \le 5 \\ \frac{(9-x)}{4}, 5 \le x \le 9 \\ 0, otherwise \end{cases}$$

$$Mag(\tilde{a}) = \frac{1}{2}\left(\int_0^1 (\bar{a} + \underline{a} + a_0) f(r) dr\right)$$

Where $(\bar{a} + \underline{a} + a_0) f(r) = (a^* + 3a_0 - a_*)r$

$$Mag(1,5,9) = \frac{1}{2}\int_0^1 (9+3(1)-5)rdr \quad = 1.75$$
$$Mag(3,7,11) = \frac{1}{2}\int_0^1 (11+3(3)-7)rdr \quad = 3.25$$

Similarly,

$$Mag(7,11,15) = 6.25$$
$$Mag(2,6,10) = 2.5$$
$$Mag(4,8,12) = 4$$
$$Mag(1,5,9) = 1.75$$
$$Mag(4,9,13) = 4$$
$$Mag(2,6,10) = 2.5$$
$$Mag(0,4,8) = 1$$
$$Mag(3,7,11) = 3.25$$
$$Mag(6,10,14) = 5.5$$

$$Mag(3,7,11) = 3.25$$
$$Mag(6,10,14) = 5.5$$
$$Mag(0,4,8) = 1$$
$$Mag(4,8,12) = 4$$
$$Mag(-1,3,7) = 0.25$$

We replace these values for this corresponding C_{ij} . We get a convenient assignment problem

$$\begin{bmatrix} 1.75 & 3.25 & 6.25 & 2.5 \\ 4 & 1.75 & 4 & 2.5 \\ 1 & 3.25 & 5.5 & 3.25 \\ 5.5 & 1 & 4 & 0.25 \end{bmatrix}$$

We solve it by using Hungarian methods to get the following optimal solution.

Step 1: Row reduction. Subtract the minimum element of each row from all elements of that row

$$\begin{bmatrix} 0 & 1.5 & 4.5 & 0.75 \\ 2.25 & 0 & 2.25 & 0.75 \\ 0 & 2.25 & 4.5 & 2.25 \\ 5.25 & 0.75 & 3.75 & 0 \end{bmatrix}$$

Step 2: Column reduction. Subtract the minimum element of each column from all elements of that column

$$\begin{bmatrix} 0 & 1.5 & 2.25 & 0.75 \\ 2.25 & 0 & 0 & 0.75 \\ 0 & 2.25 & 2.25 & 2.25 \\ 5.25 & 0.75 & 1.5 & 0 \end{bmatrix}$$

Step 3: For Making assignments, draw minimum possible horizontal and vertical lines covering all zeros.

$$\begin{bmatrix} 0 & 1.5 & 2.25 & 0.75 \\ 2.25 & 0 & 0 & 0.75 \\ 0 & 2.25 & 2.25 & 2.25 \\ 5.25 & 0.75 & 1.5 & 0 \end{bmatrix}$$

Step 4: Modify the above table by subtracting the smallest uncovered number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} 0 & 0.75 & 1.5 & 0 \\ 3 & 0 & 0 & 0.75 \\ 0 & 1.5 & 1.5 & 1.5 \\ 6 & 0.75 & 1.5 & 0 \end{bmatrix}$$

Step 5: Making assignments and drawing minimum possible horizontal and vertical lines covering all zeros.

$$\begin{bmatrix} 0 & 0.75 & 1.5 & 0 \\ 3 & 0 & 0 & 0.75 \\ 0 & 1.5 & 1.5 & 1.5 \\ 6 & 0.75 & 1.5 & 0 \end{bmatrix}$$

Step 6: Modify the above table by subtracting the smallest uncovered number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} 0 & 0 & 0.75 & 0 \\ 3.75 & 0 & 0 & 1.5 \\ 0 & 0.75 & 0.75 & 1.5 \\ 6 & 0 & 0.75 & 0 \end{bmatrix}$$

Step 7: Making assignments

$$\begin{bmatrix} 0 & 0 & 0.75 & \boxed{0} \\ 3.75 & 0 & \boxed{0} & 1.5 \\ \boxed{0} & 0.75 & 0.75 & 1.5 \\ 6 & \boxed{0} & 0.75 & 0 \end{bmatrix}$$

The fuzzy optimal total cost

$$\breve{a}_{14} + \breve{a}_{23} + \breve{a}_{31} + \breve{a}_{42} = (2,6,10) + (4,9,13) + (0,4,8) + (0,4,8) = (6,23,39)$$

$$Mag(6,23,39) = 8.5$$

Hence the fuzzy optimal total cost is 8.5.

5. Result:

In this section we will compare the fuzzy assignment cost which has been found out in example 4.1 with the assignment cost calculated by existing methods (kalaiarasi et al 2014, Jatinder pal Singh et al 2015)

	Existing Methods		Proposed Method
	Kalaiarasi et al 2014	Jatinder Pal Singh et al 2015	
Example 4.1	Assignment cost = 23	Assignment cost = 22.75	Assignment Cost = 8.5

6. Conclusion:

In this paper, the assignment cost has been considered as imprecise numbers that can be described by fuzzy numbers which are more realistic and general in nature. Here, the fuzzy assignment problem has been converted into crisp assignment problem using Method of Magnitude and Hungarian assignment has been applied to find an optimal solution. By comparing the results of the proposed method and existing method, it is shown that it is better to use the proposed methods instead of existing method.

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