Journal of Engineering Science | *Vol. XXVI, no. 1 (2019), pp. 41 - 46*
Fascicle | *Industrial Engineering* | *ISSN 2587-3474*
Topic | *Mechanical Engineering and Technologies* | *eISSN 2587-3482*

10.5281/zenodo.2640034
CZU 62-23

CLASSICAL AND MODERN KINEMATIC ANALYSIS APPLIED FOR MECHANISM STUDY

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Received: December, 16, 2018
Accepted: February, 05, 2019

Abstract. Mechanism kinematic analysis is usually used for motion study or for simulating and analyzing the movement of mechanical assemblies and the whole mechanism. In this paper we will consider kinematical analysis regarding the simplest crank mechanism without taking into account forces that cause the mechanism motion. The authors have used a calculation model and a calculation algorithm that allowed the definition of kinematic parameters of the mechanism, including crank displacements, angular velocities and acceleration, angular speeds and acceleration. All calculations were performed using a few grapho-analytical (classical) application methods and the Mathcad mathematical package. The results of the calculations are reported as numerical values and graphic presentations.

Keywords: *crank mechanism, grapho-analytival method, instant center of rotation, Mathcad software, velocity distribution.*

Introduction

Kinematics study in this paper will be performed regarding crank mechanism (Figure 1) with following initial data:

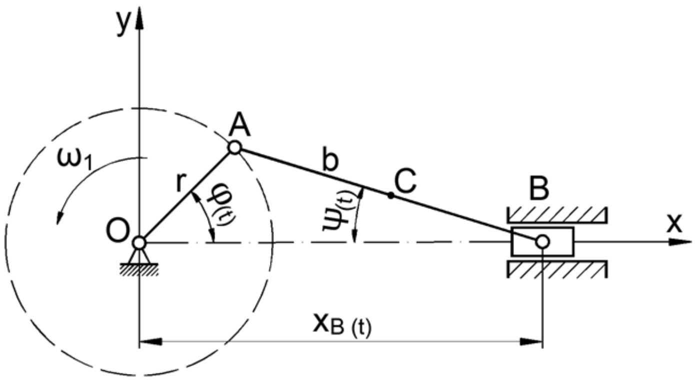

Figure 1. Studied crank mechanism.

- crank angular speed $\omega_1 = 31{,}41\,[\mathrm{s}^{-1}] = \mathrm{constant}$;
- the lengths $OA = r = 0{,}25[m]$; $AB = b = 0{,}5[m]$; $AC = 0{,}3[m]$.

To determine velocity distribution we will use several classical methods like *rotation instantaneous center method*, *the revers method*, *the projection method*, *the velocity plan and the vector equations method* at certain mechanism position when $\varphi = 30°$.

The rotation instantaneous center method

As you can see (Figure 1), point A performs rotational motion around the axis O with angular speed ω_1. In this case we can determine velocity using "Eq.(1)" [2, 4]:

$$(\perp OA, \to\omega_1):\ v_A = \omega_1 \cdot OA\ [m/s] \tag{1}$$

$$v_A = 31{,}41[s^{-1}]\cdot 0.25[m] = 7{,}85[m/s].$$

To apply *rotation instantaneous center method* we draw the mechanism (Figure 2) with scale factor $\mu_l = 0.01\left[\dfrac{m}{mm}\right]$.

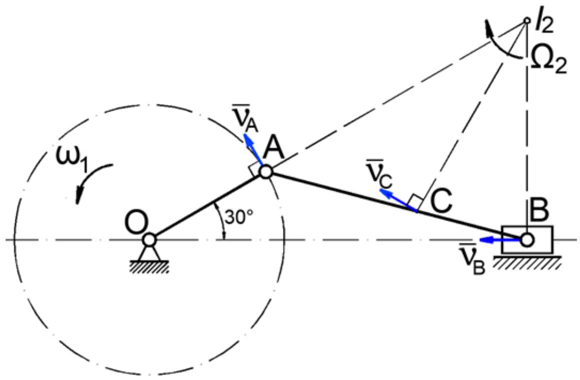

Figure 2. The rotation instantaneous center method.

Because the piston performs translational motion along the horizontal axis Ox it results $\bar{v}_B // Ox$. In order to obtain the instantaneous rotation center *I₂* we draw perpendicular line to the speed carrier $\bar{v}_A$ and $\bar{v}_B$(Figure 2).

Using "Eq.(1)" for rotational motion we can determine angular velocity Ω_2 around *I₂*:

$$\Omega_2 = \frac{v_A}{I_2A} = \omega_1\frac{OA}{I_2A} = 31{,}41[s^{-1}]\cdot\frac{25[mm]}{56[mm]} \cong 14[s^{-1}],$$

and

$$v_B = \Omega_2 \cdot I_2B = \omega_1 \cdot OA \cdot \frac{I_2B}{I_2A} = 31{,}41\ [s^{-1}]\cdot 0{,}25[m]\cdot\frac{41[mm]}{56[mm]} \cong 5{,}75[m/s].$$

Likewise we can obtain the velocity of the C point:

$$v_C = \Omega_2 \cdot I_2C = \omega_1 \cdot OA \cdot \frac{I_2C}{I_2A} = 31{,}41[s^{-1}]\cdot 0.25\ [m]\cdot\frac{41[mm]}{56[mm]} \cong 5{,}75[\text{m/s}].$$

The reverse method

Velocity of point *A* is perpendicular to *OA* and the module is equal to $v_A = \omega_1 \cdot OA$, and point *B* velocity is parallel to Ox axis. If we reverse (rotate) velocity $\bar{v}_A$ with $\dfrac{\pi}{2}$ (clockwise) we will obtain point *A'* (Figure 3) [2,6]. From point *A'* we draw a parallel line with *AB* till point *B'*. Next we reverse the *BB'* segment with $\frac{\pi}{2}$ counterclockwise and we get the velocity $\bar{v}_B$ at the scale.

To get the velocity of the *C* point, first we draw the rotation instantaneous center I_2 as in the example above and obtain *CC'* segment, which also has to be reversed with $\frac{\pi}{2}$ counterclockwise and we get the velocity $\bar{v}_C$ at the scale (Figure 3).

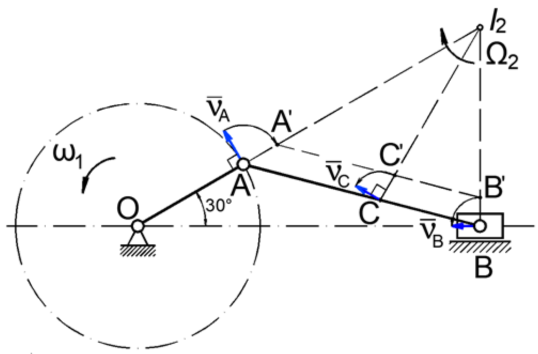

Figure 3. The reverse method.

The projection method

This method supposes that we know everything about point A velocity (module, direction, sense). From head vector $\bar{v}_A$ we take a perpendicular and get the projection *AA'* (Figure 4) [4,6]. Projections *BB'* is equal to projection *AA'*, from point *B'* we draw a perpendicular line to *Ox* axis and obtain head of the vector $\bar{v}_B$. Using the projection and the collinearity theorem we obtain the head of vector $\bar{v}_C$.

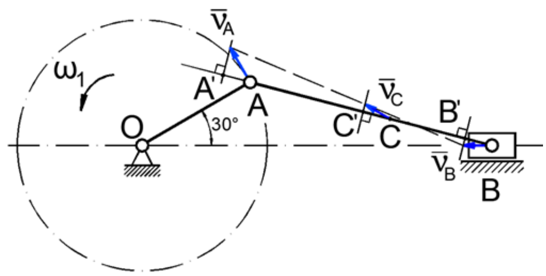

Figure 4. The projection method.

The velocity plan and the vector equations method

This method is a grafo-analytical method and it is based on Euler's velocities equations for plane-parallel motion "Eq.(2)" [4, 6]:

$$\bar{v}_B = \bar{v}_A + \bar{v}_{BA} \tag{2}$$

where $\bar{v}_{BA} = \bar{\omega} \times \overline{AB}$ is point *B* velocity towards point *A*. If we imagine that point A is fixed and point B is released by the piston, in this case we get a rotational motion of point B around A and for rotation motion we know $\bar{v}_{BA} \perp AB$.

The different velocities are represented in an arbitrary plan as vectors, with the modules reduced to the scale coefficient, *velocity scale coefficient* μ_V. In this plane, called *the velocity plan*, the null speed point is called *the velocity pole* and is marked with *p*.

In the velocity plans, relationships like "Eq.(2)" are used, which are vector equations and it is solved graphically by constructing the velocity plan (Figure 5,b). Further using this method we will determine the velocity distribution for the crank mechanism for a certain mechanism position $\varphi = 30^\circ$ (Figure 5, a).

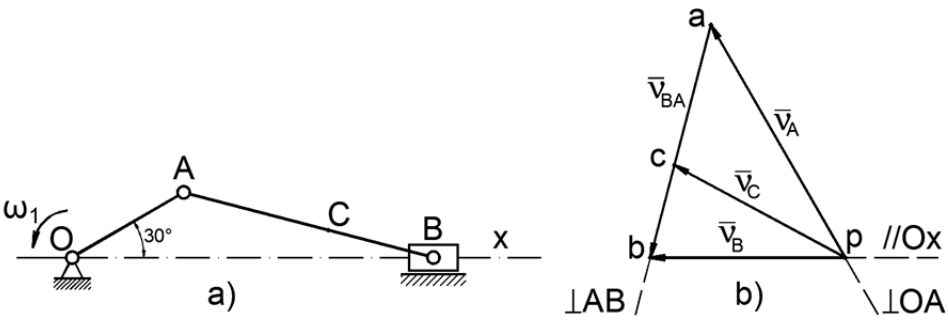

Figure 5. The velocity plan and the vector equations method.

To apply the velocity plan and the vector equations method we draw the mechanism (Figure 5, a) with scale factor $\mu_l = 0.01\left[\frac{m}{mm}\right]$.

First of all, like in the rotation instantaneous center method we will determine point *A* velocity $v_A = 31{,}41[s^{-1}] \cdot 0.25[m] = 7{,}85[m/s]$.

To start constructing velocity plan, firstly we adopt velocity scale coefficient μ_V, so that velocity $\bar{v}_A$ does not exceed 50[mm] in velocity plane (Figure 5, b):

$$\mu_V = \frac{v_A}{50[mm]} = \frac{7{,}85[m/s]}{50[mm]} \cong 0{,}15\left[\frac{m/s}{mm}\right].$$

The next step, we pass the point *A* velocity through the chosen scale coefficient μ_V:

$$\overline{pa} = \frac{v_A}{\mu_V} = \frac{7{,}85[m/s]}{0{,}15\left[\frac{m/s}{mm}\right]} \cong 52\ [mm].$$

Now we can start velocity plan construction from pole *p* (Figure 5, b). Because $\overline{pa} = \bar{v}_A = 52[mm]$ and we know that $\bar{v}_A(\perp OA, \rightarrow \omega_1)$, from point *p* we draw perpendicular line to *OA* and deploy 52[mm] regarding ω_1 direction and obtain point *a*. Next we can determine point *B* velocity using a grafo-analytical method and Euler's velocities equations for plane-parallel motion "Eq.(2)", when we consider that point *B* moves against point *A*:

$$B \rightarrow A:\ (//\,Ox)\ \underline{\bar{v}}_B = \underline{\underline{\bar{v}}}_A + \underline{\bar{v}}_{BA}\ (\perp AB),$$

now we are transposing the vector equation into the velocity plane, from velocity pole *p* we draw a parallel line to *Ox* and from point *a* perpendicular one to *AB*, at the intersection of these lines we obtain point *b* (Figure 5, b).

To determine point *C* velocity we will use theory of similarity and and we will write the similarity report:

$$\frac{AC}{AB} = \frac{ac}{ab} \Rightarrow ac = ab\frac{AC}{AB} = 46[mm]\frac{30[mm]}{50[mm]} \cong 28[mm].$$

After the velocity plan construction (Figure 5, b) you can simply determine velocities:

$$v_{BA} = ab \cdot \mu_V = 47[mm] \cdot 0{,}15\left[\frac{m/s}{mm}\right] = 7{,}05[m/s].$$

Velocities distribution using MathCad software

As initial input we will consider the same crank mechanism (Figure 1), with similar initial data like in Introduction above. In the MathCad environment first we write the equation with which we will determine the velocities and parameters that interest us [1, 3].

Angle φ variation over time *t*:

$$\varphi(t) := \omega_1 \cdot t. \tag{3}$$

Point *A* position variation over time *t*:

$$Y_A(t) := r \cdot \sin(\varphi(t)). \tag{4}$$

Angle ψ variation over time *t*:

$$\psi(t) := \operatorname{atan}\left(\frac{Y_A(t)}{b}\right). \tag{5}$$

Poin *B* motion law over time *t* (Figure 6) [3, 5]:

$$X_B(t) := r \cdot \cos(\varphi(t)) + b \cdot \cos(\psi(t)), \quad t := 0s,\ 0.1s,\ \ldots 2s. \tag{6}$$

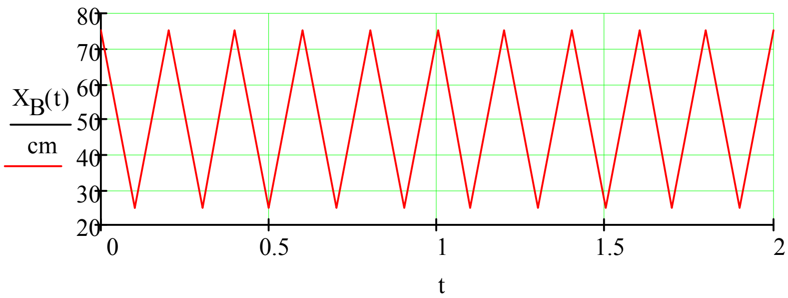

Figure 6. Point *B* motion graph obtained in MathCad.

Determination of point *B* velocity (Figure 7) [3,5]:

$$V_B(t) := \frac{d}{dt} X_B(t). \tag{7}$$

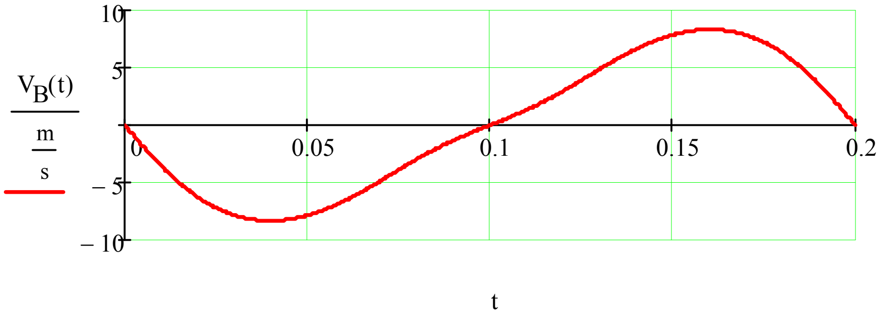

Figure 7. Point *B* velocity graph obtained in MathCad.

Determination of point *B* acceleration (Figure 8) [3, 5]:

$$a_B(t) := \frac{d}{d(t)} V_B(t). \tag{8}$$

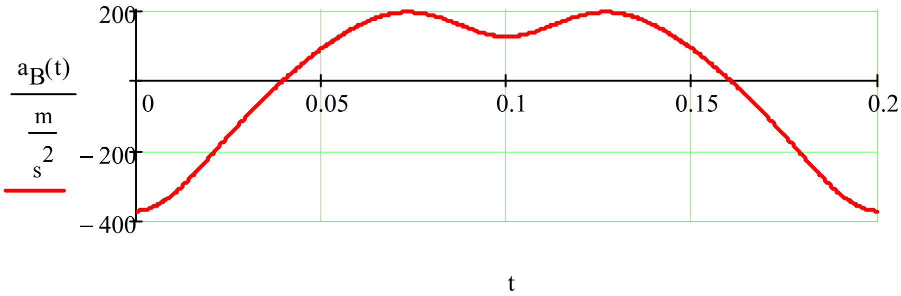

Figure 8. Point *B* acceleration graph obtained in MathCad.

Point *C* trajectory determination (Figure 9) [3, 5]:

$$X_C(t) := r \cdot \cos(\varphi(t)) + \frac{b}{2} \cdot \cos(\psi(t));\ Y_C := \frac{b}{2} \cdot \sin(\psi(t)). \tag{9}$$

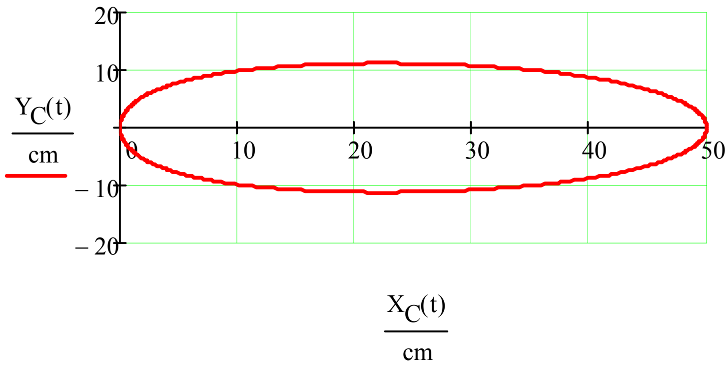

Figure 9. Point *C* trajectory obtained in MathCad.

Conclusions

After we perform kinematic study of crank mechanism with both classical method and modern one, we can conclude that using MathCad software we obtain more exact and concluding result, but in this case we have to write the proper and correct equations. For future work, we intend to demonstrate the veracity of the results obtained in Mathcad with the results obtained by the grapho-analytical way.

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