On Some Similarity Measures of Single Valued Neutrosophic Rough Sets.

K.Mohana[1], M.Mohanasundari[2]

[1] Assistant Professor, Department of Mathematics, Nirmala College For Women, Bharathiyar University, Coimbatore, 641 018, India.
[2] Assistant Professor, Department of Mathematics ,Bannari Amman Institute of Technology, Anna University, Sathyamangalam, 638 401, India.
E-mail : riyaraju1116@gmail.com[1], mohanasundari@bitsathy.ac.in[2]

Abstract**.** In this paper we have obtained the similarity measures between single valued neutrosophic rough sets by analyzing the concept of its distance between them and studied its properties. Further we have studied its similarity based on its membership degrees and studied its properties. We have also defined the cardinality of two single valued neutrosophic rough sets. A numerical example in medical diagnosis is given for the proposed similarity measure of the single valued neutrosophic rough sets which helps us to prove the usefulness and flexibility of the proposed method.

Keywords: Single valued neutrosophic rough sets, similarity measure, cardinality.

1 Introduction

Fuzzy sets are generalizations of classical (crisp) sets which is based on partial membership of the elements and this was proposed by Zadeh [32] in 1965.. In 1983,K. Atanassov [2] proposed the concept of Intuitionistic fuzzy set which is a generalization of fuzzy set theory and is based on the degree of membership and non-membership and is described in the real unit interval [0,1], whose sum also belongs to the same interval.

IFS has numerous applications in decision making problems, medical diagnosis etc. After the theory of IFS many theories have been developed which are suitable in their respective areas.

In 1995 Florentin Smarandache [27] proposed the concept of Neutrosophic logic which provides the main distinction of fuzzy and IFS. It is a logic which is based on degree of truth (T), degree of indeterminacy (I) and degree of falsity (F) and lies in the nonstandard unit interval $]0^-,1^+[$. Neutrosophic set theory deals with uncertainity factor i.e, indeterminacy factor which is independent of truth and falsity values. Neutrosophic theory is applicable to the fields which is related to indeterminacy factor i.e, in the field of image processing, medical diagnosis and decision making problem.

In 1982, Pawlak [18] introduced the concept of rough set which is based upon the approximation of sets known as lower and upper approximation of a set. These two lower and upper approximation operators based on equivalence relation.

Rough fuzzy sets, intuitionistic fuzzy rough sets, neutrosophic rough sets are introduced by combining the rough sets respectively with fuzzy, intuitionstic, neutrosophic sets. In particular rough neutrosophic set initiated by Broumi and Smarandache (2014) [5]. C. Antony Crispin Sweety & I. Arockiarani(2016)[1] studied the concept of neutrosophic rough set algebra[1]. Wang (2010) [30] proposed the concept of SVNS which is a very new hot research topic.

SVNS and rough sets both deals with inaccuracy information and both combined together to provide a new hybrid model of single valued neutrosophic rough set. Many authors [3,4,6,8,9,19,31] studied the concept of similarity and entropy between the two single valued neutrosophic sets which helps to identify whether two sets are identical or atleast to what degree they are identical by using the concept of distance formula and membership function. Similarity plays a vital role in many fields like computational intelligence, psychology and linguistics, medical diagnosis, multi-attribute decision making problems.

Smarandache.F introduced the "Neutrosophic Sets and Systems" and its applications have been spreaded in all directions at an amazing rate. Smarandache, F. & Pramanik, S. (Eds). (2016)[28] *New trends in neutrosophic theory and applications* emphases on theories, procedures, systems for decision making, medical diagnosis and also discussed the topic includes e-learning, graph theory and some more. Recently Smarandache, F. & Pramanik, S. (Eds). (2018) and Mondal, K., Pramanik, S., & Giri, B. C. (2018) [29,17] studies New trends in neutrosophic theory and applications, Fuzzy Multicriteria Decision Making Using Neutrosophic Sets which provides the innovative study and application papers from diverse viewpoints covering the areas of neutrosophic studies, such as decision making, graph theory, image processing, probability theory, topology, and some abstract papers.

Pramanik, S., Roy, R., Roy, T. K., & Smarandache, F. (2018)[24] studied multi-attribute decision making based on several trigonometric hamming similarity measures under interval rough neutrosophic environment. Pramanik, S., Roy, R., Roy, T. K. & Smarandache, F. (2017)[23] also proposed the concept of multi criteria decision making using correlation coefficient under rough neutrosophic environment. Pramanik, S., & Mondal, K. (2015)[20] Mondal, K., Pramanik, S., & Smarandache, F. (2016) [9] studied several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making.

Medical diagnosis is the process of determining which disease or condition explains a person's symptoms and signs. Similarity measures plays a efficient role in analysing the medical diagnosis problem. S. Pramanik, and K. Mondal. (2015)[12] described the cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. And also Pramanik, S., & Mondal, K. (2015)[13] studied Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis.

In this paper Section 2 gives some basic definitions of rough sets, neutrosophic sets, SVNSs and single valued neutrosophic rough sets. Section. 3 provides the distance and cardinality of two single valued neutrosophic rough sets with suitable example. In Section.4, we investigate the similarity measure of two single valued neutrosophic rough sets based on distance formulae and membership degrees. Section 5 gives a numerical example in medical diagnosis for the proposed similarity measure of single valued neutrosophic rough sets. Section 6 concludes the paper.

2 Preliminaries

In this section we recall the basic definitions of rough sets, Neutrosophic sets and single valued neutrosophic rough sets which will be used in the rest of the paper.

2.1 Definition 2.1[5]

Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non – null subset X of U, the sets $A_1(x) = \{X : [x]_R \subseteq X\}$ and $A_2(x) = \{X : [x]_R \cap X \neq \phi\}$ are called the lower approximation and upper approximation respectively of X where the pair S=(U,R) is called an approximation space. This equivalence relation R is called indiscernibility relation. The pair $A(X) = (A_1(X), A_2(X))$ is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing X.

2.2 Definition 2.2[27]

Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form, $A = \{< x : \mu_A(x), \nu_A(x), \omega_A(x) >, x \in X\}$ where the functions

$\mu, \nu, \omega : X \to \left]^{-}0,1^{+}\right[$ define respectively the degree of membership (or truth) , the degree of indeterminacy, and the degree of non-membership (or falsehood) of the element $x \in X$ to the set A with the condition,

$$^{-}0 \le \mu_A(x) + \nu_A(x) + \omega_A(x) \le 3^{+}$$

2.3 Definition 2.3[30]

Let U be a space of points (objects), with a generic element in U denoted by x. A single valued neutrosophic set (SVNS) A in U is characterized by a truth-membership function T_A , an indeterminacy- membership function I_A and a falsity membership function F_A , where $\forall x \in U,\ T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ A SVNS A can be expressed as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in U\}$

2.4 Definition 2.4[7]

A SVNS R in $U \times U$ is referred to as a single valued neutrosophic relation (SVNR) in U, denoted by

$$R = \{< (x, y) : T_R(x, y), I_R(x, y), F_R(x, y) > /(x, y) \in U \times U\}$$

where $T_R : U \times U \to [0,1],\ I_R : U \times U \to [0,1]\ and\ F_R : U \times U \to [0,1]$ represent the truth – membership function, indeterminacy-membership function and falsity-membership function of R respectively. Based on a SVNR, Yang et al.[4] gave the notion of single valued neutrosophic rough set as follows.

Let $\tilde{R}$ be a SVNR in U, the tuple $(U, \tilde{R})$ is called a single valued neutrosophic approximation space $\forall \tilde{A} \in SVNS(U)$, the lower and upper approximations of $\tilde{A}$ with respect to $(U, \tilde{R})$, denoted by $\underline{\tilde{R}}(\tilde{A})\ \ and\ \ \overline{\tilde{R}}(\tilde{A})$ are two SVNS's whose membership functions are defined as $\forall x \in U$,

$$T_{\underline{\tilde{R}}(\tilde{A})}(x) = \bigwedge_{y \in U} (F_{\tilde{R}}(x, y) \vee T_{\tilde{A}}(y)),$$

$$I_{\underline{\tilde{R}}(\tilde{A})}(x) = \bigvee_{y \in U} ((1 - I_{\tilde{R}}(x, y) \wedge I_{\tilde{A}}(y)),$$

$$F_{\underline{\tilde{R}}(\tilde{A})}(x) = \bigvee_{y \in U} (T_{\tilde{R}}(x, y) \wedge F_{\tilde{A}}(y)),$$

$$T_{\overline{\tilde{R}}(\tilde{A})}(x) = \bigvee_{y \in U} (T_{\tilde{R}}(x, y) \wedge T_{\tilde{A}}(y)),$$

$$I_{\overline{\tilde{R}}(\tilde{A})}(x) = \bigwedge_{y \in U} (I_{\tilde{R}}(x, y) \vee I_{\tilde{A}}(y)),$$

$$F_{\overline{\tilde{R}}(\tilde{A})}(x) = \bigwedge_{y \in U} (F_{\tilde{R}}(x, y) \vee F_{\tilde{A}}(y)).$$

The pair $(\underline{\tilde{R}}(\tilde{A}), \overline{\tilde{R}}(\tilde{A}))$ is called a single valued neutrosophic rough set of $\tilde{A}$ with respect to $(U, \tilde{R})$. $\underline{\tilde{R}}\ \ and\ \ \overline{\tilde{R}}$ are referred to as single valued neutrosophic lower and upper approximation operators respectively.

3 Distance between two single valued neutrosophic rough sets

In this section we define the distance between two single valued neutrosophic rough sets of $\tilde{A}$ and $\tilde{B}$ with respect to $(U, \tilde{R}_1)\ and\ \ (U, \tilde{R}_2)$ in the universe $U = \{x_1, x_2, x_3, ... x_n\}$.

3.1 Definition 3.1

Let us consider two single valued neutrosophic rough sets of $\tilde{A}$ and $\tilde{B}$ with respect to $(U,\tilde{R}_1)$ *and* $(U,\tilde{R}_2)$ in the universe $U=\{x_1, x_2, x_3, \ldots\ldots x_n\}$. Here $\underline{\tilde{R}}$ *and* $\overline{\tilde{R}}$ are referred to as the single valued neutrosophic lower and upper approximation operators respectively. Throughout this section $\tilde{A}$ and $\tilde{B}$ denote the single valued neutrosophic rough sets with respect to $(U,\tilde{R}_1)$ *and* $(U,\tilde{R}_2)$.

(i) The Hamming distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its lower approximation:

$$d_{\underline{N}}(\tilde{A},\tilde{B})=\sum_{i=1}^{n}\{|T_{\underline{\tilde{R}}(\tilde{A})}(x_i)-T_{\underline{\tilde{R}}(\tilde{B})}(x_i)|+|I_{\underline{\tilde{R}}(\tilde{A})}(x_i)-I_{\underline{\tilde{R}}(\tilde{B})}(x_i)|+|F_{\underline{\tilde{R}}(\tilde{A})}(x_i)-F_{\underline{\tilde{R}}(\tilde{B})}(x_i)|\} \tag{1}$$

(ii) The Hamming distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its upper approximation:

$$d_{\overline{N}}(\tilde{A},\tilde{B})=\sum_{i=1}^{n}\{|T_{\overline{\tilde{R}}(\tilde{A})}(x_i)-T_{\overline{\tilde{R}}(\tilde{B})}(x_i)|+|I_{\overline{\tilde{R}}(\tilde{A})}(x_i)-I_{\overline{\tilde{R}}(\tilde{B})}(x_i)|+|F_{\overline{\tilde{R}}(\tilde{A})}(x_i)-F_{\overline{\tilde{R}}(\tilde{B})}(x_i)|\} \tag{2}$$

(iii) The normalized Hamming distance of $\tilde{A}$ and $\tilde{B}$ with respect to its lower approximation:

$$l_{\underline{N}}(\tilde{A},\tilde{B})=\frac{1}{3n}\sum_{i=1}^{n}\{|T_{\underline{\tilde{R}}(\tilde{A})}(x_i)-T_{\underline{\tilde{R}}(\tilde{B})}(x_i)|+|I_{\underline{\tilde{R}}(\tilde{A})}(x_i)-I_{\underline{\tilde{R}}(\tilde{B})}(x_i)|+|F_{\underline{\tilde{R}}(\tilde{A})}(x_i)-F_{\underline{\tilde{R}}(\tilde{B})}(x_i)|\} \tag{3}$$

(iv) The normalized Hamming distance of $\tilde{A}$ and $\tilde{B}$ with respect to its upper approximation:

$$l_{\overline{N}}(\tilde{A},\tilde{B})=\frac{1}{3n}\sum_{i=1}^{n}\{|T_{\overline{\tilde{R}}(\tilde{A})}(x_i)-T_{\overline{\tilde{R}}(\tilde{B})}(x_i)|+|I_{\overline{\tilde{R}}(\tilde{A})}(x_i)-I_{\overline{\tilde{R}}(\tilde{B})}(x_i)|+|F_{\overline{\tilde{R}}(\tilde{A})}(x_i)-F_{\overline{\tilde{R}}(\tilde{B})}(x_i)|\} \tag{4}$$

(v) The Euclidian distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its lower approximation:

$$e_{\underline{N}}(\tilde{A},\tilde{B})=\sqrt{\sum_{i=1}^{n}(T_{\underline{\tilde{R}}(\tilde{A})}(x_i)-T_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2+(I_{\underline{\tilde{R}}(\tilde{A})}(x_i)-I_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2+(F_{\underline{\tilde{R}}(\tilde{A})}(x_i)-F_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2} \tag{5}$$

(vi) The Euclidian distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its upper approximation:

$$e_{\overline{N}}(\tilde{A},\tilde{B})=\sqrt{\sum_{i=1}^{n}(T_{\overline{\tilde{R}}(\tilde{A})}(x_i)-T_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2+(I_{\overline{\tilde{R}}(\tilde{A})}(x_i)-I_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2+(F_{\overline{\tilde{R}}(\tilde{A})}(x_i)-F_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2} \tag{6}$$

(vii) The normalized Euclidian distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its lower approximation:

$$q_{\underline{N}}(\tilde{A},\tilde{B})=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}(T_{\underline{\tilde{R}}(\tilde{A})}(x_i)-T_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2+(I_{\underline{\tilde{R}}(\tilde{A})}(x_i)-I_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2+(F_{\underline{\tilde{R}}(\tilde{A})}(x_i)-F_{\underline{\tilde{R}}(\tilde{B})}(x_i))^2} \tag{7}$$

(viii) The normalized Euclidian distance of two single valued neutrosophic rough sets $\tilde{A}$ and $\tilde{B}$ with respect to its upper approximation:

$$q_{\overline{N}}(\tilde{A},\tilde{B})=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}(T_{\overline{\tilde{R}}(\tilde{A})}(x_i)-T_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2+(I_{\overline{\tilde{R}}(\tilde{A})}(x_i)-I_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2+(F_{\overline{\tilde{R}}(\tilde{A})}(x_i)-F_{\overline{\tilde{R}}(\tilde{B})}(x_i))^2} \quad (8)$$

Now for equations (1) – (8) the following conditions holds:

(a) $0\le d_{\underline{N}}(\tilde{A},\tilde{B})\le 3n$, $0\le d_{\overline{N}}(\tilde{A},\tilde{B})\le 3n$ (9)

(b) $0\le l_{\underline{N}}(\tilde{A},\tilde{B})\le 1$, $0\le l_{\overline{N}}(\tilde{A},\tilde{B})\le 1$ (10)

(c) $0\le e_{\underline{N}}(\tilde{A},\tilde{B})\le \sqrt{3n}$, $0\le e_{\overline{N}}(\tilde{A},\tilde{B})\le \sqrt{3n}$ (11)

(d) $0\le q_{\underline{N}}(\tilde{A},\tilde{B})\le 1$, $0\le q_{\overline{N}}(\tilde{A},\tilde{B})\le 1$ (12)

Example 3.2

Let $U=\{x_1, x_2, x_3\}$ be the universe and $\tilde{R}_1,\tilde{R}_2 \in SVNS(U\times U)$ is given in Table 1 and Table 2

Let $\tilde{A}=\{<x_1,(0.3,0.4,0.5)>,<x_2,(0,1,0.3)>,<x_3,(0.4,0.3,0.6)>\}$

$\tilde{B}=\{<x_1,(0.2,0.8,0.1)>,<x_2,(1,0.3,1)>,<x_3,(0.5,0.3,0)>\}$ are SVNS's in U.

$\tilde{R}_1$	x_1	x_2	x_3
x_1	$(0,0.6,0.4)$	$(1,0,0.4)$	$(0.3,0.7,0.2)$
x_2	$(0,0.1,0.5)$	$(0.5,0,0.4)$	$(0.3,0.4,0.8)$
x_3	$(1,0,0.6)$	$(0.6,1,1)$	$(0,0,1)$

Table 1: SVNR $\tilde{R}_1$

$\tilde{R}_2$	x_1	x_2	x_3
x_1	$(0,0,1)$	$(0.2,0.1,0.6)$	$(1,0,0.5)$
x_2	$(0,0.1,0.3)$	$(0.5,0.4,1)$	$(0.5,1,0)$
x_3	$(1,1,0)$	$(0.4,1,1)$	$(1,0,0)$

Table 2: SVNR $\tilde{R}_2$

According to Definition 2.4 , we have

$$T_{\underline{\tilde{R}}(\tilde{A})}(x_1)=\bigwedge_{y\in U}(F_{\tilde{R}}(x_1,y)\vee T_{\tilde{A}}(y))=0.4$$

$$I_{\underline{\tilde{R}}(\tilde{A})}(x_1)=\bigvee_{y\in U}((1-I_{\tilde{R}}(x_1,y)\wedge I_{\tilde{A}}(y))=1$$

$$F_{\underline{\tilde{R}}(\tilde{A})}(x_1)=\bigvee_{y\in U}(T_{\tilde{R}}(x_1,y)\wedge F_{\tilde{A}}(y))=0.3,$$

$$T_{\overline{\tilde{R}}(\tilde{A})}(x_1)=\bigvee_{y\in U}(T_{\tilde{R}}(x_1,y)\wedge T_{\tilde{A}}(y))=0.3$$

$$I_{\overline{\tilde{R}}(\tilde{A})}(x_1)=\bigwedge_{y\in U}(I_{\tilde{R}}(x_1,y)\vee I_{\tilde{A}}(y))=0.6$$

$$F_{\overline{\tilde{R}}(\tilde{A})}(x_1)=\bigwedge_{y\in U}(F_{\tilde{R}}(x_1,y)\vee F_{\tilde{A}}(y))=0.4$$

Hence,

$$\underline{\tilde{R}}(\tilde{A})(x_1) = (0.4,1,0.3) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_1) = (0.3,0.6,0.4)$$

Similarly we can obtain,

$$\underline{\tilde{R}}(\tilde{A})(x_2) = (0.4,1,0.3) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_2) = (0.3,0.4,0.4)$$
$$\underline{\tilde{R}}(\tilde{A})(x_3) = (0.6,0.4,0.5) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_3) = (0.3,0.3,0.6)$$
$$\underline{\tilde{R}}(\tilde{B})(x_1) = (0.5,0.8,0.2) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_1) = (0.5,0.3,0.5)$$
$$\underline{\tilde{R}}(\tilde{B})(x_2) = (0.3,0.8,0.5) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_2) = (0.5,0.4,0)$$
$$\underline{\tilde{R}}(\tilde{B})(x_3) = (0.2,0.3,0.4) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_3) = (0.5,0.3,0)$$

Then the distance between $\tilde{A}$ *and* $\tilde{B}$ will be as follows :

$$d_{\underline{N}}(\tilde{A},\tilde{B}) = \sum_{i=1}^{n}\{|T_{\underline{\tilde{R}}(\tilde{A})}(x_i) - T_{\underline{\tilde{R}}(\tilde{B})}(x_i)| + |I_{\underline{\tilde{R}}(\tilde{A})}(x_i) - I_{\underline{\tilde{R}}(\tilde{B})}(x_i)| + |F_{\underline{\tilde{R}}(\tilde{A})}(x_i) - F_{\underline{\tilde{R}}(\tilde{B})}(x_i)|\}$$
$$= \sum_{i=1}^{n}\{|T_{\underline{\tilde{R}}(\tilde{A})}(x_i) - T_{\underline{\tilde{R}}(\tilde{B})}(x_i)| + |I_{\underline{\tilde{R}}(\tilde{A})}(x_i) - I_{\underline{\tilde{R}}(\tilde{B})}(x_i)| + |F_{\underline{\tilde{R}}(\tilde{A})}(x_i) - F_{\underline{\tilde{R}}(\tilde{B})}(x_i)|\}$$
$$= \quad 1.5$$
$$d_{\underline{N}}(\tilde{A},\tilde{B}) = 1.5$$

Similarly the other distances will be,

$$d_{\overline{N}}(\tilde{A},\tilde{B}) = 2$$
$$l_{\underline{N}}(\tilde{A},\tilde{B}) = 0.1666 \quad , \quad l_{\overline{N}}(\tilde{A},\tilde{B}) = 0.2222$$
$$e_{\underline{N}}(\tilde{A},\tilde{B}) = 0.5745 \quad , \quad e_{\overline{N}}(\tilde{A},\tilde{B}) = 0.86023$$
$$q_{\underline{N}}(\tilde{A},\tilde{B}) = 0.1916 \quad , \quad q_{\overline{N}}(\tilde{A},\tilde{B}) = 0.30916$$

3.3 Definition 3.3 (Cardinality)

The cardinality of a single valued neutrosophic rough set of $\tilde{A}$ with respect to $(U,\tilde{R})$ is denoted as $\underline{\tilde{R}}[c]$ *and* $\overline{\tilde{R}}[c]$, where $\underline{\tilde{R}}[c] = [\underline{\tilde{R}}(c^l), \underline{\tilde{R}}(c^u)]$ is known as single valued neutrosophic lower approximation cardinality and, $\overline{\tilde{R}}[c] = [\overline{\tilde{R}}(c^l), \overline{\tilde{R}}(c^u)]$ is known as single valued neutrosophic upper approximation cardinality.

Here $\underline{\tilde{R}}(c^l)$, $\underline{\tilde{R}}(c^u)$ denotes minimum and maximum cardinality of a single valued neutrosophic rough set with respect to lower approximation and is defined as ,

$$\underline{\widetilde{R}}(c^l)=\sum_{i=1}^{n}T_{\underline{\widetilde{R}}(A)}(x_i) \quad \text{and} \quad \underline{\widetilde{R}}(c^u)=\sum_{i=1}^{n}\{T_{\underline{\widetilde{R}}(A)}(x_i)+(1-I_{\underline{\widetilde{R}}(A)}(x_i))\} \qquad (13)$$

Here $\overline{\widetilde{R}}(c^l)$, $\overline{\widetilde{R}}(c^u)$ denotes minimum and maximum cardinality of a single valued neutrosophic rough set with respect to upper approximation and is defined as ,

$$\overline{\widetilde{R}}(c^l)=\sum_{i=1}^{n}T_{\overline{\widetilde{R}}(A)}(x_i) \quad \text{and} \quad \overline{\widetilde{R}}(c^u)=\sum_{i=1}^{n}\{T_{\overline{\widetilde{R}}(A)}(x_i)+(1-I_{\overline{\widetilde{R}}(A)}(x_i))\} \qquad (14)$$

Example 3.4

Let us consider the single valued neutrosophic rough set of $\widetilde{B}$ from Example 3.2 we have the following cardinality,

$$\underline{\widetilde{R}}(c^l)=\sum_{i=1}^{n}T_{\underline{\widetilde{R}}(A)}(x_i)$$

$$=\sum_{i=1}^{3}T_{\underline{\widetilde{R}}(A)}(x_i)$$

$$\underline{\widetilde{R}}(c^l)=1$$

$$\underline{\widetilde{R}}(c^u)=\sum_{i=1}^{n}\{T_{\underline{\widetilde{R}}(A)}(x_i)+(1-I_{\underline{\widetilde{R}}(A)}(x_i))\}$$

$$=\sum_{i=1}^{n}\{T_{\underline{\widetilde{R}}(A)}(x_i)+(1-I_{\underline{\widetilde{R}}(A)}(x_i))\}$$

$$\underline{\widetilde{R}}(c^u)=2.1$$

$$\underline{\widetilde{R}}[c]=[\underline{\widetilde{R}}(c^l),\underline{\widetilde{R}}(c^u)]=[1,2.1]$$

Similarly we can obtain,

$$\overline{\widetilde{R}}[c]=[\overline{\widetilde{R}}(c^l),\overline{\widetilde{R}}(c^u)]=[1.5,3.5]$$

4 Similarity measure between two single valued neutrosophic rough sets:

In this section we have defined the similarity measure between two single valued neutrosophic rough sets by the following two methods .

(i) Distance based similarity measure
(ii) Membership degree based similarity measure

A similarity measure between two single valued neutrosophic rough sets is a function defined as $S:\underline{N}(U)^2\rightarrow[0,1]$ and $\overline{N}(U)^2\rightarrow[0,1]$ which satisfies the following properties.

(i) $S_{\underline{N}}(\widetilde{A},\widetilde{B})\in[0,1]$ and $S_{\overline{N}}(\widetilde{A},\widetilde{B})\in[0,1]$

(ii) $S_{\underline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow\widetilde{A}=\widetilde{B}$ and $S_{\overline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow\widetilde{A}=\widetilde{B}$ (15)

(iii) $S_{\underline{N}}(\widetilde{A},\widetilde{B})=S_{\underline{N}}(\widetilde{B},\widetilde{A})$ and $S_{\overline{N}}(\widetilde{A},\widetilde{B})=S_{\overline{N}}(\widetilde{B},\widetilde{A})$

(iv) $\widetilde{A}\subset\widetilde{B}\subset\widetilde{C}\Rightarrow S_{\underline{N}}(\widetilde{A},\widetilde{C})\leq S_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S_{\underline{N}}(\widetilde{B},\widetilde{C})$ and $S_{\overline{N}}(\widetilde{A},\widetilde{C})\leq S_{\overline{N}}(\widetilde{A},\widetilde{B})\wedge S_{\overline{N}}(\widetilde{B},\widetilde{C})$

where $S_{\underline{N}}(\tilde{A},\tilde{B})$ *and* $S_{\overline{N}}(\tilde{A},\tilde{B})$ denotes the similarity measure of two single valued neutrosophic rough sets with respect to lower and upper approximation respectively.

4.1 Distance based similarity measure:

In general similarity measure or similarity function is a real-valued function that quantifies the similarity between two objects. It is the inverse of distance metrics.
Using the distance formulae it is generally defined as,

$$S^1(A,B)=\frac{1}{1+d(A,B)} \tag{16}$$

For example if we consider the Euclidian distance of two single valued neutrosophic rough sets of $\tilde{A}$ and $\tilde{B}$ with respect to its lower approximation then it's associated similarity can be calculated as,

$$S^1{}_{\underline{N}}(\tilde{A},\tilde{B})=\frac{1}{1+e_{\underline{N}}(\tilde{A},\tilde{B})}$$

Example 4.1.1

From Example 3.2 the similarity measure can be calculated as,

$$S^1{}_{\underline{N}}(\tilde{A},\tilde{B})=\frac{1}{1+e_{\underline{N}}(\tilde{A},\tilde{B})}=0.6351$$

Proposition 4.1.2

The distance based similarity measure $S^1{}_{\underline{N}}$ *and* $S^1{}_{\overline{N}}$ with respect to lower and upper approximation of two single valued neutrosophic rough sets of $\tilde{A}$ *and* $\tilde{B}$ satisfies the following properties.

$$\begin{aligned}
&\text{(i) } 0\le S^1{}_{\underline{N}}(\tilde{A},\tilde{B})\le 1 \quad and \quad 0\le S^1{}_{\overline{N}}(\tilde{A},\tilde{B})\le 1\\
&\text{(ii) } S^1{}_{\underline{N}}(\tilde{A},\tilde{B})=1\Leftrightarrow \tilde{A}=\tilde{B} \quad and \quad S^1{}_{\overline{N}}(\tilde{A},\tilde{B})=1\Leftrightarrow \tilde{A}=\tilde{B}\\
&\text{(iii) } S^1{}_{\underline{N}}(\tilde{A},\tilde{B})=S^1{}_{\underline{N}}(\tilde{B},\tilde{A}) \quad and \quad S^1{}_{\overline{N}}(\tilde{A},\tilde{B})=S^1{}_{\overline{N}}(\tilde{B},\tilde{A})\\
&\text{(iv) } \tilde{A}\subset\tilde{B}\subset\tilde{C} \Rightarrow S^1{}_{\underline{N}}(\tilde{A},\tilde{C})\le S^1{}_{\underline{N}}(\tilde{A},\tilde{B})\wedge S^1{}_{\underline{N}}(\tilde{B},\tilde{C}) \quad and \quad S^1{}_{\overline{N}}(\tilde{A},\tilde{C})\le S^1{}_{\overline{N}}(\tilde{A},\tilde{B})\wedge S^1{}_{\overline{N}}(\tilde{B},\tilde{C})
\end{aligned} \tag{17}$$

Proof:

The results (i) – (iii) holds trivially from definition. It is enough to prove only (iv).
Let us consider three single valued neutrosophic rough sets $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ with respect to $(U,\tilde{R})$ in the universe $U=\{x_1,x_2,x_3,\ldots\ldots. x_n\}$. Let $\tilde{A}\subset\tilde{B}\subset\tilde{C}$ then we have

$$T_{\underline{\tilde{R}}(\tilde{A})}(x)\le T_{\underline{\tilde{R}}(\tilde{B})}(x)\le T_{\underline{\tilde{R}}(\tilde{C})}(x);\, I_{\underline{\tilde{R}}(\tilde{A})}(x)\ge I_{\underline{\tilde{R}}(\tilde{B})}(x)\ge I_{\underline{\tilde{R}}(\tilde{C})}(x)\, and$$
$$F_{\underline{\tilde{R}}(\tilde{A})}(x)\ge F_{\underline{\tilde{R}}(\tilde{B})}(x)\ge F_{\underline{\tilde{R}}(\tilde{C})}(x)\,\forall\, x\in U$$

Now

$$|T_{\underline{\tilde{R}}(\tilde{A})}(x)-T_{\underline{\tilde{R}}(\tilde{B})}(x)|\le|T_{\underline{\tilde{R}}(\tilde{A})}(x)-T_{\underline{\tilde{R}}(\tilde{C})}(x)|\ and$$

$$|T_{\underline{\tilde{R}}(\tilde{B})}(x)-T_{\underline{\tilde{R}}(\tilde{C})}(x)|\le|T_{\underline{\tilde{R}}(\tilde{A})}(x)-T_{\underline{\tilde{R}}(\tilde{C})}(x)|\ \ will\ hold.$$

Similarly,

$$|I_{\underline{\tilde{R}}(\tilde{A})}(x)-I_{\underline{\tilde{R}}(\tilde{B})}(x)|\ge|I_{\underline{\tilde{R}}(\tilde{A})}(x)-I_{\underline{\tilde{R}}(\tilde{C})}(x)|\ and$$

$$|I_{\underline{\tilde{R}}(\tilde{B})}(x)-I_{\underline{\tilde{R}}(\tilde{C})}(x)|\ge|I_{\underline{\tilde{R}}(\tilde{A})}(x)-I_{\underline{\tilde{R}}(\tilde{C})}(x)|\ \ and\ also$$

$$|F_{\underline{\tilde{R}}(\tilde{A})}(x)-F_{\underline{\tilde{R}}(\tilde{B})}(x)|\ge|F_{\underline{\tilde{R}}(\tilde{A})}(x)-F_{\underline{\tilde{R}}(\tilde{C})}(x)|\ and$$

$$|F_{\underline{\tilde{R}}(\tilde{B})}(x)-F_{\underline{\tilde{R}}(\tilde{C})}(x)|\ge|F_{\underline{\tilde{R}}(\tilde{A})}(x)-F_{\underline{\tilde{R}}(\tilde{C})}(x)|\ \ holds$$

Thus

$$d_{\underline{N}}(\tilde{A},\tilde{B})\le d_{\underline{N}}(\tilde{A},\tilde{C})\Rightarrow S^1{}_{\underline{N}}(\tilde{A},\tilde{B})\ge S^1{}_{\underline{N}}(\tilde{A},\tilde{C})\ \ and$$

$$d_{\underline{N}}(\tilde{B},\tilde{C})\le d_{\underline{N}}(\tilde{A},\tilde{C})\Rightarrow S^1{}_{\underline{N}}(\tilde{B},\tilde{C})\ge S^1{}_{\underline{N}}(\tilde{A},\tilde{C})$$

$$\Rightarrow S^1{}_{\underline{N}}(\tilde{A},\tilde{C})\le S^1{}_{\underline{N}}(\tilde{A},\tilde{B})\wedge S^1{}_{\underline{N}}(\tilde{B},\tilde{C})$$

This is true for all the distance functions defined in equations (1) to (8)
Hence the result.

4.2 Similarity measure based on membership degrees

Another similarity measure of $S^2{}_{\underline{N}}\ and\ S^2{}_{\overline{N}}$ between two single valued neutrosophic rough sets of $\tilde{A}\ \ and\ \ \tilde{B}$ with respect to lower and upper approximation will be defined as follows:

$$S^2{}_{\underline{N}}(\tilde{A},\tilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\underline{\tilde{R}}(\tilde{A})}(x_i),T_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{I_{\underline{\tilde{R}}(\tilde{A})}(x_i),I_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{F_{\underline{\tilde{R}}(\tilde{A})}(x_i),F_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\underline{\tilde{R}}(\tilde{A})}(x_i),T_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{I_{\underline{\tilde{R}}(\tilde{A})}(x_i),I_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{F_{\underline{\tilde{R}}(\tilde{A})}(x_i),F_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}\}}$$

$$S^2{}_{\overline{N}}(\tilde{A},\tilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\overline{\tilde{R}}(\tilde{A})}(x_i),T_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{I_{\overline{\tilde{R}}(\tilde{A})}(x_i),I_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{F_{\overline{\tilde{R}}(\tilde{A})}(x_i),F_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\overline{\tilde{R}}(\tilde{A})}(x_i),T_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{I_{\overline{\tilde{R}}(\tilde{A})}(x_i),I_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{F_{\overline{\tilde{R}}(\tilde{A})}(x_i),F_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}\}}$$

Example 4.2.1

From Example 3.2 the similarity measure can be calculated as,

$$S^2{}_{\underline{N}}(\tilde{A},\tilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\underline{\tilde{R}}(\tilde{A})}(x_i),T_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{I_{\underline{\tilde{R}}(\tilde{A})}(x_i),I_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\min\{F_{\underline{\tilde{R}}(\tilde{A})}(x_i),F_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\underline{\tilde{R}}(\tilde{A})}(x_i),T_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{I_{\underline{\tilde{R}}(\tilde{A})}(x_i),I_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}+\max\{F_{\underline{\tilde{R}}(\tilde{A})}(x_i),F_{\underline{\tilde{R}}(\tilde{B})}(x_i)\}\}}$$

$$S^2{}_{\underline{N}}(\tilde{A},\tilde{B})=0.7115$$

$$S^2_{\overline{N}}(\tilde{A},\tilde{B}) = \frac{\sum_{i=1}^{n}\{\min\{T_{\overline{\tilde{R}}(\tilde{A})}(x_i), T_{\overline{\tilde{R}}(\tilde{B})}(x_i)\} + \min\{I_{\overline{\tilde{R}}(\tilde{A})}(x_i), I_{\overline{\tilde{R}}(\tilde{B})}(x_i)\} + \min\{F_{\overline{\tilde{R}}(\tilde{A})}(x_i), F_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\overline{\tilde{R}}(\tilde{A})}(x_i), T_{\overline{\tilde{R}}(\tilde{B})}(x_i)\} + \max\{I_{\overline{\tilde{R}}(\tilde{A})}(x_i), I_{\overline{\tilde{R}}(\tilde{B})}(x_i)\} + \max\{F_{\overline{\tilde{R}}(\tilde{A})}(x_i), F_{\overline{\tilde{R}}(\tilde{B})}(x_i)\}\}}$$

$$S^2_{\overline{N}}(\tilde{A},\tilde{B}) = 0.5349$$

Proposition 4.2.2

The membership degree based similarity measure $S^2_{\underline{N}}$ *and* $S^2_{\overline{N}}$ with respect to lower and upper approximation of two single valued neutrosophic rough sets of $\tilde{A}$ *and* $\tilde{B}$ satisfies the following properties.

(i) $0 \le S^2_{\underline{N}}(\tilde{A},\tilde{B}) \le 1 \quad and \quad 0 \le S^2_{\overline{N}}(\tilde{A},\tilde{B}) \le 1$

(ii) $S^2_{\underline{N}}(\tilde{A},\tilde{B}) = 1 \Leftrightarrow \tilde{A} = \tilde{B} \; and \; S^2_{\overline{N}}(\tilde{A},\tilde{B}) = 1 \Leftrightarrow \tilde{A} = \tilde{B}$ (18)

(iii) $S^2_{\underline{N}}(\tilde{A},\tilde{B}) = S^2_{\underline{N}}(\tilde{B},\tilde{A}) \; and \; S^2_{\overline{N}}(\tilde{A},\tilde{B}) = S^2_{\overline{N}}(\tilde{B},\tilde{A})$

(iv) $\tilde{A} \subset \tilde{B} \subset \tilde{C} \Rightarrow S^2_{\underline{N}}(\tilde{A},\tilde{C}) \le S^2_{\underline{N}}(\tilde{A},\tilde{B}) \wedge S^2_{\underline{N}}(\tilde{B},\tilde{C}) \; and \; S^2_{\overline{N}}(\tilde{A},\tilde{C}) \le S^2_{\overline{N}}(\tilde{A},\tilde{B}) \wedge S^2_{\overline{N}}(\tilde{B},\tilde{C})$

Proof: The results (i) – (iii) holds trivially from definition. It is enough to prove only (iv).

Let us consider three single valued neutrosophic rough sets $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ with respect to $(U,\tilde{R})$ in the universe $U = \{x_1, x_2, x_3, ... x_n\}$. Let $\tilde{A} \subset \tilde{B} \subset \tilde{C}$ then we have

$T_{\underline{\tilde{R}}(\tilde{A})}(x) \le T_{\underline{\tilde{R}}(\tilde{B})}(x) \le T_{\underline{\tilde{R}}(\tilde{C})}(x); I_{\underline{\tilde{R}}(\tilde{A})}(x) \ge I_{\underline{\tilde{R}}(\tilde{B})}(x) \ge I_{\underline{\tilde{R}}(\tilde{C})}(x) \, and$

$F_{\underline{\tilde{R}}(\tilde{A})}(x) \ge F_{\underline{\tilde{R}}(\tilde{B})}(x) \ge F_{\underline{\tilde{R}}(\tilde{C})}(x) \; \forall \, x \in U$

Now,

$T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{B})}(x) \ge T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x) \, and$

$T_{\underline{\tilde{R}}(\tilde{B})}(x) + I_{\underline{\tilde{R}}(\tilde{B})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x) \le T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x)$

$$S^2_{\underline{N}}(\tilde{A},\tilde{B}) = \frac{T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{B})}(x)}{T_{\underline{\tilde{R}}(\tilde{B})}(x) + I_{\underline{\tilde{R}}(\tilde{B})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x)} \ge \frac{T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x)}{T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x)} = S^2_{\underline{N}}(\tilde{A},\tilde{C})$$

Similarly we have,

$T_{\underline{\tilde{R}}(\tilde{B})}(x) + I_{\underline{\tilde{R}}(\tilde{B})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x) \ge T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x) \, and$

$T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x) \ge T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{B})}(x)$

$$S^2_{\underline{N}}(\tilde{B},\tilde{C}) = \frac{T_{\underline{\tilde{R}}(\tilde{B})}(x) + I_{\underline{\tilde{R}}(\tilde{B})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x)}{T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{B})}(x)} \ge \frac{T_{\underline{\tilde{R}}(\tilde{A})}(x) + I_{\underline{\tilde{R}}(\tilde{A})}(x) + F_{\underline{\tilde{R}}(\tilde{C})}(x)}{T_{\underline{\tilde{R}}(\tilde{C})}(x) + I_{\underline{\tilde{R}}(\tilde{C})}(x) + F_{\underline{\tilde{R}}(\tilde{A})}(x)} = S^2_{\underline{N}}(\tilde{A},\tilde{C})$$

$$\Rightarrow S^2_{\underline{N}}(\tilde{A},\tilde{C}) \le S^2_{\underline{N}}(\tilde{A},\tilde{B}) \wedge S^2_{\underline{N}}(\tilde{B},\tilde{C})$$

Hence the proof.

5 **Applications to Medical Diagnosis**:

In this section we present some real life applications of the similarity measure of single valued neutrosophic rough sets. Many real life practical problems consist of more uncertainty and incomplete information. To deal this problem effectively , rough neutrosophic set helps to deal with uncertainty and incompleteness.

Let us consider a medical diagnosis problem for the illustration of the proposed approach. Medical diagnosis is the process of determining which disease or condition explains a person's symptoms and signs. Diagnosis is a challenging one which consists of uncertainties and many signs & symptoms are non-specific. To handle this way of problem, rough neutrosophic set provided a good way in which several possible explanations are compared and contrasted must be perfomed by the method of similarity measure. So similarity measure helps to identify whether two sets are identical or atleast to what degree they are identical by using the concept of distance formula and membership function.

Let us consider the same example which we have discussed in earlier Section 3 in Example 3.2 and apply that example to medical diagnosis problem, let $U = \{x_1, x_2, x_3\}$ be the universe of patients. Consider the same two SVNS's A and B with respect to SVNR's $\tilde{R}_1, \tilde{R}_2 \in SVNS\,(U \times U)$ respectively which is given in Table 1 and Table 2. Let D = {Viral fever, Malaria, Typhoid} be the set of diseases and also $\tilde{R}_1, \tilde{R}_2$ denotes the relation between the patients and diseases of the SVNS's A and B respectively.

Hence, this section provides relative study among similarity measures proposed in this paper. The comparision study of similarity measures based on different distances formulae and membership degree is given in Table 3 in detail.

$$\underline{\tilde{R}}(\tilde{A})(x_1) = (0.4,1,0.3) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_1) = (0.3,0.6,0.4)$$

Similarly we can obtain,

$$\underline{\tilde{R}}(\tilde{A})(x_2) = (0.4,1,0.3) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_2) = (0.3,0.4,0.4)$$
$$\underline{\tilde{R}}(\tilde{A})(x_3) = (0.6,0.4,0.5) \quad and \quad \overline{\tilde{R}}(\tilde{A})(x_3) = (0.3,0.3,0.6)$$
$$\underline{\tilde{R}}(\tilde{B})(x_1) = (0.5,0.8,0.2) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_1) = (0.5,0.3,0.5)$$
$$\underline{\tilde{R}}(\tilde{B})(x_2) = (0.3,0.8,0.5) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_2) = (0.5,0.4,0)$$
$$\underline{\tilde{R}}(\tilde{B})(x_3) = (0.2,0.3,0.4) \quad and \quad \overline{\tilde{R}}(\tilde{B})(x_3) = (0.5,0.3,0)$$

Table 3: Similarity values

Similarity measure based on	$S_{\underline{N}}(\tilde{A},\tilde{B})$	$S_{\overline{N}}(\tilde{A},\tilde{B})$
Hamming distance	0.4	0.3333
Normalized hamming distance	0.8572	0.8182
Euclidian distance	0.6351	0.5376
Normalized euclidian distance	0.8392	0.7638
Membership degree	0.7115	0.5349

In Table 3 $S_{\underline{N}}(\tilde{A},\tilde{B})$, $S_{\overline{N}}(\tilde{A},\tilde{B})$ denotes the similarity lower and upper approximation measure of the two single valued neutrosophic rough sets respectively. In practical it represents the lower and upper approximation similarity measures between patients and diseases of two single valued neutrosophic rough sets.That is through hamming distance the similarity lower and upper measure between patients and diseases of two single valued neutrosophic rough sets A and B will be 0.4 and 0.3333 respectively.

Table 3 represents that each method has its own way to calculate the similarity measure and also any method can be preferrable to calculate the similarity measure between two single valued neutrosophic rough sets.

6 Conclusion

Single valued neutrosophic set (SVNS) is an instance of NS and it is an extension of fuzzy set and IFS. Compare to previous traditional models like fuzzy set, IFS, NS, crisp set , it provides more precise, compatible and flexible in comparison. By combining the concept of SVNS with rough set a new hybrid model of single valued neutrosophic rough set was introduced and now-a-days it is a very new hot research topic. In this paper we have defined the notion of similarity between two single valued neutrosophic rough sets based on distance formulae and membership degrees. We have also studied some properties on them and proved some prepositions and a numerical example is given in medical diagnosis for the proposed similarity measure concept.

References

[1] C. Antony Crispin Sweety & I. Arockiarani, Neutrosophic Rough Set Algebra, International Journal of Mathematics Trends and Technology, Volume 38(3),2016.

[2] K.Atanasov, Intuitionistic Fuzzy Sets, Fuzzy Sets & Systems 20 (1986), 87-96.

[3] S.Broumi and F.Smarandache, Several similarity measures of neutrosophic sets, Neutrosophic Sets and Systems 1(1) (2013), 54-62.

[4] S.Broumi and F.Smarandache, Cosine similarity measures of interval valued neutrosophic sets, Neutrosophic Sets and Systems 5 (2013), 15-20.

[5] Broumi S, Smarandache F (2014) Rough Neutrosophic sets. Ital J Pure Appl Math 32:493-502.

[6] S.M.Chen and P.H.Hsiao, A Comparison of similarity measures of fuzzy values, Fuzzy Sets and Systems 72(1995), 79-89.

[7] Hai-Long Yang, A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model, Soft Comput (2017) 21:6253-6267.

[8]J.M.Merigo and M.Casanovas, Decision making with distance measures and induced aggregation operators, Computers and Industrial Engineering 60(1) (2011), 66-76.

[9] J.M.Merigo and M.Casanovas, Induced aggregation operators in the Euclidean distance and its application in financial decision making, Expert Systems with Applications 38(6)(2011), 7603-7608.

[10] Mondal, K., & Pramanik, S. (2015). Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. *Critical Review, 11*, 26-40.

[11] Mondal, K. & Pramanik, S. (2015). Decision making based on some similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems 10*, 46-57.

[12]Mondal, K., & Pramanik, S. (2015). Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. *Neutrosophic Sets and Systems 8*, 16-22.

[13]Mondal, K., &Pramanik, S. (2015). Rough neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems, 7*, (2015), 8-17.

[14]Mondal, K., Pramanik, S. & Smarandache, F. (2016). Rough neutrosophic TOPSIS for multi-attribute group decision making. *Neutrosophic Sets and Systems, 13*, 105-117.

[15]Mondal, K., Pramanik, S. & Smarandache, F. (2016). Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. *Neutrosophic Sets and Systems, 13*, 3-17.

[16] Mondal, K., Pramanik, S., & Smarandache, F. (2016). Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and application (pp. 93-103). Brussels, Belgium: Pons Editions.

[17]Mondal, K., Pramanik, S., & Giri, B. C. (2018). Rough neutrosophic aggregation operators for multi-criteria decision-making. In C. Kahraman & I. Otay (Eds.): C. Kahraman and ˙ I. Otay (eds.), Fuzzy Multicriteria Decision Making Using Neutrosophic Sets, Studies in Fuzziness and Soft Computing 369.

[18] Z.Pawlak, Rough sets, International Journal of Computer and Information Sciences 11(5) (1982), 341-356.

[19] Pinaki Majumdar, On similarity and entropy of neutrosophic sets, Journal of Intelligent & Fuzzy systems 26(2014) 1245-1252.

[20] Pramanik, S., & Mondal, K. (2015). Some rough neutrosophic similarity measures and their application to multi attribute decision making. *Global Journal of Engineering Science and Research Management, 2* (7), 61-74.

[21]S. Pramanik, and K. Mondal. (2015). Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory, 4*, 90-102.

[22]Pramanik, S., & Mondal, K. (2015). Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research, 2*(1), 212-220.

[23]Pramanik, S., Roy, R., Roy, T. K. & Smarandache, F. (2017). Multi criteria decision making using correlation coefficient under rough neutrosophic environment. *Neutrosophic Sets and Systems, 17*, 29-36.

[24]Pramanik, S., Roy, R., Roy, T. K., & Smarandache, F. (2018). Multi-attribute decision making based on several trigonometric hamming similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems, 19*, 110-118.
[25]Pramanik, S., Roy, R., & Roy, T. K. (2018). Multi criteria decision making based on projection and bidirectional projection measures of rough neutrosophic sets.
[26]Pramanik, S., Roy, R., & Roy, T. K. (2018). Multi attribute decision making strategy on projection and bidirectional projection measures of interval rough neutrosophic sets. *Neutrosophic Sets and Systems,* 19, 101-109.
[27] F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set & Logic. American Research Press, Rehoboth, 1999.
[28]Smarandache, F. & Pramanik, S. (Eds). (2016). *New trends in neutrosophic theory and applications*, Vol.2. Brussels: Pons Editions.
[29]Smarandache, F. & Pramanik, S. (Eds). (2018). *New trends in neutrosophic theory and applications*. Brussels: Pons Editions.
[30] Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistruct 4:410-413.
[31] S.Ye and J.Ye, Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis, Neutrosophic Sets and Systems 6 (2014), 49-54.
[32] L.Zadeh , Fuzzy sets, Information and Control 8 (1965), 87-96.

Received: January 2, 2019. Accepted: February 25, 2019