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Squeezing properties of degenerate Three and Two-level Lasers with Non-degenerate Sub-harmonic Light

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ABSTRACT

We study the squeezing properties of the light generated by degenerate three and two-level lasers with non-degenerate sub-harmonic light and the cavity modes are coupled to a two-mode ordinary vacuum reservoir via a single port mirror We obtained c-number langevin equations associated with the normal ordering using the pertinent master equation. Making use of the solutions of the c-number langevin equations. The result shows that the two-mode light produced by the system is in squeezed. Moreover, it is found that 64% of squeezing below the coherent state at $\eta = 0.04$ for $A = 100$, $\kappa = 0.8$, $A_a = 1$, and $\eta_a = 0.5$. The effect of the two-level laser and parametric oscillator are to decrease and increase the properties of the quadrature squeezing.

Keyword: *Lasers properties, sub-harmonic light, quadrature squeezing.*

INTRODUCTION

Light has played a special role in our attempt to understand nature quantum mechanically. Squeezing is one of the non-classical features of light that has attracted a great deal of interest. In squeezed light the noise in one quadrature is below the coherent-state level at the expense of enhanced fluctuations in the other quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation. Squeezed light

has potential applications in low-noise optical communication and weak signal detection. Hence it is vital to find new optical devices or to combine the existing ones to generate highly squeezed and bright light [1]-[15].

It has been predicted that a three-level laser under certain conditions can produce squeezed light. In a cascade three-level laser, three-level atoms in a cascade configuration are injected into a cavity coupled to a vacuum reservoir via a single-port mirror. The injected atoms may initially be prepared in a coherent superposition of the top and bottom levels and/or these levels may be coupled by strong coherent light after they are injected into the cavity. The superposition or the coupling of the top and bottom levels is responsible for the interesting non-classical features of the generated light [1]-[7] and [11]-[15]. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, two photons are generated. If the two photons have the same frequency, the three-level atom is called degenerate otherwise it is called non-degenerate.

A parametric oscillator has been considered as an important source of squeezed light. It is one of the most interesting and well characterized optical devices in quantum optics. In this device a pump photon interacts with a nonlinear crystal inside a cavity and is down converted into two highly correlated photons. If these photons have the same frequency the device is called a degenerate parametric oscillator, otherwise it is called a non-degenerate

parametric oscillator [1]-[3],[10].

E. Alebachew and K. Fesseha in their study have considered a degenerate three-level laser whose cavity contains a degenerate parametric amplifier, with the top and bottom levels of the three-level atoms coupled by the pump mode emerging from the parametric amplifier. They obtained using the master equation the stochastic differential equations of the system. Applying the solutions of the resulting equations, they calculated the quadrature variance and squeezing spectrum. Moreover, using the same solutions, they determined the mean photon number and the photon number distribution. The research have shown that the light generated by such quantum optical system is in a squeezed state, with the maximum intracavity squeezing attainable being 93% and the parametric amplifier increases the squeezing significantly over and above the squeezing achievable due to the coupling of the top and bottom levels by the pump mode. In addition, there is perfect squeezing of the output light for $\beta=\alpha = 0$ and for any values of A and κ. They found that the presence of the parametric amplifier leads to a significant increase in the mean photon number for small values of β.

Objective

General objective

- *To study the squeezing properties of light produce by two and three level laser whose cavity contain non-degenerate sub-harmonic light.*

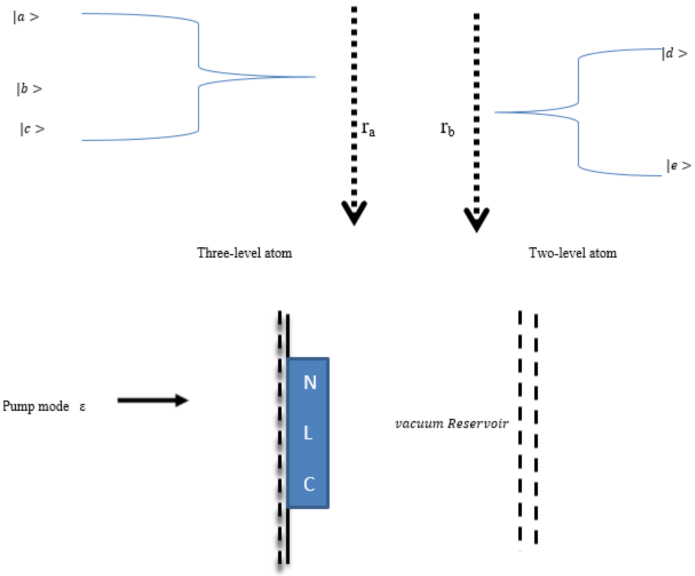
Specific objective

- *To drive master equation*
- *To determine steady state solution using matrix.*
- *To obtain the solution of the c-number langevin equations.*
- *To drive normally ordering c-number langavien equations (the cavity mode variables) of the cavity mode variables.*

METHODS

We use the cavity mode variables to calculate the quantities of interest which are obtained from the solution of the c-number langevin equations associated with the steady state solution using matrix. Applying the resulting cavity mode variables, we calculate the quadrature variances and squeezed spectrum.In this chapter we seek to obtain the equation of evolution of the density operator for

a degenerate three-level and two-level lasers with non-degenerate parametric oscillator and the cavity modes are coupled to a two-mode ordinary vacuum reservoir via a single port mirror as seen in Fig-1. We first derive the equation of evolution of the density operators for three-level laser, two-level laser, non-degenerate parametric oscillator, and cavity modes coupled to a vacuum reservoir applying the Linear, Adiabatic and bornapproximation schemes, separately. Then with aid of the resulting equations we develop the master equation for the system under consideration.



Operator Dynamics

The interaction of a degenerate three-level atom with a cavity mode can be described by the Hamiltonian

.... $\hat{H} = i\lambda_1\left[(|a\rangle\langle b| + |b\rangle\langle c|)\,\hat{a} - \hat{a}^{\dagger}(|b\rangle\langle a| + |c\rangle\langle b|)\right]$.

Where λ_1 is the coupling constant for the interaction of the three-level atom with cavity mode, â and â† are the annihilation and creation operators for the cavity mode, respectively.
Considering that the atoms to be initially in coherent superposition of the top and bottom states

$$|\Psi_A(0)\rangle = c_a|a\rangle + c_c|c\rangle$$

The density operator for a single atom initially has the form

$$\hat{\rho}_A(0) = \rho^{(0)}_{aa}|a\rangle\langle a| + \rho^{(0)}_{ac}|a\rangle\langle c| + \rho^{(0)}_{ca}|c\rangle\langle a| + \rho^{(0)}_{cc}|c\rangle\langle c|$$

Where

$$\rho^{(0)}_{aa} = c^*_a c_a \qquad \text{………………………4}$$

Eqs. (4) is the probability of finding the atom in the upper level,

$$\rho^{(0)}_{cc} = c^*_c c_c \qquad \text{……………..…..….5}$$

Eqs. (5) is the probability of finding the atom in the lower level, and

$$\rho^{(0)}_{ac} = \sqrt{\rho^{(0)}_{aa}\rho^{(0)}_{cc}}\,e^{i\phi_1} \qquad \text{… ….…………………….6}$$

Suppose $\rho_{AC}(t, tj)$ be the density operator for a single atom plus the cavity mode at
time t with the three-level atom injected at time tj, such as $t - \tau \le tj \le t$. Then the
density operator for all atoms in the cavity plus the cavity modes at time t can be written
as

$$\hat{\rho}_{AC}(t) = r_a \sum_j \hat{\rho}_{AC}(t, t_j)\triangle t_j \qquad \text{………………7}$$

Where $r_a$is the rate at which the atoms injected into the cavity and $r_a \Delta t_j$is the number
of atoms injected into the cavity in the time interval Δt_j. converting the summation into
integration in the limit $\Delta t_j \to 0$, we see that

$$\hat{\rho}_{AC}(t) = r_a \int_{t-\tau}^{t} \hat{\rho}_{AC}(t, t')dt' \qquad \text{……………… …….8}$$

And differentiating with respect to t, there follows

$$\frac{d}{dt}\hat{\rho}_{AC}(t) = r_a\left\{\hat{\rho}_{AC}(t,t) - \hat{\rho}_{AC}(t, t-\tau)\right\} + r_a\int_{t-\tau}^{t}\frac{\partial}{\partial t}\hat{\rho}_{AC}(t,t')dt' \qquad \text{……..9}$$

We observe that $\rho_{AC}(t, t)$ is the density operator for cavity mode plus an atom injected
at time t and $\rho_{AC}(t, t - \tau)$ represents the density operator for an atom plus cavity mode
at time t with the atom being removed from the cavity at this time. Therefore, these
density operators can be put in the form

$$\hat{\rho}_{AC}(t,t) = \hat{\rho}_A(0)\hat{\rho}(t) \qquad \text{…………………………..…10}$$

$$\hat{\rho}_{AC}(t, t-\tau) = \hat{\rho}_A(t-\tau)\hat{\rho}(t), \qquad \text{……………………………………………….……….11}$$

with$\rho(t)$ being the density operator for the cavity mode alone. Then in view of Eqs. (10) and (11),

We see that

$$\frac{d}{dt}\hat{\rho}_{AC}(t) = r_a\left(\hat{\rho}_A(0) - \hat{\rho}_A(t-\tau)\right)\hat{\rho}(t) + r_a\int_{t-\tau}^{t}\frac{\partial}{\partial t}\hat{\rho}_{AC}(t,t')dt' \qquad \text{……..……..12}$$

In the absence of damping of the cavity mode by vacuum reservoir, $\rho_{AC}(t, t0)$ evolves in time according to Heisenberg picture is

$$\frac{\partial}{\partial t}\hat{\rho}_{AC}(t,t') = -i\left[\hat{H}, \hat{\rho}_{AC}(t,t')\right] \qquad \text{……………….…….13}$$

With the aid of Eqs. (8) and (13), one can put Eq. (12) in the form

$$\frac{d}{dt}\hat{\rho}_{AC}(t) = r_a\left(\hat{\rho}_A(0) - \hat{\rho}_A(t-\tau)\right)\hat{\rho}(t) - i\left[\hat{H}, \hat{\rho}_{AC}(t)\right] \qquad \text{…………..14}$$

Furthermore, applying the trace operation over the atomic variables, we have

$$\frac{d\hat{\rho}(t)}{dt} = -iTr_A\left[\hat{H}, \hat{\rho}_{AC}(t)\right] \qquad \text{……………………15}$$

Employing Eq. (1), we can write in the form

$$\frac{d\hat{\rho}}{dt} = \lambda_1\left(\hat{a}\hat{\rho}_{ba} + \hat{a}\hat{\rho}_{cb} - \hat{a}^{\dagger}\hat{\rho}_{ab} - \hat{a}^{\dagger}\hat{\rho}_{bc} - \hat{\rho}_{ba}\hat{a} - \hat{\rho}_{cb}\hat{a} + \hat{\rho}_{ab}\hat{a}^{\dagger} + \hat{\rho}_{bc}\hat{a}^{\dagger}\right) \quad \text{……...16}$$

in which the matrix element $\rho_{\alpha\beta}$are defined by

$$\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}_{AC}(t)|\beta\rangle \quad \text{…………………..….17}$$

With $\alpha, \beta = a, b, c$

We next proceed to determine the matrix elements involved in Eq. (16). Multiplying Eq. (14) on the left by $<\alpha|$ and on the right by $|\beta>$, we see that

$$\frac{d}{dt}\hat{\rho}_{\alpha\beta} = r_a\langle\alpha|\left(\hat{\rho}_A(0) - \hat{\rho}_A(t-\tau)\right)|\beta\rangle\hat{\rho}(t) - i\langle\alpha|\left[\hat{H}, \hat{\rho}_{AC}(t)\right]|\beta\rangle - \gamma_1\hat{\rho}_{\alpha\beta}. \quad \text{……….18}$$

The last term in Eq. (18) is added to include the decay of atoms due to spontaneous emission and γ_1 represents the atomic decay rate, which is assumed to be the same for both levels from level $|a>$to level $|b>$and from level $|b>$to $|c>$. We also assume that the atoms are removed after they have decayed to a level other than levels $|b>$and $|c>$. We then have

$$\langle\alpha|\hat{\rho}_A(t-\tau)|\beta\rangle = 0. \quad \text{……………………………..19}$$

Using Eqs. (1), (3), and (19), we can write as

$$\begin{aligned}\frac{d\hat{\rho}_{\alpha\beta}}{dt} &= r_a\left(\rho_{aa}^{(0)}\delta_{\alpha a}\delta_{a\beta} + \rho_{ac}^{(0)}\delta_{\alpha a}\delta_{c\beta} + \rho_{ca}^{(0)}\delta_{\alpha c}\delta_{a\beta} + \rho_{cc}^{(0)}\delta_{\alpha c}\delta_{c\beta}\right)\hat{\rho} \\ &+ \lambda_1\Big(\hat{a}\hat{\rho}_{b\beta}\delta_{\alpha a} + \hat{a}\hat{\rho}_{c\beta}\delta_{\alpha b} - \hat{a}^{\dagger}\hat{\rho}_{a\beta}\delta_{\alpha b} - \hat{a}^{\dagger}\hat{\rho}_{b\beta}\delta_{\alpha c} \\ &- \hat{\rho}_{\alpha a}\hat{a}\delta_{b\beta} - \hat{\rho}_{\alpha b}\hat{a}\delta_{c\beta} + \hat{\rho}_{\alpha b}\hat{a}^{\dagger}\delta_{a\beta} + \hat{\rho}_{\alpha c}\hat{a}^{\dagger}\delta_{b\beta}\Big) - \gamma_1\hat{\rho}_{\alpha\beta}.\end{aligned} \quad \text{…..20}$$

From which follows that

$$\frac{d\hat{\rho}_{ab}}{dt} = \lambda_1\left(\hat{a}\hat{\rho}_{bb} - \hat{\rho}_{aa}\hat{a} + \hat{\rho}_{ac}\hat{a}^{\dagger}\right) - \gamma_1\hat{\rho}_{ab}, \quad \text{………………21}$$

$$\frac{d\hat{\rho}_{bc}}{dt} = \lambda_1\left(\hat{a}\hat{\rho}_{cc} - \hat{\rho}_{bb}\hat{a} - \hat{a}^{\dagger}\hat{\rho}_{ac}\right) - \gamma_1\hat{\rho}_{bc}, \quad \text{………22}$$

$$\frac{d\hat{\rho}_{aa}}{dt} = r_a\rho_{aa}^{(0)}\hat{\rho} + \lambda_1\left(\hat{a}\hat{\rho}_{ba} + \hat{\rho}_{ab}\hat{a}^{\dagger}\right) - \gamma_1\hat{\rho}_{aa}, \quad \text{……….23}$$

$$\frac{d\hat{\rho}_{cc}}{dt} = r_a\rho_{cc}^{(0)}\hat{\rho} - \lambda_1\left(\hat{a}^{\dagger}\hat{\rho}_{bc} + \hat{\rho}_{cb}\hat{a}\right) - \gamma_1\hat{\rho}_{cc}, \quad \text{…….24}$$

$$\frac{d\hat{\rho}_{ac}}{dt} = r_a\rho_{ac}^{(0)}\hat{\rho} + \lambda_1\left(\hat{a}\hat{\rho}_{bc} - \hat{\rho}_{ab}\hat{a}\right) - \gamma_1\hat{\rho}_{ac}, \quad \text{…….25}$$

$$\frac{d\hat{\rho}_{bb}}{dt} = \lambda_1\left(\hat{a}\hat{\rho}_{cb} - \hat{a}^{\dagger}\hat{\rho}_{ab} - \hat{\rho}_{ba}\hat{a} + \hat{\rho}_{bc}\hat{a}^{\dagger}\right) - \gamma_1\hat{\rho}_{bb}. \quad \text{………...26}$$

Dropping the $\lambda_1$term inEqs. (23) – (26) and applying the adiabatic approximation scheme, we get

$$\hat{\rho}_{aa} = \frac{r_a\rho_{aa}^{(0)}}{\gamma_1}\hat{\rho}, \quad \text{…………………………………….…..27}$$

$$\hat{\rho}_{cc} = \frac{r_a\rho_{cc}^{(0)}}{\gamma_1}\hat{\rho}, \quad \text{………………………..……………..28}$$

$$\hat{\rho}_{ac} = \frac{r_a\rho_{ac}^{(0)}}{\gamma_1}\hat{\rho}, \quad \text{……………………………………….29}$$

$$\hat{\rho}_{bb} = 0. \quad \text{………………………………….…………30}$$

Employing Eqs. (27) - (30) in Eq. (21) and (22), we have

$$\frac{d\hat{\rho}_{ab}}{dt} = \frac{\lambda_1 r_a}{\gamma_1}\left(\rho_{ac}^{(0)}\hat{\rho}\hat{a}^{\dagger} - \rho_{aa}^{(0)}\hat{\rho}\hat{a}\right) - \gamma_1\hat{\rho}_{ab}, \quad \text{…………31}$$

$$\frac{d\hat{\rho}_{bc}}{dt} = \frac{\lambda_1 r_a}{\gamma_1}\left(\rho_{cc}^{(0)}\hat{a}\hat{\rho} - \rho_{ac}^{(0)}\hat{a}^{\dagger}\hat{\rho}\right) - \gamma_1\hat{\rho}_{bc} \quad \text{……………32}$$

Applying the adiabatic approximation scheme once more, we get

$$\hat{\rho}_{ab} = \frac{\lambda_1 r_a}{\gamma_1^2}\left(\rho_{ac}^{(0)}\hat{\rho}\hat{a}^{\dagger} - \rho_{aa}^{(0)}\hat{\rho}\hat{a}\right) \quad \text{…………………33}$$

$$\hat{\rho}_{bc} = \frac{\lambda_1 r_a}{\gamma_1^2}\left(\rho_{cc}^{(0)}\hat{a}\hat{\rho} - \rho_{ac}^{(0)}\hat{a}^{\dagger}\hat{\rho}\right) \quad \text{…….……….34}$$

Finally, using Eqs. (33) and (34) in Eq. (16), the time evolution of the density operator for degenerate three-level laser takes the form

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= \frac{A\rho_{aa}^{(0)}}{2}\left(2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}\right) + \frac{A\rho_{cc}^{(0)}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\right) \\ &- \frac{A\rho_{ac}^{(0)}}{2}\left(2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}\right) - \frac{A\rho_{ca}^{(0)}}{2}\left(2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2\right),\end{aligned} \quad \text{…...35}$$

$$A = \frac{2r_a\lambda_1^2}{\gamma_1^2} \quad \text{………………………………………..36}$$

is the linear gain co-efficient.

Similarly

The interaction of a two-level atom with a cavity mode can be described by the Hamiltonian

$$\hat{H} = i\lambda_2\left(|d\rangle\langle e|\hat{b} - \hat{b}^{\dagger}|e\rangle\langle d|\right) \quad \text{……………….………37}$$

Applying the same procedure we have

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= \frac{A_a\rho_{dd}^{(0)}}{2}\left(2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}\right) + \frac{A_a\rho_{ee}^{(0)}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\right) \\ &+ A_c\rho_{de}^{(0)}\left(\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{\rho}\right) + A_c\rho_{ed}^{(0)}\left(\hat{a}\hat{\rho} - \hat{\rho}\hat{a}\right),\end{aligned} \quad \text{…….38}$$

Eq. (38) represents the time evolution of the density operator for two-level laser.

Where

$$A_a = \frac{2r_b\lambda_2^2}{\gamma_2^2} \quad \text{.....................................39}$$

is the linear gain co-efficient of the two-level atom, and

$$A_c = \frac{r_b\lambda_2}{\gamma_2}. \quad \text{......... ....40}$$

Evolution of the reduced density operator for the cavity modes coupled to a two-mode vacuum reservoir. We denote the total density operator for the cavity plus reservoir by ˆ χ (t). The time evolution of this density operator is

$$\frac{d\hat{\chi}(t)}{dt} = -i\left[\hat{H}_{sR}, \hat{\chi}(t)\right] \quad \text{...................41}$$

After finding the formal solution and applying trace operation, we get

$$\frac{d\hat{\rho}(t)}{dt} = -i\left[\langle\hat{H}_{sR}(t)\rangle_R, \hat{\rho}(0)\right] - \int Tr_R\left(\left[\hat{H}_{sR}(t), \left[\hat{H}_{sR}(t'), \hat{\rho}(t')R\right]\right]\right)dt'. \quad \text{.....42}$$

The interaction Hamiltonian for the cavity mode with the

$$\hat{H}_{sR}(t) = i\sum_k \lambda_k\left(\hat{a}^\dagger\hat{c}_k e^{i(\omega_a-\omega_k)t} - \hat{a}\hat{c}_k^\dagger e^{-i(\omega_a-\omega_k)t}\right) + i\sum \eta_j\left(\hat{b}^\dagger\hat{d}_j e^{i(\omega_b-\omega_j)t} - \hat{b}\hat{d}_j^\dagger e^{-i(\omega_b-\omega_j)t}\right). \quad \text{......43}$$

$$\frac{d\hat{\rho}}{dt} = \kappa_1\left(2\hat{a}\hat{\rho}(t)\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}(t) - \hat{\rho}(t)\hat{a}^\dagger\hat{a}\right) + \kappa_2\left(2\hat{b}\hat{\rho}(t)\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho}(t) - \hat{\rho}(t)\hat{b}^\dagger\hat{b}\right).$$

This equation represents the evolution of the reduced density operator for cavity modes coupled to a two-modes vacuum reservoir

With the pump mode treated classically, non-degenerate sub-harmonic parametric oscillator is described by the Hamiltonian

$$\hat{H} = i\varepsilon\left(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b}\right), \quad \text{........................45}$$

whereε is the coupling constant proportional to the amplitude of the pump mode. The time evolution of the density operator corresponding to this Hamiltonian is

$$\frac{d\hat{\rho}(t)}{dt} = \varepsilon\left(\hat{a}^\dagger\hat{b}^\dagger\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{b}^\dagger + \hat{\rho}\hat{a}\hat{b} - \hat{a}\hat{b}\hat{\rho}\right) \quad \text{........46}$$

With the help of Eqs. (20), (35), and (36), we write

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= \frac{1}{2}\left(A\rho_{aa}^{(0)} + A_a\rho_{dd}^{(0)}\right)\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right) - \frac{1}{2}A\rho_{ac}^{(0)}\left(2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}\right)\\ &+ \frac{1}{2}\left(\kappa_1 + A\rho_{cc}^{(0)} + A_a\rho_{ee}^{(0)}\right)\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right) - \frac{1}{2}A\rho_{ac}^{(0)}\left(2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2\right)\\ &+ A_c\rho_{de}^{(0)}\left(\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho} + \hat{a}\hat{\rho} - \hat{\rho}\hat{a}\right) + \frac{1}{2}\kappa_2\left(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}\right)\\ &\cdot\cdot + \varepsilon\left(\hat{\rho}\hat{a}\hat{b} - \hat{a}\hat{b}\hat{\rho} + \hat{a}^\dagger\hat{b}^\dagger\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{b}^\dagger\right) \quad \text{.................47}\end{aligned}$$

Here we have taken that $\rho(0)$ ac = $\rho(0)$ $ca$and $\rho(0)$ de = $\rho(0)$ ed. Eq. (2.112) represents the equation of evolution for degenerate three- and two-level lasers with non sub-harmonic parametric oscillator and the cavity modes are coupled to vacuum reservoir.

C-number Langevin Equation

In this section we seek to derive the c-number langevin equations associated with the normal ordering, the correlation properties of the noise forces and the solution of the c-number langevin equations. We now proceed to derive to the c-number langevin equation associated with the normal ordering. Employing Eq. (47) and
, we see that

$$\frac{d\langle\hat{A}\rangle}{dt} = Tr\left(\hat{A}\frac{d\hat{\rho}}{dt}\right) \quad \text{.........48}$$

$$\begin{aligned}\frac{d}{dt}\langle\hat{a}\rangle &= \frac{1}{2}\left(A\rho_{aa}^{(0)} + A_a\rho_{dd}^{(0)}\right)Tr\left(2\hat{a}\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}^2\hat{a}^\dagger\hat{\rho} - \hat{a}\hat{\rho}\hat{a}\hat{a}^\dagger\right)\\ &+ \frac{1}{2}\left(\kappa_1 + A\rho_{cc}^{(0)} + A_a\rho_{ee}^{(0)}\right)Tr\left(2\hat{a}^2\hat{\rho}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{a}\hat{\rho} - \hat{a}\hat{\rho}\hat{a}^\dagger\hat{a}\right)\\ &- \frac{1}{2}A\rho_{ac}^{(0)}Tr\left(2\hat{a}\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}\hat{a}^{\dagger 2}\hat{\rho} - \hat{a}\hat{\rho}\hat{a}^{\dagger 2} + 2\hat{a}^2\hat{\rho}\hat{a} - \hat{a}^3\hat{\rho} - \hat{a}\hat{\rho}\hat{a}^2\right)\\ &+ A_c\rho_{de}^{(0)}Tr\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho} + \hat{a}^2\hat{\rho} - \hat{a}\hat{\rho}\hat{a}\right) + \frac{1}{2}\kappa_2 Tr\left(2\hat{a}\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{a}\hat{b}^\dagger\hat{b}\hat{\rho} - \hat{a}\hat{\rho}\hat{b}^\dagger\hat{b}\right) \quad \text{..48}\end{aligned}$$

Applying the cyclic property of the trace operation and the commutation relation,

We get

$$\frac{d}{dt}\langle\hat{a}\rangle = -\frac{1}{2}\mu\langle\hat{a}\rangle + \varepsilon\langle\hat{b}^\dagger\rangle - \nu_{de}, \quad \text{...................49}$$

Where

$$\mu = \kappa_1 + A\eta_{ac} + A_a\eta_{de}, \quad \text{...............................50}$$

$$\nu_{de} = A_c\rho_{de}^{(0)}. \quad \text{..51}$$

$$\eta_{ac} = \rho_{cc}^{(0)} - \rho_{aa}^{(0)}. \quad \text{.......................................52}$$

$$\eta_{de} = \rho_{ee}^{(0)} - \rho_{dd}^{(0)}, \quad \text{.......................................53}$$

Following the same procedure, it can be easily verified that

$$\frac{d}{dt}\langle\hat{a}^2\rangle = -\mu\langle\hat{a}^2\rangle + 2\varepsilon\langle\hat{b}^\dagger\hat{a}\rangle - 2\nu_{de}\langle\hat{a}\rangle + \nu_{ac}. \quad \text{.......54}$$

$$\frac{d}{dt}\langle\hat{b}^2\rangle = -\kappa_2\langle\hat{b}^2\rangle + 2\varepsilon\langle\hat{a}^\dagger\hat{b}\rangle \quad \text{............................55}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger\hat{a}\rangle = -\mu\langle\hat{a}^\dagger\hat{a}\rangle + \varepsilon\left(\langle\hat{a}\hat{b}\rangle + \langle\hat{a}^\dagger\hat{b}^\dagger\rangle\right) - \nu_{de}\left(\langle\hat{a}^\dagger\rangle + \langle\hat{a}\rangle\right) + A\rho_{aa}^{(0)} + A_a\rho_{dd}^{(0)}, \quad \text{...56}$$

$$\frac{d}{dt}\langle\hat{b}^\dagger\hat{b}\rangle = -\kappa_2\langle\hat{b}^\dagger\hat{b}\rangle + \varepsilon\left(\langle\hat{a}\hat{b}\rangle + \langle\hat{a}^\dagger\hat{b}^\dagger\rangle\right) \quad \text{.........57}$$

$$\frac{d}{dt}\langle\hat{a}\hat{b}\rangle = -\frac{1}{2}(\mu+\kappa_2)\langle\hat{a}\hat{b}\rangle + \varepsilon\left(\langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{b}^\dagger\hat{b}\rangle\right) - \nu_{de}\langle\hat{b}\rangle + \varepsilon, \quad \ldots\ldots.58$$

$$\frac{d}{dt}\langle\hat{b}^\dagger\hat{a}\rangle = -\frac{1}{2}(\mu+\kappa_2)\langle\hat{b}^\dagger\hat{a}\rangle + \varepsilon\left(\langle\hat{a}^2\rangle + \langle\hat{b}^\dagger\rangle\right) - \nu_{de}\langle\hat{b}^\dagger\rangle \quad \ldots\ldots\ldots..59$$

Where

$$\nu_{ac} = A\rho_{ac}^{(0)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.60$$

We note that Eqs. (49) and (54) - (59) are in the normal order and the c-number equations corresponding to these equations are

$$\frac{d}{dt}\langle\alpha\rangle = -\frac{1}{2}\mu\langle\alpha\rangle + \varepsilon\langle\beta^*\rangle - \nu_{de}, \quad \ldots\ldots\ldots\ldots\ldots\ldots61$$

$$\frac{d}{dt}\langle\beta\rangle = -\frac{1}{2}\kappa_2\langle\beta\rangle + \varepsilon\langle\alpha^*\rangle, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..62$$

$$\frac{d}{dt}\langle\alpha^2\rangle = -\mu\langle\alpha^2\rangle + 2\varepsilon\langle\beta^*\hat{\alpha}\rangle - 2\nu_{de}\langle\alpha\rangle + \nu_{ac} \quad \ldots\ldots\ldots..63$$

$$\frac{d}{dt}\langle\beta^2\rangle = -\kappa_2\langle\beta^2\rangle + 2\varepsilon\langle\alpha^*\beta\rangle \quad \ldots\ldots\ldots\ldots\ldots\ldots.64$$

$$\frac{d}{dt}\langle\beta^*\beta\rangle = -\kappa_2\langle\beta^*\beta\rangle + \varepsilon\left(\langle\alpha\beta\rangle + \langle\alpha^*\beta^*\rangle\right)$$

$$\frac{d}{dt}\langle\alpha^*\alpha\rangle = -\mu\langle\alpha^*\alpha\rangle + \varepsilon\left(\langle\alpha\beta\rangle + \langle\alpha^*\beta^*\rangle\right) - \nu_{de}\left(\langle\alpha^*\rangle + \langle\alpha\rangle\right) + A\rho_{aa}^{(0)} + A_a\rho_{dd}^{(0)} \quad \ldots66$$

$$\frac{d}{dt}\langle\beta^*\alpha\rangle = -\frac{1}{2}(\mu+\kappa_2)\langle\beta^*\alpha\rangle + \varepsilon\left(\langle\alpha^2\rangle + \langle\beta^*\rangle\right) - \nu_{de}\langle\beta^*\rangle \quad \ldots\ldots\ldots\ldots\ldots67$$

$$\frac{d}{dt}\langle\alpha\beta\rangle = -\frac{1}{2}(\mu+\kappa_2)\langle\alpha\beta\rangle + \varepsilon\left(\langle\alpha*\alpha\rangle + \langle\beta^*\beta\rangle\right) - \nu_{de}\langle\beta\rangle + \varepsilon, \quad \ldots\ldots\ldots\ldots\ldots\ldots68$$

On the basis of Eqs. (61) and (62), we can write

$$\frac{d}{dt}\alpha(t) = -\frac{1}{2}\mu\alpha(t) + \varepsilon\beta^* - \nu_{de} + f_\alpha(t). \quad \ldots\ldots..69$$

$$\frac{d}{dt}\beta(t) = -\frac{1}{2}\kappa_2\beta(t) + \varepsilon\alpha^* + f_\beta(t). \quad \ldots\ldots\ldots...70$$

In which $f\alpha(t)$ and $f\beta(t)$ are noise forces. The formal solutions of Eqs. (69) and (70) can be put in the form

$$\alpha(t) = \alpha(0)e^{-\frac{1}{2}\mu t} + \int_0^t dt' e^{-\frac{1}{2}\mu(t-t')}\left(\varepsilon\beta^*(t') - \nu_{de} + f_\alpha(t')\right) \quad \ldots\ldots\ldots71$$

$$\beta(t) = \beta(0)e^{-\frac{\kappa_2}{2}t} + \int_0^t dt' e^{-\frac{\kappa_2}{2}(t-t')}\left(\varepsilon\alpha^*(t') + f_\beta(t')\right) \quad \ldots\ldots72$$

We now proceed to determine the properties of the noise forces. We note that Eq. (61) and Eq. (69) as well as the expectation value of Eq. (62) and Eq.(72) will have the same form providing that

$$\langle f_\alpha(t)\rangle = \langle f_\beta(t)\rangle = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..73$$

Using the relation $\frac{d}{dt}\langle\alpha^2\rangle = 2\langle\alpha\frac{d\alpha}{dt}\rangle$ along with Eq. (69), we see that

$$\frac{d}{dt}\langle\alpha^2\rangle = -\mu\langle\alpha^2\rangle + \varepsilon\langle\hat{\beta}^*\hat{\alpha}\rangle - 2\nu_{de}\langle\alpha\rangle + \langle\alpha(t)f_\alpha(t)\rangle. \quad \ldots\ldots..74$$

Comparing Eqs.(74) and (63)

$$\langle\alpha(t)f_\alpha(t)\rangle = \frac{1}{2}\nu_{ac}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots....75$$

Employing Eqs.(71), we have

$$\langle\alpha(0)f_\alpha(t)\rangle e^{-\frac{1}{2}\mu t} + \int_0^t dt' e^{-\frac{1}{2}\mu(t-t')}\left\{\varepsilon\langle\beta^*(t')f_\alpha(t)\rangle - \nu_{de}\langle f_\alpha(t)\rangle + \langle f_\alpha(t')f_\alpha(t)\rangle\right\} = \frac{1}{2}\nu_{ac}.$$

Taking into account Eq. (73) and the fact that a noise force at a certain instant does not affe $\langle\alpha(t')f_\alpha(t)\rangle = \langle\beta(t')f_\beta(t)\rangle = 0$ variables at earlier time,
for , Eq. (76) reduces to

$$\int_0^t dt' e^{-\frac{1}{2}\mu(t-t')}\langle f_\alpha(t)f_\alpha(t')\rangle = \frac{1}{2}\nu_{ac}. \quad \ldots\ldots\ldots\ldots\ldots\ldots77$$

Or this equation can be rewritten in the form

$$\int_0^t dt' e^{-\frac{1}{2}\mu(t-t')}\langle f_\alpha(t)f_\alpha(t')\rangle = \int_0^t dt' e^{-\frac{1}{2}\mu(t-t')}\nu_{ac}\delta(t-t'). \quad \ldots\ldots78$$

It then follows that

$$\langle f_\alpha(t)f_\alpha(t')\rangle = \nu_{ac}\delta(t-t'). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..79$$

We obtains allthe correlation properties of the noise forces, $f_\alpha(t)$ and $f_\beta(t)$ associated with the normal ordering and applying he complex conjugate of can be written in the form

$$\frac{d}{dt}Y(t) = -\frac{1}{2}MY(t) + F(t) + V; \quad \ldots\ldots\ldots\ldots\ldots\ldots80$$

Applying the normalization condition: $u_{11}^2 + u_{21}^2 = 1$

Calculate the quadrature variances of the cavity modes employing the solutions of c-number langevin equations and the correlation properties of the noise forces. For two and single mode light.

$$[\hat{c}_+, \hat{c}_-] = 2i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..81$$

The variances of the quadratures represented by the operators defined by (81) can be expressed in terms of c-number variables associated with the normal ordering as

$$\Delta c_\pm^2 = 1 \pm \langle\gamma_\pm(t), \gamma_\pm(t)\rangle. \quad \ldots\ldots\ldots...\ldots\ldots\ldots\ldots.82$$

Where

$$\gamma_{\pm}(t) = \frac{1}{\sqrt{2}}\left(\alpha^*(t) \pm \alpha(t) + \beta^*(t) \pm \beta(t)\right) \qquad \ldots\ldots\ldots 83$$

On account of Eqs(83) we can write

$$\langle\gamma_{\pm}(t),\gamma_{\pm}(t)\rangle = \frac{1}{2}\Big(\langle\alpha(t),\alpha(t)\rangle + \langle\beta(t),\beta(t)\rangle + 2\langle\alpha(t),\beta(t)\rangle$$
$$\pm\langle\alpha^*(t),\alpha(t)\rangle \pm \langle\beta^*(t),\beta(t)\rangle \pm 2\langle\alpha^*(t),\beta(t)\rangle\Big) + c.c., \qquad \ldots\ldots\ldots\ldots 84$$

in which *c.c.* stands for the complex conjugate the other terms.

We next to proceed to calculate the expectation values involved in Eq.(3.8). Taking into account α(x) ,β(x), their conjugate and the assumption that the cavity is initially in vacuum state, we easily find that

$$\langle\alpha\rangle = \nu_1(t). \qquad \ldots\ldots\ldots 85$$

$$\langle\alpha^*(t)\alpha(t)\rangle = \langle|p_1(t)\alpha(0) + q(t)\beta(0) + \nu_1(t) + Z_1(t)|^2\rangle \qquad \ldots\ldots 86$$

Taking into account the assumption that the cavity is initially vacuum along with the fact that a noise force at a certain time does not affect the cavity mode variables at earlier time, this equation reduces to

$$\langle\alpha^*(t)\alpha(t)\rangle = \langle Z_1^*(t)Z_1(t)\rangle + \nu_1^2(t) \qquad \ldots\ldots\ldots 87$$

Applying

$$\langle\alpha^*\alpha\rangle = \int_0^t \left\{p_1^2(t-t')f_{\alpha^*\alpha} + 2\varepsilon p_1(t-t')q(t-t')\right\}dt' + \nu_1^2(t) \qquad \ldots\ldots\ldots 88$$

Where

$$f_{\alpha^*\alpha} = A\rho_{aa}^{(0)} + A_a\rho_{dd}^{(0)}. \qquad \ldots\ldots\ldots 89$$

Carrying out integration we get

$$\langle\alpha^*(t)\alpha(t)\rangle = \frac{W_+^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_+}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) + \frac{W_-^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_-}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{4W_+W_-f_{\alpha^*\alpha} - 16\varepsilon^2 W_+ - 16\varepsilon^2 W_-}{4\varphi^2(\lambda_- + \lambda_+)}\left(1-e^{-1(\lambda_- + \lambda_+)t/2}\right) + \nu_1^2(t). \qquad \ldots\ldots\ldots 90$$

$$\langle\alpha^*(t),\alpha(t)\rangle = \frac{W_+^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_+}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) + \frac{W_-^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_-}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{4W_+W_-f_{\alpha^*\alpha} - 16\varepsilon^2 W_+ - 16\varepsilon^2 W_-}{4\varphi^2(\lambda_- + \lambda_+)}\left(1-e^{-1(\lambda_- + \lambda_+)t/2}\right). \qquad \ldots\ldots\ldots 91$$

$$\langle\beta^*(t),\beta(t)\rangle = \frac{16\varepsilon^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_-}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) + \frac{16\varepsilon^2 f_{\alpha^*\alpha} - 8\varepsilon^2 W_+}{4\varphi^2\lambda_-}e^{-\lambda_- t}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{64\varepsilon^2 f_{\alpha^*\alpha} - 16\varepsilon^2 W_+ - 16\varepsilon^2 W_-}{4\varphi^2(\lambda_- + \lambda_+)}\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right) \qquad \ldots\ldots\ldots 93$$

$$\langle\alpha(t),\beta(t)\rangle = \frac{4\varepsilon W_+ f_{\alpha^*\alpha} - W_-W_+\varepsilon - 16\varepsilon^3}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) \qquad \ldots\ldots 93$$
$$+\frac{4\varepsilon W_- f_{\alpha^*\alpha} - W_+W_-\varepsilon - 16\varepsilon^3}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{8\varepsilon W_+ f_{\alpha^*\alpha} + 8\varepsilon W_- f_{\alpha^*\alpha} - 2W_+^2\varepsilon - 2W_-^2\varepsilon - 64\varepsilon^3}{4\varphi^2(\lambda_+ + \lambda_-)}\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right)$$

$$\langle\alpha(t),\alpha(t)\rangle = \frac{\text{[illegible]}}{4\varphi^2\lambda_+}\left(1-e^{\text{[illegible]}}\right) + \frac{\text{[illegible]}}{4\varphi^2\lambda_-}\left(1-e^{\text{[illegible]}}\right)$$
$$-\frac{4W_+W_-\nu_{ac}}{4\varphi^2(\lambda_+ + \lambda_-)}\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right), \qquad \ldots\ldots\ldots 94$$

$$\langle\beta(t),\beta(t)\rangle = \frac{16\varepsilon^2\nu_{ac}}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) + \frac{16\varepsilon^2\nu_{ac}}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{64\varepsilon^2\nu_{ac}}{4\varphi^2(\lambda_+ + \lambda_-)}\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right), \qquad \ldots\ldots\ldots 95$$

$$\langle\alpha^*(t),\beta(t)\rangle = \frac{4\varepsilon W_+\nu_{ac}}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right) + \frac{4\varepsilon W_-\nu_{ac}}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-\frac{8\varepsilon W_+\nu_{ac} + 8\varepsilon W_-\nu_{ac}}{4\varphi^2(\lambda_+ + \lambda_-)}\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right). \qquad \ldots\ldots 96$$

Using Eqs. (92), (93), (94), (95), (96), and (3.20) in (87), we get

$$\langle\gamma_{\pm}(t),\gamma_{\pm}(t)\rangle = \frac{(W_+ \pm 4\varepsilon)^2(\nu_{ac} \pm f_{\alpha^*\alpha}) - 2\varepsilon(W_+ \pm 4\varepsilon)(W_- \pm 4\varepsilon)}{4\varphi^2\lambda_+}\left(1-e^{-\lambda_+ t}\right)$$
$$+\frac{(W_- \pm 4\varepsilon)^2(\nu_{ac} \pm f_{\alpha^*\alpha}) - 2\varepsilon(W_- \pm 4\varepsilon)(W_+ \pm 4\varepsilon)}{4\varphi^2\lambda_-}\left(1-e^{-\lambda_- t}\right)$$
$$-4\frac{(W_+ \pm 4\varepsilon)(W_- \pm 4\varepsilon)(\nu_{ac} \pm f_{\alpha^*\alpha}) - \varepsilon((W_+ \pm 4\varepsilon)^2 + (W_- \pm 4\varepsilon)^2)}{4\varphi^2(\lambda_+ + \lambda_-)}$$
$$\times\left(1-e^{-\frac{1}{2}(\lambda_+ + \lambda_-)t}\right) \qquad \ldots\ldots\ldots 97$$

With the aid of Eqs. (87) and (97), we write at steady state

$$\Delta c_{\pm}^2 = 1 \pm \frac{(W_+ \pm 4\varepsilon)^2(\nu_{ac} \pm f_{\alpha^*\alpha}) - 2\varepsilon(W_+ \pm 4\varepsilon)(W_- \pm 4\varepsilon)}{4\varphi^2\lambda_+}$$
$$\pm\frac{(W_- \pm 4\varepsilon)^2(\nu_{ac} \pm f_{\alpha^*\alpha}) - 2\varepsilon(W_- \pm 4\varepsilon)(W_+ \pm 4\varepsilon)}{4\varphi^2\lambda_-}$$
$$\mp 4\frac{(W_+ \pm 4\varepsilon)(W_- \pm 4\varepsilon)(\nu_{ac} \pm f_{\alpha^*\alpha}) - \varepsilon((W_+ \pm 4\varepsilon)^2 + (W_- \mp 4\varepsilon)^2)}{4\varphi^2(\lambda_+ + \lambda_-)}. \qquad \ldots\ldots\ldots *$$

RESULT AND DISCUSSION

Eq. (*) represents the quadrature variances of a two-mode light produced by degenerate three and two-level Lasers with non-degenerate sub-harmonic Light and the cavity modes are coupled to a vacuum reservoir. Fig 3.2 represents the variances of the minus quadrature of Eq. (*) versus η for $\kappa = 0.8$, $\varepsilon = 0.3$, $A_a = 1$, $\eta_a = 0.5$, and for $A = 25$ (solid), $A = 50$(dash) ,$A = 75$ (dotted), and $A = 100$ (dash-dotted). The figure indicates that the degree of squeezing increases with the linear gain coefficient and perfect squeezing can be obtained for large values of the linear gain coefficient and small values of η. Moreover, the minimum value of the quadrature variance described by Eq. (*) for $A = 100$, $\kappa = 0.8$, $A_a = 10$, and $\eta_a = 0.5$ is found to be $\Delta c^2_- = 0.36$ and occurs at $\eta = 0.04$. The result implies that the maximum squeezing for the above values is 64% below the coherent state level. We also observe from the plots that the degree of squeezing decreases as Aincreases for large value of η.

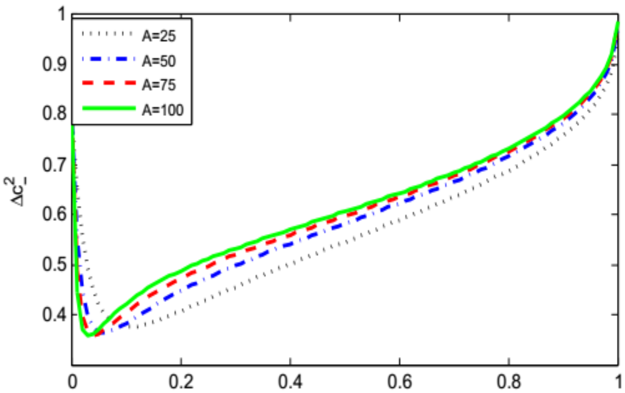


Fig. 3.2: Plots of the quadrature variance Eq. (*) versus η for κ = 0.8, ε = 0.3, Aa= 1, ηa= 0.5 and for different values of the linear gain coefficient.

In Fig 3.3 we plot the variances of the minus quadrature of Eq. (*) versus η for A = 100, A_a= 10, κ = 0.8, η_a= 0.5 and for ε = 0.00 (dash-dotted), ε = 0.20 (dotted), ε = 0.30 (dash), and ε = 0.40 (solid). As the plot show that the degree of squeezing increase with ε.

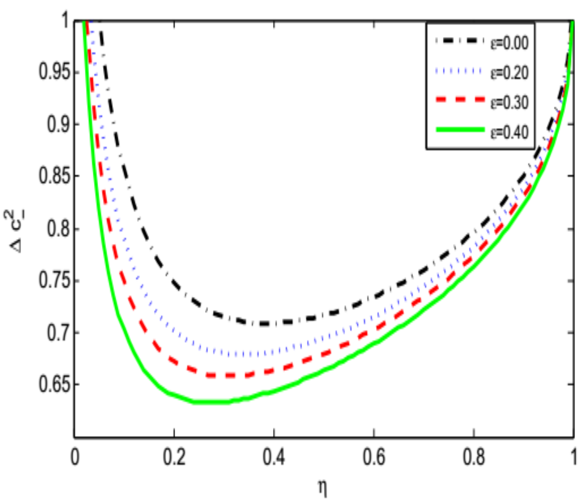


Fig. 3.3: Plots of the quadrature variances Eq. (3.22) versus η for A = 100, Aa= 10, κ = 0.8, ηa= 0.5 and for different values of amplitude proportional to the pump mode.

Fig 3.4 is the plots of the minus quadrature variances of Eq. (*) versus η for A = 100, ε = 0.3, κ = 0.8, η_a= 0.5, and for A_a= 0 (solid), A_a= 5 (dash), A_a= 10 (dotted), and A_a= 15 (dash-dotted). We see from the figure that the degree of squeezing increase as $A_a$decreases. The result shows that the two-level laser distracts the squeezing properties of the two-mode light.

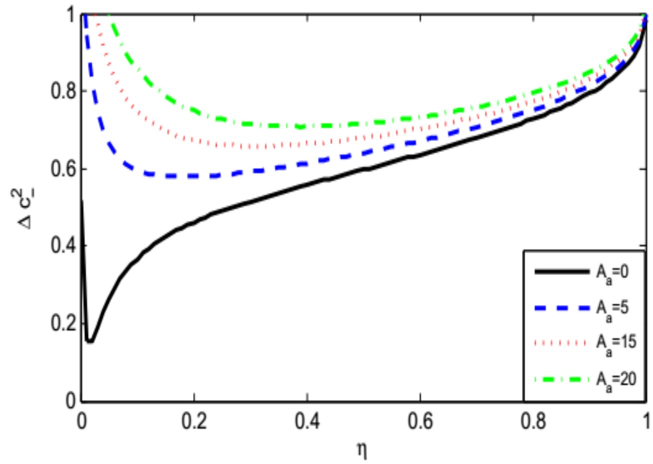


Fig. 3.4: Plots of the quadrature variance Eq. (*) versus η for A = 100, ε = 0.3, κ = 0.8, η_a= 0.5, and for different values of the linear gain coefficient of the two level atoms.

In Fig 3.4 we plot the variance of the minus quadrature of Eq. (*) versus η for A = 100, A_a= 10, κ = 0.8, ε = 0.3 and for η_a= 0.00 (dash-dotted), η_a= 0.20 (dotted), η_a= 0.30 (dash), and η_a= 0.4(solid). We see from the figure that the degree of squeezing increases with η_a. This is due to the cavity mode produced by the two-level atom decreases.

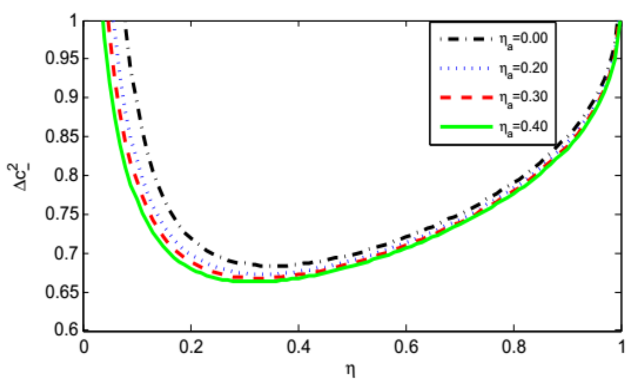


Fig. 3.4: Plots of the quadrature variance Eq. (*) versus η for A = 100, A_a= 10, κ = 0.8, ε = 0.3 and for different values of η_a**.**

CONCLUSION

We study the squeezing properties of the light produced by degenerate three and two-level lasers with non-degenerate sub-harmonic light. We have obtained, using the pertinent of master equation, c-number langevin equations associated with the normal ordering. Applying the solutions of the resulting

c-number langevin equations and the correlation properties noise force, we calculated the quadrature variances and the EPR-type variables. It is found that the two-mode light produced by the system under consideration in squeezed state. The squeezing occurs in the minus quadrature. The degree of squeezing increases with the linear gain coefficient for small values of η *and* decreases as the linear gain coefficient increases for large value of η.

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