Low-Mass Stars and Getting Started with MESA!

Lars Bildsten (Lecturer), Evan Bauer (TA)

- Review some simple physics of stars and their early phase of Kelvin-Helmholtz contraction
- Focus will be on low masses (<0.3), both because its a focus for later lectures and because it is easy!
- Will compare to analytics where possible, and highlight some astronomical uses.

Stars 101!

- Hydrostatic balance demands a certain required temperature in the interior.
- That hot core leads to energy loss to the exterior at a rate fixed by the physics of the heat transfer (i.e. radiative diffusion or convection)
- Until some other energy source can be tapped (e.g. nuclear fusion), that energy loss leads to slow gravitational contraction to smaller radii!


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Good Place to see Slow Gravitational Contraction in Action: Young Clusters

- Many stars are born together in clusters, with typical ‘birthday’ spreads of 1-10 Myrs
- Different mass stars cool at different rates, something we can hopefully observe and use to some astronomical end (e.g. determining masses, or, using theory, ages!)
- Also provides tests of low-mass stellar evolution and contraction to the Main Sequence

Hydrostatic Balance

Since sound waves travel around a young star on times of days to weeks, there is plenty of time for hydrostatic balance to apply:

$$\rightarrow \frac{dP}{dr} = -\rho g, \text{ where } g(r) = \frac{Gm(r)}{r^2}$$

We now cheat, and assume that at a typical place in the star, m=total mass M, r=radius R, then hydrostatic balance gives

$$\frac{P}{R} \sim \rho \frac{GM}{R^2}, \text{combine with ideal gas } P \approx \frac{\rho k_B T}{m_p}$$

Where m_p=proton mass (everything is ionized). This allows us to find the relation between the central temperature, T_c, and the Mass and Radius as well as the central density

$$k_B T_c \approx \frac{GMm_p}{R} \qquad T_c \propto M^{2/3} \rho^{1/3}$$

More on Hydrostatic Balance

$$k_B T_c \approx \frac{GMm_p}{R}$$

As R shrinks (e.g. as the star contracts from the large cloud it started in), the core temperature rises! This is the same as what happens to a particle in orbit (the Kepler problem), as it loses energy (radiates!), it moves in (radius shrinks!), and moves faster (higher temperature!). Welcome to the world of negative heat capacities.

Minilab 1: Kelvin-Helmholtz Contraction

We're going to try to crowd-source a logarithmically distributed range of masses between $M = 0.03\,M_\odot$ and $M = 0.3\,M_\odot$. Below the lower bound of this mass range, it gets a little tricky to construct pre main sequence models with `MESA`, so we'll leave that for a later lab. In the `&controls` section of `inlist_brown_dwarf`, find the line

```
    initial_mass = 0.05
```

Generate a random number between -1.5 and -0.5, take 10 to the power of the result, and modify the `initial_mass` line accordingly:

Be sure to generate your random number in log space!!!

We're now ready to run the model (`./rn`). Report the following information on the spreadsheet:

- initial mass
- core temperature when $R = 1.0\,R_\odot$
- core temperature when $R = 0.3\,R_\odot$
- core temperature when $R = 0.1\,R_\odot$

What's the "Real Answer" ?

- Stars in this mass range are convective from surface to core.
- Convection is so efficient that the whole star has, more or less, one value of the entropy, implying that

$$PV^{\gamma} = \text{constant} \rightarrow P \propto \rho^{5/3}$$

- Integrating hydrostatic balance and mass conservation over such a relation yields

$$k_B T_c = 0.54 \frac{G M \mu m_p}{R}$$

$$\mu \approx 0.6 = \text{mean molecular weight}$$

A Paper from a Class 20 years ago!

THE ASTROPHYSICAL JOURNAL, 482:442–447, 1997 June 10

LITHIUM DEPLETION IN FULLY CONVECTIVE PRE–MAIN-SEQUENCE STARS

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Received 1996 September 11; accepted 1997 January 7

$$\rho_c = 8.44\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right)^3 \text{ g cm}^{-3} ,$$

$$T_c = 7.41 \times 10^6\left(\frac{\mu_{\text{eff}}}{0.6}\right)\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right) \text{ K} ,$$

Whats that?

Evan Show Results of Lab 1

What Went Wrong at Low Masses?

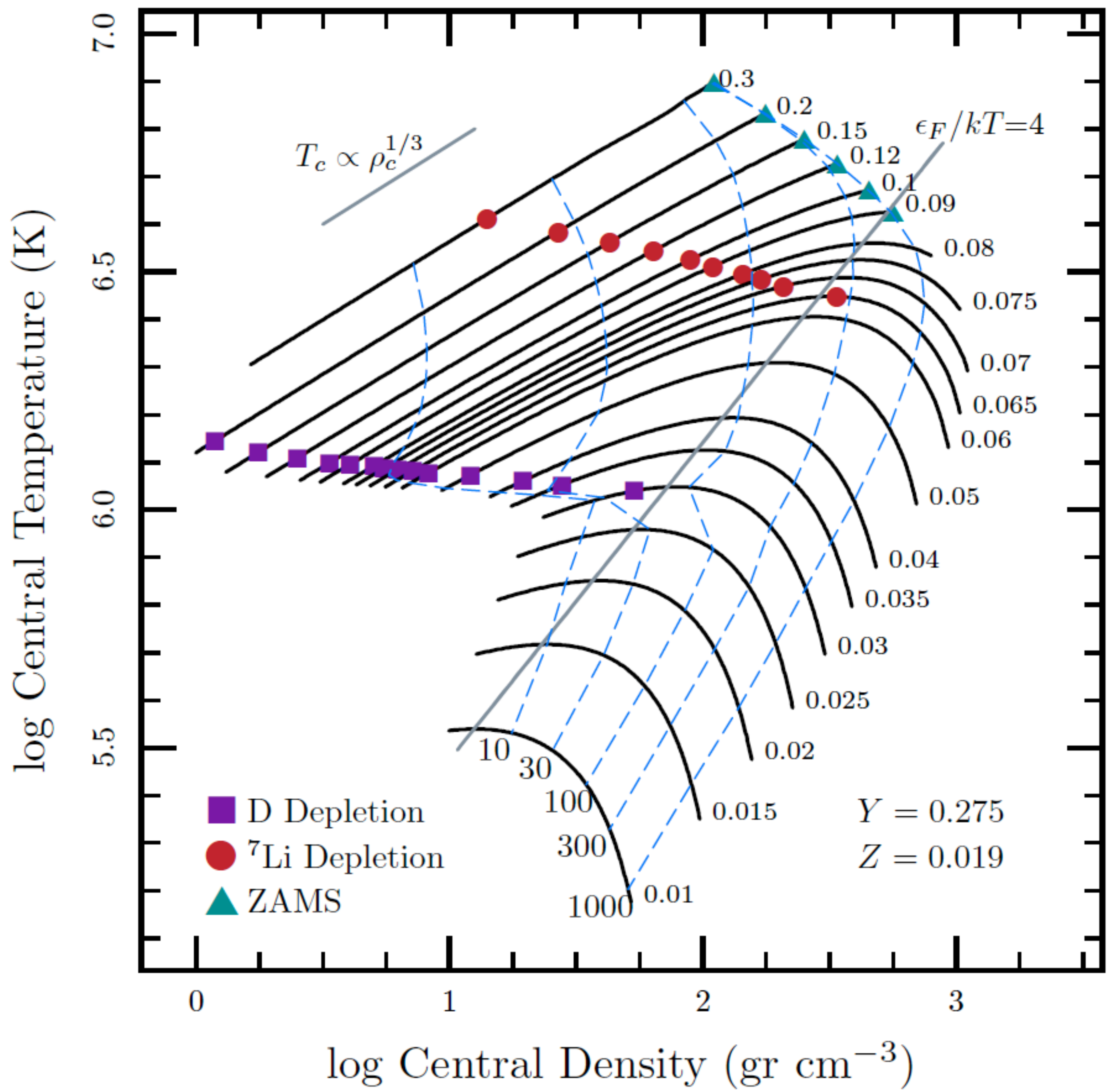
- We assumed that the gas supplies a pressure set by the ideal gas EOS

$$P = \frac{\rho k_B T}{\mu m_p} \rightarrow T_c \propto M^{2/3} \rho_c^{1/3}$$

- As the density increases, the electrons become degenerate when

$$\lambda \approx \frac{\hbar}{p} \approx \frac{\hbar}{(m_e k_B T_c)^{1/2}} > a \sim \frac{1}{n_e^{1/3}}$$

$$\text{implying onset of degeneracy when } T_c \propto \rho_c^{2/3}$$



$T_c \propto \rho_c^{1/3}$
$\epsilon_F/kT=4$
0.3
0.2
0.15
0.12
0.1
0.09
0.08
0.075
0.07
0.065
0.06
0.05
0.04
0.035
0.03
0.025
0.02
0.015
0.01
10
30
100
300
1000
D Depletion
^{7}Li Depletion
ZAMS
$Y = 0.275$
$Z = 0.019$
log Central Temperature (K)
log Central Density (gr cm^{-3})

Generic Kelvin-Helmholtz Contraction

Prior to ignition of any nuclear energy source, the loss of energy (at luminosity L) leads to a slow contraction of the star. If the Sun were powered this way, it would change its radius on a time

$$t_{\mathrm{Kelvin}} \approx \frac{GM^2/R}{L} \approx 10^7 \mathrm{yr}$$

What happens to a young star that is fully convective?

Contraction of a Fully Convective Pre-Main Sequence Star

- Hydrostatic balance always holds!
- Heat is internally transported via convective motions at << sound speed
- Constant entropy of efficient convection allows for a simple ‘connect the dots’ of surface to core. Need for creating photons at the surface to radiate => closed solution at nearly constant surface temperature (the Hayashi Track!)..

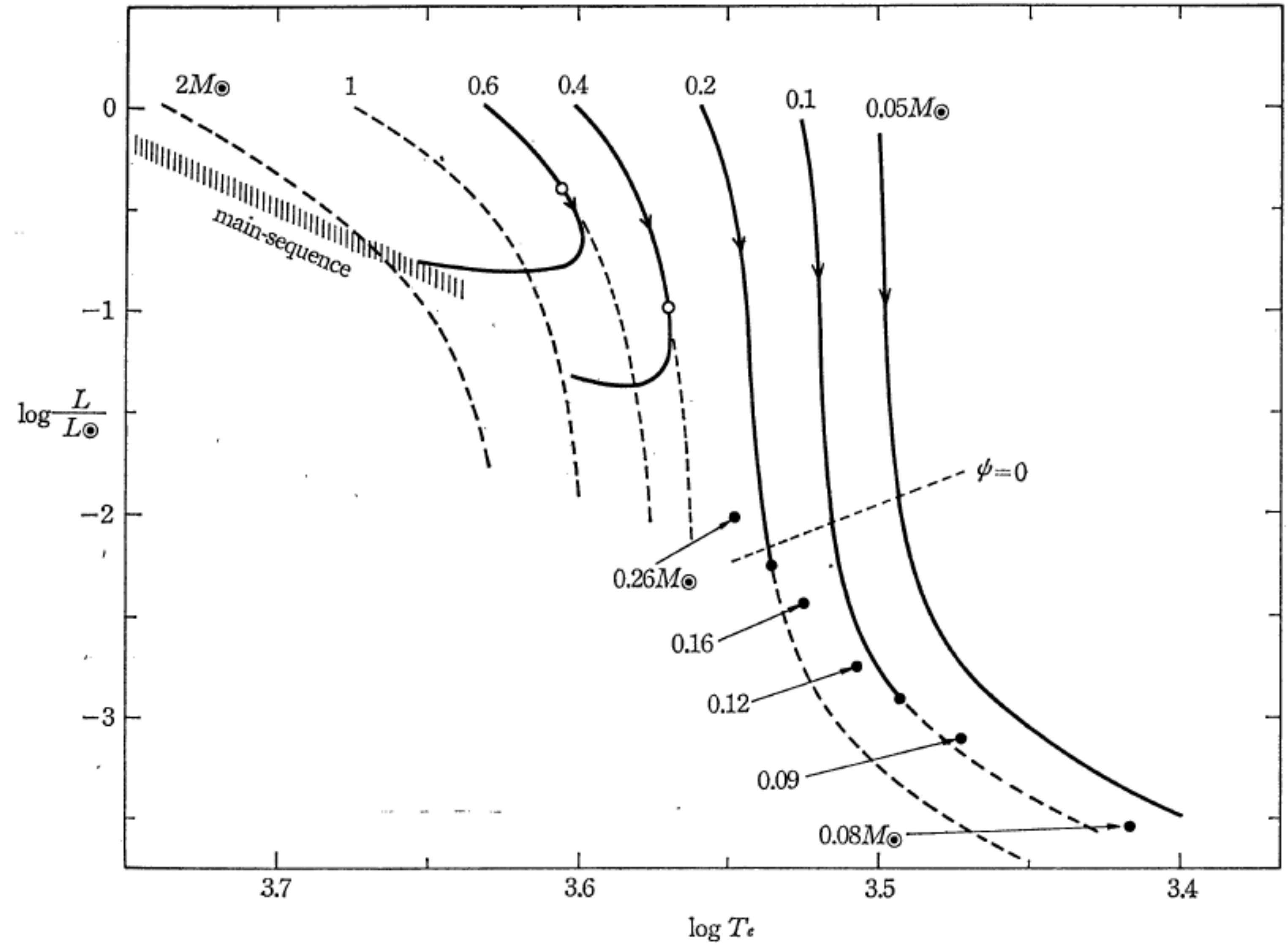


Fig. 2. Curves of $E=45.48$ and evolutionary tracks of contracting stars in the HR diagram. The solid curves show the tracks of contracting pre-main-sequence stars. The dashed curves together with the solid curves above the open and closed circles represent the surface condition for $E=45.48$. The closed circles represent the zero-age main-sequence stars which are wholly convective. The dotted curve $\psi=0$ represents the stage of incipient degeneracy in the stellar interior.

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$$\frac{R}{R_\odot} = 0.850\left(\frac{M}{0.1\ M_\odot}\right)^{2/3}\left(\frac{3000\ \mathrm{K}}{T_{\mathrm{eff}}}\right)^{4/3}\left(\frac{\mathrm{Myr}}{t}\right)^{1/3},$$

$$\frac{L}{L_\odot} = 5.25 \times 10^{-2}\left(\frac{M}{0.1\ M_\odot}\right)^{4/3}\left(\frac{T_{\mathrm{eff}}}{3000\ \mathrm{K}}\right)^{4/3}\left(\frac{\mathrm{Myr}}{t}\right)^{2/3}$$

Nuclear Burning

Quantum mechanics allowed for tunneling into the nucleus giving the fusion reactions for Hydrogen=>Helium:

$$p + p \rightarrow D + e^{+} + \nu_e$$

$$p + D \rightarrow {}^{3}\mathrm{He} + \gamma$$

$${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + p + p$$

This reaction rate is temperature sensitive, so once a certain T_c is reached, the energy generation rate can matche that lost. This fixes the stellar radius R (proportional to M), and lifetime set by the fuel available and stellar luminosity at that moment.

Minilab 2: Isochrones

Remember to turn off any other stopping conditions such as `photosphere_r_lower_limit` that you may have put in before. Now run and report these values from the terminal output at the end of the run in the Day 1 Minilab 2 tab of the spreadsheet:

- `Mass`
- `lg_L`
- `Teff`
- `lg_Tcntr`
- `lg_Dcntr`

Edit the stopping condition to run again up to ages of 100 Myr, 300 Myr, and 1 Gyr. Report the same four entries as before in the spreadsheet for each age. You should be able to save time by restarting from a recent photo each time you change to a new `max_age`.

2 With Burning

Recall that we turned off burning back in minilab 1. Now let's turn it back on by commenting out the line about burning:

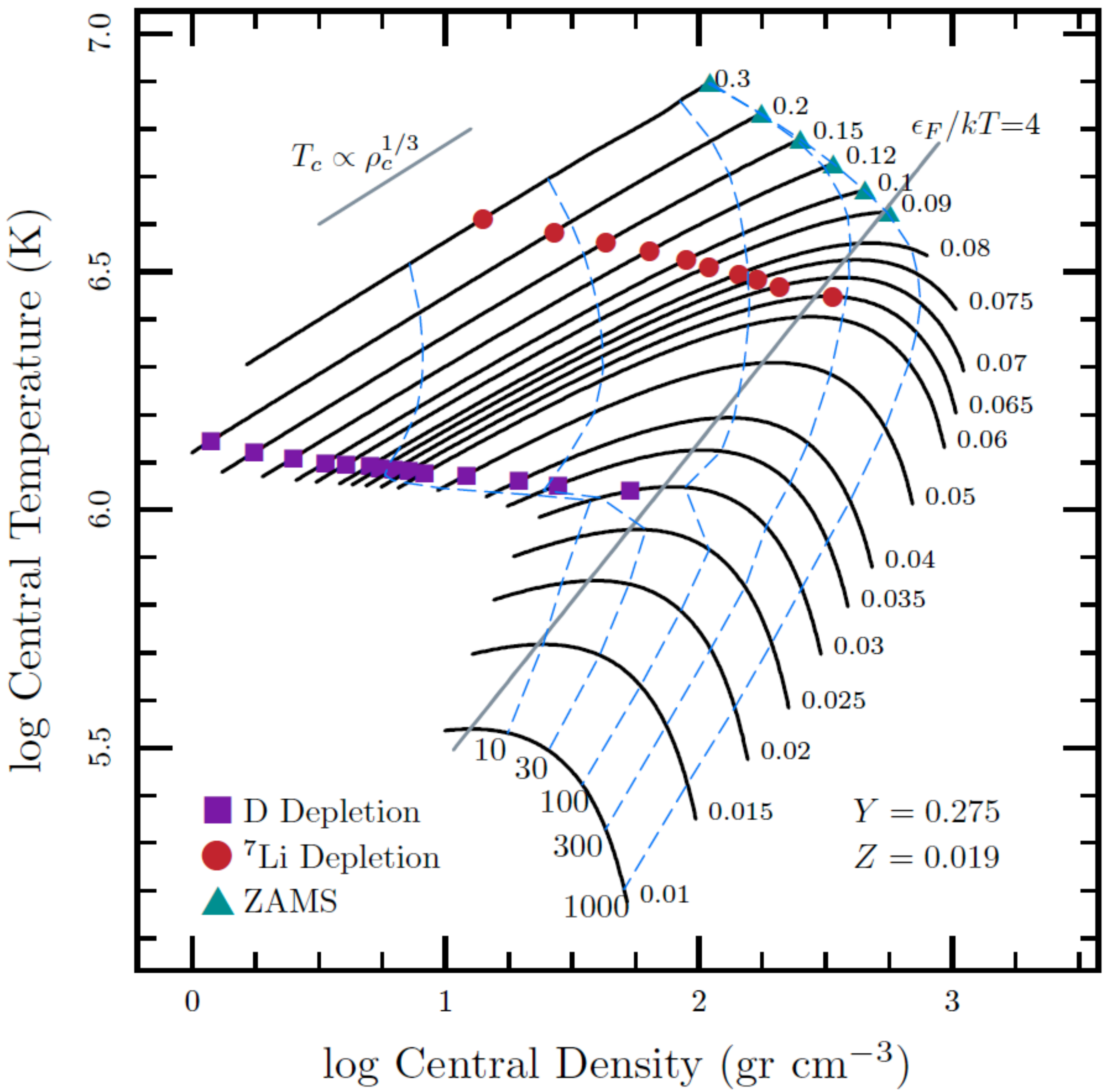
```
! max_abar_for_burning = 0
```

With burning on, go through the same steps as you did in part one for ages of 100 Myr, 300 Myr, and 1 Gyr. In addition to the quantities reported on the spreadsheet before, also report

- `lg_Lnuc`

Watch as the plots are populated with points for all the different masses that have been run.

Evan Show Lab 2 Results



log Central Temperature (K)
log Central Density (gr cm^{-3})
7.0
6.5
6.0
5.5
0
1
2
3
$T_c \propto \rho_c^{1/3}$
$\epsilon_F/kT=4$
0.3
0.2
0.15
0.12
0.1
0.09
0.08
0.075
0.07
0.065
0.06
0.05
0.04
0.035
0.03
0.025
0.02
0.015
0.01
10
30
100
300
1000
D Depletion
^{7}Li Depletion
ZAMS
$Y = 0.275$
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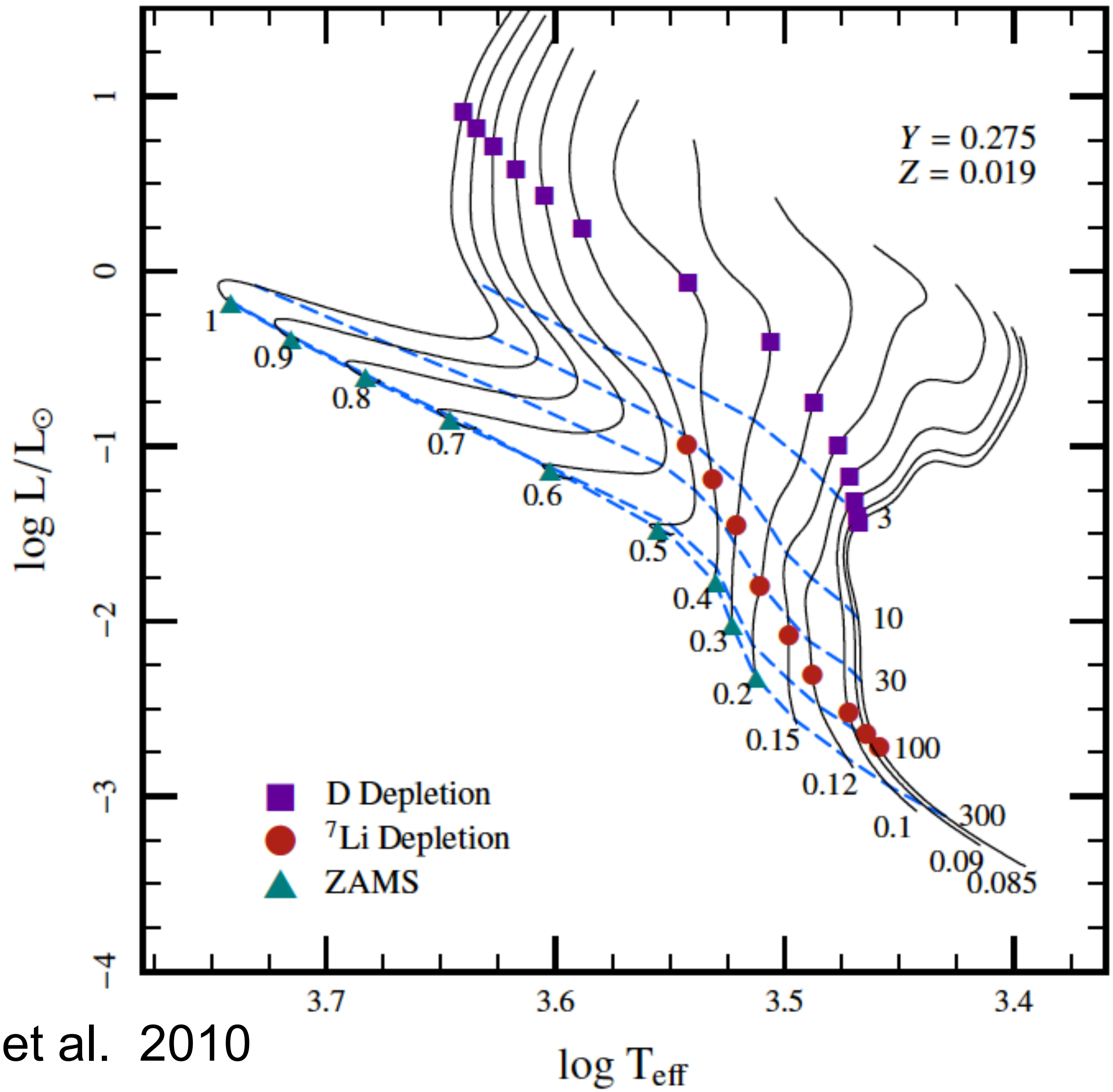
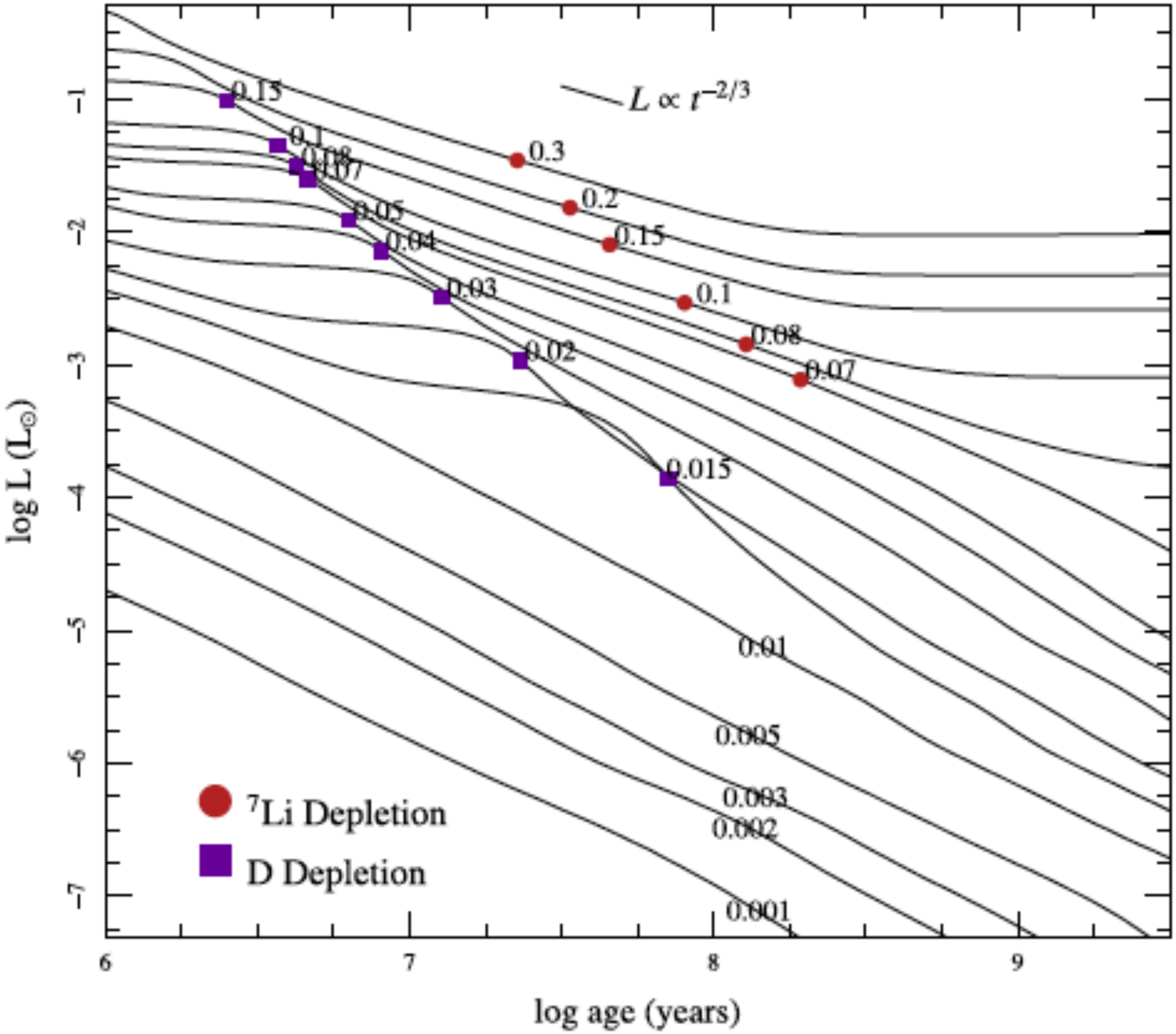


Paxton et al. 2010



$L \propto t^{-2/3}$
0.15
0.1
0.08
0.07
0.05
0.04
0.03
0.02
0.015
0.3
0.2
0.15
0.1
0.08
0.07
0.01
0.005
0.003
0.002
0.001
^{7}Li Depletion
D Depletion
log L ($L_\odot$)
log age (years)

???

What’s a Brown Dwarf??

- As evident, there is a critical mass defined by the bifurcation for a star that just gets hot enough to fuse and stay bright ‘forever’ vs. that which ‘misses’ and perpetually cools.
- Brown dwarf is an object that cools forever….now a plethora of them known, but a big deal when finally found.

Progress of Theoretical Physics, Vol. 30, No. 4, October 1963

Evolution of Stars of Small Masses in the Pre-Main-Sequence Stages

Chushiro HAYASHI and Takenori NAKANO

Department of Nuclear Science, Kyoto University, Kyoto

(Received June 12, 1963)

The structure of the outer envelope with an H-ionization zone and an H_2-dissociation zone is investigated for Population I stars of small masses ($2M_{\odot} \gtrsim M \gtrsim 0.05M_{\odot}$), which have low luminosities ($L_{\odot} \gtrsim L \gtrsim 10^{-3}L_{\odot}$) and low effective temperatures (6000 °K $\gtrsim T_e \gtrsim$ 2500 °K), in order to find the surface condition for the internal structure of these stars. The effective temperature of a star which is wholly convective and which has an H_2-dissociation zone is found to be nearly constant in the wide range of its luminosity.

Using stellar models composed of a radiative core and a convective envelope together with the above surface condition, the evolution of contracting stars is calculated up to the onset of hydrogen burning and the results are compared with the observed red dwarf stars. It is found that the stars on the zero-age main-sequence have radiative cores for $M > 0.26M_{\odot}$ but they are wholly convective for $0.26M_{\odot} \gtrsim M \gtrsim 0.08M_{\odot}$. The stars less massive than $0.08M_{\odot}$ are found to contract toward the configurations of high electron-degeneracy without hydrogen burning.

Minilab 3: Brown Dwarf Edge

Time to pick a new mass

1 Standard Abundances

We are now going to look at a narrower mass range than before. Generate a random number between $M = 0.06\,M_\odot$ and $M = 0.10\,M_\odot$, and change the initial mass in your inlist:

```
initial_mass = <your random mass between 0.06,0.10>
```

Change the maximum age to 1 Gyr:

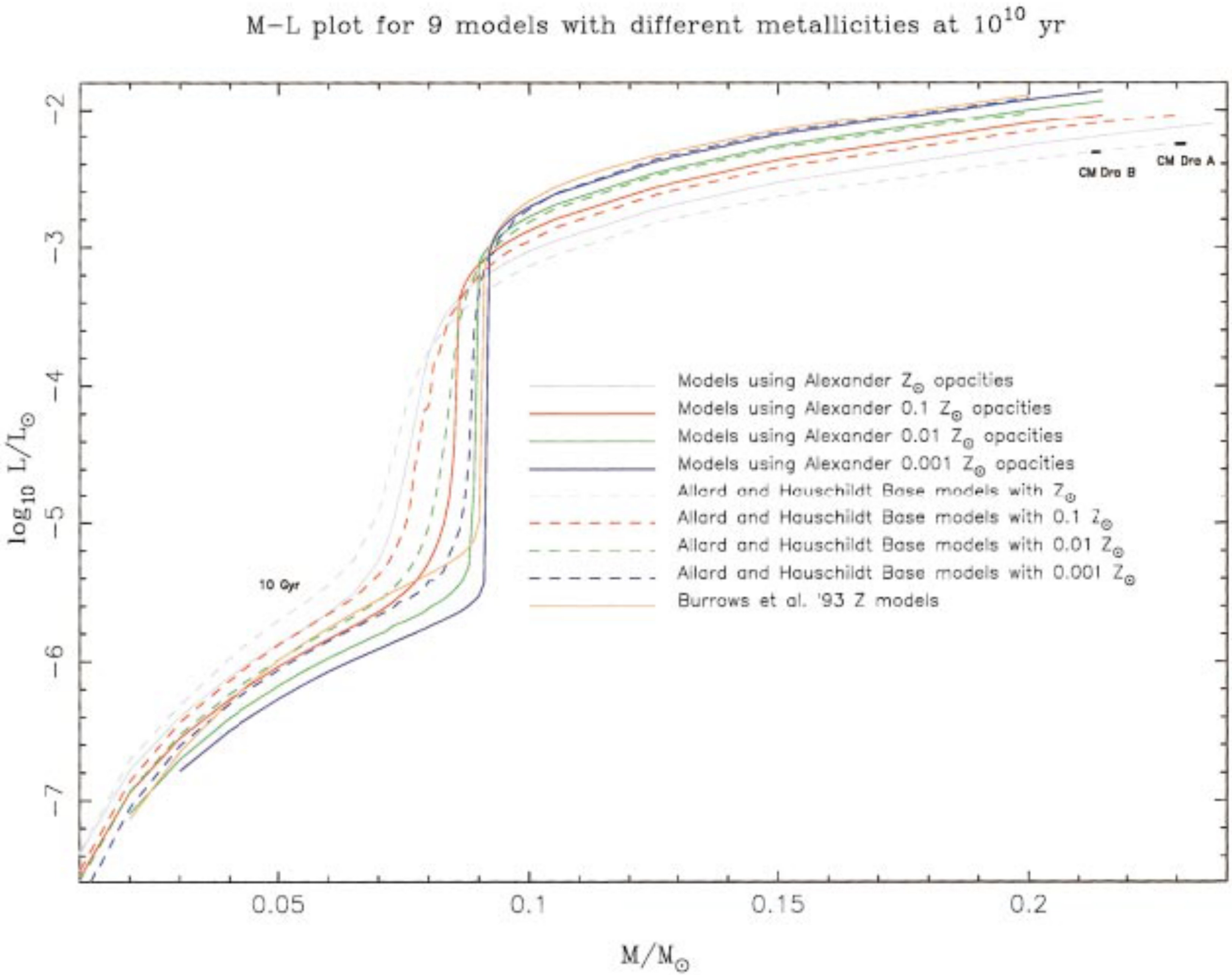
```
max_age = 1d9
```

Report the luminosity at the end of the run on the spreadsheet. Now change the maximum age to 10 Gyr:

```
max_age = 10d9
```

Restart from a recent photo, and again report the luminosity on the spreadsheet when the run finishes. We should now have

- Mass
- Luminosity at 1 Gyr
- Luminosity at 10 Gyr


M–L plot for 9 models with different metallicities at 10^{10} yr
$\log_{10} L/L_\odot$
$M/M_\odot$
-2
-3
-4
-5
-6
-7
0.05
0.1
0.15
0.2
10 Gyr
CM Dra B
CM Dra A
Models using Alexander $Z_\odot$ opacities
Models using Alexander 0.1 $Z_\odot$ opacities
Models using Alexander 0.01 $Z_\odot$ opacities
Models using Alexander 0.001 $Z_\odot$ opacities
Allard and Hauschildt Base models with $Z_\odot$
Allard and Hauschildt Base models with 0.1 $Z_\odot$
Allard and Hauschildt Base models with 0.01 $Z_\odot$
Allard and Hauschildt Base models with 0.001 $Z_\odot$
Burrows et al. '93 Z models

Talk First about Normal Helium Fraction Results

III. THE EDGE OF THE MAIN SEQUENCE

The properties of stars at the main sequence edge are a function of the helium fraction ($Y_\alpha \sim 0.25-0.28$), metallicity, and the opacity of the clouds of silicate grains that characterize L dwarf atmospheres with $T_{\rm eff}$'s near 1400–2000 K (see Sec. VII). A large grain opacity (perhaps due to smaller average particle size), high helium fractions, and higher metallicity lead to lower edge masses (HBMM), edge $T_{\rm eff}$'s, and edge luminosities. The higher helium fraction leads to a more compact object, with a larger central temperature and density, all else being equal. Larger opacities translate into optically thicker atmospheric blankets that do two things: (1) they produce higher central temperatures by steepening the temperature gradient and (2) they decrease the energy leakage rate (luminosity) from the surface. The former enhances the thermonuclear rate while the latter makes it easier to achieve main-sequence power balance at a lower mass. Hence the HBMM at solar metallicity and $Y_\alpha = 0.25$ is $\sim$0.07–0.074 $M_\odot$ (Hayashi and Nakano, 1963; Kumar, 1963), with a $T_{\rm eff}$ of 1700–1750 K and an $L_{\rm edge}$ of $\sim 6.0 \times 10^{-5}\,L_\odot$, while the HBMM at zero metallicity is $\sim 0.092\,M_\odot$, with a $T_{\rm eff}$ of $\sim$3600 K and an $L_{\rm edge}$ of $\sim 1.3 \times 10^{-3}\,L_\odot$ (Saumon *et al.*, 1994). The derivative, $\partial M_{\rm edge}/\partial Y_\alpha$, is approximately equal to $-0.1\,M_\odot$. However, as implied above, uncertainties in

2 Helium Rich

Now let's change the initial abundances and see what changes. This time we'll start with models that are helium rich ($Y = 0.78$) and hydrogen poor ($X = 0.20$). Add this to the **&controls** section of your inlist:

```
initial_y = 0.78
```

MESA will automatically adjust the hydrogen abundance to account for the increased presence of helium. You'll notice that your inlist also specifies `initial_z = 0.02`, which sets the total metal abundance to $Z = Z_\odot$ with solar ratios. The hydrogen mass fraction that **MESA** will set is then

$$X = 1.0 - Y - Z$$

Repeat the steps from the previous part, and once again report on the spreadsheet

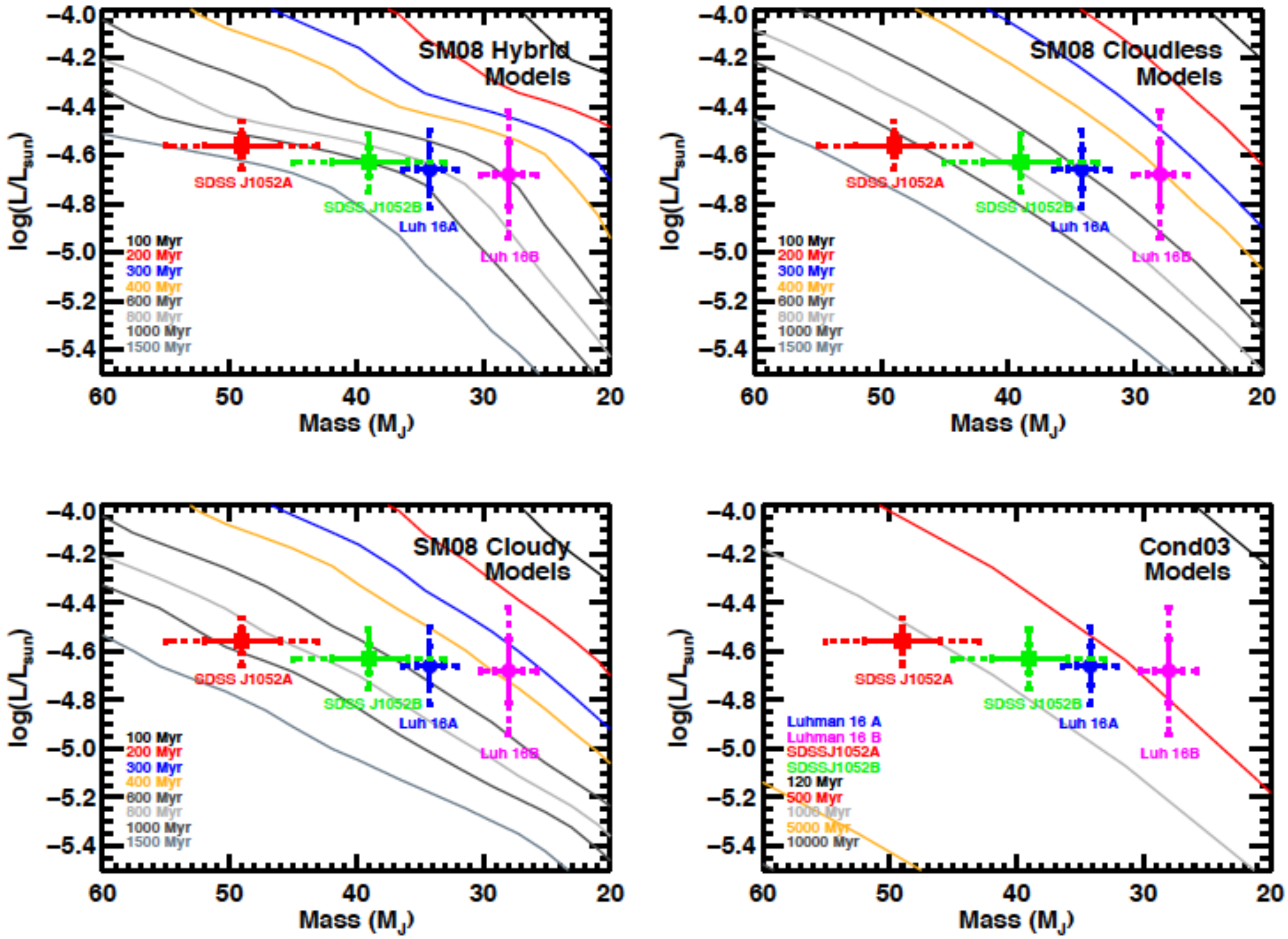
- Mass
- Luminosity at 1 Gyr
- Luminosity at 10 Gyr

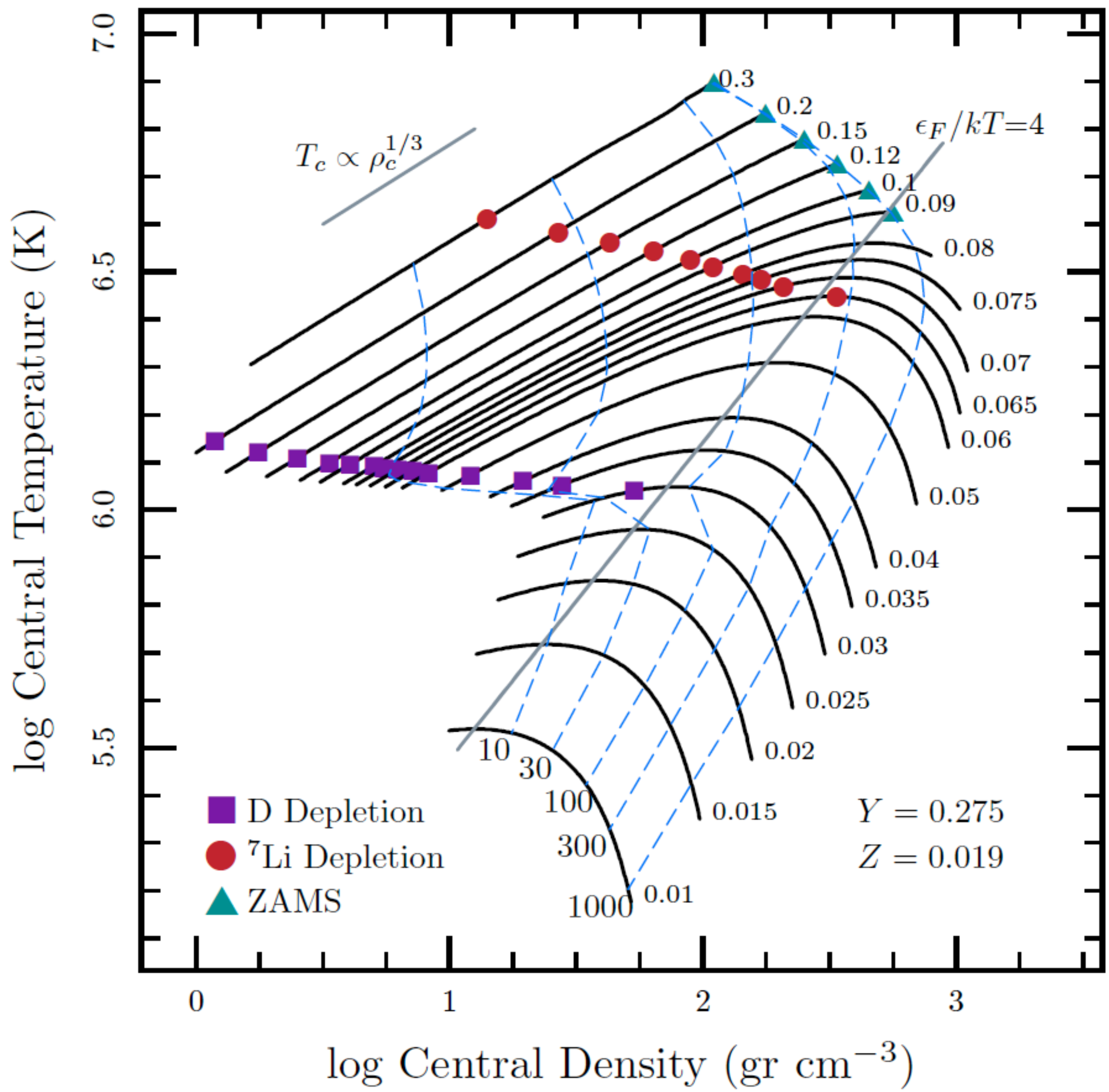
- Do we agree with the Burrows prediction??

Bonus work if you wish!!

Individual, Model-Independent Masses of the Closest Known Brown Dwarf Binary to the Sun

E. Victor Garcia[1], S. Mark Ammons[1], Maissa Salama[2], Ian Crossfield[3], Eduardo Bendek[4], Jeffrey Chilcote[5], Vincent Garrel[6], James R. Graham[7], Paul Kalas[7], Quinn Konopacky[8], Jessica R. Lu[2], Bruce Macintosh[9], Eduardo Marin[6], Christian Marois[10,11] Eric Nielsen[9,12], Benoît Neichel[13], Don Pham[1], Robert J. De Rosa[7], Dominic M. Ryan[7], Maxwell Service[2], Gaetano Sivo[6]





$T_c \propto \rho_c^{1/3}$
$\epsilon_F/kT{=}4$
0.3
0.2
0.15
0.12
0.1
0.09
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0.065
0.06
0.05
0.04
0.035
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0.015
0.01
10
30
100
300
1000
D Depletion
^{7}Li Depletion
ZAMS
$Y = 0.275$
$Z = 0.019$
log Central Temperature (K)
log Central Density (gr cm^{-3})

Deuterium “Burning”: A Coincidence & Lesson

$$\mathrm{p} + \mathrm{D} \rightarrow {}^{3}\mathrm{He} + \gamma \quad \text{releases 5.5 MeV}$$

- Deuterium is present in the ISM due to both production in the Big Bang + other processes at a value of D/H=2e-5.
- At the time when burning lifetime equals contraction, the core is at 10^6 K.

$$\text{Thermal Energy Density} = \frac{3\rho k_B T_c}{2\mu m_p}$$

Nuclear Energy Release can Slow Contraction

- Remember, contraction is occurring so as to tap gravitational energy to match luminosity. Nuclear energy release per volume is

$$\text{Nuclear Energy Density} = 2 \times 10^{-5} Q n_H$$

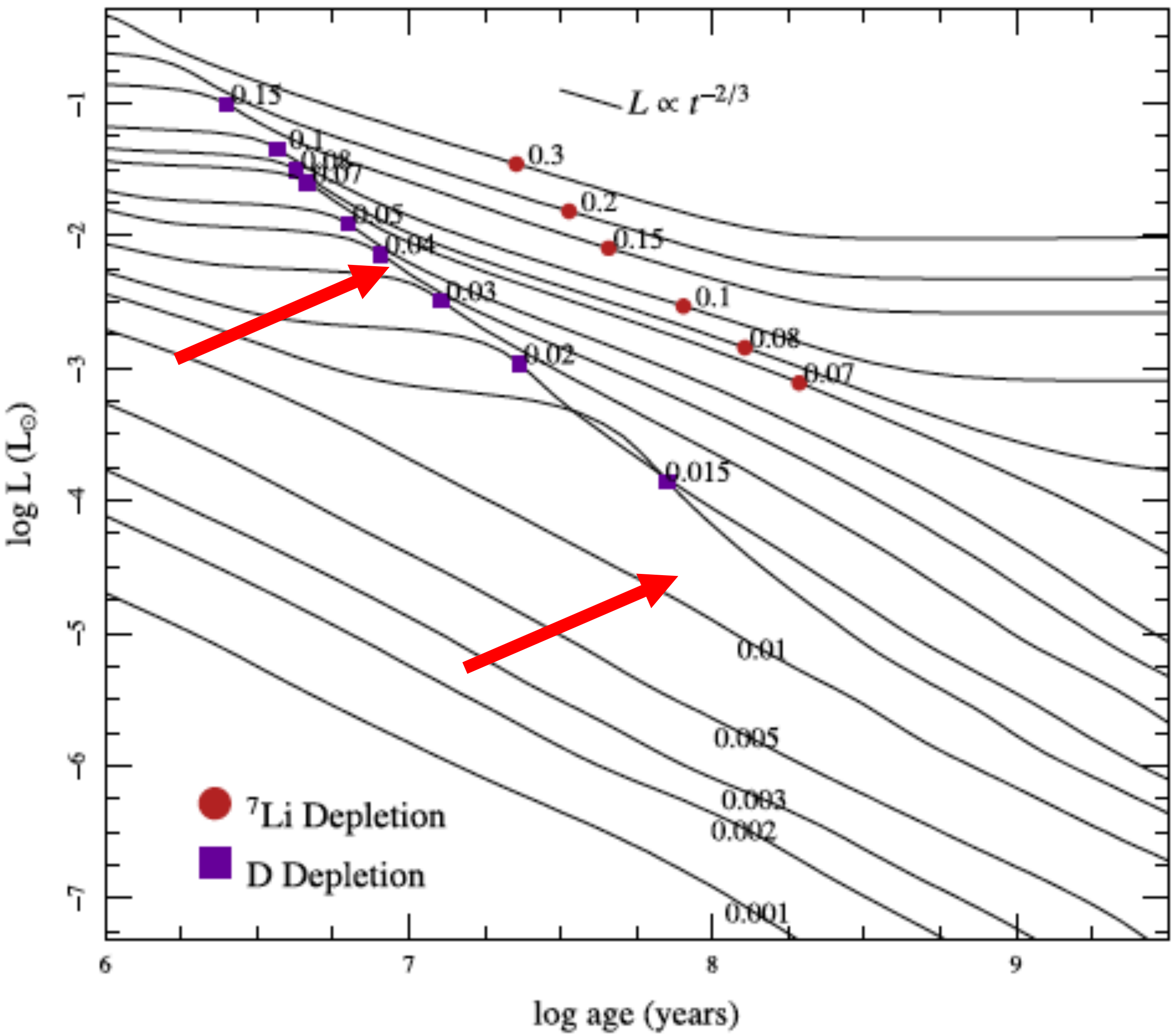
$$\frac{E_{\text{nuclear}}}{E_{\text{thermal}}} \approx \frac{2 \times 10^{-5} X Q}{3 k_B T_c / 2\mu} \approx \frac{0.35}{T_6}$$

So, when at a temperature of 10^6 K, it appears to have 35% more thermal energy. Slight slowing…

Minilab 4: Deuterium Main Sequence

You may have noticed in the previous minilab that metallicity was set with solar abundance ratios. This isn't quite the right thing to do on the pre main sequence, since some isotopes can be depleted by burning before the main sequence, where we are observing metal abundances in the Sun today. In fact, some isotopes can even burn in objects that never reach main sequence hydrogen burning! Here, we'll look at deuterium. In `inlist_brown_dwarf`, find and uncomment these lines:

Show Results



$L \propto t^{-2/3}$
0.15
0.1
0.08
0.07
0.05
0.04
0.03
0.02
0.015
0.3
0.2
0.15
0.1
0.08
0.07
0.01
0.005
0.003
0.002
0.001
^{7}Li Depletion
D Depletion
log L ($L_\odot$)
log age (years)

Let's Live on Deuterium!

Uncomment the `log_star_age` line. Now run the model, and see if you can identify the effects of deuterium on the cooling track. If you want to watch the burning and abundance profiles as well, you can add this to your `pgstar`:

```
Profile_Panels3_win_flag = .true.
```

Now set the deuterium abundance to be higher. Try setting `initial_h2` to `2d-4` or even `2d-3`. Run the model again and watch the `pgstar` output. What is the effect of increasing deuterium abundance on the cooling track?

Mon. Not. R. Astron. Soc. **380**, 712–732 (2007) doi:10.1111/j.1365-2966.2007.12106.x

A wide deep infrared look at the Pleiades with UKIDSS: new constraints on the substellar binary fraction and the low-mass initial mass function★

N. Lodieu,[1,2]† P. D. Dobbie,[3,2] N. R. Deacon,[4] S. T. Hodgkin,[5] N. C. Hambly[6] and R. F. Jameson[2]