Mini-Lab 1: Creating Sub-Neptune Size Planets in MESA

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It is a testament to the versatility of MESA that a code initially designed for "Stellar Astrophysics", can simulate H/He envelopes surround small planets down to masses only a couple times that of the Earth. Creating an initial model for a small planet takes several steps, following, in a rough sense, a method of successive approximations to get from a rather "star-like" starting point, to a planet-like end point. In this mini-lab, we will go through some of the basic steps required to create an initial starting model (for a planet of a desired core mass and envelope mass). Then, we will make a mass-radius diagram for small planets of various compositions.

Through out the day, when converting from Earth and Jupiter based units to cgs, let's be sure to use the same numerical values for these astronomical constants as mesa uses. from `const_def.f90`:

```
real(dp) :: m_earth ! = 5.9764d27 ! earth mass (g)
real(dp) :: r_earth ! = 6.37d8 ! earth radius (cm)
real(dp) :: au ! = 1.495978921d13 ! astronomical unit (cm)

real(dp) :: m_jupiter ! = 1.8986d30 ! jupiter mass (g)
real(dp) :: r_jupiter ! = 6.9911d9 ! jupiter mean radius (cm)
real(dp) :: semimajor_axis_jupiter ! = 7.7857d13 ! jupiter semimajor axis (cm)

real(dp) :: msol ! = 1.9892d33  ! solar mass (g)
real(dp) :: rsol ! = 6.9598d10 ! solar radius (cm)
real(dp) :: lsol ! = 3.8418d33  ! solar luminosity (erg s^-1)
```

Note, in the lab below, action items are highlighted in red.

a) Create an Initial Model

Extract the folder `rogers_minilabs`, clean and compile. Note, the work folder for this mini lab was adapted from the `make_planets` test suite, and is based upon Chen & Rogers (2016).

The starting point, is to create a new adiabatic contracting initial model, using `create_initial_model`. This creates a (coreless) H/He planet with an adiabatic temperature profile through out. This procedure works down to $\sim 0.1\ M_J$. Run `inlist_1a_create` will create a $\sim 30\ M_\oplus$ model that is near the lower mass limit of the initial models that we can form in this way. We're aiming to use MESA to model "Sub-Neptune Planets". Since Neptune has a mass of 17 $M_\oplus$, we will have to eventually decrease the mass of our model by a factor of 2 or more, to truly reach the "Sub-Neptune" regime – we'll get to this in step c).

You can run `inlist_1a_create` by copying `inlist_1a_create` to `inlist`:

```
cp inlist_1a_create inlist
./rn
```

Note, you don't need to change the `radius_in_cm_for_create_initial_model` (or any other parameters in this inlist). Don't worry! You'll have to make changes to all the inlists from steps b) to e).

b) Adding a Core

The next step is to add an inert core to our model. This core represents the "heavy element interior" of the planet. The core also provides a deeper gravitational potential well that will help to keep the planet envelope bound (hopefully) when we later reduce the mass of the planet.

We will use the flag in MESA `relax_core` to add a core to the planet. The core mass is set with `new_core_mass` and the radius is set by mean density `core_ave_rho`. (Alternatively, one can also use `relax_R_center` and set the core radius. An energy generation rate can be set by `core_ave_eps` (we will ignore this for now, keeping `core_ave_eps = 0`, but come back to this in a little bit). Let's consider planets with heavy element compositions similar to the Earth. We would expect the density of an Earth-composition core to increase with core mass due to compression, roughly as $(R/R_\oplus) = (M/M_\oplus)^{0.27}$.

Each group will be responsible for simulating a different planet core mass: $M_{\rm core} = 3, 4, 5, 6, 7, 8, 9, 10, 12, 14\ M_\oplus$. Use the scaling above to calculate the mean density of your assigned core mass, and then use `inlist_1b_core` (with the requisite updates) to generate a cored model planet with your desired core mass and density. Using:

```
num_trace_history_values = 2
trace_history_value_name(1) = 'm_center_gm'
trace_history_value_name(2) = 'r_center_cm'
```

the planet core mass is output to the terminal so that you can watch as MESA gradually adjusts the properties of the core.

For book keeping purposes, in addition to changing the input core mass and density you may want to change the following lines in `inlist_1b_core`:

```
save_model_filename = "planet_1b_core_15ME.mod"
star_history_name = "history_1b_core_15ME"
```

to replace 15ME with your assigned core mass value (e.g., a model with a 10 $M_\oplus$ core would be saved as `planet_1b_core_10ME.mod`). You should similarly adjust the corresponding lines in all subsequent inlists in today's labs.

c) Relax Mass

The next step, is to reduce the mass of the planet from its current value of $\sim 30\ M_\oplus$, down to our desired value(s).

What should your target `new_mass` be? Our planets consist of an inert heavy element core of mass $M_{\rm core}$ with a H/He envelope of mass $M_{\rm env}$ on top. We will explore planets with four different values for the envelope mass fraction: $f_{\rm env} = M_{\rm env}/M_p = 0.003$, 0.01, 0.03, and 0.1. Each group should work together to create models relaxed down to the planet masses, $M_p = M_{\rm core} + M_{\rm env} = M_{\rm core}/(1 - f_{\rm env})$ corresponding to each of these four envelope mass fractions, for their assigned $M_{\rm core}$. Each person at a table can take a different $f_{\rm env}$ among the list of 4 above. Ultimately, by the conclusion of this lab, we will make a planet mass-radius diagram to see how planet size depends on mass, for each of these bulk compositions.

We'll use the `relax_mass` option in the inlist to reduce the masses of our modeled planets. This gradually changes the mass of the planet (or star), using a wind. Specify the desired target mass (in $M_\odot$ units) in the `new_mass` inlist option. A template inlist is provided in `inlist_1c_reducemass`.

Don't forget to change the `saved_model_filename`, `save_model_filename` and `star_history_name` to reflect your particular value of $M_{\rm core}$ and $f_{\rm env}$. For example, a model with a 15 $M_\oplus$ core and $f_{\rm env} = 0.1$ would be saved as `planet_1b_core_15ME_0.1.mod`.

Note, for future reference, another approach to reducing the mass of a planet is `relax_mass_scale`. Instead of using a wind, this procedure gradually rescales the mass of the planet or star without changing composition as function of m/mstar.

d) Set Initial Entropy

Awesome! Now we have a freshly baked batch of planets with our desired grid of masses and compositions. The last step before evolving these planets is to try to standardize their starting conditions after all that `relax_mass`-ing and `relax_core`-ing.

Let's specific the planet's interior entropy in order to set their starting conditions. Let's say that the "central" entropy (in our case at the base of the envelope) is $S_c = 9k_\mathrm{B}$/baryon at $t = 0$ for our planets. Double check your `history_1c_reducemass` to see what the `center_entropy` is (it's listed in units k_B/baryon) for each of the models saved at the end of the last step. You should find that the central entropy of your last saved model is lower than our target $S = 9k_\mathrm{B}$/baryon.

To set the initial entropy of our planets, we need to puff our planets up! We do this by adding an "artificial luminosity" in the core using `relax_initial_L_center`. We need to pick an "artificial luminosity" that's high enough to reverse the planet's expansion and to cause them to expand up to our desired entropy, but not so high such that we blow the envelope off our poor planet. We've found that `new_L_center =` 2.0×10^{27} ergs s^{-1} works over the range of planets in this mini lab. Then we use the stopping criterion `center_entropy_limit` to get MESA to quit and to save a model once our specified entropy limit is exceeded.

A template inlist `inlist_1d_setS` is provided. Modify this inlist and run to set the central entropy of the planet that you saved in the last step. Add the following lines to your inlist so that you use the terminal output to see if your model is headed toward the target S_c.

```
num_trace_history_values = 1
trace_history_value_name(1) = 'center_entropy'
```

e) Evolve

Finally, we have a grid of planet models with specified M_core, f_env and S_c. Phew! Now it's time to evolve these!

For this, we have another template inlist `inlist_1e_evolve` that you can modify for your specified planet mass and composition.

The main features to notice in this inlist, are the following:

- It uses `relax_initial_L_center` again to turn off the artificial core luminosity, and to relax the core luminosity to a more physical (lower) value. Use an core energy generation rate of $\epsilon_\mathrm{core} = 5 \times 10^{-8}$ erg g^{-1} s^{-1} to estimate a plausible value for L_core for your assigned value of M_core. This ϵ_core roughly corresponds Earth's current production of heat from decay of long-lived radio isotopes ($\sim$30TW per Earth mass).
- It resets the initial age of the planet to $t = 0$ using `set_initial_age`.
- The maximum age stopping criterion is extended to 5 Gyr.

Modify `inlist_1e_evolve` to evolve your planet for 5 Gyr. Then input the final radius of your planet (at 5 Gyr) in the Google spread sheet. These points will be used to crowd source a planet mass-radius diagram.

f) Bonus Question

What if we specified that we wanted an initial entropy of $S_c = 7k_{\mathrm{B}}$/baryon at $t = 0$? How would we modify the "Set Initial Entropy" step in that case? Create an inlist to specify an initial entropy of $S_c = 7k_{\mathrm{B}}$/baryon for your planet model. Then, repeat the evolve step with the $S = 7k_{\mathrm{B}}$/baryon models. How similar/different are the radii of the 7 and $9k_{\mathrm{B}}$/baryon at $t \sim 0$, at 10 Myr, at 1 Gyr, and 5 Gy?

References

Chen, H., & Rogers, L. A. 2016, ApJ, 831, 180