Joint Acoustic and Electrical Measurements for Unfrozen Water Saturation Estimate - A Review

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ABSTRACT: The previous laboratory study of joint electrical resistivity and acoustic velocity measurements is reviewed for both consolidated and unconsolidated permafrost in this paper. The relation of logarithm of resistivity log(R) and P-wave velocity V_p is a concave function. An increase of temperature, fine content and salinity results in a decrease of both acoustic velocity and electrical resistivity. Electrical resistivity is sensitive to salinity, while acoustic velocity changes substantially near thawing temperature.

The joint measurement results could be used to estimate unfrozen water saturation (UWS) based on Archie's law, weighted equation (WE) or Kuster-Toksoz equations (KT). However, the estimated UWS from different methods is not always consistent. The difference can be up to 20%. It might be due to the fact that UWS is not the only parameter influencing the electrical and acoustic properties. In order to obtain consistent UWS, a joint model that combines the electrical effective medium theory (EMT) and the acoustic self-consistent approximation (SCA) is proposed. In this method, UWS and aspect ratio which describes particles shape are found simultaneously from the joint SCA–EMT model. Most of the results from the proposed method are between that of Archie's law and WE method, which indicates that the electrical method might overestimate UWS and acoustic method might underestimate it.

KEY WORDS: Unfrozen water saturation, Electrical resistivity, Sonic velocity, Effective medium theory

1 INTRODUCTION

Frozen geomaterials are essentially multiphase materials, where water-ice phase transition is a gradual process due to capillarity, osmosis, and adsorption (Watanabe and Mizoguchi, 2002). This phase transition can change the mechanical, hydraulic and thermal properties of the soil up to several orders of magnitude.

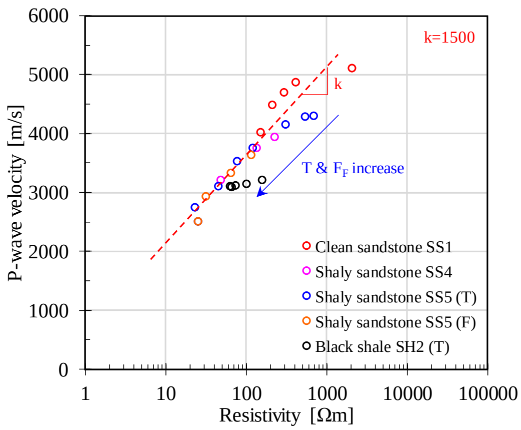
Different methods, such as calorimetric and dilatometric methods and more recent techniques such as nuclear magnetic resonance (NMR) and time-domain reflectometer (TDR), are used to estimate UWS in labs (Dillon and Andersland 1966, Smith and Tice 1988). In terms of field investigation, electrical resistivity tomography (ERT) and seismic refraction tomography are widely applied, since they are not sensitive to noise and they have relatively large penetration depth (Kneisel et al. 2008). Reliable estimation of UWS is dependent on the models which map the resistivity and velocity results into the material properties. Archie's law, WE method and KT model are the most popular models for estimating UWS. However, these methods might provide different estimation of the UWS.

In this paper, we tried to find a more reliable estimation of UWS by introducing a joint model. We focus on joint measurements of electrical resistivity and acoustic velocity. Firstly, the relation of P-wave velocity V_p vs. logarithm of resistivity log(R) are analyzed based on previous laboratory results. Then, we use Archie's law, WE method and KT model to estimate the UWS. Finally, a joint model is proposed to obtain a consistent estimation of UWS from a joint acoustic and electrical measurement.

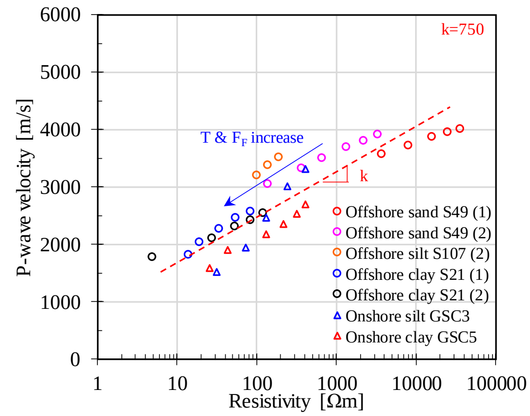
2 LAB TEST RESULTS REVIEW

There are limited works in the current literature on the joint electrical and acoustic measurements comparing with massive applications of single geophysical methods. King and his colleagues (King 1977, Pandit and King 1979, King et al. 1982, King et al. 1988) carried out one of the most systematic laboratory experiments using a joint measurement system. Sondergeld and Rai (2007) and Wu et al. (2017) used joint measurements to monitor the freezing and thawing process of saturated saline Berea sandstone. The in-phase change of acoustic velocity and resistivity is found during their study.

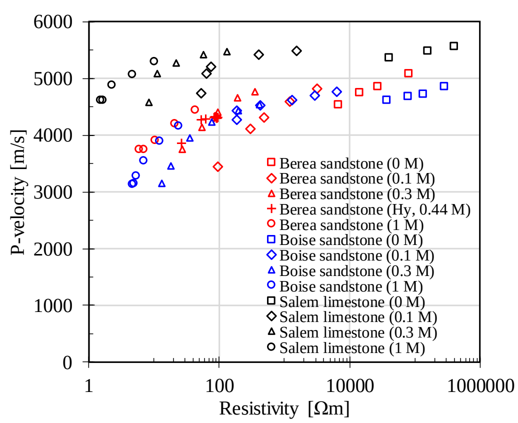
The results reported by (King 1977, Pandit and King 1979, King et al. 1982, King et al. 1988) are summarized in figure 1. We focus on the relation between acoustic velocity and resistivity with variation of salinity (0~1 molar (M) concentrations of NaCl solution), frozen soil type (consolidated and unconsolidated) and temperature (-15 °C~ 0 °C). In the figure 1 (a) and (b), measurement results shift from high resistivity and velocity to relatively low resistivity and velocity by increasing temperature (T) and fine particle fraction (F_F). Surprisingly, the ratio between velocity change ΔV and the change of log(R) is roughly constant for the certain range of salinity and temperature. The ratio is around 1500 for consolidated sandstone and 750 for unconsolidated permafrost. Figure 1 (c) and (d) show that the effect of salinity is more pronounced for the resistivity than the acoustic velocity. When the temperature of the porous medium approaches the thawing point, the ratio $\Delta V/\Delta \log(R)$ becomes larger, since acoustic velocity is more sensitive near thawing temperature. The relation of V_p vs. log(R) is a concave function. In conclusion, there is a similar effect of temperature, fine content and salinity on both acoustic velocity and electrical resistivity.



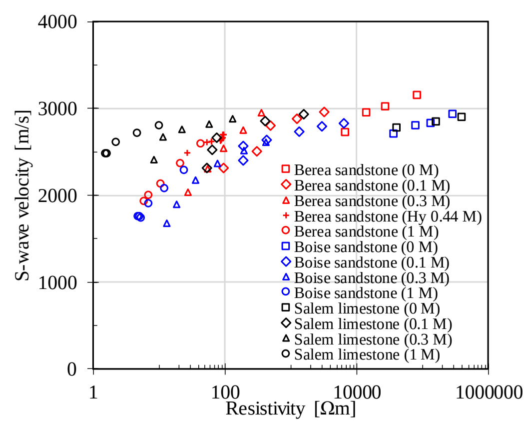
(a) Frozen rock-V_p vs. R (King 1977)



(b) Frozen soil-V_p vs. R (King et al. 1982)



(c) Salinity effect-V_p vs. R (Pandit and King 1979)



(d) Salinity effect-V_s vs. R (Pandit and King 1979)

Figure 1. Relation of acoustic velocity and electrical resistivity in lab tests (F: freezing; T: thawing)

3 ESTIMATION OF THE UNFROZEN WATER SATURATION

In this section, the focus is on the estimation of UWS from the joint measurement results. We start by calculating the UWS based on the Archie's law, WE method and KT model. The goal is to verify the consistency of the frequently applied models and to propose a joint model.

3.1 Comparison between electrical and acoustic models

Electrical conductivity is a measurement of charge mobility in response to the electrical field and it generally depends on the ionic concentration, porosity, surface conduction, percolation and anisotropy in the mixture of soil and water (Santamarina et al. 2001). In saturated frozen soils, the resistivity of the fluid, especially for electrolyte fluid, is much lower than the resistivity of ice and solid particles. Thus, resistivity decreases by one or even several orders with the increase of UWS, although ionic concentration decreases. The trend is similar for the relation of acoustic velocity and UWS, because the bulk and shear modulus of ice are higher than these of water. Ice as a cementing material highly enhances the shear modulus of the skeleton for unconsolidated permafrost. In summary, resistivity and wave velocity are shown in-phase change with UWS in figure 1.

Electrical model

Archie's law is suggested to estimate UWS for frozen soils (Daniels et al. 1976, King et al. 1988),

$$R_\alpha = a(R_w)_\alpha \phi^{-m} S_w^{-n} \quad ; \quad \alpha = i, f \tag{1}$$

where i and f refer to unfrozen and frozen state, respectively; R_w is the water resistivity in the pore space; a, m and n are empirical constants, S_w is the (unfrozen) water saturation (UWS), ϕ is the porosity.

Estimating UWS for cases (c) in figure 1: In these cases R_i and R_f are both given in the paper (Pandit and King 1979). The water resistivity in frozen soil $(R_w)_f$ is related to bulk water resistivity above the freezing point $(R_w)_i$, which for weak electrolyte solutions gives (Pandit and King 1979):

$$\left(R_w\right)_f = S_w \cdot \left(R_w\right)_i \tag{2}$$

Introducing Eq. [2] into [1], UWS relates to the ratio of frozen resistivity R_f to unfrozen resistivity R_i by

$$\frac{R_f}{R_i} = S_w^{(1-n)} \tag{3}$$

The parameter n follows the empirical relation introduced by (Pandit and King 1979):

$$n = 0.9 \log\left(\frac{R_f}{R_i}\right) + 2.3 \tag{4}$$

Then, UWS can be calculated from equation (3) according to provided values of R_i and R_f.

Estimating UWS for cases (b) in figure 1: King et al. (1982) reported the water salinity C_w (ppm) and R_f for these cases. C_w can be used for calculating $(R_w)_i$ at the temperature of 0 °C using Arps relation (Arps 1953):

$$(R_w)_{\text{at T}=20^oC} = 0.0123 + \frac{3627.5}{C_w^{\ 0.955}} \qquad [5]$$

$$(R_w)_{\text{at T}=0^oC} = (R_w)_{\text{at T}=20^oC} \frac{(20^oC + 21.5)}{(0^oC + 21.5)} \qquad [6]$$

R_i is calculated from Archie's eq. [1], having S_w=1 at unfrozen condition. Here a = 1, m = 1.5, 1.75, 2 and n = 2.5, 3.3, 5.8 for sand, silt and clay according to the previous study (Jackson et al. 1978, Edwards et al. 1988, King et al. 1988). Finally we can calculate the S_w in the Eq. [3] according to the estimates of R_i and measurements of R_f.

Acoustic model

Estimating UWS for cases (c) in figure 1: The weighted equation (eq. [9]) is a weighted average of results from Wood equation (eq. [7]) and Time average equation (eq. [8]) for the UWS estimation of consolidated permafrost:

$$\frac{1}{\rho V_{p1}^2} = \frac{\phi S_w}{\rho_w V_w^{\ 2}} + \frac{\phi(1-S_w)}{\rho_i V_i^2} + \frac{(1-\phi)}{\rho_s V_s^{\ 2}} \qquad [7]$$

$$\frac{1}{V_{p2}} = \frac{\phi S_w}{V_w} + \frac{\phi(1-S_w)}{V_i} + \frac{(1-\phi)}{V_s} \qquad [8]$$

where ρ_w, ρ_i, ρ_s and ρ are density of water, ice, solid and frozen soil and V_w, V_i and V_s are P wave velocity of water, ice and solid.

$$\frac{1}{V_f} = \frac{W\phi(1-S_w)^n}{V_{p1}} + \frac{1-W\phi(1-S_w)^n}{V_{p2}} \qquad [9]$$

where V_{p1} and V_{p2} are P-wave velocity by Wood equation and Time average equation, respectively, and V_f is the P wave velocity estimation of frozen soil. W and n are empirical factors. Lee et al. (1996) suggests n = 1 or 1/2 and W = 1 to describe the elastic behavior of permafrost.

Estimating UWS for cases (b) in figure 1: King et al. (1988) extended the KT model and proved that it could approximately estimate the effective bulk and shear moduli K and G for unconsolidated permafrost by:

$$\frac{K}{K_m} = \frac{1 + \{4G_m(K_i - K_m)/(3K_i + 4G_m)K_m\}c}{1 - \{3(K_i - K_m)/(3K_i + 4G_m)\}c} \qquad [10]$$

$$
\frac{G}{G_m}=\frac{6G_i\left(K_m+2G_m\right)+\left(9K_m+8G_m\right)\left\{\left(1-c\right)G_m+cG_i\right\}}{G_m\left(9K_m+8G_m\right)+6\left(K_m+2G_m\right)\left\{\left(1-c\right)G_i+cG_m\right\}} \qquad [11]
$$

where the subscripts *m* and *i* refer to matrix and inclusion; *c* is the fraction of the inclusions. In both WE method and KT model, the physical properties of the three phases are introduced in Table 1 (King et al. 1988).

Table 1. Physical properties of the three phases

Phase	*K* (GPa)	*G* (GPa)	ρ (kg/m^3)	V_p (m/s)
Quartz	44.0	37.0	2700	5980
Water	2.2	0.0	1000	1500
Ice	8.4	3.7	920	3800

We show UWS results from the joint measurements of consolidated permafrost (Pandit and King 1979) in figure 2 (a)-(c) and unconsolidated permafrost (King et al. 1982) in (d)-(f).

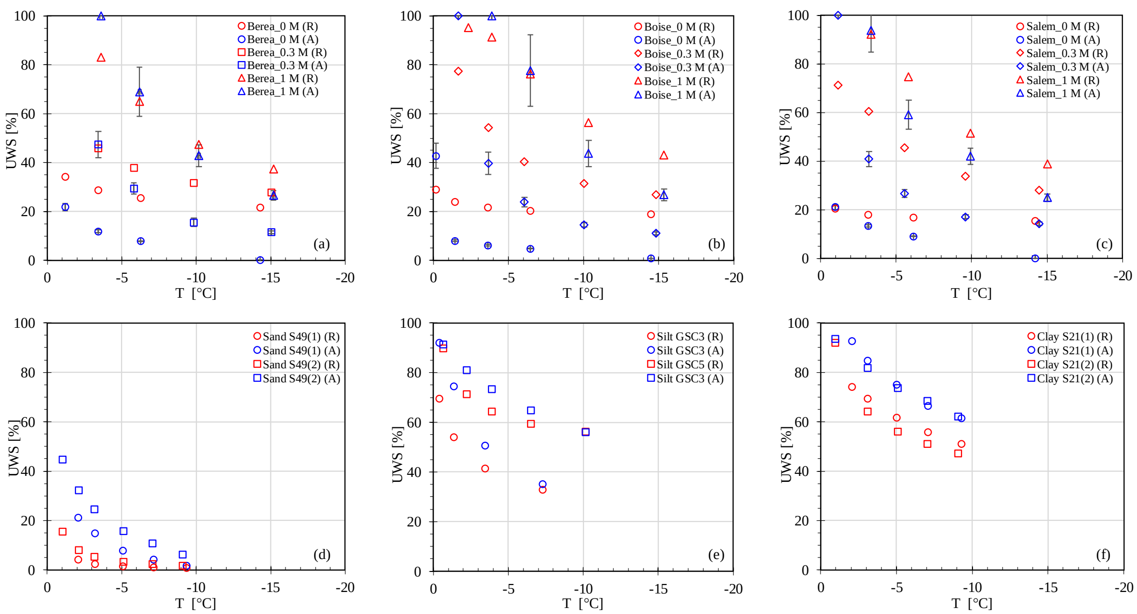


Figure 2. UWS estimates of consolidated and unconsolidated permafrost (Berea: Berea sandstone; Boise: Boise sandstone; Salem: Salem limestone; 0~1 M: Salinity; R: Resistivity results; A: Acoustic results)

According to Figure 2, estimations of UWS from resistivity and acoustic velocity are not consistent. The maximum difference of UWS is up to 20%. With the increase of salinity, the estimates become more consistent. Results based on Archie's law are higher than the WE model in consolidated permafrost, but lower than the KT model in unconsolidated permafrost. The inconsistency of estimation motivates further experiments and the development of joint EMT model.

3.2 A joint EMT method

The shape factor of particle (aspect ratio) might influences the results of electrical resistivity and acoustic velocity. Considering this effect in the model will improve the accuracy. We propose to solve the aspect ratio α (grain shape factor) and UWS (S_w) simultaneously by applying the electrical effective medium theory (EMT), developed by Schmeling (1985), and the self-consistent approximation (SCA), developed by Berryman (1980, 1995). The original papers show clear derivation and explanation of the theory. A short presentation of the mathematical forms and calibration procedures are given below.

<u>EMT for electrical resistivity</u>: In this method, the electrical conductivity of a mixture σ_m (inverse of resistivity R) is calculated as:

$$\sigma_m = \sigma_{HS}{}^p \sigma_{iso}{}^{(1-p)} \qquad [12]$$

where σ_{HS} is the Hashin-Shtrikman conductivity, σ_{iso} is the isolated conductivity, and p is the probability function that could be estimated by

$$p = \frac{n(\alpha,\beta)}{n_{\max}} \qquad \text{when } n \le n_{\max} \qquad [13]$$

$$p = 1 \qquad \text{when } n \ge n_{\max} \qquad [14]$$

where $n(\alpha,\beta) = (5.65 + 1.72/\alpha)\beta$, α is the aspect ratio and β is the fluid fraction.

σ_{HS} could be approximated by

$$\sigma_{HS} = \frac{2}{3}\beta\sigma_f + (1-\beta)\sigma_s \qquad [15]$$

where σ_f is the fluid conductivity and σ_s is the solid or ice conductivity. Here it is assumed that ice and solid conductivity are equal, hence a three-phase medium could be simplified into a two-phase medium. This assumption is acceptable since solid and ice conductivities are much lower than fluid conductivity. In this paper, σ_f is determined by calibrating equation (12) at an unfrozen state of the soil. Ice and solid conductivity are assumed as 100 kΩm.

σ_{iso} could be approximated by

$$\sigma_{iso} = 1/3(\sigma_1 + \sigma_2 + \sigma_3) \qquad [16]$$

where $\sigma_i = \sigma_s \frac{(1-\beta)(k_i - 1)\sigma_s + (k_i - (k_i - 1)(1-\beta))\sigma_f}{(k_i - 1 + \beta)\sigma_s + (1-\beta)\sigma_f}$ $\qquad i$=1, 2, 3

$$k_1 = k_2 = -\frac{2h^3}{h-(1+h^2)\tan^{-1}(h)},\ k_3 = -\frac{h^3}{(1+h^2)(\tan^{-1}(h)-h)} \text{ and } h=\sqrt{\alpha^{-2}-1}$$

Self-consistent approximation Model for wave velocity: Berryman SCA model (Eq. [17]) is related to the scattering theory based on minimization of multi-scattering effect. It can be used for high inclusion concentration.

$$\sum_{i=1}^{N} \beta_i (K_i - K_{SC}) P_i = 0 \qquad [17]$$

where β_i and K_i are the volume fraction and bulk modulus of each component. P_i is the geometric coefficient which depends on the aspect ratio α (Berryman 1980, 1995). In this paper, it is assumed that the sandstone is completely frozen in a non-saline condition at the temperature of -15 °C.

Both the electrical EMT and acoustic SCA contains two same variables, S_w (water saturation) and aspect ratio (α), which are independent of the measurement method. α and S_w are solved together at the given pairs of resistivity and acoustic velocity (R, V_p). For example, the measurement results $R = 26871$ Ωm, $V_p = 4862$ m/s and $R = 14039$ Ωm, $V_p = 4758$ m/s when salinity, $C_w = 0$ M, and temperature $T = -6.2$ and -3.4 °C for Berea sandstone. The solution is the crossing point of the two curves from EMT and SCA models in Figure 3. In figure 4, we compare the estimated UWS based on the the joint method and single geophysical methods.

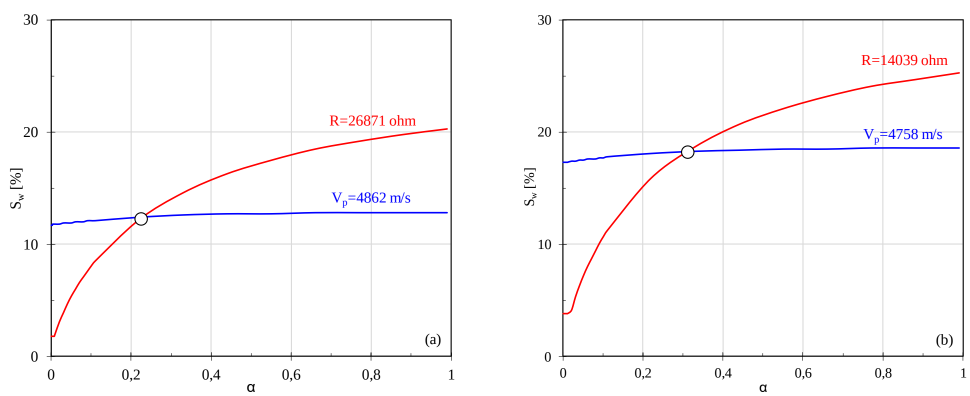


Figure 3. Numerical solution of (α, S_w) according to R = 26871 Ωm, V_p = 4862 m/s (a) and R = 14039 Ωm, V_p = 4758 m/s (b)

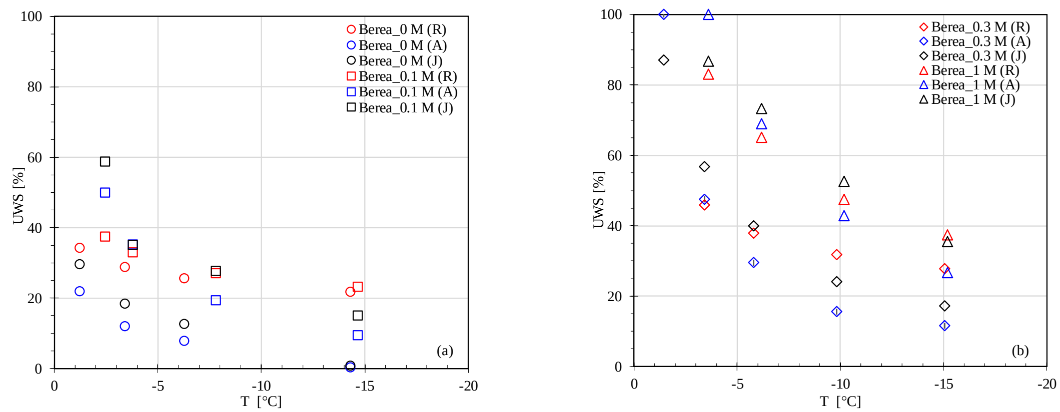


Figure 4. UWS estimates of EMT-SCA joint methods (Berea: Berea sandstone; 0~1 M: Salinity; R: Resistivity results; A: Acoustic results; J: Joint method results)

Surprisingly, the most estimates from the joint method are between the estimates of Archie's law and the WE model, especially when they are of significant different. It might indicate that the electrical method might overestimate UWS and the acoustic method might underestimate UWS. In a practical point of view, we may suggest that the average of the estimations from Archie's law and WE model is more reasonable.

4 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

We firstly review the previous laboratory studies of joint electrical resistivity R and acoustic velocity (V_p) for both unconsolidated and consolidated permafrost. It is found that the increase of temperature, salinity and fine content decreases both acoustic velocity and electrical resistivity. Resistivity of frozen soil is sensitive to salinity. The acoustic velocity becomes more sensitive near the thawing temperature.

Archie's law, WE method and KT model are used to estimate UWS from electrical and acoustic measurements, respectively. However, these estimates are not consistent and the maximum difference between them is up to 20%. One of the main reasons is that the physical properties are not only dependent on UWS, resulting in a need to involve geometrical parameters such as aspect ratio.

Finally, we combine an acoustic SCA model with an electrical EMT model to solve aspect ratio α and water saturation S_w together, according to joint acoustic and electrical measurements. UWS estimates from the joint SCA-EMT are most likely between estimates from Archie's law and the WE model. This result suggests the average of UWS estimated from Archie's law and WE model might be more reasonable.

The joint model proposed in this paper might not be the most proper one since they are derived from different theories. However, it is still quite meaningful trial to develop the joint methods for more accurate estimation of UWS.

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