

# Pre-Main Sequence of Low Mass Stars - I

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MESA Summer School 2013

I

## Outline - this week

page 2

- from the Hayashi Track to the ZAMS
- Energetics (today)
  - gravitational energy
  - pre-natal nuclear fusion (Lab 1)
- Other (observable) properties (Thursday)
  - rotation (Lab 2)
  - pulsation

# 'Initial' conditions

- what I won't talk about (much) but that does matter
  - initial collapse (see Palla & Stahler)
  - magnetic fields
  - disks / jets
- where we'll start
  - uniform composition
  - cool enough for no nuclear energy/transformation
  - spherical
  - non-magnetic, some rotation

# Mass conservation

- ASSUME - spherical symmetry EVERYWHERE
  - $P, T, \rho$ , etc. all functions of  $r$  only
  - define  $M_r$  as mass contained within radius  $r$   
mass of a (thin!) spherical shell:

$$dM_r = M_{r+dr} - M_r = 4\pi r^2 \rho(r) dr$$

- or, simply rearranging,

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$$

- this also provides a 'coordinate transformation'

$$\frac{dF}{dr} = 4\pi r^2 \rho(r) \frac{dF}{dM_r}$$

# Mechanical Equilibrium

- gravitational force downwards:

$$\rho(r)g(r) = \rho(r)\frac{GM_r}{r^2}$$

- pressure (force) imbalance upwards

$$P(r) - P(r + dr) = - \left( \frac{dP}{dr} \right) dr$$

- ... equation of motion (vertical)

$$\rho(r)\frac{d^2r}{dt^2} = -\rho(r)\frac{GM_r}{r^2} - \frac{dP}{dr}$$

- at equilibrium, the above = 0, so we have HSE:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho(r) \quad \text{or} \quad \frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$$

## mechanical structure - special case

- HSC and Continuity contain only  $P$  and  $\rho$
- consider a functional relation between  $P$  and  $\rho$  only, i.e.

- $P(r) = K \rho^\gamma(r)$

- with such an Equation of State (EOS)

- $\frac{dP}{dr} = \frac{GM_r}{r^2} \frac{1}{K^{1/\gamma}} P^{1/\gamma}(r)$

- $\frac{dM_r}{dr} = 4\pi r^2 \frac{1}{K^{1/\gamma}} P^{1/\gamma}(r)$

# Hayashi's Tracks

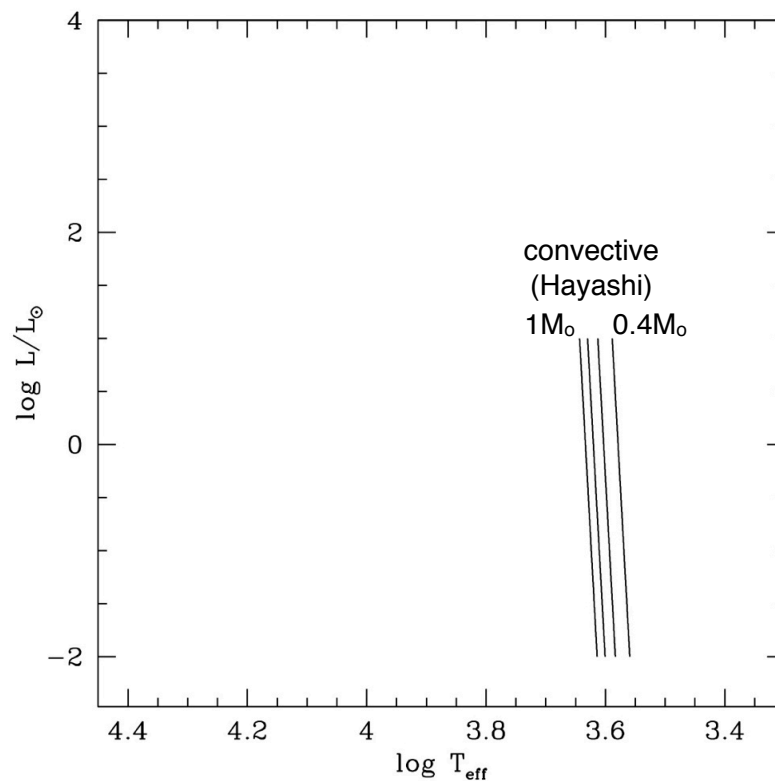
page 7

- C. Hayashi (1961) realized that pre-main sequence stars were **convective**
- Polytropes of index  $n=1.5$  ( $\gamma=5/3$ )
- limit on gradient  $d\ln T/d\ln P \sim 0.4$
- $P_c \sim M^2/R^4$  lower limit to  $T_{\text{surf}}$  is set by  $M, R$
- $L = 4\pi R^2 \sigma T_e^4$ 
  - fix  $L$ , there's a minimum  $T_e$  for a given  $R$
  - use polytropes, add in an opacity scaling, fill in the details to find

$$T_{\text{eff}} \approx 4200 \mu^{13/51} \left( \frac{M}{M_{\odot}} \right)^{7/51} \left( \frac{L}{L_{\odot}} \right)^{1/102}$$

## convective tracks

page 8



# Henyeey (radiative) tracks

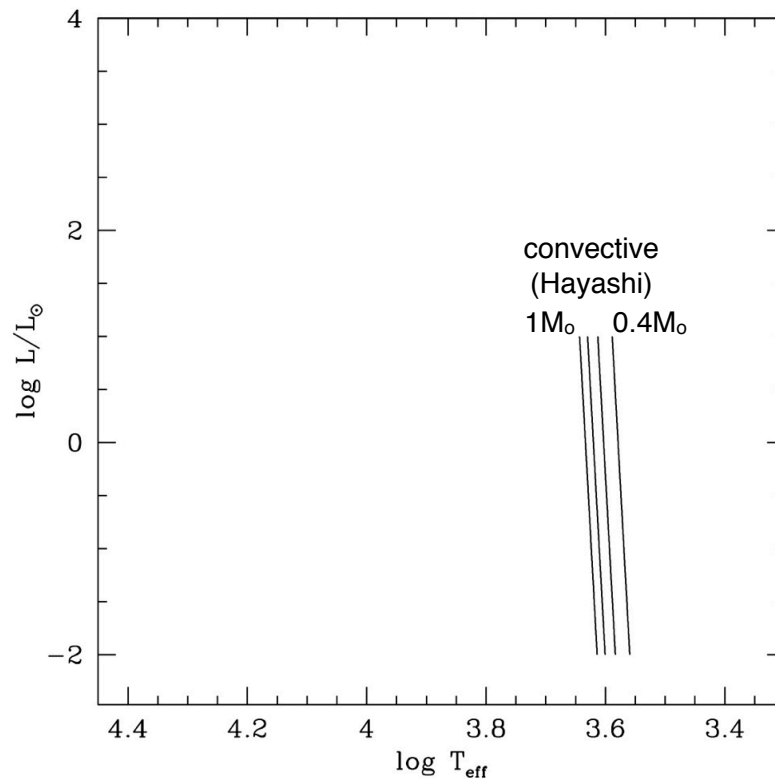
page 9

- below some luminosity,  $d\ln T/d\ln P$  is smaller, and the structure can be **radiative**
- continued contraction - continued core heating
- smaller T gradient  $\rightarrow$  heat flow constrained by core, not by surface
- $T_{\text{eff}}$  must rise, Luminosity also rises
- an exercise in homology yields:

$$T_{\text{eff}} \approx 4200 \left( \frac{M}{M_{\odot}} \right)^{-5.5} \left( \frac{L}{L_{\odot}} \right)^{1.25}$$

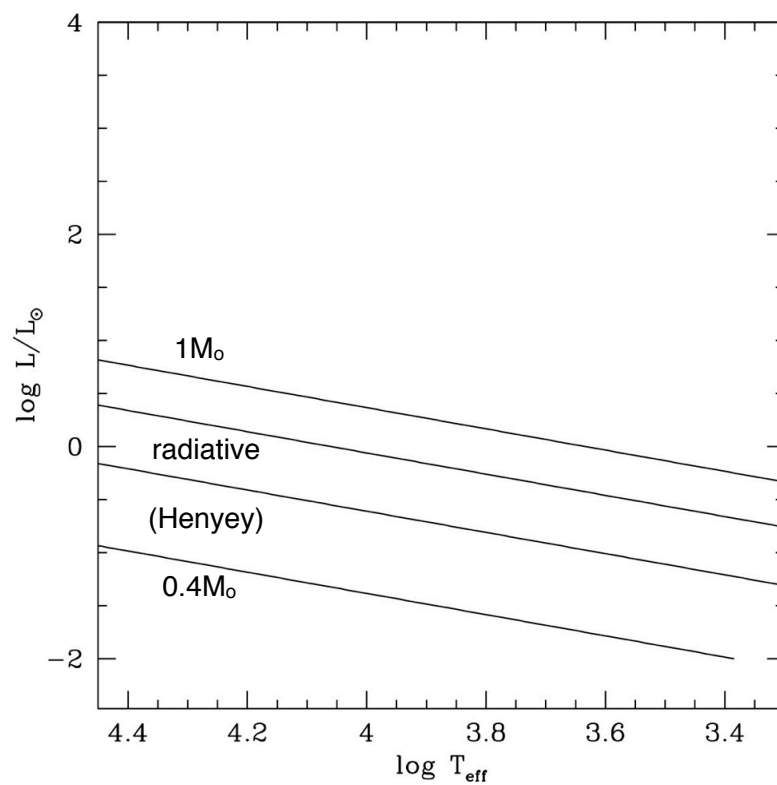
## convective tracks

page 10



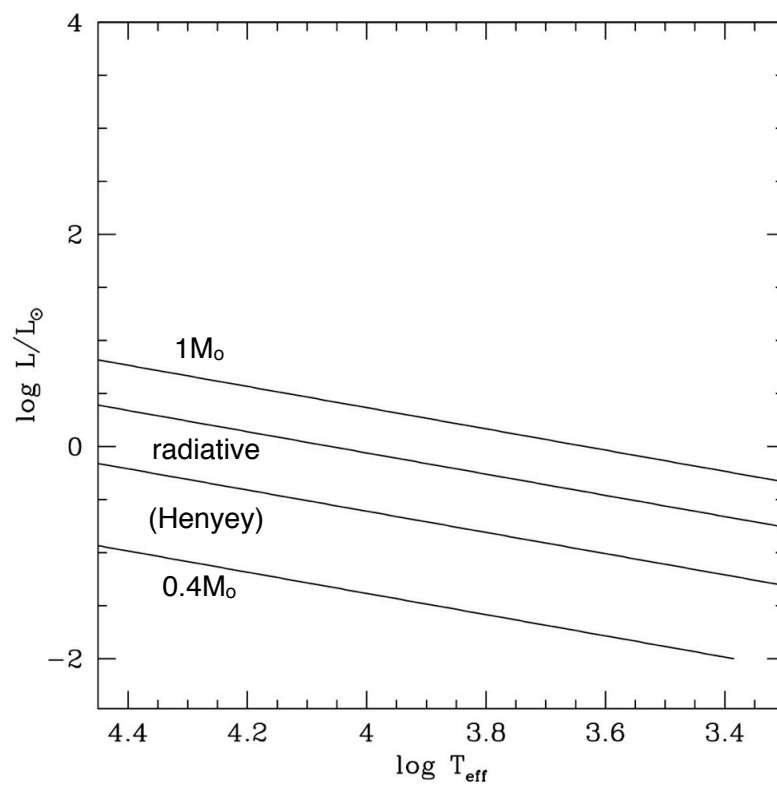
# radiative tracks

page 11



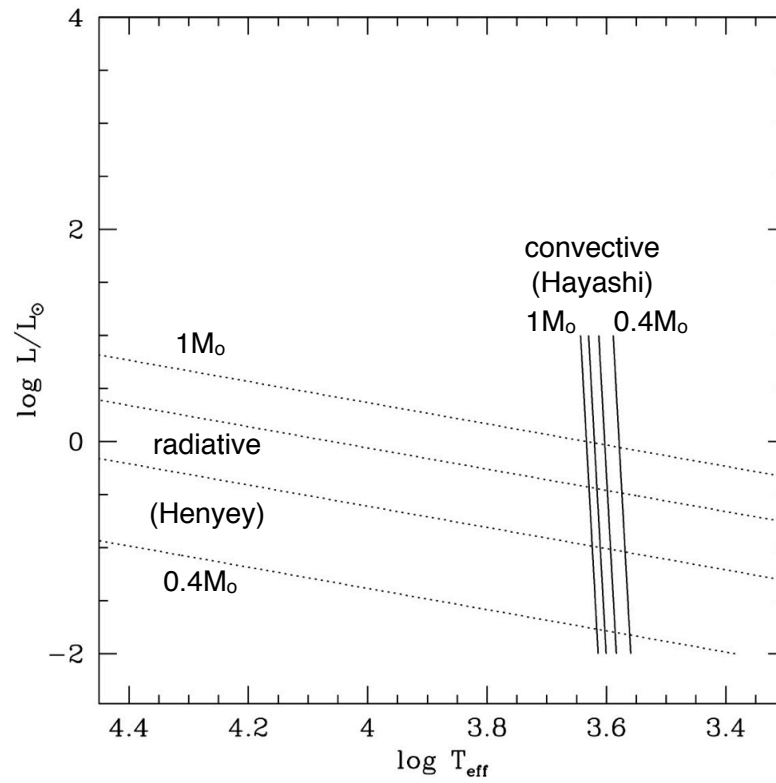
# radiative tracks

page 12



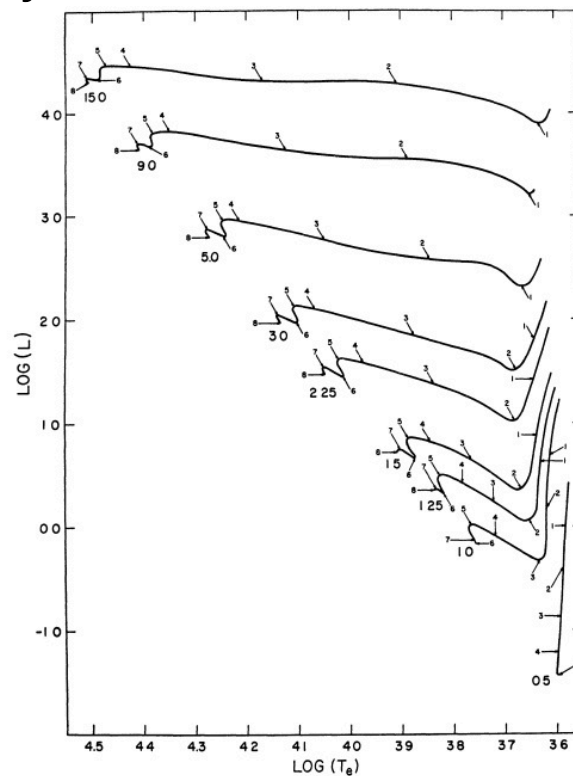
# convective and radiative tracks

page 13

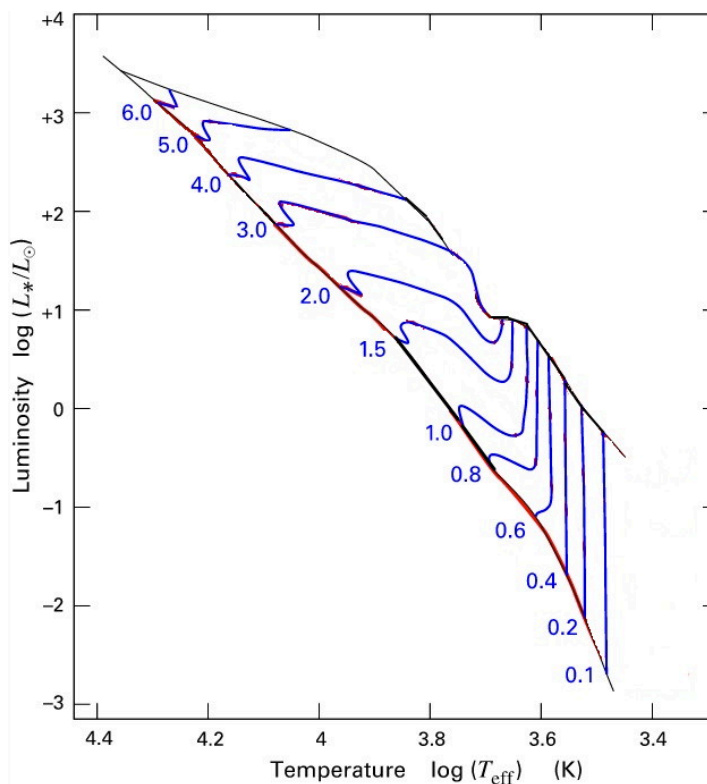


## early PMS models - Iben 1965

page 14



## 'modern' tracks



from Stahler & Palla  
(2008)

## Energy conservation

- If no energy lost or produced in a zone:

$$L_r = L_{r+dr} \quad \text{- or -} \quad L_{r+dr} - L_r = 0$$

- more generally

$$L_{r+dr} - L_r = \left( -\frac{dQ}{dt} + \epsilon \right) dM_r$$

heat gain / loss rate per gram:

$$\frac{dQ}{dt} = \frac{\partial E}{\partial t} - P \frac{\partial}{\partial t} \left( \frac{1}{\rho} \right)$$

internal energy change   PdV work

(nuclear) energy production /  
loss rate per gram:

$$\epsilon(\rho, T, X_i)$$

- nuclear energy production
- neutrino losses

- $\frac{dQ}{dt} = T \frac{\partial S}{\partial t}$

- the only explicit time-dependence in equations of stellar structure



# the thermal time scale

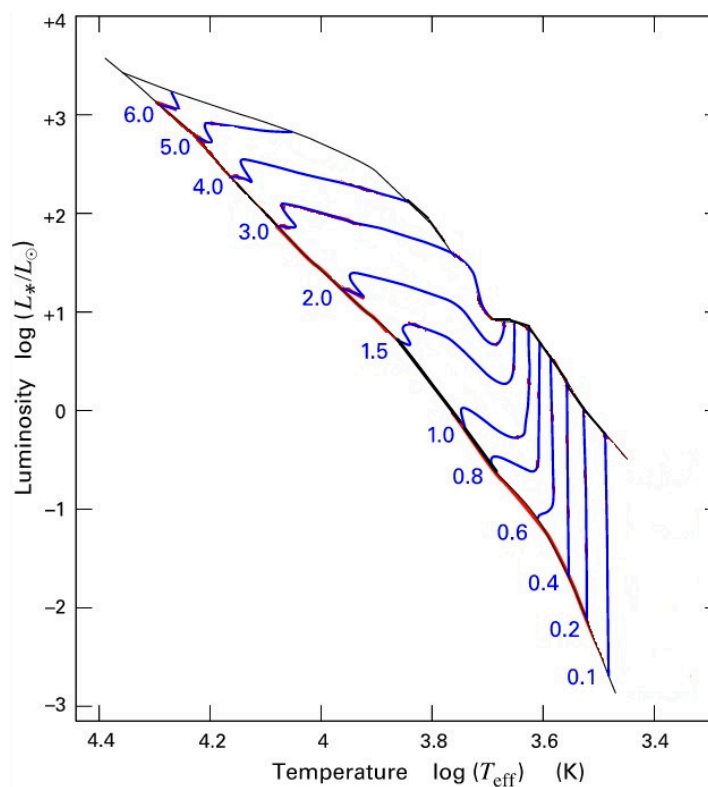
- rate of energy loss =  $dE/dt \sim L$
- available thermal energy  
 $E_{\text{therm}} = \langle c_v T \rangle M \sim GM^2/R$
- thermal time scale =  $E_{\text{therm}} / dE/dt$
- this, too, is a long time in general

$$t_{\text{therm}} \approx \frac{1}{L} \frac{GM^2}{R}$$

$$\approx 6 \times 10^7 \text{ yr} \left( \frac{M}{M_{\odot}} \right)^2 \left( \frac{R}{R_{\odot}} \right)^{-1} \left( \frac{L}{L_{\odot}} \right)^{-1}$$

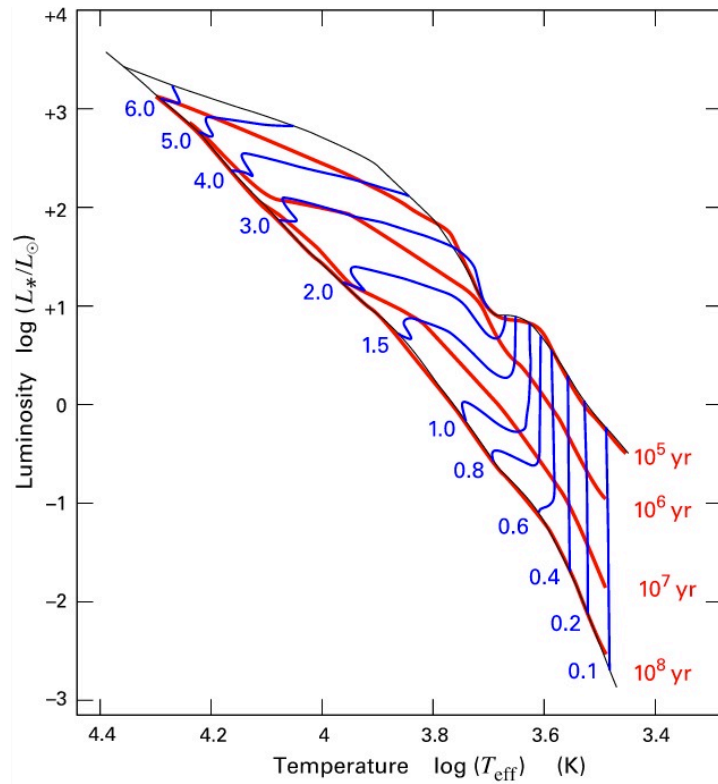
- $10 L_{\odot}$ ,  $3 R_{\odot}$ ,  $t_{\text{therm}} = 2 \times 10^6 \text{ yr}$

## Evolutionary tracks



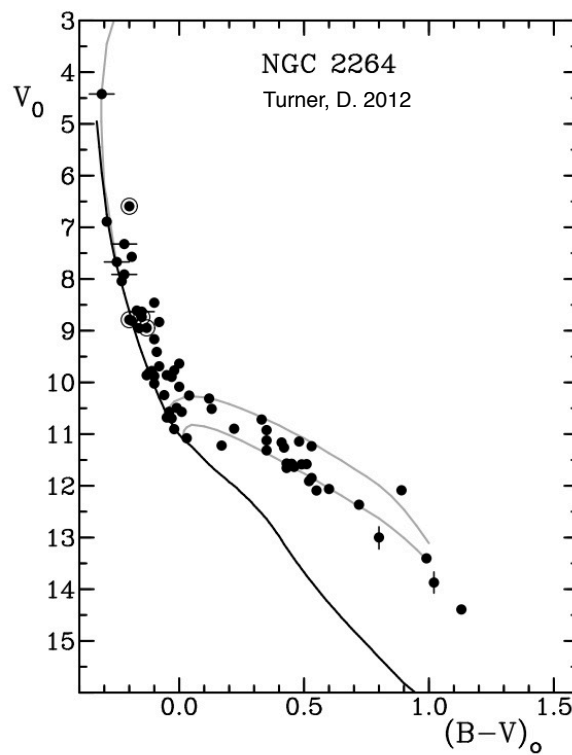
from Stahler & Palla  
(2008)

# Evolutionary tracks / isochrones

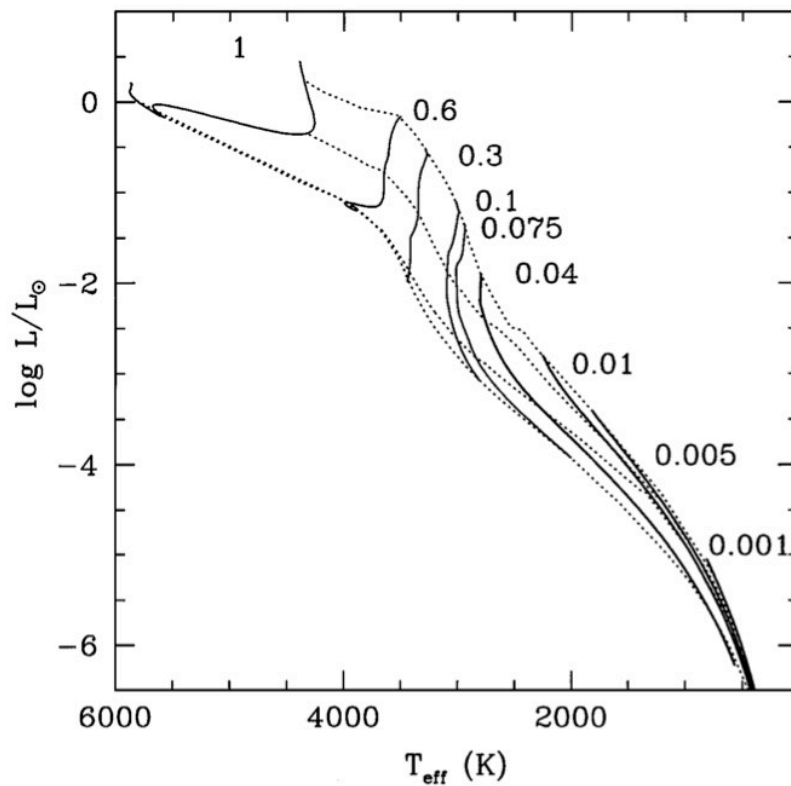


from Stahler & Palla  
(2008)

## NGC 2264

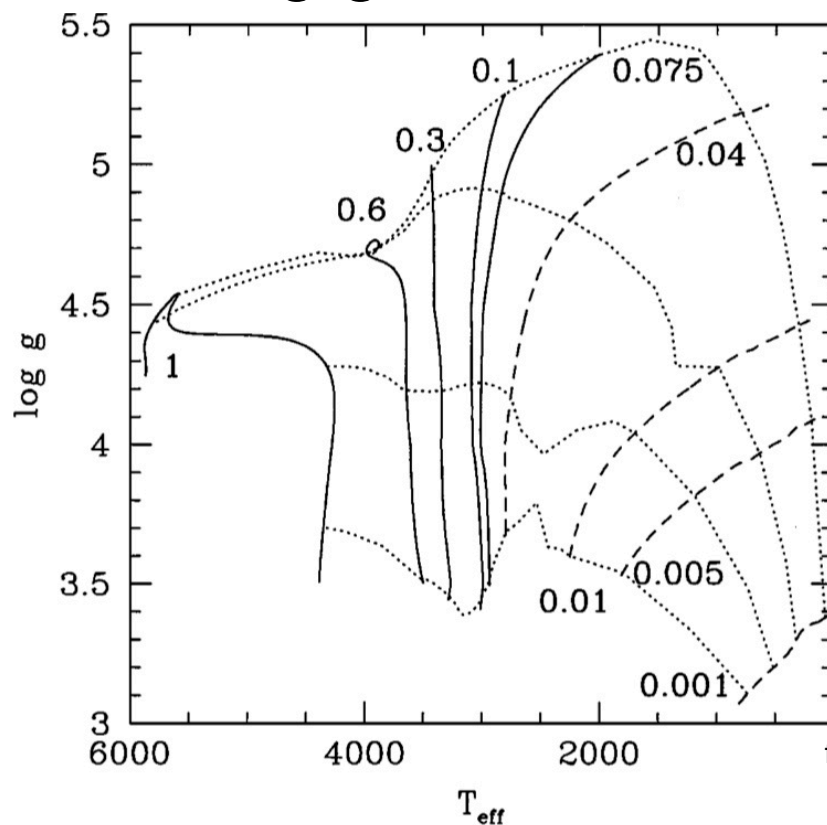


# Low Mass Evolutionary tracks / isochrones



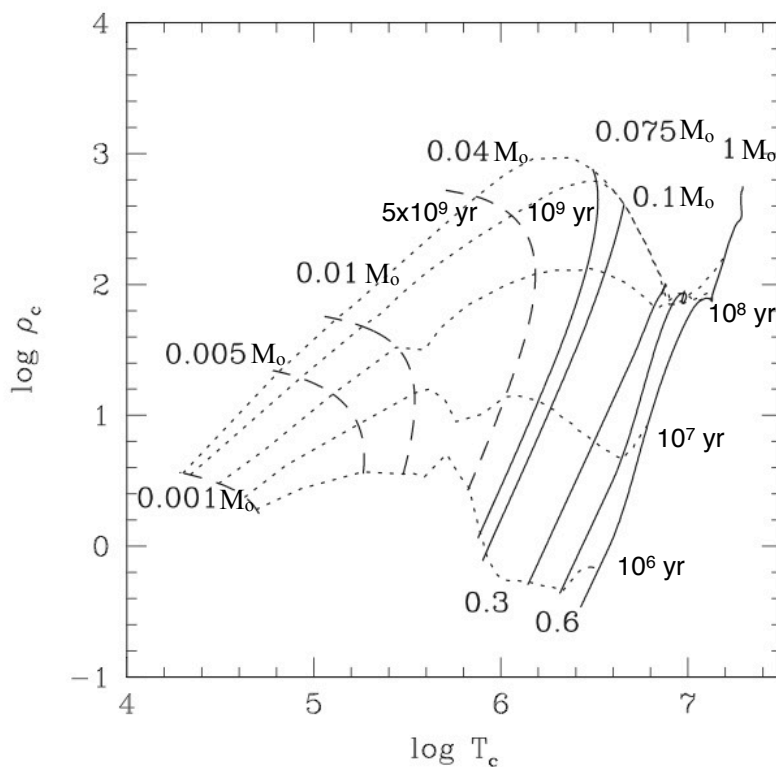
from Chabrier &  
Baraffe (2000)

## log g vs. $T_{\text{eff}}$



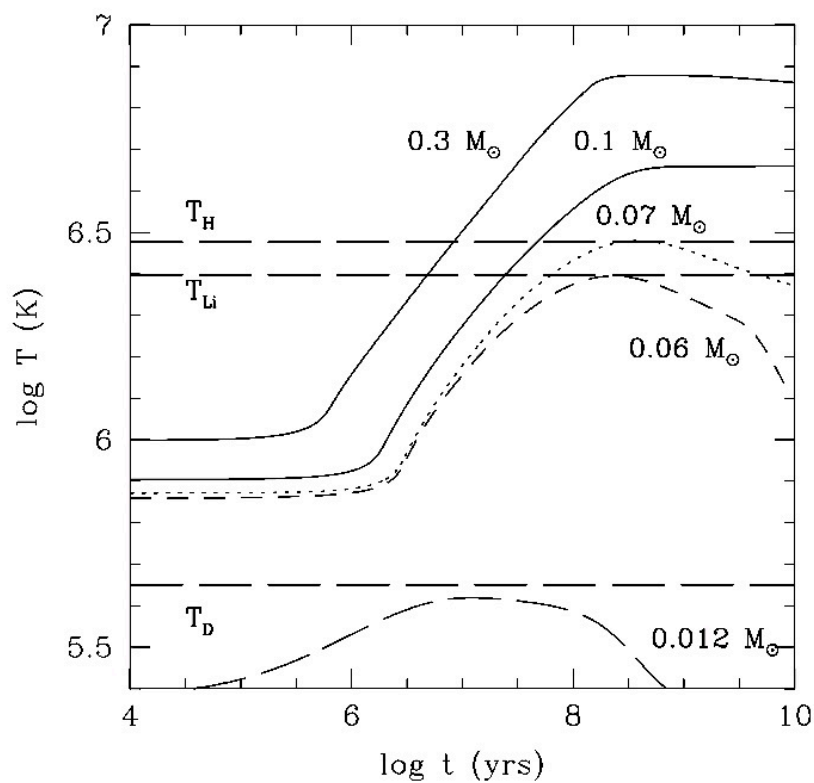
from Chabrier &  
Baraffe (2000)

# $\rho_c$ vs $T_c$



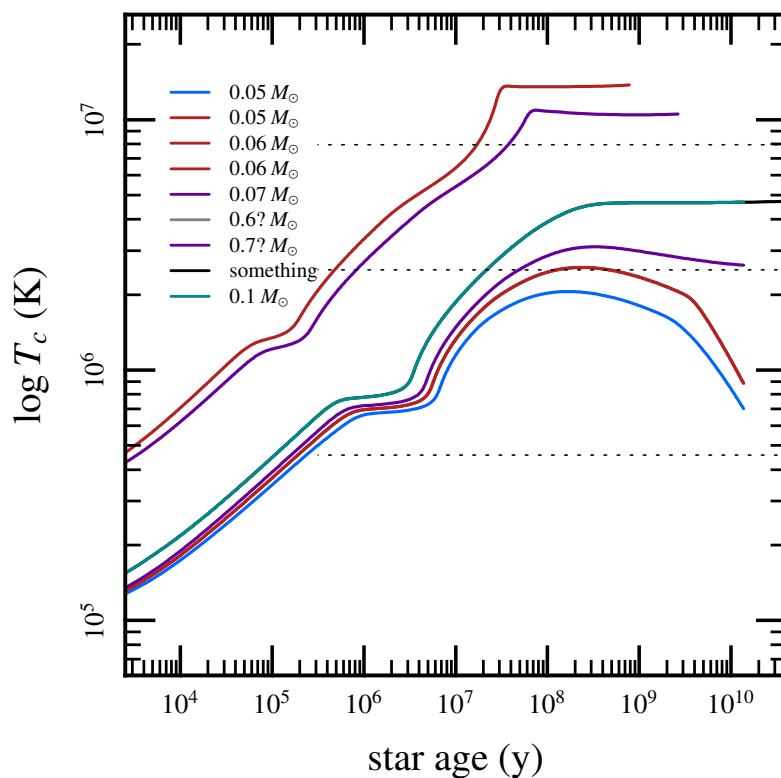
from Chabrier & Baraffe (2000)

## central temperature behavior

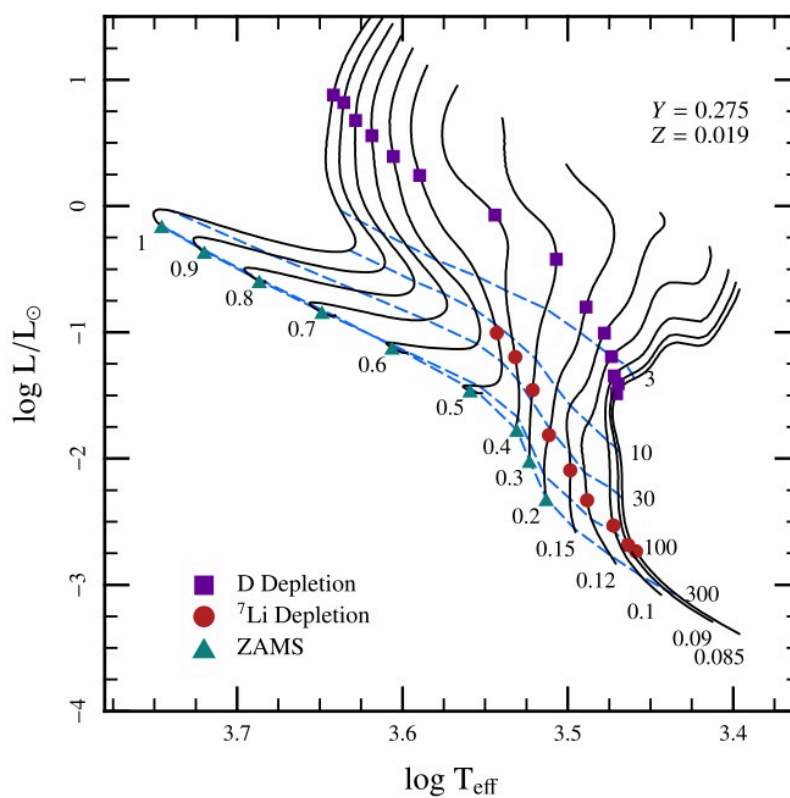


from Chabrier & Baraffe (2000)

# central temperature behavior



# MESA Evolutionary tracks / isochrones



MESA

## mini-lab 1: Tc vs T in PMS models

- download *work\_pms\_minilab1.zip*
- unzip in a good place
- run *./mk*
- edit *inlist* to point to *inlist\_create\_pms*
- edit *inlist\_create\_pms* to create a starting model with a mass of:
  - your birthday date = 1-8, do 0.05M<sub>o</sub>
  - your birthday date = 9-16, do 0.06M<sub>o</sub>
  - your birthday date = 17-24, do 0.07M<sub>o</sub>
  - your birthday date = 25-31, do 0.10M<sub>o</sub>
- you'll need to specify the mass itself, and the output model filename to match the mass, in the obvious places in *inlist\_create\_pms*

## mini-lab 1: Tc vs T in PMS models

- edit *inlist* to point to *inlist\_evolve*
- edit *inlist\_evolve* to point to the starting model you just made
- run *star* via *./rn*
- watch the output whiz by
- the file *LOGS\_pms\_\_minilab1/history.data* contains what you need to plot log Tc vs log age - the header of the file lists the history columns
- Look at your plot. Add horizontal lines at the expected burning temperatures of Li, <sup>2</sup>H, and H.
- Does your model eventually burn Li? <sup>2</sup>H? Hydrogen?
- Send your results as a two-column simple text file (column 1 = age, column 2 = log Tc) to Chris (Chris [cmankovich@ucsc.edu](mailto:cmankovich@ucsc.edu)) who will magically produce a nice plot

# Lab 1

- Objectives

- identify and characterize the first nucleosynthesis epochs in pre-main sequence models
- explore observational consequences of deuterium and lithium burning in PMS models
- clarify (or not) the lower mass limit for 'hydrogen burning' stars by setting criteria for whether or not a model experiences meaningful nucleosynthesis

- and beyond

- explore a range of other input parameters
- compare with observations