Early-Universe Simulations of the Cosmological Axion: Density Field Construction for *Supplementary Data*

This note describes the procedure by which the density field and its time derivative may be constructed from the included axion field configuration data. This note also provides a primer on the interpretation of the data, which is provided in simulation units, in terms of physical scales.

1 Interpretation in Physical Units

The simulation for which we provide data was evolved in units defined relative to the axion mass. Consequently, the physical interpretation of the simulation output must be made within the context of a defined axion mass m_a. We choose to interpret this simulation for an axion of mass $25.2\,\mu$eV ($f_a = 2.27 \times 10^{11}$ GeV) based on our extrapolations for the predicted dark matter axion mass.

The relevant values for the physical interpretation of the simulation data provided here are $H_1 = 3.01\,\text{neV}$ and $\hat{\eta} = R_F/R_1 = 10^6$. H_1 is the Hubble expansion rate at the time at which the axion began to oscillate in the simulation (H_1 is defined as the Hubble parameter at the temperature T_1 where $H_1 \equiv H(T_1) = m_a(T_1)$) and $\hat{\eta}$ is the ratio of the scale factor at the end of the simulation to the scale factor at the time axion oscillations began. At this final scale factor, the final temperature is $T \approx 4.5\,\text{keV}$ and the physical length is given by $4\hat{\eta}/H_1 \approx 263,000\,\text{km}$.

2 The Dataset

The relevant data are provided in three independent `HD5F` files. They are:

1. The axion field in `Field_Data.hdf5`, accessed by the key `Axion_Field`
2. The first derivative of the axion field with respect to conformal time in `First_Deriv_Data.hdf5`, accessed by the key `Axion_Field_First_Derivative`
3. The second derivative of the axion field with respect to conformal time in `Second_Deriv_Data.hdf5`, accessed by the key `Axion_Field_Second_Derivative`.

For the purposes of the construction of the density field and its first derivative, the second derivative will be negligible. The data are provided as a 3-dimensional array of 1024^3 elements, which represents an eightfold downsampling from the 2048^3 grid on which simulations were performed. The high-resolution datasets may be made available at request.

3 Density Field Construction

From the axion field a and its first conformal time derivative a', to good approximation, the density field and the first derivative of the density field with respect to metric time can be computed by

$$\rho = \frac{m_a^2 f_a^2}{2}\left(\frac{a'^2}{\hat{\eta}_c^n \hat{\eta}^2} + a^2\right) \quad (1)$$

$$\dot{\rho} = -\frac{m_a^2 f_a^2 H_1}{\hat{\eta}^4 \hat{\eta}_c^n} a'^2\,, \quad (2)$$

with $\hat{\eta}_c = 3.6$, $\hat{\eta} = 10^6$, and $n = 6.68$.

The simulations for which we have provided data were not performed for an axion which produced the correct relic abundance. To correct this, the density and its derivative constructed above should be rescaled by a factor of 1.04×10^{-2} so that the correct relic abundance is achieved in order to predict accurate minihalo masses.