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# On Multi-Criteria Decision Making problem via Bipolar Single-Valued Neutrosophic Settings

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**Abstract**. In this paper, the idea bipolar single-valued neutrosophic (BSVN) set was introduced. We also introduce bipolar single-valued neutrosophic topological space and some of its properties were characterized. Comparing Bipolar single-valued neutrosophic sets with score function, certainty function and accuracy function .Bipolar single-valued neutrosophic weighted average operator ($A_\omega$) and bipolar single-valued neutrosophic weighted geometric operator ($G_\omega$) were developed and based on Bipolar single-valued neutrosophic set, a multiple decision making problem were evaluated through an example to select the desirable one.

**Keywords:** Bipolar single-valued neutrosophic set, bipolar single-valued neutrosophic topological space, bipolar single-valued neutrosophic average operator, bipolar single-valued neutrosophic geometric operator, score, certainty and accuracy functions.

## 1. Introduction

Fuzzy Logic resembles the human decision making methodology.Zadeh [39] who was considered as the Father of Fuzzy Logic introduced the fuzzy sets in 1965 and it is a tool in learning logical subject. He put forth the concept of fuzzy sets to deal with contrasting types of uncertainties. Using single value $\mu_A(x)\in[0, 1]$ , the degree of membership of the fuzzy set is in classic fuzzy, which is defined on a universal scale, they cannot grasp convinced cases where it is hard to define $\mu_A$ by one specific value.

Intuitionistic fuzzy sets which was proposed by Atanassov [2] is the extension of Zadeh's Fuzzy Sets to overthrown the lack of observation of non-membership degrees. Intuitionistic fuzzy sets generally tested in solving multi-criteria decision making problems. Intuitionistic fuzzy sets detailed into the membership degree, non-membership degree and simultaneously with degree of indeterminancy.

Neutrosophic is the base for the new mathematical theories derives both their classical and fuzzy depiction. Smarandache [4,5] introduced the neutrosophic set . Neutrosophic set has the capability to induce classical sets, fuzzy set, Intuitionistic fuzzy sets. Introduceing the components of the neutrosophic set are True(T), Indeterminacy(I), False(F) which represent the membership, indeterminacy, and non-membership values respectively.The notion of classical set, fuzzy set [17], interval-valued fuzzy set [39], Intuitionistic fuzzy [2], etc were generalized by the neutrosophic set. Majumdar & Samant [19] recommended the Single-valued neutrosophic sets (SVNSs), which is a variation of Neutrosophic Sets. Wang, et.al [38] describe an example of neutrosophic set and sgnify single valued Neutrosophic set (SVNs).They give many properties of Single-Valued Neutrosophic Set, which are associated to the operations and relations by Single-Valued Neutrosophic Sets.The correlation coefficient of SVNSs placed on the development of the correlation coefficient of Intuitionistic fuzzy sets and tested that the cosine similarity measure of SVNS is a special case of the correlation coefficient and correlated it to single valued neutrosophic multicriteria decision-making problems which was presented by Jun Ye [7]. For solving multi-criteria decision-making problems, he overworked similarity measure for interval valued neutrosophic set. Single valued neutrosophic sets (SVNSs) can handle the undetermined and uncertain information and also symbolize, which fuzzy sets and Intuitionistic fuzzy sets cannot define and finalize.

Turksen [37] proposed the Interval-valued fuzzy set is similar as Intuitionistic fuzzy set. The concept is to hook the anxiety of class of membership . Interval- valued fuzzy set need an interval value $[\mu_A^L(a), \mu_A^U(a)]$ with $0\leq\mu_A^L(a)\leq\mu_A^U(a)\leq1$ to represent the class of membership of a fuzzy set A. But it is not suffient to take only the membership function, but also to have the non-membership function .

Bipolar fuzzy relations was given by Bosc and Pivert [3] where a pair of satisfaction degrees is made with each tuple. In 1994, an development of fuzzy set termed bipolar fuzzy was given by Zhang [40].By the notion of fuzzy sets, Lee [16] illustrate bipolar fuzzy sets. Manemaran and Chellappa [20] provide some applications in groups are called the bipolar fuzzy groups, fuzzy d-ideals of groups under (T-S) norm. They also explore few properties of the groups and the relations. Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras were researched by K. J. Lee[17]. Multiple attribute decision-making method situated on single-valued neutrosophic was granted by P. Liu and Y. Wang[18].

In bipolar neutrosophic environment, bipolar neutrosophic sets(BNS) was developed by Irfan Deli [6] and et.al. The application based on multi-criteria decision making problems were also given by them in bipolar neutrosophic set. To collect bipolar neutrosophic information, they defined score, accuracy, and certainty functions to compare BNS and developed bipolar neutrosophic weighted average (BNWA) and bipolar neutrosophic weighted geometric (BNWG) operators. In the study, a Multi Criteria Decision Making approach were discussed on the basis of score, accuracy, and certainty functions, bipolar Neutrosophic Weighted Average and bipolar Neutrosophic Weighted Geometric operators were calculated. Fuzzy neutrosophic sets and its Topological spaces was introduced by I.Arockiarani and J.Martina Jency [1].

Positive and Negative effects count on Decision making . Multiple decision-making problems have gained very much attention in the area of systemic optimization, urban planning, operation research, management science and many other fields. Correlation Coefficient between Single Valued Neutrosophic Sets and its Multiple Attribute Decision Making Method given by Jun Ye [7]. A Neutrosophic Multi-Attribute Decision making with Unknown Weight data was investigated by Pranab Biswas, Surapati Pramanik, Bibhas C. Giri[30]**.** Neutrosophic Tangent Similarity Measure and its Application was given by Mondal, Surapati Pramanik [11]. Many of the authors[8-14,21,22,24-29,31,32,33,35,36] studied and examine different and variation of neutrosophic set theory in Decision making problems.

Here, we introduce bipolar single-valued neutrosophic set which is an expansion of the fuzzy sets, Intuitionistic fuzzy sets, neutrosophic sets and bipolar fuzzy sets. Bipolar single-valued neutrosophic topological spaces were also proposed. Bipolar single-valued neutrosophic topological spaces characterized a few of its properties and a numerical example were illustrated. Bipolar single-valued neutrosophic sets were compared with score function, certainty function and accuracy function. Then,the bipolar single-valued Neutrosophic weighted average operator ($A_\omega$) and bipolar single-valued neutrosophic weighted geometric operator ($G_\omega$) are developed to aggregate the data.To determine the application and the performance of this method to choose the best one, atlast a numerical example of the method was given.

## 2 Preliminaries

**2.1 Definition [34]:** Let $X$ be a non-empty fixed set. $A$ neutrosophic set B is an object having the form $B=\{<x,\mu_B(x),\sigma_B(x),\gamma_B(x)>x\in X\}$ Where $\mu_B(x),\sigma_B(x)$ and $\gamma_B(x)$ which represent the degree of membership function , the degree of indeterminacy and the degree of non-membership respectively of each element $x\in X$ to the set B .

**2.2 Definition [38]:** Let a universe X of discourse. Then $A_{NS}=\{<x,F_A(x),T_A(x)I_A(x)>x\in X\}$ defined as a single-valued neutrosophic set where truth-membership function $T_A:X\rightarrow[0,1]$,an indeterminacy-membership function $I_A$: $X\rightarrow[0,1]$ and a falsity-membership function $F_A$: $X \rightarrow [0,1]$.No restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0\le\sup T_A(x) \le\sup I_A(x) \le\sup F_A(x) \le 3$. $\tilde{A}=<T, I, F>$ is denoted as a single-valued neutrosophic number.

**2.3 Definition [23]:** Let two single-valued neutrosophic number be $\tilde{A}_1=<T_1, I_1, F_1>$ and $\tilde{A}_2=<T_2, I_2, F_2>$ . Then,the operations for NNs are defined as follows:

i. $\lambda \tilde{A} =<1-(1-T_1)^\lambda,I_1^\lambda,F_1^\lambda>$
ii. $\tilde{A}_1^\lambda =<(T_1^\lambda,1-(1-I_1)^\lambda,1-(1-F_1)^\lambda>$
iii. $\tilde{A}_1+\tilde{A}_2=<T_1+T_2-T_1T_2,I_1I_2,F_1F_2>$
iv. $\tilde{A}_1.\tilde{A}_2=<T_1T_2,I_1+I_2-I_1I_2,F_1+F_2-F_1F_2>$

**2.4 Definition [15]:** Let a single-valued neutrosophic number be $\tilde{B}_1=<T_1, I_1, F_1>$. Then, SNN are defined as

i. score function $s(\tilde{B}_1) = (T_1+1- I_1+ 1-F_1)/3$;
ii. accuracy function $a(\tilde{B}_1) = T_1-F_1$;
iii. certainty function $c(\tilde{B}_1) = T_1$.

**2.5 Definition [23]:** Let two single-valued neutrosophic number be $\tilde{B}_1=<T_1, I_1, F_1>$ and $\tilde{B}_2=<T_2, I_2, F_2>$. The comparison method defined as:

i. if $s(\tilde{B}_1) > s(\tilde{B}_2)$, then $\tilde{B}_1$ is greater than $\tilde{B}_2$, that is, $\tilde{B}_1$ is superior to $\tilde{B}_2$, denoted by $\tilde{B}_1>\tilde{B}_2$.
ii. if $s(\tilde{B}_1) = s(\tilde{B}_2)$ and $a(\tilde{B}_1)>a(\tilde{B}_2)$, then $\tilde{B}_1$ is greater than $\tilde{B}_2$, that is, $\tilde{B}_1$ is superior to $\tilde{B}_2$, denoted by

$\tilde{B}_1 < \tilde{B}_2$.

iii.if s($\tilde{B}_1$) = s($\tilde{B}_2$) and a($\tilde{B}_1$)=a($\tilde{B}_2$) and c($\tilde{B}_1$) >c($\tilde{B}_2$), then $\tilde{B}_1$ is greater than $\tilde{B}_2$, that is, $\tilde{B}_1$ is superior to $\tilde{B}_2$, denoted by $\tilde{B}_1 > \tilde{B}_2$.

iv.if s($\tilde{B}_1$) = s($\tilde{B}_2$) and a($\tilde{B}_1$)=a($\tilde{B}_2$) and c($\tilde{B}_1$)=c($\tilde{B}_2$), then $\tilde{B}_1$ is equal to $\tilde{B}_2$, that is, $\tilde{B}_1$ is indifferent to $\tilde{B}_2$, denoted by $\tilde{B}_1 = \tilde{B}_2$.

**2.6 Definition [6]:** In *X*, a bipolar neutrosophic set B is defined in the form

$$B=<x, (T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)):x\in X>$$

Where $T^+$, $I^+$, $F^+$: X→ [1, 0] and $T^-$ , $I^-$ , $F^-$ : X [-1, 0].The positive membership degree denotes the truth membership $T^+(x)$, indeterminate membership $I^+$ (x) and false membership $F^+$ (x) of an element x∈X corresponding to the set *A* and the negative membership degree denotes the truth membership $T^-(x)$, indeterminate membership $I^-(x)$ and false membership $F^-(x)$ of an element x∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set .

**2.7 Definition [39, 2]:** Each element had a degree of membership (T) in the fuzzy set . The Intuitionistic fuzzy set on a universe, where the degree of membership $\mu_B(x) \in[0,1]$ of each element x∈X to a set B, there was a degree of non-membership $\nu_B(x)\in[0,1]$,such that $\forall$ x∈X, $\mu_B(x) + \nu_B(x) \le 1$.

**2.8 Definition [15, 20]:** Let a non-empty set be X. Then, $B_{BF}$= {<x, $\mu^+_B(x)$, $\mu^-_B(x)$>: x∈X}is a bipolar-valued fuzzy set denoted by $B_{BF}$ ,where $\mu^+_B$: X → [0, 1] and $\mu^-_B$: X → [0, 1]. The positive Membership degree $\mu^+_B(x)$ denotes the satisfaction degree of an element x to the property corresponding to $B_{BF}$ and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of x to some implicit counter property of $B_{BF}$ .

In this section, we give the concept bipolar single-valued neutrosophic set and its operations. We also developed the bipolar single-valued neutrosophic weighted ($A_\omega$) average operator and geometric operator ($G_\omega$). Some of it is quoted from [2, 5, 7, 10, and 14].

## 3. Bipolar single-valued Neutrosophic set(BSVN):

**3.1 Definition :** A Bipolar Single-Valued Neutrosophic set (BSVN) S in X is defined in the form of

$$\mathbf{BSVN\ (S) =<v,(T_{BSVN}^+,T_{BSVN}^-),(I_{BSVN}^+,I_{BSVN}^-),(F_{BSVN}^+,F_{BSVN}^-):v\in X>}$$

where $(T_{BSVN}^+,I_{BSVN}^+,F_{BSVN}^+)$:X→[0,1] and $(T_{BSVN}^-,I_{BSVN}^-,F_{BSVN}^-)$:X→ [-1,0] .In this definition, there $T_{BSVN}^+$ and $T_{BSVN}^-$ are acceptable and unacceptable in past. Similarly $I_{BSVN}^+$ and $I_{BSVN}^-$ are acceptable and unacceptable in future. $F_{BSVN}^+$ and $F_{BSVN}^-$ are acceptable and unacceptable in present respectively.

**3.2 Example** : Let X={$s_1$,$s_2$,$s_3$}.Then a bipolar single-valued neutrosophic subset of *X* is

$$S=\begin{Bmatrix} <s_1,(0.1,-0.1),(0.2,-0.3),(0.3,-0.5)> \\ <s_2,(0.2,-0.3),(0.4,-0.4),(0.6,-0.5)> \\ <s_3,(0.2,-0.8),(0.6,-0.4),(0.7,-0.7)> \end{Bmatrix}$$

**3.3 Definition :** Let two bipolar single-valued neutrosophic sets $BSVN_1(S)$ and $BSVN_2(S)$ in X defined as $BSVN_1(S)=<v,(T_{BSVN}^+(1),T_{BSVN}^-(1)),(I_{BSVN}^+(1),I_{BSVN}^-(1)),(F_{BSVN}^+(1),F_{BSVN}^-(1)):v\in X>$ and $BSVN_2(S)=<v,(T_{BSVN}^+(2),T_{BSVN}^-(2)),(I_{BSVN}^+(2),I_{BSVN}^-(2)),(F_{BSVN}^+(2),F_{BSVN}^-(2)):v\in X>$. Then the operators are defined as follows:

**(i) Complement**

$BSVN^c(S)$ = {< v,(1-$T_{BSVN}^+$), (-1-$T_{BSVN}^-$), (1-$I_{BSVN}^+$), (-1-$I_{BSVN}^-$), (1-$F_{BSVN}^+$), (-1-$F_{BSVN}^-$): v∈X >}

**(ii) Union of two BSVN**

$BSVN_1(S)$ U $BSVN_2(S)$ =

$$\left\langle \begin{matrix} \max(T^+_{BSVN}(1),T^+_{BSVN}(2)),\min(I^+_{BSVN}(1),I^+_{BSVN}(2)),\min(F^+_{BSVN}(1),F^+_{BSVN}(2)) \\ \max(T^-_{BSVN}(1),T^-_{BSVN}(2)),\min(I^-_{BSVN}(1),I^-_{BSVN}(2)),\min(F^-_{BSVN}(1),F^-_{BSVN}(2)) \end{matrix} \right\rangle$$

**(iii) Intersection of two BSVN**

BSVN$_1$(S) ∩BSVN$_2$(S) =

$$\left\langle \begin{array}{l} \min(T^+_{BSVN}(1),T^+_{BSVN}(2)),\max(I^+_{BSVN}(1),I^+_{BSVN}(2)),\max(F^+_{BSVN}(1),F^+_{BSVN}(2)) \\ \min(T^-_{BSVN}(1),T^-_{BSVN}(2)),\max(I^-_{BSVN}(1),I^-_{BSVN}(2)),\max(F^-_{BSVN}(1),F^-_{BSVN}(2)) \end{array} \right\rangle$$

**3.4 Example :** Let X={s$_1$,s$_2$,s$_3$}.Then the bipolar single-valued neutrosophic subsets S$_1$ and S$_2$ of *X* ,

$$S_1=\left\{ \begin{array}{l} <s_1,(0.1,-0.1),(0.2,-0.3),(0.3,-0.5)> \\ <s_2,(0.2,-0.3),(0.4,-0.4),(0.6,-0.5)> \\ <s_3,(0.2,-0.8),(0.6,-0.4),(0.7,-0.7)> \end{array} \right\} \text{ and } S_2=\left\{ \begin{array}{l} <s_1,(0.2,-0.1),(0.3,-0.5),(0.4,-0.5)> \\ <s_2,(0.3,-0.3),(0.3,-0.5),(0.4,-0.6)> \\ <s_3,(0.5,-0.3),(0.6,-0.3),(0.8,-0.7)> \end{array} \right\}$$

(i) Complement of S$_1$ is $S_1^c=\left\{ \begin{array}{l} <s_1,(0.9,-0.9),(0.8,-0.7),(0.7,-0.5> \\ <s_2,(0.8,-0.7),(0.6,-0.6),(0.4,-0.5> \\ <s_3,(0.8,-0.2),(0.4,-0.6),(0.3,-0.3> \end{array} \right\}$

(ii) Union of S$_1$ and S$_2$ is S$_1$US$_2$= $\left\{ \begin{array}{l} <s_1,(0.2,-0.1),(0.2,-0.5),(0.3,-0.5> \\ <s_2,(0.3,-0.3),(0.3,-0.5),(0.4,-0.6)> \\ <s_3,(0.5,-0.3),(0.6,-0.4),(0.7,-0.7)> \end{array} \right\}$

(iii) Intersection of S$_1$ and S$_2$ is S$_1$∩S$_2$= $\left\{ \begin{array}{l} <s_1,(0.1,-0.1),(0.3,-0.3),(0.4,-0.5)> \\ <s_2,(0.2,-0.3),(0.4,-0.4),(0.6,-0.5)> \\ <s_3,(0.2,-0.8),(0.6,-0.3),(0.8,-0.7)> \end{array} \right\}$

**3.5 Definition :** Let two bipolar single-valued neutrosophic sets be BSVN$_1$(S) and BSVN$_2$(S) in X defined as BSVN$_1$(S)=<v,($T_{BSVN}^+(1)$,$T_{BSVN}^-(1)$),($I_{BSVN}^+(1)$,$I_{BSVN}^-(1)$),($F_{BSVN}^+(1)$,$F_{BSVN}^-(1)$):v∈X> and BSVN$_2$(S)=<v, ($T_{BSVN}^+(2)$,$T_{BSVN}^-(2)$),($I_{BSVN}^+(2)$,$I_{BSVN}^-(2)$), ($F_{BSVN}^+(2)$,$F_{BSVN}^-(2)$):v∈X>.

Then S$_1$=S$_2$ if and only if

$T_{BSVN}^+(1) = T_{BSVN}^+(2)$ , $I_{BSVN}^+(1) = I_{BSVN}^+(2)$ , $F_{BSVN}^+(1) = F_{BSVN}^+(2)$ ,
$T_{BSVN}^-(1) = T_{BSVN}^-(2)$ , $I_{BSVN}^-(1) = I_{BSVN}^-(2)$ , $F_{BSVN}^-(1) = F_{BSVN}^-(2)$ for all v∈X.

**3.6 Definition :** Let two bipolar single-valued neutrosophic sets be BSVN$_1$ and BSVN$_2$ in X defined as BSVN$_1$(S)=<v,($T_{BSVN}^+(1)$,$T_{BSVN}^-(1)$),($I_{BSVN}^+(1)$,$I_{BSVN}^-(1)$),($F_{BSVN}^+(1)$,$F_{BSVN}^-(1)$):v∈X> and BSVN$_2$(S)=<v, ($T_{BSVN}^+(2)$,$T_{BSVN}^-(2)$),($I_{BSVN}^+(2)$,$I_{BSVN}^-(2)$), ($F_{BSVN}^+(2)$,$F_{BSVN}^-(2)$):v∈X>.

Then S$_1$⊆S$_2$ if and only if

$T_{BSVN}^+(1) \le T_{BSVN}^+(2)$ , $I_{BSVN}^+(1) \ge I_{BSVN}^+(2)$ , $F_{BSVN}^+(1) \ge F_{BSVN}^+(2)$ ,
$T_{BSVN}^-(1) \le T_{BSVN}^-(2)$ , $I_{BSVN}^-(1) \ge I_{BSVN}^-(2)$ , $F_{BSVN}^-(1) \ge F_{BSVN}^-(2)$ for all v∈X.

**4. Bipolar single-valued Neutrosophic Topological space:**

**4.1 Definition :** A bipolar single-valued neutrosophic topology on a non-empty set X is a τ of BSVN sets satisfying the axioms

(i) $0_{BSVN}$ , $1_{BSVN}$ ∈ τ
(ii) S$_1$ ∩ S$_2$ ∈ τ for any S$_1$,S$_2$ ∈ τ
(iii) US$_i$ ∈ τ for any arbitrary family {S$_i$ :i∈ j} ∈ τ

The pair (X,τ) is called BSVN topological space. Any BSVN set in τ is called as BSVN open set in X. The complement $S^c$ of BSVN set in BSVN topological space (X,τ) is called a BSVN closed set.

**4.2 Definition :** Null or Empty bipolar single-valued neutrosophic set of a Bipolar single-valued Neutrosophic set S over X is said to be if <v, (0, 0), (0, 0), (0, 0)> for all v∈X and it is denoted by $0_{BSVN}$.

**4.3 Definition :** Absolute Bipolar single-valued neutrosophic set denoted by $1_{BSVN}$ of a Bipolar single-valued Neutrosophic set S over X is said to be if <v, (1,-1), (1,-1), (1,-1)> for all v∈X.

**4.4 Example :** Let X={$s_1$,$s_2$,$s_3$} and τ= {$0_{BSVN}$,$1_{BSVN}$,P,Q,R,S}Then a bipolar single-valued neutrosophic subset of *X* is

$$P=\begin{Bmatrix} < s_1,(0.3,-0.5),(0.4,-0.2),(0.5,-0.3) > \\ < s_2,(0.3,-0.6),(0.7,-0.1),(0.4,-0.4) > \\ < s_3,()0.2,-0.7,(0.4,-0.3),(0.4,-0.1) > \end{Bmatrix} \quad Q=\begin{Bmatrix} < s_1,(0.5,-0.2),(0.5,-0.2),(0.3,-0.2) > \\ < s_2,(0.3,-0.4),(0.4,-0.2),(0.4,-0.2) > \\ < s_3,(0.3,-0.2),(0.4,-0.3),(0.4,-0.4) > \end{Bmatrix}$$

$$R=\begin{Bmatrix} < s_1,(0.5,-0.2),(0.4,-0.2),(0.3,-0.3) > \\ < s_2,(0.3,-0.4),(0.4,-0.2),(0.4,-0.4) > \\ < s_3,(0.3,-0.2),(0.4,-0.3),(0.4,-0.4) > \end{Bmatrix} \quad S=\begin{Bmatrix} < s_1,(0.3,-0.5),(0.5,-0.2),(0.5,-0.2) > \\ < s_2,(0.3,-0.6),(0.7,-0.1),(0.4,-0.2) > \\ < s_3,(0.2,-0.7),(0.4,-0.3),(0.4,-0.1) > \end{Bmatrix}$$

Then (X,τ) is called BSVN topological space on X.

**4.5 Definition :** Let (X,τ) be a BSVN topological space and
BSVN (S) =<v, ($T_{BSVN}^+$,$T_{BSVN}^-$), ($I_{BSVN}^+$,$I_{BSVN}^-$),($F_{BSVN}^+$,$F_{BSVN}^-$):v∈X> be a BSVN set in X. Then the closure and interior of A is defined as

Int (S) = U {F: F is a BSVN open set (BSVNOs) in X and F⊆S}

Cl (S) = ∩ {F: F is a BSVN closed set (BSVNCs) in X and S⊆F}.

Here cl(S) is a BSVNCs and int (S) is a BSVNOs in X.

(a) S is a BSVNCs in X iff cl (S) =S.
(b) S is a BSVNOs in X iff int (S) =S.

**4.6 Example :** Let X={$s_1$,$s_2$,$s_3$} and τ={$0_{BSVN}$,$1_{BSVN}$,P,Q,R,S}. Then a bipolar single-valued neutrosophic subset of *X* is

$$P=\begin{Bmatrix} < s_1,(0.3,-0.5),(0.4,-0.2),(0.5,-0.3) > \\ < s_2,(0.3,-0.6),(0.7,-0.1),(0.4,-0.4) > \\ < s_3,(0.2,-0.7),(0.4,-0.3),(0.4,-0.1) > \end{Bmatrix} Q=\begin{Bmatrix} < s_1,(0.5,-0.2),(0.5,-0.2),(0.3,-0.2) > \\ < s_2,(0.3,-0.4),(0.4,-0.2),(0.4,-0.2) > \\ < s_3,(0.3,-0.2),(0.4,-0.3),(0.4,-0.4) > \end{Bmatrix}$$

$$R=\begin{Bmatrix} < s_1,(0.5,-0.2),(0.4,-0.2),(0.3,-0.3) > \\ < s_2,(0.3,-0.4),(0.4,-0.2),(0.4,-0.4) > \\ < s_3,(0.3,-0.2),(0.4,-0.3),(0.4,-0.4) > \end{Bmatrix} S=\begin{Bmatrix} < s_1,(0.3,-0.5),(0.5,-0.2),(0.5,-0.2) > \\ < s_2,(0.3,-0.6),(0.7,-0.1),(0.4,-0.2) > \\ < s_3,(0.2,-0.7),(0.4,-0.3),(0.4,-0.1) > \end{Bmatrix}$$

$$T=\begin{Bmatrix} < s_1,0.7,0.3,0.3,-0.5,-0.2,-0.4 > \\ < s_2,0.6,0.6,0.3,-0.3,-0.5,-0.5 > \\ < s_3,0.5,0.2,0.3,-0.5,-0.5,-0.6 > \end{Bmatrix}$$

Then int(T)=P and cl(T)=$1_{BSVN}$.

**4.7 Proposition :** Let BSVNTS of (X,τ) and S,T be BSVN's in X. Then the properties hold:

i. int (S) ⊆ S and S ⊆ cl(S)
ii. S⊆ T ⇒int(S) ⊆ int(T)
 S⊆ T ⇒cl(S) ⊆ cl(T)

iii. $int(int(S))=int(S)$
$cl(cl(S))=cl(S)$
iv. $int(S\cap T)=int(S)\cap int(T)$
$cl(SUT)=cl(S)Ucl(T)$
v. $int(1_{BSVN})=1_{BSVN}$
$cl(0_{BSVN})=0_{BSVN}$

**Proof:** The proof is obvious.

**4.8 Proposition :** Let BSVN sets of $S_i$'s and T in X, then $S_i \subseteq T$ for each $i\in J \Rightarrow$ (a). $US_i \subseteq T$ and (b). $T \subseteq \cap S_i$.
**Proof:** (a).Let $S_i \subseteq B$ (i.e) $S_1 \subseteq B$, $S_2 \subseteq B$,….., $S_n \subseteq B$.

$\Rightarrow \{T_{BSVN}^{+}(S_1) \leq T_{BSVN}^{+}(T)$ , $T_{BSVN}^{-}(S_1) \leq T_{BSVN}^{-}(T)$ , $I_{BSVN}^{+}(S_1) \geq I_{BSVN}^{+}(T)$ , $I_{BSVN}^{-}(S_1) \geq I_{BSVN}^{-}(T)$,
$F_{BSVN}^{+}(S_1) \geq F_{BSVN}^{+}(T)$ , $F_{BSVN}^{-}(S_1) \geq F_{BSVN}^{-}(T)$ ,$T_{BSVN}^{+}(S_2) \leq T_{BSVN}^{+}(T)$ , $T_{BSVN}^{-}(S_2) \leq T_{BSVN}^{-}(T)$ ,
$I_{BSVN}^{+}(S_2)\geq I_{BSVN}^{+}(T)$,$I_{BSVN}^{-}(S_2)\geq I_{BSVN}^{-}(T)$,$F_{BSVN}^{+}(S_2)\geq F_{BSVN}^{+}(T)$,$F_{BSVN}^{-}(S_2)\geq F_{BSVN}^{-}(T)$…...................,
$T_{BSVN}^{+}(S_n)\leq T_{BSVN}^{+}(T)$, $T_{BSVN}^{-}(S_n) \leq T_{BSVN}^{-}(T)$ , $I_{BSVN}^{+}(S_n) \geq I_{BSVN}^{+}(T)$ , $I_{BSVN}^{-}(S_n) \geq I_{BSVN}^{-}(T)$,
$F_{BSVN}^{+}(S_n) \geq F_{BSVN}^{+}(T)$ , $F_{BSVN}^{-}(S_n) \geq F_{BSVN}^{-}(T)$ }

$\Rightarrow \max\{(T_{BSVN}^{+}(S_1), T_{BSVN}^{+}(S_2) ,...,T_{BSVN}^{+}(S_n)),( T_{BSVN}^{-}(S_1) ,T_{BSVN}^{-}(S_2) ,..,T_{BSVN}^{-}(S_n) )\}\leq(T_{BSVN}^{+}(T),T_{BSVN}^{-}(T))$
$\min\{(I_{BSVN}^{+}(S_1) ,I_{BSVN}^{+}(S_2) ,\ldots,I_{BSVN}^{+}(S_n) ), (I_{BSVN}^{-}(S_1) ,I_{BSVN}^{-}(S_2) ,\ldots,I_{BSVN}^{-}(S_n) )\} \geq (I_{BSVN}^{+}(T) ,I_{BSVN}^{-}(T))$
$\min\{(F_{BSVN}^{+}(S_1) ,F_{BSVN}^{+}(S_2) ,\ldots,F_{BSVN}^{+}(S_n)),(F_{BSVN}^{-}(S_1),F_{BSVN}^{-}(S_2) ,\ldots,F_{BSVN}^{-}(S_n) )\} \geq (F_{BSVN}^{+}(T),F_{BSVN}^{-}(T))$

where $UA_i$ = <x, $\max\{(T_{BSVN}^{+}(S_1), T_{BSVN}^{+}(S_2) ,...,T_{BSVN}^{+}(S_n) ),( T_{BSVN}^{-}(S_1) ,T_{BSVN}^{-}(S_2) ,..,T_{BSVN}^{-}(S_n) )\}$
$\min\{(I_{BSVN}^{+}(S_1) ,I_{BSVN}^{+}(S_2) ,\ldots,I_{BSVN}^{+}(S_n) ), (I_{BSVN}^{-}(S_1) ,I_{BSVN}^{-}(S_2) ,\ldots,I_{BSVN}^{-}(S_n) )\}$
$\min\{(F_{BSVN}^{+}(S_1) ,F_{BSVN}^{+}(S_2) ,\ldots,F_{BSVN}^{+}(S_n) ), (F_{BSVN}^{-}(S_1) ,F_{BSVN}^{-}(S_2) ,\ldots,F_{BSVN}^{-}(S_n) )\}$ >
$\therefore$ $U S_i \subseteq T$ .Hence proved.

(b)Let $T\subseteq S_i$ (i.e) $T\subseteq S_1$, $T\subseteq S_2$, … $T\subseteq S_i$.

$\Rightarrow<$ $T_{BSVN}^{+}(T) \leq T_{BSVN}^{+}(S_1)$ , $T_{BSVN}^{-}(T) \leq T_{BSVN}^{-}(S_1)$ , $I_{BSVN}^{+}(T) \geq I_{BSVN}^{+}(S_1)$ , $I_{BSVN}^{-}(T) \geq I_{BSVN}^{-}(S_1)$,
$F_{BSVN}^{+}(T) \geq F_{BSVN}^{+}(S_1)$ , $F_{BSVN}^{-}(T) \geq F_{BSVN}^{-}(S_1)$ , $T_{BSVN}^{+}(T) \leq T_{BSVN}^{+}(S_2)$ , $T_{BSVN}^{-}(T) \leq T_{BSVN}^{-}(S_2)$,
$I_{BSVN}^{+}(T) \geq I_{BSVN}^{+}(S_2)$ , $I_{BSVN}^{-}(T) \geq I_{BSVN}^{-}(S_2)$,$F_{BSVN}^{+}(T) \geq F_{BSVN}^{+}(S_2)$ ,$F_{BSVN}^{-}(T) \geq F_{BSVN}^{-}(S_2)$ ,……………….,
$T_{BSVN}^{+}(T)\leq T_{BSVN}^{+}(S_n)$ , $T_{BSVN}^{-}(T) \leq T_{BSVN}^{-}( S_n)$ ,$I_{BSVN}^{+}(T) \geq I_{BSVN}^{+}( S_n)$ , $I_{BSVN}^{-}(T) \geq I_{BSVN}^{-}( S_n)$,
$F_{BSVN}^{+}(T) \geq F_{BSVN}^{+}(S_n)$ , $F_{BSVN}^{-}(T) \geq F_{BSVN}^{-}( S_n)$ }>

$\Rightarrow(T_{BSVN}^{+}(T),T_{BSVN}^{-}(T)) \leq \min\{(T_{BSVN}^{+}(S_1),T_{BSVN}^{+}(S_2) ,...,T_{BSVN}^{+}(S_n) ),( T_{BSVN}^{-}(S_1) ,T_{BSVN}^{-}(S_2) ,..,T_{BSVN}^{-}(S_n) )\}$
$(I_{BSVN}^{+}(T) ,I_{BSVN}^{-}(T)) \geq \max \{( I_{BSVN}^{+}(S_1) ,I_{BSVN}^{+}(S_2) ,\ldots,I_{BSVN}^{+}(S_n) ), (I_{BSVN}^{-}(S_1) ,I_{BSVN}^{-}(S_2) ,\ldots,I_{BSVN}^{-}(S_n) )\}$
$(F_{BSVN}^{+}(T),F_{BSVN}^{-}(T))\geq \max\{(F_{BSVN}^{+}(S_1) ,F_{BSVN}^{+}(S_2) ,\ldots,F_{BSVN}^{+}(S_n) ),(F_{BSVN}^{-}(S_1) ,F_{BSVN}^{-}(S_2) ,\ldots, F_{BSVN}(S_n))\}$

Where $\cap A_i$= <x, $\min\{(T_{BSVN}^{+}(S_1), T_{BSVN}^{+}(S_2) ,...,T_{BSVN}^{+}(S_n) ),( T_{BSVN}^{-}(S_1) ,T_{BSVN}^{-}(S_2) ,..,T_{BSVN}^{-}(S_n) )\}$
$\max\{(I_{BSVN}^{+}(S_1) ,I_{BSVN}^{+}(S_2) ,\ldots,I_{BSVN}^{+}(S_n) ), (I_{BSVN}^{-}(S_1) ,I_{BSVN}^{-}(S_2) ,\ldots,I_{BSVN}^{-}(S_n) )\}$
$\max\{(F_{BSVN}^{+}(S_1) ,F_{BSVN}^{+}(S_2) ,\ldots,F_{BSVN}^{+}(S_n) ), (F_{BSVN}^{-}(S_1) ,F_{BSVN}^{-}(S_2) ,\ldots,F_{BSVN}^{-}(S_n) )\}$ >
$\therefore$ $T \subseteq \cap S_i$ .Hence proved.

**4.9 Proposition :** Let $S_i$'s and T are BSVN sets in X then (i). $(US_i)^c = \cap S_i^c$ , (ii). $(\cap S_i)^c = US_i^c$ and
(iii). $(S^c)^c = S$.
**Proof:** (i) Let $US_i$ = <x, $\max\{(T_{BSVN}^{+}(S_1), T_{BSVN}^{+}(S_2) ,...,T_{BSVN}^{+}(S_n) ),( T_{BSVN}^{-}(S_1) ,T_{BSVN}^{-}(S_2) ,..,T_{BSVN}^{-}(S_n))\}$
$\min\{(I_{BSVN}^{+}(S_1) ,I_{BSVN}^{+}(S_2) ,\ldots,I_{BSVN}^{+}(S_n) ), (I_{BSVN}^{-}(S_1) ,I_{BSVN}^{-}(S_2) ,\ldots,I_{BSVN}^{-}(S_n) )\}$
$\min\{(F_{BSVN}^{+}(S_1) ,F_{BSVN}^{+}(S_2) ,\ldots,F_{BSVN}^{+}(S_n) ),(F_{BSVN}^{-}(S_1) ,F_{BSVN}^{-}(S_2) ,..,F_{BSVN}^{-}(S_n))\}$ >

$(US_i)^c$ = <x,$\min\{(1-T_{BSVN}^{+}(S_1),1-T_{BSVN}^{+}(S_2) ,..,1-T_{BSVN}^{+}(S_n) ),(-1- T_{BSVN}^{-}(S_1) ,-1-T_{BSVN}^{-}(S_2) ,..,-1- T_{BSVN}^{-}(S_n))\}$
$\max\{(1-I_{BSVN}^{+}(S_1) ,1-I_{BSVN}^{+}(S_2) ,\ldots,1-I_{BSVN}^{+}(S_n) ), (-1-I_{BSVN}^{-}(S_1) ,-1-I_{BSVN}^{-}(S_2) ,\ldots,-1-I_{BSVN}^{-}(S_n))\}$
$\max\{(1-F_{BSVN}^{+}(S_1) ,1-F_{BSVN}^{+}(S_2) ,..,1-F_{BSVN}^{+}(S_n) ), (-1-F_{BSVN}^{-}(S_1) ,-1-F_{BSVN}^{-}(S_2) ,..,-1-F_{BSVN}^{-}(S_n))$ >
------------------>(1)

$S_i^c$ = <x,$(1-T_{BSVN}^{+}(S_1),1- T_{BSVN}^{+}(S_2) ,...,1-T_{BSVN}^{+}(S_n) ),(-1- T_{BSVN}^{-}(S_1) ,-1-T_{BSVN}^{-}(S_2) ,..,-1-T_{BSVN}^{-}(S_n))$
$(1-I_{BSVN}^{+}(S_1) ,1-I_{BSVN}^{+}(S_2) ,\ldots,1-I_{BSVN}^{+}(S_n) ), (-1-I_{BSVN}^{-}(S_1) ,-1-I_{BSVN}^{-}(S_2) ,\ldots,-1-I_{BSVN}^{-}(S_n) )$
$(1-F_{BSVN}^{+}(S_1) ,1-F_{BSVN}^{+}(S_2) ,\ldots,1-F_{BSVN}^{+}(S_n) ), (-1-F_{BSVN}^{-}(S_1) ,-1-F_{BSVN}^{-}(S_2) ,\ldots,-1-F_{BSVN}^{-}(S_n)$>

$\cap S_i^c =$ <x,min {(1-$T_{BSVN}^{+}(S_1)$,1-$T_{BSVN}^{+}(S_2)$ ,...,1-$T_{BSVN}^{+}(S_n)$ ),(-1- $T_{BSVN}^{-}(S_1)$ ,-1-$T_{BSVN}^{-}(S_2)$ ,..,-1-$T_{BSVN}^{-}(S_n)$ )}
max{(1-$I_{BSVN}^{+}(S_1)$ ,1-$I_{BSVN}^{+}(S_2)$ ,…,1-$I_{BSVN}^{+}(S_n)$ ), (-1-$I_{BSVN}^{-}(S_1)$ ,-1-$I_{BSVN}^{-}(S_2)$ ,…,-1-$I_{BSVN}^{-}(S_n)$ )}
max{(1-$F_{BSVN}^{+}(S_1)$ ,1-$F_{BSVN}^{+}(S_2)$ ,..,1-$F_{BSVN}^{+}(S_n)$ ), (-1-$F_{BSVN}^{-}(S_1)$ ,-1-$F_{BSVN}^{-}(S_2)$ ,..,-1-$F_{BSVN}^{-}(S_n)$}>
------------------->(2)

From (1) and (2), $(US_i)^c = \cap S_i^c$ .Hence proved.

(ii). Similar as proof of (i).

(iii). Let S=< ($T_{BSVN}^{+}(S)$,$T_{BSVN}^{-}(S)$), ($I_{BSVN}^{+}(S)$, $I_{BSVN}^{-}(S)$),($F_{BSVN}^{+}(S)$, $F_{BSVN}^{-}(S)$)> be a BSVN set in X, then
$S^c$ = < (1-$T_{BSVN}^{+}(S)$, -1-$T_{BSVN}^{-}(S)$),(1-$I_{BSVN}^{+}(S)$, -1-$I_{BSVN}^{-}(S)$),(1-$F_{BSVN}^{+}(S)$, -1-$F_{BSVN}^{-}(S)$>
$(S^c)^c$ = < ($T_{BSVN}^{+}(S)$, $T_{BSVN}^{-}(S)$),($I_{BSVN}^{+}(S)$, $I_{BSVN}^{-}(S)$),($F_{BSVN}^{+}(S)$, $F_{BSVN}^{-}(S)$)>
$(S^c)^c$ = S. Hence proved.

## 5. Bipolar single-valued Neutrosophic Number (BSVNN)

**5.1 Definition :** Let two bipolar single-valued neutrosophic number(BSVNN) be
$\tilde{S}_1$=<$T_{BSVN}^{+}(1)$,$T_{BSVN}^{-}(1)$),($I_{BSVN}^{+}(1)$,$I_{BSVN}^{-}(1)$),($F_{BSVN}^{+}(1)$,$F_{BSVN}^{-}(1)$> and
$\tilde{S}_2$=<$T_{BSVN}^{+}(2)$,$T_{BSVN}^{-}(2)$),($I_{BSVN}^{+}(2)$,$I_{BSVN}^{-}(2)$), ($F_{BSVN}^{+}(2)$,$F_{BSVN}^{-}(2)$> . Then the operations are

i. $\lambda\tilde{S}_1$=<1-(1-$T_{BSVN}^{+}(1))^{\lambda}$, -(-$T_{BSVN}^{-}(1))^{\lambda}$, ($I_{BSVN}^{+}(1))^{\lambda}$, -(-$I_{BSVN}^{-}(1))^{\lambda}$, ($F_{BSVN}^{+}(1))^{\lambda}$, -(1-(1-(-$F_{BSVN}^{-}(1)))^{\lambda}$)>

ii. $\tilde{S}_1^{\lambda}$ =<($T_{BSVN}^{+}(1))^{\lambda}$, -(1-(1-(-$T_{BSVN}^{-}(1)))^{\lambda}$), 1-(1-$I_{BSVN}^{+}(1))^{\lambda}$, -(-$I_{BSVN}^{-}(1))^{\lambda}$, 1-(1-$F_{BSVN}^{+}(1))^{\lambda}$, -(-$F_{BSVN}^{-}(1))^{\lambda}$>

iii. $\tilde{S}_1+\tilde{S}_2$= <$T_{BSVN}^{+}(1)$+ $T_{BSVN}^{+}(2)$- $T_{BSVN}^{+}(1)$ $T_{BSVN}^{+}(2)$,- $T_{BSVN}^{-}(1)$ $T_{BSVN}^{-}(2)$,
$I_{BSVN}^{+}(1)$ $I_{BSVN}^{+}(2)$, - (- $I_{BSVN}^{-}(1)$ - $I_{BSVN}^{-}(2)$ - $I_{BSVN}^{-}(1)$ $I_{BSVN}^{-}(2)$),
$F_{BSVN}^{+}(1)$ $F_{BSVN}^{+}(2)$, -(- $F_{BSVN}^{-}(1)$ - $F_{BSVN}^{-}(2)$ - $F_{BSVN}^{-}(1)F_{BSVN}^{-}(2)$)>

iv. $\tilde{S}_1.\tilde{S}_2$ =< $T_{BSVN}^{+}(1)$ $T_{BSVN}^{+}(2)$,-(- $T_{BSVN}^{-}(1)$- $T_{BSVN}^{-}(2)$- $T_{BSVN}^{-}(1)T_{BSVN}^{-}(2)$),
$I_{BSVN}^{+}(1)$+ $I_{BSVN}^{+}(2)$- $I_{BSVN}^{+}(1)$ $I_{BSVN}^{+}(2)$,- $I_{BSVN}^{-}(1)$ $I_{BSVN}^{-}(2)$,
$F_{BSVN}^{+}(1)$+ $F_{BSVN}^{+}(2)$- $F_{BSVN}^{+}(1)$ $F_{BSVN}^{+}(2)$,- $F_{BSVN}^{-}(1)$ $F_{BSVN}^{-}(2)$>

**5.2 Definition :** Let a bipolar single-valued neutrosophic number(BSVNN) be
$\tilde{S}_1$=<$T_{BSVN}^{+}(1)$,$T_{BSVN}^{-}(1)$),($I_{BSVN}^{+}(1)$,$I_{BSVN}^{-}(1)$),($F_{BSVN}^{+}(1)$,$F_{BSVN}^{-}(1)$>.Then

i. score function: s($\tilde{S}_1$)=( $T_{BSVN}^{+}(1)$+1- $I_{BSVN}^{+}(1)$+1- $F_{BSVN}^{+}(1)$+1+$T_{BSVN}^{-}(1)$- $I_{BSVN}^{-}(1)$-$F_{BSVN}^{-}(1)$)/6
ii. accuracy function: a($\tilde{S}_1$)= $T_{BSVN}^{+}(1)$-$F_{BSVN}^{+}(1)$+$T_{BSVN}^{-}(1)$-$F_{BSVN}^{-}(1)$
iii. certainty function : c($\tilde{S}_1$)= $T_{BSVN}^{+}(1)$-$F_{BSVN}^{+}(1)$

**5.3 Definition :** The two bipolar single-valued neutrosophic numbers (BSVNN) are compared
$\tilde{S}_1$=<$T_{BSVN}^{+}(1)$,$T_{BSVN}^{-}(1)$),($I_{BSVN}^{+}(1)$,$I_{BSVN}^{-}(1)$),($F_{BSVN}^{+}(1)$,$F_{BSVN}^{-}(1)$>
$\tilde{S}_2$=< $T_{BSVN}^{+}(2)$,$T_{BSVN}^{-}(2)$),($I_{BSVN}^{+}(2)$,$I_{BSVN}^{-}(2)$), ($F_{BSVN}^{+}(2)$,$F_{BSVN}^{-}(2)$> can be defined as

i. If s($\tilde{S}_1$)>s($\tilde{S}_2$), $\tilde{S}_1$ is superior to $\tilde{S}_2$ ,(i.e.) $\tilde{S}_1$ is greater than $\tilde{S}_2$ denoted as $\tilde{S}_1>\tilde{S}_2$.
ii. If s($\tilde{S}_1$)=s($\tilde{S}_2$) and $\tilde{S}$ ($\tilde{S}_1$)> $\tilde{S}$ ($\tilde{S}_2$), $\tilde{S}_1$is superior to $\tilde{S}_2$,(i.e.) $\tilde{S}_1$ is greater than $\tilde{S}_2$ denoted as $\tilde{S}_1<\tilde{S}_2$.
iii. If s($\tilde{S}_1$)=s($\tilde{S}_2$) and $\tilde{S}$ ($\tilde{S}_1$)= $\tilde{S}$ ($\tilde{S}_2$) and c($\tilde{S}_1$)>c($\tilde{S}_2$), $\tilde{S}_1$ is greater than $\tilde{S}_2$,that is $\tilde{S}_1$ is superior to $\tilde{S}_2$,denoted as $\tilde{S}_1>\tilde{S}_1$.
iv. If s($\tilde{S}_1$)=s($\tilde{S}_2$) and $\tilde{S}$ ($\tilde{S}_1$)= $\tilde{S}$ ($\tilde{S}_2$) and c($\tilde{S}_1$)=c($\tilde{S}_2$), $\tilde{S}_1$ is equal to $\tilde{S}_2$,that is $\tilde{S}_1$ is indifferent to $\tilde{S}_2$ ,denoted as $\tilde{S}_1=\tilde{S}_1$.

**5.4 Definition :** Let a family of bipolar single-valued neutrosophic numbers(BSVNN) be $\tilde{S}_j$=<$T_{BSVN}^{+}(j)$, $T_{BSVN}^{-}(j)$),($I_{BSVN}^{+}(j)$,$I_{BSVN}^{-}(j)$),($F_{BSVN}^{+}(j)$,$F_{BSVN}^{-}(j)$>(j=1,2,3,…,n). A mapping $A_{\omega}$ :$F_n \rightarrow F$ is called bipolar single-valued Neutrosophic weighted average ($BSVNWA_{\omega}$) operator if satisfies

$A_\omega(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n) = \sum_{j=1}^{n} \omega_j \tilde{s}_j = < 1-\prod_{j=1}^{n}(1-T_{BSVN}^{+}(j))^{\omega_j}, -\prod_{j=1}^{n}(-T_{BSVN}^{-}(j))^{\omega_j}, \prod_{j=1}^{n} I_{BSVN}^{+}(j)^{\omega_j},$

$-(1-\prod_{j=1}^{n}(1-(-I_{BSVN}^{-}))^{\omega_j}), \prod_{j=1}^{n} F_{BSVN}^{+}(j)^{\omega_j}, -(1-\prod_{j=1}^{n}(1-(-F_{BSVN}^{-}))^{\omega_j})>$

Here $\omega_j$ is the weight of $\tilde{s}_j$ (j=1,2,…n), $\sum_{j=1}^{n}\omega_j=1$ and $\omega_j \in [0,1]$.

**5.5 Definition :** Let a family of bipolar single-valued neutrosophic numbers(BSVNN) be $\tilde{s}_j$ =<$T_{BSVN}^{+}(j)$, $T_{BSVN}^{-}(j)$),($I_{BSVN}^{+}(j)$,$I_{BSVN}^{-}(j)$)),($F_{BSVN}^{+}(j)$,$F_{BSVN}^{-}(j)$>(j=1,2,3,…,n) . A mapping $G_\omega: F_n \rightarrow F$ is called bipolar single-valued neutrosophic weighted geometric($BSVNWG_\omega$) operator if it satisfies

$G_\omega(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n) = \prod_{j=1}^{n} \tilde{s}_j^{\,\omega_j} = < \prod_{j=1}^{n} T_{BSVN}^{+}(j))^{\omega_j}, -(1-\prod_{j=1}^{n}(1-(-T_{BSVN}^{-}(j)))^{\omega_j}),$

$1-\prod_{j=1}^{n}(1-I_{BSVN}^{+}(j))^{\omega_j}, -\prod_{j=1}^{n}(-I_{BSVN}^{-}))^{\omega_j}, 1-\prod_{j=1}^{n}(1-F_{BSVN}^{+}(j)^{\omega_j}, -\prod_{j=1}^{n}(-F_{BSVN}^{-})^{\omega_j}>$ where $\omega_j$ is the weight of $\tilde{s}_j$ (j=1,2,…n), $\sum_{j=1}^{n}\omega_j=1$ and $\omega_j \in [0,1]$.

**5.6. Decision making problem:**

Here, with bipolar single-valued neutrosophic data, we develop decision making problem based on $A_\omega$ operator Suppose the set of alternatives is S = {$S_1$,$S_2$,...$S_m$} and the set of all criterions (or attributes) are G= {$G_1$,$G_2$,….,$G_n$}.Let $\omega=(\omega_1,\omega_2,….\omega_n)^T$ be the weight vector of attributes such that $\sum_{j=1}^{n}\omega_j=1$ and $\omega_j \geq 0$ (j=1,2,…n) and $\omega_j$ assign to the weight of attribute $G_j$.An alternative on criterions is calculated by the decision maker and the assess values are represented by the design of bipolar single-valued neutrosophic numbers.

Assume the decision matrix $(\tilde{S}_{ij})_{m\times n}$ =(<$T_{BSVN}^{+}(ij)$,$T_{BSVN}^{-}(ij)$)),($I_{BSVN}^{+}(ij)$,$I_{BSVN}^{-}(ij)$)),($F_{BSVN}^{+}(ij)$,$F_{BSVN}^{-}(ij)$>$)_{mxn}$ contributed by the decision maker, for Alternative $S_i$ with criterion $G_j$ ,the bipolar single-valued neutrosophic number is $\tilde{S}_{ij}$. The conditions are $T_{BSVN}^{+}(ij)$,$T_{BSVN}^{-}(ij)$,($I_{BSVN}^{+}(ij)$,$I_{BSVN}^{-}(ij)$,$F_{BSVN}^{+}(ij)$,$F_{BSVN}^{-}(ij)$ $\in$ [0,1] such that $0 \leq T_{BSVN}^{+}(ij) - T_{BSVN}^{-}(ij) + I_{BSVN}^{+}(ij) - I_{BSVN}^{-}(ij) + F_{BSVN}^{+}(ij) - F_{BSVN}^{-}(ij) \leq 6$ for i=1,2,3,…m and j=1,2,…n.

**Algorithm:**

**STEP 1:** Construct the decision matrix by the decision maker.

$(\tilde{S}_{ij})_{m\times n}$ =(<$T_{BSVN}^{+}(ij)$,$T_{BSVN}^{-}(ij)$)),($I_{BSVN}^{+}(ij)$,$I_{BSVN}^{-}(ij)$)),($F_{BSVN}^{+}(ij)$,$F_{BSVN}^{-}(ij)$>$)_{mxn}$

**STEP 2:** Compute $\tilde{S}_i = A_\omega(\tilde{S}_{i1}, \tilde{S}_{i2}, … \tilde{S}_{in})$ for each i=1,2,….m.

**STEP 3:** Using the set of overall bipolar single-valued neutrosophic number of $\tilde{S}_i$ (i=1,2,…m),calculate the score values $\tilde{S}(\tilde{S}_i)$.

**STEP 4:** Rank all the structures of $\tilde{S}_i$ (i=1,2,…m) according to the score values.

**Example (5.7):** A patient is intending to analyze which disease is caused to him. Four types of diseases $S_i$(i=1,2,3,4) are Cancer, Asthuma, Hyperactive, Typhoid. The set of symptoms are $G_1$=cough, $G_2$=Headache,$G_3$=stomach pain,$G_4$=blood cloting. To evaluate the 4 diseases (alternatives) $S_i$(i=1,2,3,4) under the above four symptoms(attributes) using the bipolar single-valued neutrosophic values.The weight vector of the attributes $G_j$ (j=1, 2, 3, 4) is ω= $(0.25,0.35,0.20,0.20)^T$.

**STEP 1:** The decision matrix provided by the patient is constructed as below:

| $S_i$ /$G_i$ | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $S_1$ | (0.3,-0.5)(0.4,-0.4)(0.4,-0.2) | (0.3,-0.3)(0.5,-0.2)(0.3,-0.4) | (0.6,-0.4)(0.4,-0.3)(0.3,-0.5) | (0.1,-0.3)(0.6,-0.4)(0.5,-0.3) |
| $S_2$ | (0.3,-0.4)(0.7,-0.5)(0.4,-0.5) | (0.1,-0.3)(0.2,-0.4)(0.3,-0.5) | (0.3,-0.5)(0.2,-0.4)(0.1,-0.3) | (0.4,-0.2)(0.2,-0.3)(0.1,-0.2) |
| $S_3$ | (0.3,-0.4)(0.4,-0.5)(0.5,-0.6) | (0.1,-0.2)(0.2,-0.3)(0.3,-0.4) | (0.5,-0.4)(0.4,-0.5)(0.5,-0.6) | (0.1,-0.3)(0.2,-0.4)(0.3,-0.6) |
| $S_4$ | (0.3,-0.2)(0.2,-0.1)(0.1,-0.2) | (0.3,-0.1)(0.4,-0.2)(0.5,-0.3) | (0.2,-0.3)(0.4,-0.7)(0.7,-0.8) | (0.1,-0.3)(0.2,-0.5)(0.3,-0.7) |

**STEP 2: C**ompute $\tilde{S}_i=A_\omega(\tilde{S}_{i1}, \tilde{S}_{i2}, \tilde{S}_{i3}, \tilde{S}_{i4})$ for each i=1,2,3,4;

$\tilde{S}_1$=< (0.3,-0.4) (0.5,-0.3) (0.4,-0.4)>
$\tilde{S}_2$=< (0.2,-0.3) (0.3,-0.4) (0.2,-0.4)>
$\tilde{S}_3$=< (0.2,-0.3) (0.3,-0.4) (0.4,-0.5)>
$\tilde{S}_4$=< (0.2,-0.2) (0.3,-0.4) (0.3,-0.5)>

**STEP 3:** The score value of $\tilde{S}(\tilde{S}_i)$ (i=1, 2, 3, 4) are computed for the set of overall bipolar single-valued neutrosophic number .

$\tilde{S}(\tilde{S}_1)=0.45$
$\tilde{S}(\tilde{S}_2)=0.53$
$\tilde{S}(\tilde{S}_3)=0.51$
$\tilde{S}(\tilde{S}_4)=0.55$

**STEP 4:** According to the score values rank all the software systems of $S_i$ (i=1, 2, 3, and 4)

$$\mathbf{S_4 > S_2 > S_3 > S_1}$$

Thus $S_4$ is the most affected disease (alternative) . Typhoid($S_4$) is affected to him.

**Conclusion:**

In this paper, bipolar single-valued neutrosophic sets were developed. Bipolar single-valued neutrosophic topological spaces were also introduced and characterized some of its properties. Further score function, certainty function and accuracy functions of the Bipolar single-valued neutrosophic were given. We proposed the average and geometric operators ($A_\omega$ and $G_\omega$) for bipolar single-valued neutrosophic information. To calculate the integrity of alternatives on the attributes taken, a bipolar single-valued neutrosophic decision making approach using the score function, certainty function and accuracy function were refined.

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