

PVTree: simulating L-system fractals in 3D for solar energy generation

F. Bull^b, R. Cain^{a*}, E. Dimitrokali^a, G. Jones^b, J. Mackrill^c,
B. Morgan^b, Y.A. Ramachers^{b†} and K. Uzzell^b

^aWMG, University of Warwick, Coventry CV4 7AL, UK

^bDept. of Physics, University of Warwick, Coventry CV4 7AL, UK

^cImperial College London, London SW7 2AZ, UK

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Abstract

A first quantitative study of employing tree geometry structures from Lindenmayer systems for solar power harvesting is given. This computational study uses the Geant4 Monte-Carlo simulation framework for its flexibility in handling arbitrarily complex geometries combined with realistic physics simulation capabilities, in this case optical photon transport. An attempt is made to use realistic solar irradiance calculations for the simulation and study nature-inspired tree geometries as alternatives to flat panel photovoltaic structures. This concept of a photovoltaic (PV) tree, PVTree, targets a step change in public perception of photovoltaic power production technology.

1 Introduction

In a scenario with rapidly decreasing costs for photovoltaic (PV) energy harvesting technology, new research questions on the deployment and implementation of the technology in real world applications open up. One particular concept we base our research on has been pioneered in [1], i.e. considering three-dimensional PV structures as opposed to existing flat-panel installations. The new conceptual element we put forward is to consider design inspired by nature, biomimicry, in the generic natural form of

*Present address: Loughborough Design School, Loughborough University, LE11 3TU, UK

†Corresponding author; email: y.a.ramachers@warwick.ac.uk

tree structures, dubbed PVTree [2], rather than freely computer generated, artificial structures as in [1, 3] or solar tree type structures as in [4].

1.1 Assumptions

The assumption of cheap PV technology, i.e. physical realizations of elements receiving and transforming light into a different form of energy, typically electrical energy, is a necessary prerequisite when considering 3DPV [1]. The significantly increased energy per footprint area, a measure of energy density, for 3DPV is countered by a significantly increased number of PV elements, i.e. these PV elements can not be the dominant cost factor. This generic research result in [1] is even more important when considering structures such as trees with many individual leaves, adding up to large total areas. Our research hence rests on the projection of ever decreasing costs for PV elements, without relying on any specific technology in particular, be it silicon based, organic or otherwise.

The benefits of considering 3DPV and PVTrees in particular in our view outweigh the risk of assuming a hypothetical future cheap PV device supply. Moving away from flat panel solar power results in (a) more homogeneous power production over time, see below and [3], beneficial for power distribution systems, (b) increased utilization of reflected and diffuse light with static installations and most significantly, (c) a strongly reduced use of land for energy production due to the greatly reduced footprint of an installation. In [3] it is claimed that 3DPV delivers maximal energy per footprint area as opposed to flat panels delivering maximal energy per active material area. Joining these general benefits of 3DPV with increased public acceptance of solar power installations, for instance in the shape of PVTrees, could result in a major impact on solar power uptake.

1.2 Why trees?

Given the assumptions discussed above, how could a future deployment of solar power harvesting technology look like? Is there a realistic scenario more successful in increasing usage of solar PV than the existing flat panel structures? Our answer to these questions consists of taking inspiration from the environment and consider natural realizations of 3D photovoltaic structures such as trees. We foresee a potential impact on the discussion of aesthetics of current solar PV installations, particularly in domestic settings where retro-fitting flat panels is often the only option. PVTrees would offer an alternative form factor, moving towards vertical, space-saving solutions rather than horizontal space-hungry solutions.

Trees have to balance multiple requirements such as nutrients transport and water management in addition to photosynthesis. Our PVTree concept

would only have to maximise light absorption over the course of a year hence might well evolve into a structure different to natural trees. Nonetheless, it is expected that natural tree structures serve well as a first approximation. This expectation is partly based on aesthetics and public acceptance [2] and partly on physics. A tree structure with leaves as light receivers oriented in most directions offers passive sun-tracking for parts of the canopy as a function of time as well as the chance of constantly benefitting from indirect light over the course of a year.

Note that independence of PV technology implementations permits us to focus entirely on receiving light as the main first stage in all PV technologies. The efficiency of producing the desired end product, gas or electrical current or any other outcome, is merely a multiplying global factor for the computational study. Specific PV technology solutions enter calculations in the form of materials and their optical properties.

Required calculations on this project can be structured as three separate challenges, two of which are discussed in this article while the third challenge is work in progress:

1. Computational geometry to represent tree-like structures on the computer. These trees should contain active and passive elements with respect to light interactions, i.e. PV sensitive, active, surfaces representing PV devices as leaves and structural, passive, parts such as branches and the trunk. Additionally, the geometry description should permit the freedom to define a realistic environment for the tree such as obstacles, various ground conditions or other trees.
2. Light production and propagation has to be implemented which includes creating realistic sun spectra for arbitrary locations and environments and subsequent optical calculations or full simulations of photon transport. Realistic implementations of materials at this stage would also aid the reliability of results.
3. Individual results would be optimised using the computer in two complementary ways: Given a tree geometry and its parameters, an optimised set of parameters would be sought using a particle swarm algorithm [5] due to the relatively high dimensionality of the parameter space for each individual tree geometry. Additionally, the computational tree description method permits the use of genetic algorithms to eventually vary entire tree structures, creating new parameter spaces to explore.

2 L-systems for Monte-Carlo geometry

Arbitrary tree-like structures are described here in the form of Lindenmayer systems, L-systems [6, 7]. We use the Geant4 particle transport simulation

framework [8] to model photon transport in arbitrary geometries and materials. A Lindenmayer system [6] represents a simple and concise method to create fractal structures. It is a re-writing system which starts with a simple combination of letters, called an axiom, to replace each letter according to fixed rules, productions, creating longer sequences of letters. The set of letters and productions defines a deterministic context-free L-system.

The connection to geometry and plant-structures in particular originates from translating each element of the set of letters into graphical commands, each of which may be a function of parameter values. A graphical command to move forward for instance could take the value of a length. A versatile set of letters, commands, strictly follows the discussion in the original text [7] and contains move, rotate and branching commands acting on three independent space vectors in order to create 3D plant-like structures as fractals. The iterative re-writing process creates growing command sequences and therefore more complex structures at each iteration. The uncontrolled growth of (virtual) structures implies that an automated process to remove geometry overlaps is also required before a structure can be considered sufficiently realistic to be examined. Running currently without genetic algorithm modifications of L-systems, predefined seeds allow exploration of specific tree geometries and their parameters. Already implemented structures are classified as monopodial, sympodial, ternary and helical, see Fig. 1. A variable flat panel geometry for validation is also an available choice of structure. It is constructed such that it has full freedom to rotate and tilt in any direction but the panel size is fixed to a 1 m² square.

Each tree structure can be populated with variable leaf structures, again with their own parameter lists, regulating form and more importantly, size. Individual leaves are defined as sensitive on both sides, resembling organic leaves as opposed to common one-sided photovoltaic elements. This possibly unusual choice, just like the entire PVTree concept, relies on the assumed low cost PV technology with the added assumption of low weight for an individual PV element, i.e. a leaf. Thin-film photovoltaics of any kind would be the main candidate technology in this case, assuming that it can be combined with a light-weight packaging technology as opposed to relatively heavy and rigid glass packaging.

The next step towards genetic algorithm optimisation promises many more variations as discussed for plant structures in [9]. This particular application of genetic algorithms on L-systems has attracted interest in a variety of applications in plant science [10] and computer graphics [11].

In summary, the main elements and optical properties of the structures presented in this article are as follows:

- World structure: Half-sphere, optically transparent boundary except for the circular floor defined as 50% diffuse (Lambertian) reflective, non-dispersive.

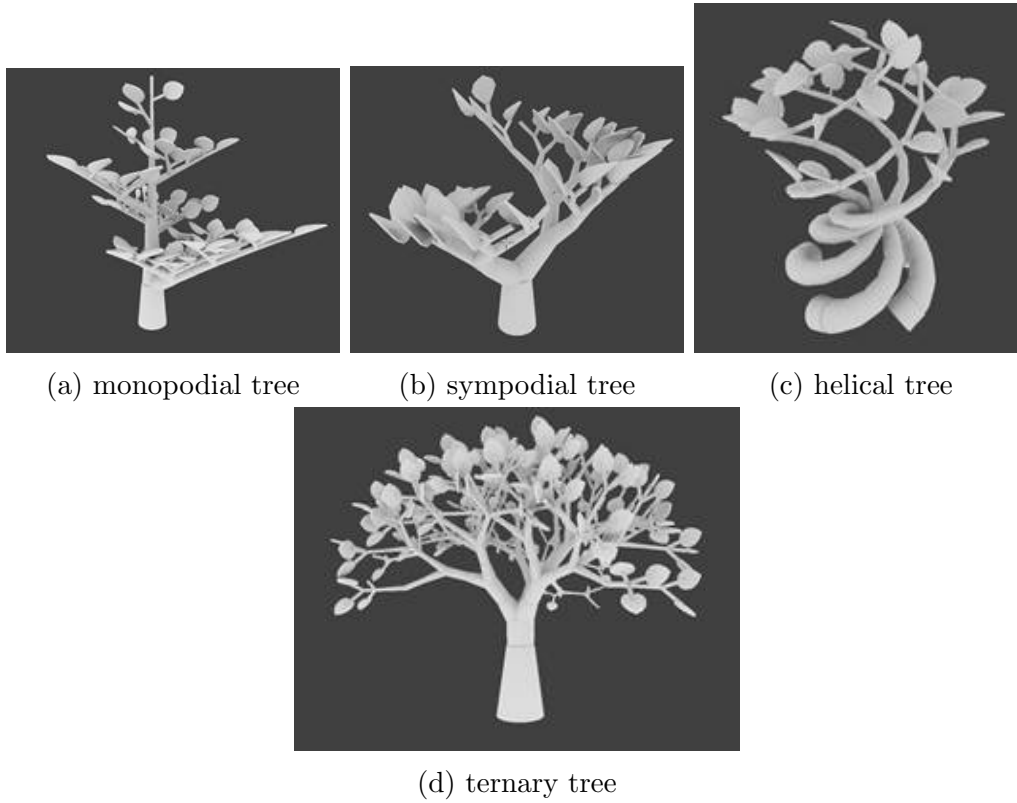


Figure 1: L-system plant structures used for this project: (a) the monopodial tree structure, (b) the sympodial, (c) the helical and (d) the ternary tree. The display shows each tree type with cordate leaf structures. Each of these L-systems correspond to a specific axiom and replacement rule, see text for discussion. Note that the pictures show 3D printed examples, used for exhibitions and outreach and not computer generated graphics.

- L-system: Centred position in world, stem and branches solid, 100% specular reflective, non-dispersive.
- Leaves: Three-layer structures located at each end of any branch unless overlapping with any other volume. The outer two layers simulate packaging material defined as 95% transparent, non-dispersive, refractive index of typical plastics at 1.49. The inner layer is the PV element with 100% photon absorption efficiency in the volume, enabling to scale to realistic efficiencies depending on technology later. Other optical properties resemble non-dispersive silicon.

These material properties represent a compromise. They are considered sufficiently detailed to enable robust results from the simulations without being

too realistic, favouring one technology over another. For a specific application setting and technology choice it would be straightforward to specify environment structures and material properties in much greater detail, in particular dispersive properties have been neglected for this study.

3 Solar irradiance simulation model

Once a geometry has been fixed, all optical calculations are conducted as full photon transport simulations in Geant4. This system permits detailed photon transport physics in arbitrary geometries and materials with a fully configurable source [8]. While ray tracing software would be the faster solution to photon transport, see [4], the freedom to specify complete geometries in code including static environments, materials and their optical properties as well as the Sun left the Geant4 framework as the ideal solution to our requirements.

So far, most of the effort was spent on modelling solar irradiation as realistically as possible for a given location and climate data on Earth over the course of several years. Direct normal irradiance as well as diffuse and global irradiance solar spectra have been obtained using the SMARTS code [12]. The angular distribution for diffuse light across the sky-dome is modelled according to the Hosek, Wilkie model [13]. The direct normal irradiance instead is modelled as a parallel bundle of light rays, large enough to illuminate the entire relevant structure from the position of the Sun as function of time.

Simulating the huge number of individual photons contained in sunlight is not realistic for a photon transport code. The Monte-Carlo principle, however, permits the use of weighted event scoring as a variance reduction technique. This has been implemented as follows: the SMARTS code delivers for a given location and time the total values for irradiance contributions as well as the spectral distributions. For direct normal irradiance the integral in Watt per source area is converted to total power and distributed in equal fractional weights in units of Watt amongst the number of photons to be simulated. Each photon receives an individual wavelength, randomly sampled from the SMARTS spectrum, in order to enable correct transport optics such as reflection and refraction. However, the scoring of a photon registers the weighting factor, not the energy. The assumption underpinning this variance reduction scheme is that each simulated photon represents an entire photon bundle according to the amount of power assigned to it as a weighting factor.

Validation of the Monte-Carlo simulation method and tools entails firstly detailed checks of each part of the geometry separately and secondly convergence tests of the entire transport simulation for a given geometry, see

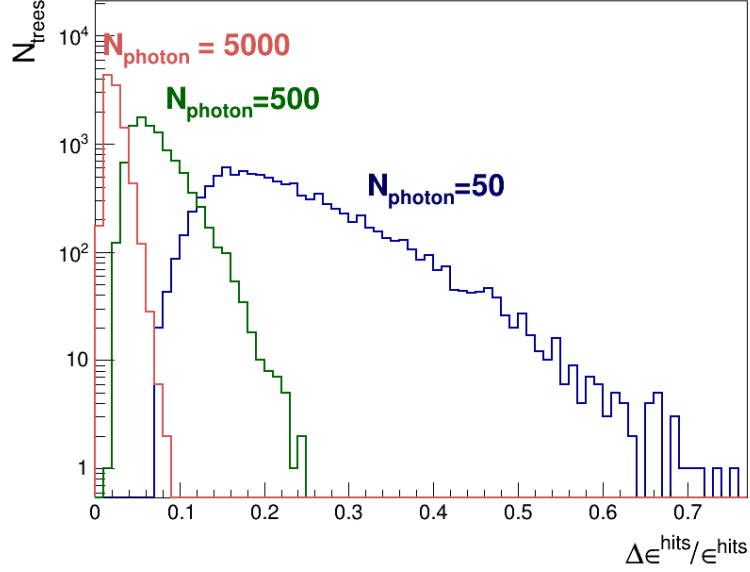


Figure 2: Shown here is the relative hit efficiency uncertainty distribution for three different total photon number simulations. The simulation models a full day in 10 time segments for a large number of flat panel geometries and simulation repetitions. As simulated photon numbers increase systematic uncertainties of the number of photon hits, the simulation efficiency in this case, decrease as expected, i.e. the simulation converges.

Fig. 2. Convergence tests also aid drawing up requirements on the simulation to achieve a given systematic uncertainty bound.

The balance between computational efficiency and realistic modelling of the Sun, see above, resulted in the compromise of implementing direct normal irradiance in the form of a finite radius disc centred at the position of the Sun and emitting parallel rays of photons. The size of the disc is defined as larger than the projected radius of the maximum extent of the PVTree. This compromise illuminates the entire tree geometry without illuminating the entire scene, the simulated world, which would be more realistic but inefficient.

However, diffuse irradiance is modelled as originating from the half-sphere with the angular distribution according to the Hosek, Wilkie model [13], see Fig. 3. The compromise for this component of sunlight is that all diffuse light-rays point at random locations on the floor, i.e. randomly intercepting PVTree leaves or not. A fully-featured simulation would describe the diffuse irradiation for each sensitive element, each leaf, individually with

its own adapted angular distribution. This refinement is considered a necessary step once weather conditions resulting in dominant diffuse irradiance are implemented in the code. So far, the diffuse irradiance component from SMARTS is always calculated as sub-dominant, i.e. all results below are obtained under blue-sky conditions with some allowance for haze.

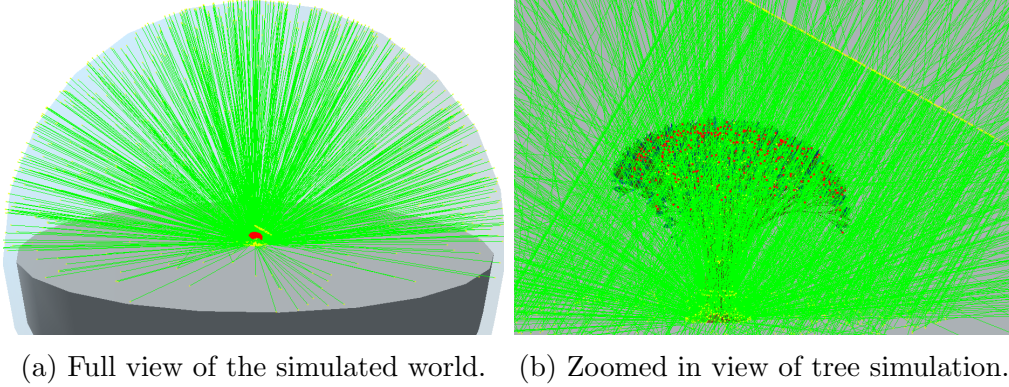


Figure 3: Shown are two example views of the full simulated world containing a single tree. The full view of the simulated scene displays the distribution of simulated diffuse light (green lines) starting on the sky-dome. The zoomed in view displays the tree and registered hits on leaves (red dots) as well as the simulated direct normal irradiance source defined as disc source at the location of the Sun, illuminating the entire tree.

The challenge to incorporate diffuse irradiance is then to describe atmospheric conditions built into phenomenological models such as [13]. One well documented method can be found in the Perez model [14] where each parameter is connected to atmospheric physics observables. This is not the case for [13] but its mathematical modelling appears to improve on the Perez model which is the reason for adopting [13] in this work. However, in [13] some effort has been made to connect their parameters to the Perez model parameters. For instance, it is possible to link their turbidity to the Perez brightness parameter which has the advantage that it can be calculated directly from irradiance (diffuse to global) values from SMARTS. Ground-reflective albedo does not exist in [14] or in SMARTS as a concept hence was taken from climate data at a given location for this study.

4 Simulation processes

The simulation provides numerical results on the amount of light received for a fixed tree structure in a fixed environment. Time dependence of such a given scene is implemented by changing the light source, the Sun, in

discrete time steps during the day and subsequently interpolating between these steps. This process can result in a daily simulation or repeated over the course of an entire year yield a full annual simulation. The solar position calculation uses the Solpos code [15]. In addition to time its required input is the simulated location, here the University of Warwick campus in Coventry, UK.

Once time and position of the Sun are known, selected climate data is retrieved as input to SMARTS. Currently, the code has access to European climate data from 2005 to 2015 in form of the World Meteorological Organisation standard gridded binary (GRIB) file format. The data has been retrieved from the European Centre for Medium-Range Weather Forecasts (ECMWF) website¹[16]. Climate data of interest so far includes: temperature, atmospheric pressure and column densities of water and ozone. Work in progress on the simulation will implement a cloud model in order to improve on the diffuse light contribution to results and cloud cover is a retrievable value of the climate data hence worth noting here.

Once the SMARTS code delivered the three required irradiance spectra, direct normal, diffuse and global, the Sun as photon generator is fixed and the simulation of photon transport can begin. The SMARTS spectra cover the whole solar wavelength range from 280 nm to 4000 nm. Their respective integrals determine the probabilities for the event generator to start a photon either as diffuse from the sky-dome, see above, or as direct normal. The simulation variance reduction method determines the weighting factor assigned to each simulated photon, see discussion above, but the photon energy is randomly sampled from the respective wavelength spectrum. This is significant for two reasons: (a) keeping all material properties dispersion free at the moment means the photon wavelength is ignored for optical processes but realistic, dispersive materials can easily be implemented subsequently without changes to the code and (b) allowing the full spectral range from the Sun simplifies standard efficiency calculations for realistic PV technologies. Typically measured efficiency values result from full Sun simulator devices emitting solar-like spectra of wavelengths including photons with insufficient energy to cause a photovoltaic effect. It is hence not considered a waste of computational resources simulating wavelengths which would for most if not all technologies have no effect. A 100% efficient light receiving material as implemented here in the layered leaf structure then permits to simply scale results on received solar power to realistic values by a global efficiency value specific to a given technology.

With the event generator set up, each individual simulation run can be started with a fixed number of photons, for production runs typically 10^5 photons. The number of simulated photons secures low systematic uncertainties, see Fig. 2, and is affordable in time. The main bottleneck for the

¹<http://www.ecmwf.int/research>

simulation is the construction of higher iteration tree structures involving leaf numbers beyond roughly a thousand.

A full simulation run for a single day simulation comprises therefore a variable number of discrete time steps, chosen by the user, subdividing the length of day time between sunrise and sunset as calculated for the chosen day in the year. Each such step calculates SMARTS spectra and generates the full set of photons, transports them until all either escape the world setting or are absorbed. Registered photons, hits, are collected to later sum up their total energy deposited during the day. In case of a full annual simulation all the previous steps are repeated for a chosen number of discrete times during a year. That concludes a full run for a given tree geometry. The simulation would then proceed by producing another tree and repeat.

Computational tree production has been automated thanks to the use of L-systems. Each individual tree can be mapped to a set of simple parameter values as input to the L-system production rules. Table 1 lists the number of parameter values for each tree type considered. Each parameter value has a default setting and upper and lower bounds. These span a high-dimensional parameter space. Each automated tree construction randomly samples this parameter space. Ongoing work to employ a particle swarm algorithm [5] for sampling should improve on the random collection.

Table 1: Shown are the number of variable parameters required to specify each of the tree structures considered in this study.

PVTree type	Flat	Monopodial	Sympodial	Helical	Ternary
Number of parameters	4	10	9	15	10

Every simulation run records the entire set of parameter values, tree characteristics and simulation results to persistent storage. Useful tree characteristics have been identified as number of leaves, total sensitive area, tree 'crown' area or footprint, i.e. the projected size of each tree canopy on the floor, here the x-y-plane, and the tree height.

5 Results

5.1 Figure of merit definition

The results of this study depend on the definition of what distinguishes a good PVTree from a bad PVTree. This definition should enable numerical optimization and hence ideally be summarised as a number. Two optimization target classes can be identified in this context:

- The simplest attempt would be to consider the energy density equivalent ratio of energy received divided by the total photovoltaic element area. The target would be to maximise this ratio. The more light harvesting area is required to produce a fixed amount of energy the worse the ratio and hence the device. The flat panel structure is the ideal geometry in this case and the simulations using the validation flat panel geometry confirm this conclusion. This optimization target however assumes implicitly the dominance of light harvesting elements to the resource cost. The fewer elements as cost are needed for a fixed gain, the energy output, the better. If the photovoltaic elements are considered negligible as cost, the ratio becomes meaningless.
- If the photovoltaic elements are considered negligible as cost but land-use and possibly public acceptance become the dominant resource cost then different optimization targets gain in importance. This is the explicit assumption in this study. A convenient measure of the impact of trees on their immediate environment in ecophysiology is the leaf area index (LAI) [17]. This number can simply be defined as ratio between the leaf area, here the total light harvesting area, divided by the projected ground area of the extent of the entire tree, the size of its 'crown'. This measure hence introduces land-use by a PVTree explicitly.

One wouldn't construct a second PVTree closer to the first than the projected ground extent of each to avoid overlapping leaves in crowns, i.e. unnecessary shading. In order to include again the total energy harvested, the investment dividend, the optimization parameter is defined for the following results as product of LAI and total energy. The target again would be to maximise this number. A large number for PVTrees means that many photovoltaic elements harvest as much light as possible at minimal cost of land. Either of the two factors can balance the other. Either the LAI isn't outstanding but the structure yields maximum energy for instance due to optimal orientation or the energy value isn't outstanding but the land-use is minimal for the amount of energy harvested. The optimal outcome for each tree would be to get the best of both factors.

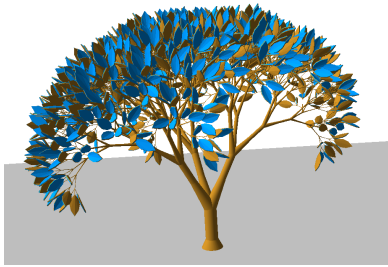
For the following results the quality factor or figure of merit adopts the definition given in the second point above, i.e. LAI times energy is to be maximised. Unsurprisingly, the most natural appearing PVTree structure, the ternary structure, is ranked first in all parameter scans while the flat panel comes out last. All other tree structures are placed in between these two extremes.

5.2 Flat panel validation

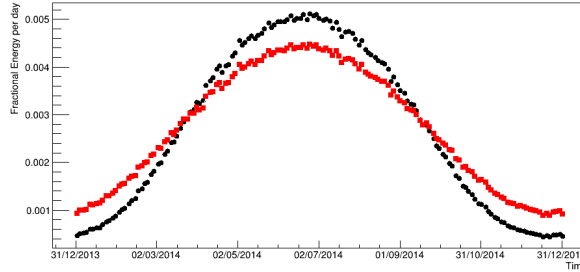
Results for the validation structure, the flat panel, are reported in table Tab. 2. The four parameter describing this structure allow it to rotate on two axes and reach a variable height. However, like for all the tree structures, once it is fixed, it remains fixed throughout the simulation. As a result, the SMARTS calculated annual integral energy per unit area (solar irradiation) of 3256.5 kWh/m^2 (at the chosen location, see above) can not be fully recovered and losses of about 30% [18] are to be expected. For purely geometrical reasons, the variability of the Sun's orbit over the year means that any fixed flat panel will not be oriented optimally all the time like a panel which could track the sun at all times during the day. This expectation is validated for the flat panel in the simulation which receives 72% of the available energy at our default location.

5.3 Single leaf branches

The first set of PVTree results deals with simpler tree structures featuring a single leaf mounted at the end of each final branch of the L-system, see Figs. 1, 5. Historically this was the easiest way of constructing PVTrees in code. This construction concept was expected to show important general features of PVTrees before refinements, see below, were considered. The best PVTree after full annual simulations of several thousand randomly chosen trees is shown in Fig. 4.



(a) Best ternary tree.



(b) Comparison flat panel to ternary.

Figure 4: (a) Picture of the currently best ternary PVTree structure from annual simulations. Note that the colours here are included purely for reasons of display. The grey background shows the simulation floor and horizon for the chosen viewpoint. (b) Comparison of the annual simulation performance of a flat panel (black) and the best ternary PVTree (red). Shown is the fractional energy deposition per day over the course of a year, showing the smaller contrast of the ternary tree between winter and summer. Climate data for the year 2014 is used.

One might expect that random sampling of even high-dimensional parameter spaces for relatively ordered structures such as these does not hold too many surprises even when searching with a full optimization algorithm like particle swarm. An optimal ternary tree better than the above will likely look similar as do those other trees randomly sampled and similarly 'good' to the one shown in Fig. 4. All the best trees and a collection of their characteristics are listed in Tab. 2 while Fig. 4(a) shows the best ternary tree and Fig. 5 shows the best alternative tree structures.

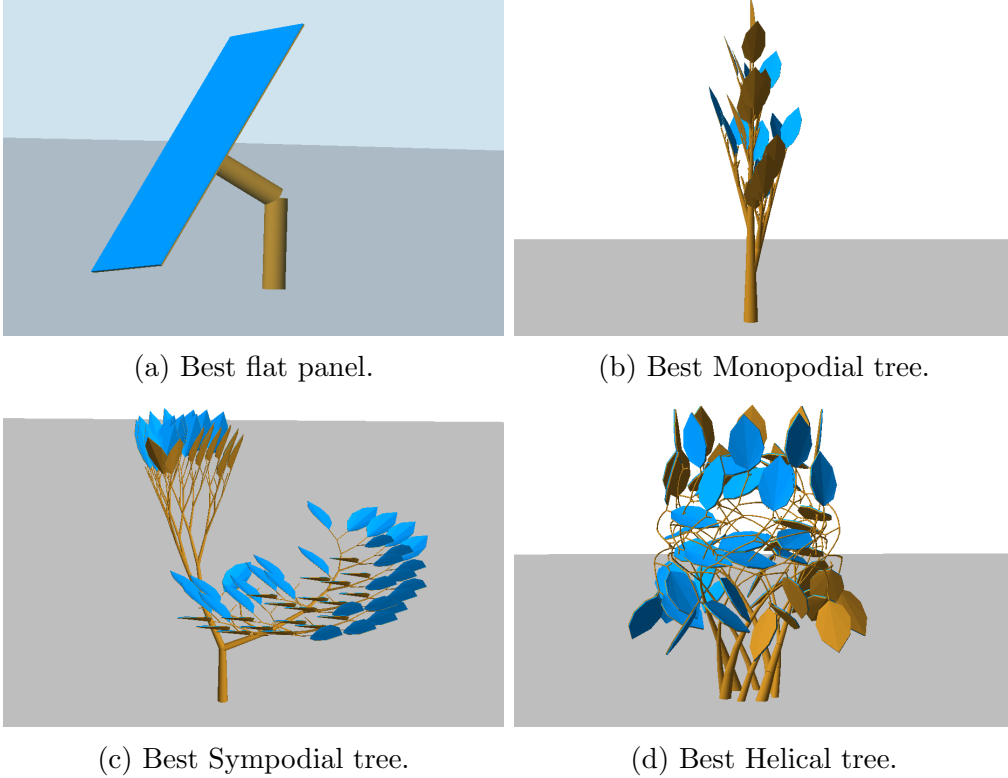


Figure 5: Each panel shows the best tree from random sampling scans of four PVTree structures from annual simulations used in the study in addition to the ternary PVTree structure. For discussion, see text.

However, inspection of Tab. 2 and Fig. 5 brings up an important point to consider which was ignored so far. Public acceptance is key to any new technology with a strongly visible impact. It is unlikely that any purely computer-designed structure will find broad public appeal and application in public spaces [2]. Feedback would be paramount in order to steer the design towards public acceptance if PVTrees stand any chance of ever being implemented.

Table 2: Shown are characteristics of each of the best tree structures for single leaf per branch PVTrees from annual simulations after randomly sampling their respective parameter spaces. Note that this study does not account for efficiencies of a specific photovoltaic technology, merely absorbed light at 100% efficiency is reported in the energy figure. The table is ordered according to tree-type ranking with the figure of merit, see text, from left to right with the best tree overall on the right.

PVTree type	Flat	Monopodial	Sympodial	Helical	Ternary
number of leaves	1	62	73	97	1035
total sensitive area [m ²]	1.0	29.24	30.47	34.7	137.4
footprint [m ²]	0.17	17.4	11.6	7.6	26.0
height [m]	0.85	9.2	9.9	6.3	7.5
total energy [kWh]	2,346.5	41,345.5	30,884.4	41,920.7	131,496.0
Figure of merit	13,803	69,479	81,124	191,298	694,906

5.4 Multi leaf branches

Allowing multiple leaves on branches of a PVTree increases the complexity of the structure by several new degrees of freedom. Which branches should have leaves and in which geometrical relation should they be attached to a branch.

As a first attempt, in addition to the previously mounted leaf at the end of each branch, additional leaves are placed along the length of each branch from each iteration with exception of the trunk. Two leaves are placed on opposite sides of a given branch, repeatedly along the branch if its length permits it. No rotation of leaf pairs or staggering of leaves along a branch has been implemented which could be another, natural, way of placing leaves along a branch.

Results can be inspected in Figs. 6 and Tab. 3. The ternary tree again dominates as the highest figure of merit structure but interesting characteristics of other tree structures are also noteworthy. For instance the helical tree is consistently scoring highly due to the low footprint achievable. Overall, moving these artificial structures slightly closer to natural trees with multiple leaves on branches improves the performance for all structures when counting performance as given by our figure of merit.

PVTree forest

The final case study using the PVTree concept considers a collection of trees, a PVTree forest. The relevance of this case is first of all to flag up that such a forest could be considered as a solar power station which should greatly reduce the required footprint on the landscape for comparable output to the

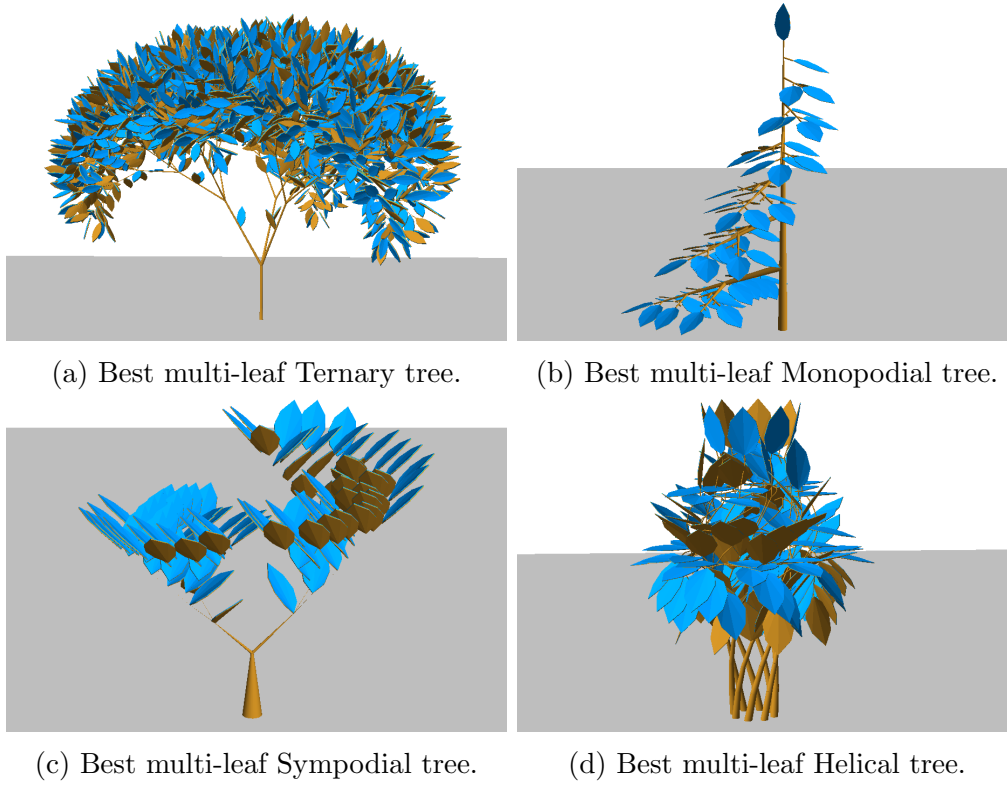


Figure 6: Each panel shows the best multi leaf tree from random sampling scans of four PVTree structures from annual simulations used in the study.

proverbial farmer’s field covered in flat solar panels.

Secondly, such a tree collection permits a study on the effect of indirect, scattered light on PVTree light collection. So far, all results discussed above refer to single trees standing in isolation on a plane.

Our results only consider the simplest geometric arrangement of PVTrees in a grid structure, closely spaced at minimum distance, avoiding any branch or leaf overlaps. Two cases were simulated, a 5×5 grid with identical PVTrees as a small installation and a 30×30 grid to maximize any indirect light harvesting effects compared to the smaller grid.

Table 4 displays results on figures of merit (LAI times total energy) for all PVTree types considered in the above listed forest examples. Again, best configurations on display result from random scans in PVTree parameter spaces hence non-identical tree geometries result compared to previous simulations of single trees. The purpose of this case study is to examine the potential benefit of indirect light harvesting in a forest structure. The larger forest configuration features 36 times the number of PVTrees of the

Table 3: Same as Tab. 2 but for PVTrees featuring multiple leaves per branch.

PVTree type	Monopodial	Sympodial	Helical	Ternary
number of leaves	77	104	188	2553
total sensitive area [m ²]	33.9	41.7	85.0	659.0
footprint [m ²]	11.7	9.2	2.52	75.5
height [m]	9.1	5.6	6.5	12.8
total energy [kWh]	46,405	35,152	50,044	456,904
Figure of merit	134,456	159,330	1,687,992	3,988,076

Table 4: Figures of merit for two sizes of PVTree forests made from identical copies of multi-leaf trees of given type; for discussion, see text.

Forest 5 × 5 grid	
PVTree type	Figure of merit
Monopodial	8.4e+07
Sympodial	7.6e+07
Helical	8.8e+08
Ternary	1.4e+09
Forest 30 × 30 grid	
PVTree type	Figure of merit
Monopodial	4.1e+10
Sympodial	4.3e+10
Helical	4.3e+11
Ternary	4.8e+11

smaller configuration. If indirect light would play no role then one could expect roughly that number as ratio of figures of merit from these forest configurations since each should favour similar best PVTree solutions.

However, inspection of Tab. 4 shows that forests of PVTrees outperform the simple scaling expectation, even when allowing for fluctuations since these are each made from non-identical PVTrees (for each given tree type). Ratios ranging from more than 300 to nearly 600 are observed, demonstrating the benefits of indirect light harvesting in this environment. The caveat on these ratio numbers is that any change of configurations will have an impact on these numbers hence they should not be considered as generic for PVTree forests. Already changing the reflectance of the floor should show a significant impact as would any change to the perfect mirror finish on trunks and branches. Nevertheless, arranging PVTrees in a forest collection will show added value compared to single PVTrees.

5.5 Example application

The ternary trees in Tab. 2 and Tab. 3 can serve as illustrative examples to reply to a regular, practical question. If the photovoltaic light-weight technology delivers a 10% efficiency, conservatively, how many trees would be needed in a back garden to supply a residential dwelling? The single leaf ternary tree footprint of 26 m^2 is non-negligible but the height of 7.5 m is generally not excessive in a residential setting and permits using some space below the tree canopy. Such a single PVTree would supply 13.15 MWh in a year which almost covers the entire energy demand of an average medium household in the UK [19] (gas: 12.5 MWh; electricity: 3.1MWh). Speculating about future low-cost photovoltaics might as well include future domestic gas production technology from sun-light.

The multi-leaf ternary tree, however, would be more suitable for a village green or a park with a footprint of 75.5 m^2 , producing in this example close to 45.7 MWh. The compact multi-leaf helical solution instead would fit in a residential setting in case the more artistic design aspect is acceptable with its 2.5 m^2 footprint, 6.5 m height and 5.0 MWh production in this example.

The ternary tree structure enjoys the highest approval ratings in all surveys undertaken so far with the helical tree a close second. However, such surveys depend on the specific physical provocation and might well change if a sample group is presented with different trees of the same type, see the differences between the types in Fig. 1, Fig. 5 and Fig. 6. Therefore the key feature for any future computational studies will be to translate feedback from perception and acceptance studies into parameter constraints for computer modelling of PVTrees. This interdisciplinary effort is work in progress.

6 Conclusion

A versatile computational tool to study novel photovoltaic tree structures, PVTrees, has been created and validated with first results presented in this document. The assumption of future low-cost photovoltaic elements removes design paradigms on the best deployment of light-harvesting technology, permitting the search for alternatives to flat panel PV such as PVTrees and their design.

Based on the low-cost assumption, other factors could become more dominant influences on what constitutes a successful photovoltaic installation. Here it is proposed that the cost of land for installations and public perception will become more influential. This proposal is implemented in a figure of merit for rating PVTree results as leaf area index times total energy received, the latter being the result of a complete simulation run for a given PVTree.

PVTrees could provide an answer to both future challenges, land cost and perception, by firstly representing a three-dimensional PV installation, i.e. minimising land-use like natural trees do, and secondly providing a versatile structure with a chance of public acceptance. They offer either a natural appearance, biomimicry, or a more sculptural, artistic and artificial appearance, either of which feature different approval ratings depending on audience and environmental setting [2].

No working PVTree has been manufactured so far. However, several challenges and features can already be listed. Solar energy storage is likely to play at least as prominent a role for any installation as light harvesting. The PVTree offers a natural storage volume in the trunk of the structure. Likewise, a modular manufacturing process is naturally favoured due to L-systems being fractal, i.e. repeating fundamental structures at different length scales. Leaves should be light-weight and modular for simple exchange when damaged, restored or upgraded. Technology for light-weight thin-film PV in light-weight plastic packaging is expected to become commercially viable in the near future. It is this element above all else which is assumed to become a negligible cost factor in order for PVTrees to present an attractive new form of photovoltaic design and installation.

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References

- [1] Myers, B., Bernardi, M. and Grossman, J.C., Three-dimensional photovoltaics, *Appl. Phys. Lett.* 96 (2010) 071902.
- [2] Dimitrakali, E. et al., Moving away from flat solar panels to PVTrees: exploring ideas and people’s perceptions, *Procedia Engineering* 118 (2015) 1208–1216
- [3] Bernardi, M. et al., Solar energy generation in three dimensions, *Energy and Environmental Science* 5 (2012) 6880-6884.
- [4] Verma, N.N. and Mazumder, S., Monte-Carlo simulation of sunlight transport in solar trees for effective sunlight capture, *Transact. ASME, J. of Solar Energy Eng.* 137(4) (2015) 021015-1.

- [5] Kiranyaz, S., Ince, T. and Gabbouj, M., Particle Swarm Optimization, in Multidimensional particle swarm optimization for machine learning and pattern recognition (Springer, Berlin, 2014) 45–82.
- [6] Lindenmayer, A., Mathematical models for cellular interaction in development, *J. Theor. Bio.* 18 (1968) 280.
- [7] Prusinkiewicz, P. and Lindenmayer, A. The algorithmic beauty of plants, Springer (1993).
- [8] Agostinelli, S. et al., *Nucl. Instr. Meth.* A506 (2003) 250.
- [9] Ochoa, G., On genetic algorithms and Lindenmayer systems, in Parallel Problem Solving from Nature - PPSN V, vol. 1498, Lecture Notes in Computer Science, Berlin, Springer, 1998.
- [10] Boudon, F. et al., L-Py: an L-system simulation framework for modelling plant architecture development based on a dynamic language, *Frontiers in Plant Science* 3 (2012) 76.
- [11] Hornby, G.S. and Pollack, J.B., Evolving L-systems to generate virtual creatures, *Comp. and Graphics* 25 (2001) 1041.
- [12] Gueymard, C.A., Parameterized Transmittance Model for Direct Beam and Circumsolar Spectral Irradiance, *Solar Energy* 71 (2001) 325–346.
- [13] Hosek, L. and Wilkie, A., An analytic model for full spectral sky-dome radiance, *ACM Trans. on Graphics (TOG)* 31(4) (2012) 1–9.
- [14] Perez, R., Seals, R. and Michalsky, J. All-weather model for sky luminance distribution - preliminary configuration and validation, *Solar Energy* 50 (1993) 235.
- [15] Rymes, M., Solpos 2.0 Documentation (2000) <http://rredc.nrel.gov/solar/codesandalgorithms/solpos/aboutsolpos.html> and references therein.
- [16] Dee, D.P. et al., The ERA-Interim reanalysis: configuration and performance of the data assimilation system. *Q.J.R. Meteorol. Soc.*, 137 (2011) 553–597
- [17] Breda, N.J.J., Ground-based measurements of leaf area index: a review of methods, instruments and current controversies, *J. Experim. Botany* 54 (2003) 2403–2417.
- [18] Lubitz, W.D., Effect of manual tilt adjustments on incident irradiance on fixed and tracking solar panels, *Applied Energy* 88 (2011) 1710–1719.
- [19] Office of Gas and Electricity Markets UK (OFGEM), Typical domestic consumption values, report 2015, <http://www.ofgem.gov.uk>