Mixed Delay Constraints in Wyner's Soft-Handoff Network

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***Abstract*—Wyner's *soft-handoff* network with mixed delay constraints is considered when neighbouring receivers can cooperate over rate-limited links. Each source message is a combination of independent "fast" and "slow" bits, where the former are subject to a stringent decoding delay. Inner and outer bounds on the capacity region are derived, and the multiplexing gain region is characterized when only transmitters or only receivers cooperate.**

I. Introduction

Wireless communication networks have to accommodate different types of data traffics with different latency constraints. In particular, delay-sensitive video-applications represent an increasing portion of data traffic. On the other hand, modern networks can increase data rates by means of cooperation between terminals or with helper relays. However, cooperation typically introduces additional communication delays, and is thus not applicable to delay-sensitive applications. In this paper, we analyze the rates of communication that can be attained over an interference network with either transmitter or receiver cooperation, and where parts of the messages cannot profit from this cooperation because they are subject to stringent delay constraints. Mixed delay constraints in wireless networks have previously been studied in [1]–[3]. In particular, [1] proposes a broadcasting approach over a single-antenna fading channel to communicate a stream of "fast" messages, which have to be sent over a single coherence block, and a stream of "slow" messages, which can be sent over multiple blocks. A similar approach was taken in [3] but for a broadcast scenario with K users. Instead of superposing "slow" on "fast" messages, this latter work proposes a scheduling approach to give preference to the communication of "fast" messages. The closely related setup of *unreliable conferencing*, where a part of the message needs to be decoded without using the conferencing link, was introduced in [4].

For simplicity, in this paper, we focus on Wyner's *soft-handoff* model [5]–[7] with K interfering transmitter and receiver pairs. Each transmitter sends a pair of independent source messages called "fast" and "slow" messages. Each receiver decodes the "fast" message immediately and only based on its own channel outputs. Before decoding its "slow" message, it can communicate with its immediate neighbours over conferencing links during a given maximum number of rounds [8] and subject to a rate-constraint. It then decodes the "slow" message based on its own channel outputs and the cooperation messages received from its neighbours. In the case of only transmitter conferencing, receivers decode both messages only based on their own channel outputs; transmitters can hold a conferencing communication that depends only on the "slow" messages but not on the "fast" messages.

We propose inner and outer bounds on the capacity region of the soft-handoff network with receiver conferencing. We also characterize the multiplexing gain region of the setups with only transmitter conferencing or only receiver conferencing. The multiplexing gain regions of the two scenarios coincide, and thus show a duality between transmitter and receiver conferencing in the high signal-to-noise ratio regime.

Our results also indicate that the sum-rate of "fast" and "slow" messages is approximately constant when "fast" messages are sent at small rate. In this regime, the stringent decoding delay of part of the messages does not cause a loss in overall performance. When "fast" messages have large rates, this is not the case. In this regime, increasing the rate of "fast" messages by Δ, requires that the rate of "slow" messages be reduced by approximately $2 \cdot \Delta$.

II. Problem Setup

Consider a wireless communication system as in Fig. 1 with K interfering transmitter (Tx) and receiver (Rx) pairs $1, \ldots, K$ that are aligned on a line. Transmitters and receivers are each equipped with a single antenna, and channel inputs and outputs are real valued. Interference is short-range so that the signal sent by Tx k is observed only by Rx k and Rx $k+1$. As a result, the time-t channel output at Rx k is

$$Y_{k,t} = X_{k,t} + \alpha X_{k-1,t} + Z_{k,t}, \tag{1}$$

where $X_{k,t}$ and $X_{k-1,t}$ are the symbols sent by Tx k and $k-1$ at time t, respectively; $\{Z_{k,t}\}$ are independent and identically distributed (i.i.d.) standard Gaussians for all k and t; $\alpha \neq 0$ is a fixed real number smaller than 1; and $X_{0,t} = 0$ for all t.

Each Tx k wishes to send a pair of independent source messages $M_k^{(F)}$ and $M_k^{(S)}$ to Rx k. The "fast" source message $M_k^{(F)}$ is uniformly distributed over the set $\mathcal{M}_k^{(F)} := \{1, \ldots, \lfloor 2^{nR_k^{(F)}} \rfloor\}$ and needs to be decoded subject to a stringent delay constraint, as we explain shortly. The "slow" source message $M_k^{(S)}$ is uniformly distributed over $\mathcal{M}_k^{(S)} :=$

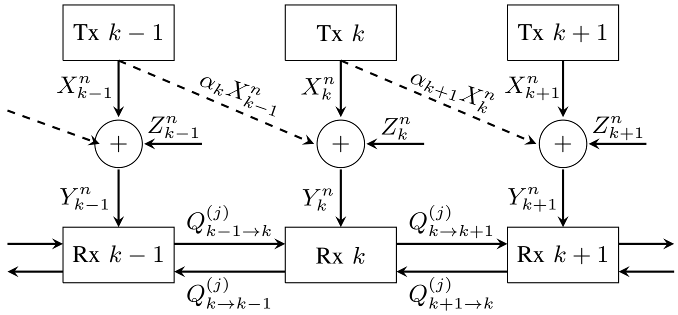

Fig. 1. System model

$\{1,\dots,\lfloor 2^{nR_k^{(S)}}\rfloor\}$ and is subject to a less stringent decoding delay constraint. Here, n denotes the blocklength of transmission and $R_k^{(F)}$ and $R_k^{(S)}$ are the rates of transmissions of the "fast" and the "slow" messages. All source messages are independent of each other and of all channel noises.

Tx k computes its channel inputs $X_k^n := (X_{k,1},\dots,X_{k,n})$ as a function of the pair $(M_k^{(F)}, M_k^{(S)})$:

$$X_k^n = f_k^{(n)}\big(M_k^{(F)}, M_k^{(S)}\big), \tag{2}$$

for some function $f_k^{(n)}$ on appropriate domains that satisfies the average block-power constraint

$$\frac{1}{n}\sum_{t=1}^{n} X_{k,t}^2 \le P, \quad \text{a.s.}, \quad \forall\, k \in \{1,\dots,K\}. \tag{3}$$

Receivers decode in two phases. During the first *fast-decoding phase*, each Rx k decodes the "fast" source message $M_k^{(F)}$ based on its own channel outputs $Y_k^n := (Y_{k,1},\dots,Y_{k,n})$. So, it produces:

$$\hat{M}_k^{(F)} = g_k^{(n)}\big(Y_k^n\big) \tag{4}$$

for some decoding function $g_k^{(n)}$ on appropriate domains.

In the subsequent *slow-decoding phase*, the receivers first communicate to each other over orthogonal conferencing links, and then they decode their intended "slow" messages based on their own channel outputs and the conferencing messages received from their neighbours. Only neighbouring receivers can exchange conferencing messages, and conferencing is limited to a maximum number of $\mathsf{D}_{\max}$ rounds and to rate-constraint π. In conferencing round $j \in \{1,2,\dots,\mathsf{D}_{\max}\}$, Rx k sends the conferencing message $Q_{k\to k-1}^{(j)}$ to its left neighbour, Rx $k-1$, and the conferencing message $Q_{k\to k+1}^{(j)}$ to its right neighbour, Rx $k+1$. These conferencing messages can depend on the outputs Y_k^n and on the conferencing messages that Rx k received in the previous rounds. So, for $\tilde{k} \in \{k-1, k+1\}$:

$$Q_{k\to\tilde{k}}^{(j)} = \psi_{k,\tilde{k}}^{(n)}\Big(Y_k^n, Q_{k-1\to k}^{(1)}, Q_{k+1\to k}^{(1)}, \dots, Q_{k-1\to k}^{(j-1)}, Q_{k+1\to k}^{(j-1)}\Big), \tag{5}$$

for an encoding function $\psi_{k,\tilde{k}}^{(n)}$ on appropriate domains. The $\mathsf{D}_{\max}$ messages sent over a conferencing link in each direction are subject to a rate constraint π. So, for all $k \in \{1,\dots,K\}$ and $\tilde{k} \in \{k, k+1\}$:

$$\sum_{j=1}^{\mathsf{D}_{\max}} H(Q_{k\to\tilde{k}}^{(j)}) \le \pi \cdot n. \tag{6}$$

After the last conferencing round $\mathsf{D}_{\max}$, each Rx k decodes its desired "slow" message as

$$\hat{M}_k^{(S)} := b_k^{(n)}\Big(Y_k^n, Q_{k-1\to k}^{(1)}, Q_{k+1\to k}^{(1)}, \dots, Q_{k-1\to k}^{(\mathsf{D}_{\max})}, Q_{k+1\to k}^{(\mathsf{D}_{\max})}\Big) \tag{7}$$

by means of a decoding function $b_k^{(n)}$ on appropriate domains.

The main interest in this paper is in the achievable sum-rates of "fast" and "slow" messages. Given a maximum conferencing rate π and a maximum allowed power P, the pair of (average) rates $(R^{(F)}, R^{(S)})$ is called *achievable*, if there exists a sequence (in n) of encoding and decoding functions so that

$$\frac{1}{K}\sum_{k=1}^{K} R_k^{(F)} = R^{(F)} \qquad \text{and} \qquad \frac{1}{K}\sum_{k=1}^{K} R_k^{(S)} = R^{(S)}, \tag{8}$$

and the probability of decoding error

$$P_e^{(n)} := \Pr\Bigg[\bigcup_{k\in\{1,\dots,K\}} \Big\{\hat{M}_k^{(F)} \neq M_k^{(F)} \text{ or } \hat{M}_k^{(S)} \neq M_k^{(S)}\Big\}\Bigg]$$

tends to 0 as $n \to \infty$.

Definition 1: Given power constraint $P > 0$ and maximum conferencing rate π, the *capacity region* $\mathsf{C}(P,\pi)$ is the closure of the set of all rate pairs $(R^{(F)}, R^{(S)})$ that are achievable.

We will particularly be interested in the high signal-to-noise ratio (SNR) regime, and thus in the set of achievable *multiplexing gains* when the conferencing capacity also scales logarithmically in the SNR. Given a conferencing prelog $\mu \ge 0$, the *pair of multiplexing gains* $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ is called *achievable*, if for each K there exists a sequence of rates $\{R_K^{(F)}(P), R_K^{(S)}(P)\}_{P>0}$ so that

$$\mathsf{S}^{(F)} := \overline{\lim_{K\to\infty}}\ \overline{\lim_{P\to\infty}}\ \frac{R_K^{(F)}}{\frac{1}{2}\log(1+P)}, \tag{9}$$

$$\mathsf{S}^{(S)} := \overline{\lim_{K\to\infty}}\ \overline{\lim_{P\to\infty}}\ \frac{R_K^{(S)}}{\frac{1}{2}\log(1+P)}, \tag{10}$$

and for each K and $P > 0$ the pair $(R_K^{(F)}(P), R_K^{(S)}(P))$ is achievable with conferencing rate at most $\pi = \mu \cdot \frac{1}{2}\log P$.

Definition 2: Given a conferencing-prelog μ, the closure of the set of all achievable multiplexing gains $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ is called *multiplexing gain region* and denoted $\mathcal{S}^\star(\mu)$.

III. Main Results

Our first result is an inner bound on the capacity region. It is based on two schemes. The first scheme assumes $\pi \le R^{(F)}$. Each transmitter uses a 3-layer superposition code, where it sends its "fast" message in the lowest two layers and its "slow" message in the upper-most layer. Each receiver immediately decodes its intended "fast" message based only on its channel outputs and then sends the part encoded in the lower-most layer to its right neighbour. To decode its intended "slow" message, it first pre-subtracts the interference caused by the lower-most layer of the superposition codeword sent by the transmitter to its left. This scheme uses only a single conferencing round.

The second scheme assumes $\pi > R^{(F)}$ and also exchanges parts of "slow" source messages over the conferencing links. Each transmitter employs a $\mathsf{D}_{\max}+1$-layer superposition code, where the lower-most layer encodes the "fast" message and all higher layers encode parts of the "slow" message. As before, each receiver decodes its intended "fast" message immediately based on its channel outputs. It then sends this decoded message over the conferencing link to its left neighbour during the first conferencing round. Subsequently, after each conferencing round $j = 1, \ldots, \mathsf{D}_{\max}$, each receiver cancels the interference from the layer-j codeword sent by the transmitter to its left and then decodes the layer $j+1$-part of its intended message. It sends the decoded message part and the conferencing message that it obtained in the previous rounds to its right neighbour.

Theorem 1 (Capacity Inner Bound): The capacity region $\mathsf{C}(P, \pi)$ includes all rate-pairs $(R^{(F)}, R^{(S)})$ that satisfy

$$R^{(F)} \leq \min\left\{I(U_2; Y), I(U_2; Y|U_1) + \pi\right\} \tag{11a}$$

and

$$R^{(F)} + R^{(S)} \leq \frac{1}{K} \sum_{k=1}^{K} \Big[I(X; Y, U_1'|U_1) + \min\left\{I(U_2; Y), I(U_2; Y|U_1) + \pi\right\}\Big], \tag{11b}$$

where triples (U_1, U_2, X) and (U_1', U_2', X') are i.i.d. according to some probability distribution $P_{U_1 U_2 X}$ that satisfies the Markov chain $U_1 \to U_2 \to X$, and where $Y = X + \alpha X' + Z$ with Z standard Gaussian independent of $(U_1, U_2, X, U_1', U_2', X')$.

The capacity region $\mathsf{C}(P, \pi)$ also includes all rate-pairs $(R^{(F)}, R^{(S)})$ that satisfy

$$R^{(F)} \leq I(U; Y) \tag{12a}$$

$$R^{(F)} + R^{(S)} \leq I(U; Y) + I(V_1; Y, U'|U) + \sum_{d=2}^{\mathsf{D}_{\max}-1} I(V_d; Y, V_{d-1}'|V_{d-1}) + I(X; Y, X'|V_{\mathsf{D}_{\max}-1}), \tag{12b}$$

where the tuples $(U, V_1, \ldots, V_{\mathsf{D}_{\max}-1}, X)$ and $(U', V_1', \ldots, V_{\mathsf{D}_{\max}-1}', X')$ are i.i.d. according to some probability distribution $P_{UV_1 \ldots V_{\mathsf{D}_{\max}-1} X}$ satisfying the Markov chain $U \to V_1 \to V_2 \to \ldots \to V_{\mathsf{D}_{\max}-1} \to X$ and the rate constraint

$$I(U; Y) + I(V_1; Y, U'|U) + \sum_{d=2}^{\mathsf{D}_{\max}-1} I(V_d; Y, V_{d-1}'|V_{d-1}) \leq \pi, \tag{13}$$

and where $Y = X + \alpha X' + Z$ with Z independent standard Gaussian.

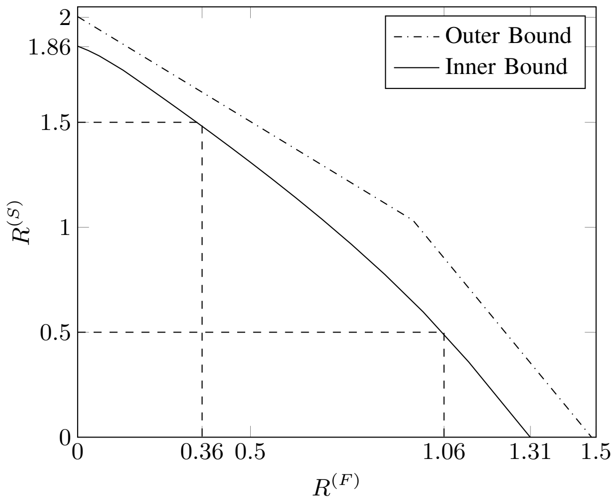

Fig. 2. Capacity outer bound in Theorem 2 and inner bound in Theorem 1 for $P = 5$, $\alpha = 0.2$, $\pi = 0.346$, and $\mathsf{D}_{\max} = 16$.

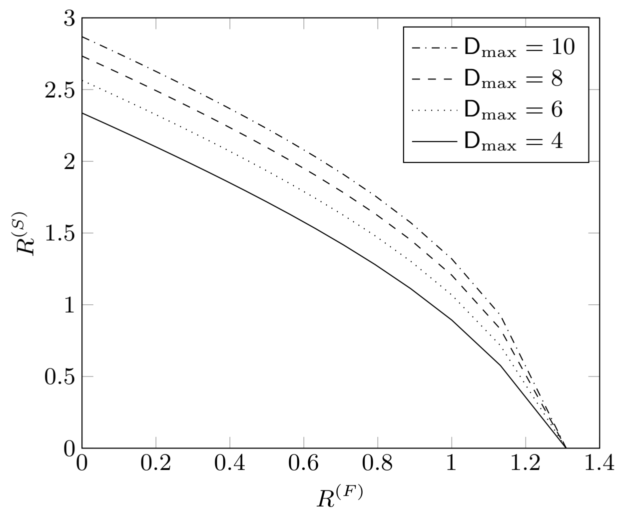

Fig. 3. Capacity inner bound in Theorem 1 for $\pi = 2$, $P = 5$, $\alpha = 0.2$ and different values of $\mathsf{D}_{\max}$.

Proof: See [9]. ∎

Theorem 2 (Capacity Outer Bound): Any achievable rate pair $(R^{(F)}, R^{(S)})$ satisfies the following two conditions:

$$R^{(F)} + R^{(S)} \leq \frac{\left(\left\lceil \frac{K-1}{2} \right\rceil + 1\right)}{K} \cdot \frac{1}{2} \log\left(1 + (1+\alpha^2)P\right) + \frac{\left\lfloor \frac{K-1}{2} \right\rfloor}{K} \cdot \max\{-\log|\alpha|, 0\} + \frac{\left\lfloor \frac{K}{2} \right\rfloor}{K} \cdot \frac{1}{2} \log(1+\alpha^2) + \frac{K-1}{K} \cdot \pi, \tag{14}$$

$$2R^{(F)} + R^{(S)} \leq \frac{K-1}{2K} \Big(\frac{1}{2} \log\left((1 + (1+\alpha^2)P)(1+\alpha^2)\right) + 2\max\{-\log|\alpha|, 0\}\Big) + \frac{1}{K} \log(1+P). \tag{15}$$

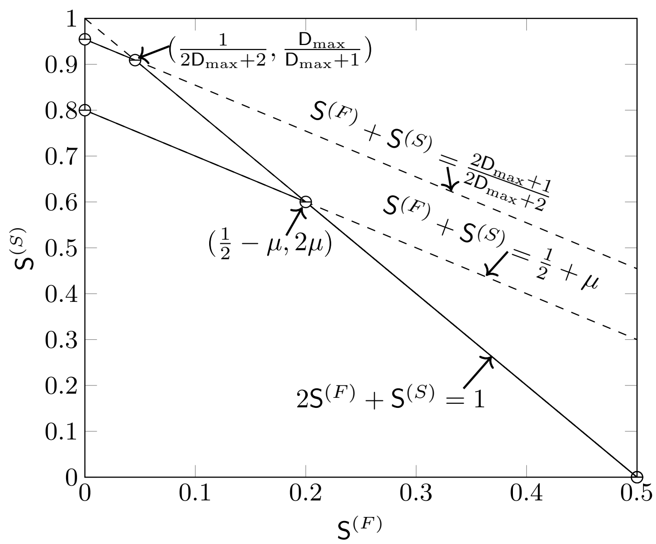

Fig. 4. Region $\mathcal{S}^\star(\mu)$ for $\mathsf{D}_{\max} = 10$ and $\mu = 0.3$ or $\mu \geq 10/22$.

Proof: See Section V. ■

Fig. 2 illustrates the outer bound on the capacity-region in Theorem 2 and the inner bound in Theorem 1 when this latter is evaluated for jointly Gaussian distributions on the inputs and the auxiliaries. For small values of $R^{(F)}$, both the lower and the upper bounds decrease with slope -1. For large values of $R^{(F)}$, they decrease with slope -2.

Inner and outer bounds in Theorems 1 and 2 are generally not tight at high SNR. New bounds are required to obtain the following theorem. In particular, a new inner bound based on a scheme that periodically silences transmitters so as to avoid that interference propagates too far.

Theorem 3 (Multiplexing Gain): The multiplexing gain region $\mathcal{S}^\star(\mu)$ is the set of all nonnegative pairs $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ satisfying

$$2\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq 1 \tag{16}$$

$$\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq \min\left\{\frac{1}{2} + \mu, \; \frac{2\mathsf{D}_{\max}+1}{2\mathsf{D}_{\max}+2}\right\}. \tag{17}$$

Proof: Omitted due to space limitations. See [9]. ■

Figure 4 shows the multiplexing gain region for different values of μ. We notice that for $\mathsf{S}^{(F)} \leq \frac{1}{2} - \mu$, the slope of the boundary of the region is -1, and for $\mathsf{S}^{(F)} > \frac{1}{2} - \mu$, it is -2.

Remark 1: An analogous result can be obtained for the setup where each receiver can send conferencing messages only to its left neighbour or only to its right neighbour. The multiplexing gain region is characterized by (16) and

$$\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq \min\left\{\frac{1}{2} + \frac{\mu}{2}, \; \frac{\mathsf{D}_{\max}+1}{\mathsf{D}_{\max}+2}\right\}. \tag{18}$$

Notice that despite the asymmetry of the network, the result is the same for conferencing to left or right neighbours.

IV. Transmitter-Conferencing

We consider a related setup where transmitters can send conferencing messages but not the receivers. Transmitter conferencing is limited to $\mathsf{D}_{\max}$ rounds and the exchanged messages can only depend on the "slow" messages but not on the fast messages. This models a setup where the transmitters learn the "slow" messages in advance before they communicate to the receivers, whereas "fast" messages arrive at the transmitters just shortly before this communication.

In each round $j \in \{1, \ldots, \mathsf{D}_{\max}\}$, Tx k produces the two conferencing messages $T^{(j)}_{k\to k-1}$ and $T^{(j)}_{k\to k+1}$, where

$$T^{(j)}_{k\to\tilde{k}} = \xi^{(n)}_{k\to\tilde{k}}\big(M_k^{(S)}, T^{(1)}_{k-1\to k}, \ldots, T^{(j-1)}_{k-1\to k}, T^{(1)}_{k+1\to k}, \ldots, T^{(j-1)}_{k+1\to k}\big) \tag{19}$$

for some function $\xi^{(n)}_{k\to\tilde{k}}$ on appropriate domains. It sends these messages over the conferencing links to its left and right neighbours. As before, the conferencing links are rate-limited to rate π. So, for all $k \in \{1, \ldots, K\}$ and $\tilde{k} \in \{k-1, k+1\}$:

$$\sum_{j=1}^{\mathsf{D}_{\max}} H(T^{(j)}_{k\to\tilde{k}}) \leq \pi \cdot n. \tag{20}$$

Each Tx k then computes its channel inputs as

$$X_k^n = \tilde{f}_k^{(n)}\big(M_k^{(F)}, M_k^{(S)}, T^{(1)}_{k-1\to k}, \ldots, T^{\mathsf{D}_{\max}}_{k-1\to k}, T^{(1)}_{k+1\to k}, \ldots, T^{\mathsf{D}_{\max}}_{k+1\to k}\big) \tag{21}$$

subject to the power constraint in (3).

Each Rx k decodes the two messages $M_k^{(F)}, M_k^{(S)}$ only based on its channel outputs Y_k^n:

$$\big(\hat{M}_k^{(F)}, \hat{M}_k^{(S)}\big) = \tilde{g}_k^{(n)}\big(Y_k^n\big). \tag{22}$$

Capacity region and multiplexing gain region $\tilde{\mathcal{S}}^\star(\mu)$ are defined analogously as for receiver conferencing.

Theorem 4 (Only Transmitter-Conferencing): Given $\mu \geq 0$, the multiplexing gain region $\tilde{\mathcal{S}}^\star(\mu)$ is the set of all nonnegative pairs $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ that satisfy (16) and (17).

Proof: Omitted due to space limitations. See [9]. ■

Remark 2: Our results exhibit a duality between transmitter and receiver conferencing. They yield the same multiplexing gain region.

V. Proof of Theorem 2

For convenience of notation, define for any $k \in \{1, \ldots, K\}$:

$$M_k := (M_k^{(F)}, M_k^{(S)}). \tag{23}$$

Due to space limitations, we only prove Inequality (14). By Fano's Inequality and the independence of the messages, we have for any $k \in \{1, \ldots, K-1\}$:

$$\begin{aligned}
&R_k^{(F)} + R_k^{(S)} + R_{k+1}^{(F)} \\
&\leq \frac{1}{n}\Big[I(M_k^{(F)}; Y_k^n | M_{k-1}) \\
&\qquad + I(M_k^{(S)}; Y_1^n, \ldots, Y_K^n | M_1, \ldots, M_{k-1}, M_k^{(F)}, M_{k+1}, \ldots, M_K) \\
&\qquad + I(M_{k+1}^{(F)}; Y_{k+1}^n | M_{k-1}, M_{k+1}^{(S)})\Big] + \frac{\epsilon_n}{n} \\
&\stackrel{(a)}{=} \frac{1}{n}\Big[I(M_k^{(F)}, M_k^{(S)}; Y_k^n | M_{k-1})
\end{aligned}$$

$$
\begin{aligned}
&+I(M_k^{(S)}; Y_{k+1}^n | Y_k^n, M_k^{(F)}, M_{k-1}, M_{k+1}) \\
&\qquad +I(M_{k+1}^{(F)}; Y_{k+1}^n | M_{k-1}, M_{k+1}^{(S)})\Big] + \frac{\epsilon_n}{n} \\
&\le \frac{1}{n}\Big[h(X_k^n + Z_k^n) - h(Z_k^n) + h(\alpha X_k^n + Z_{k+1}^n | X_k^n + Z_k^n) \\
&\qquad -h(Z_{k+1}^n) + h(Y_{k+1}^n | M_{k+1}^{(S)}) - h(\alpha X_k^n + Z_{k+1}^n)\Big] \\
&\quad + \frac{\epsilon_n}{n} \\
&\overset{(b)}{\le} \frac{1}{2}\log(1 + (1+|\alpha|^2)P) + \frac{1}{2}\log(1+\alpha^2) \\
&\quad + \max\{-\log|\alpha|, 0\} + \frac{\epsilon_n}{n}. \qquad (24)
\end{aligned}
$$

Here, (a) follows by the chain rule and because given source messages M_{k-1} and M_{k+1}, the triple (M_k, Y_k^n, Y_{k+1}^n) is independent of the rest of the outputs and the source messages, and because M_{k+1} is independent of the tuple (M_{k-1}, M_k, Y_k^n); (b) is obtained by rearranging terms, by the entropy-maximizing property of the Gaussian distribution, and the following two bounds:

$$
\begin{aligned}
h(\alpha X_k^n + Z_{k+1}^n | X_k^n + Z_k^n) &\le h(Z_{k+1}^n - \alpha Z_k^n) \\
&= \frac{1}{2}\log((2\pi e)(1+\alpha^2)) \qquad (25)
\end{aligned}
$$

and

$$
h(X_k^n + Z_k^n) - h(\alpha X_k^n + Z_{k+1}^n) \le \max\{0, -\log|\alpha|\}. \qquad (26)
$$

Following similar steps, one can prove that for $k \in \{1, K\}$:

$$
R_k^{(F)} + R_k^{(S)} \le \frac{1}{2}\log(1+P) + \frac{\epsilon_n}{n}. \qquad (27)
$$

We sum up the bound in (24) for all values of $k \in \{1, \ldots, K-1\}$, and combine it with (27). Taking $n \to \infty$, establishes the desired converse bound.

VI. Proof of Theorem 3

The converse follows directly from Theorem 2 and [8, Theorem 2]. Achievability is proved in the following.

When transmitting only "fast" messages or only "slow" messages, the setup in this paper coincides with the setup in [8] with 0 transmitter conferencing rounds and either 0 or $\mathsf{D}_{\max}$ receiver conferencing rounds. Thus, by [8], the following two multiplexing gain pairs are achievable:

$$
\Big(\mathsf{S}^{(F)} = \frac{1}{2},\ \mathsf{S}^{(S)} = 0\Big), \qquad (28)
$$

$$
\Big(\mathsf{S}^{(F)} = 0,\ \mathsf{S}^{(S)} = \min\Big\{\frac{1}{2} + \mu,\ \frac{2\mathsf{D}_{\max}+1}{2\mathsf{D}_{\max}+2}\Big\}\Big). \qquad (29)
$$

Recall that in the case of only receiver conferencing, the coding scheme in [8] periodically silences every $2\mathsf{D}_{\max}+2$nd transmitter. This splits the network into smaller subnets of $2\mathsf{D}_{\max}+1$ active transmitters and $2\mathsf{D}_{\max}+2$ receivers, where each active transmitter can send a message at prelog 1. A close inspection of the coding scheme in [8] reveals that the decoding of the source messages sent at the left-most transmitter of each subnetwork does not rely on the conferencing messages. We can thus easily adopt the coding scheme in [8] to our setup with "fast" and "slow" messages by letting the left-most transmitter of any subnetwork send a "fast" message and all other active transmitters send "slow" messages. With conferencing prelog $\mu_{\max} = \frac{\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2}$, this scheme achieves the pair

$$
\Big(\mathsf{S}^{(F)} = \frac{1}{2\mathsf{D}_{\max}+2},\ \mathsf{S}^{(S)} = \frac{2\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2}\Big). \qquad (30)
$$

For a given conferencing prelog $\mu \le \mu_{\max}$, we timeshare this scheme with the scheme achieving (28). As a result, for all $\mu \le \mu_{\max}$, the following pair of multiplexing gains is achievable:

$$
\mathsf{S}^{(F)} := \beta \cdot \frac{1}{2\mathsf{D}_{\max}+2} + (1-\beta)\cdot\frac{1}{2} = \frac{1}{2} - \mu \qquad (31a)
$$

$$
\mathsf{S}^{(S)} := \beta \cdot \frac{2\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2} + (1-\beta)\cdot 0 = 2\mu. \qquad (31b)
$$

Timesharing finally the schemes achieving the pairs in (28), (29) and (30) establishes the direct part of the theorem.

VII. Outlook

Extending these results to a setup with both transmitter and receiver conferencing seems interesting. In particular, our preliminary results indicate that the gains with both types of conferencing are more pronounced than with only transmitter or only receiver conferencing. Particular interest will also be on duality-aspects of transmitter and receiver conferencing with respect to the multiplexing gain.

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