Higgs, dark sector and the vacuum: From Nambu-Goldstone bosons to massive particles via the hydrodynamics of a doped vacuum.

Marco Fedi *

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Abstract

Here the physical vacuum is treated as a superfluid, fundamental quantum scalar field, coinciding with dark energy and doped with particle dark matter, able to produce massive particles and interactions via a hydrodynamic reinterpretation of the Higgs mechanism. Here the Nambu-Goldstone bosons are circularly polarized phonons around the edge of the Brillouin zone of vacuum's quasi-lattice and they give mass to particles by triggering quantized vortices, whose dynamics reproduces any possible spin. Doped vortices also exert hydrodynamic forces which may correspond to fundamental interactions.

Keywords— quantum vacuum; dilatant vacuum; dark energy; dark matter; Higgs mechanism; spin; fundamental interactions.

1 Introduction

Dark energy [1], which is $\sim 69\%$ of the universe's mass-energy, can coincide with the physical vacuum itself, this is clear for instance in the cosmological constant, $\Lambda = \kappa\rho_{vac}$, in which κ is Einstein's constant and ρ_{vac} is the energy density of the vacuum, whose units correspond to pressure ($\mathrm{J/m^3} = \mathrm{Pa}$), hence justifying the repulsive action of dark energy. One can describe the virtual pairs forming and annihilating in quantum vacuum – considered as a fundamental, scalar, quantum field – as vortex-antivortex pairs of vacuum's quanta, via a mechanism analogous to the Higgs mechanism, where phonons in the superfluid vacuum are the Nambu-Goldstone bosons, which here trigger quantized vortices and the mass-acquisition process, due to the interaction with diffused particle dark matter [2], which acts as a dopant of the superfluid vacuum and that could be the reason for vacuum dilatancy, described and proven in [22]. Hence, is the Higgs field really something different or along with the dark sector and quantum vacuum we are using different names to refer to the same thing? Dilatant vacuum [22] could refer to the possible apparent viscosity of the Higgs field. And is Higgs' tachyonic condensate the special Bose-Einstein condensate (BEC) described for the vacuum? [3, 4, 5, 6, 7] If a Higgs-like mass-acquisition process can be described in a superfluid, doped vacuum, we are then allowed to suppose that the process is unique and that the Higgs field coincides with the dark sector and quantum vacuum themselves. The resulting picture is that of a quantum scalar field whose hydrodynamics drives the processes of spontaneous symmetry breaking and mass acquisition, and also produces particles' structures, interactions and spin. In this framework, dark matter, as a dopant of the vacuum, participates in the mass-acquisition process, making the vortices heavier, and, according to their chirality, it also allows them to exert attractive and repulsive forces, which might correspond to fundamen-

*Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR), Rome, Italy; LIVAC, livac-research.org
E-mail: marco.fedi@istruzione.it

tal interactions. Driven by the hydrodynamics of physical vacuum.

2 Spontaneous symmetry breaking as a hydrodynamic process

Let ϕ be a complex scalar field obeying the constraint $\phi^*\phi = v^2(constant)$ and including in its Lagrangian density a potential term, $\lambda(\phi^*\phi - v^2)^2$, with $\lambda \to \infty$. We observe a Goldstone sombrero potential with a circle of minima (vacuum states). By $\phi = ve^{i\theta}$ we redefine the field as a real scalar field θ with no constraint and this can correspond to a spin-0 boson, in this case a Nambu-Goldstone boson, $v\theta$. The $U(1)$ symmetry transformation corresponds to an energy shift of $\partial\theta = \varepsilon$ and does not preserve the vacuum state $|0\rangle$, which is then degenerate once the symmetry breaking occurs. The Lagrangian density reads

$$\begin{aligned}\mathscr{L} &= -\frac{1}{2}(\partial^\mu\phi^*)\partial_\mu\phi + m^2\phi^*\phi = \\ &= -\frac{1}{2}(ive^{-i\theta}\partial^\mu\theta)(ive^{i\theta}\partial_\mu\theta) + m^2v^2 = \\ &= -\frac{1}{2}v^2(\partial^\mu\theta)(\partial_\mu\theta) + m^2v^2.\end{aligned}$$

Since the superfluid field of the physical vacuum is taken into account, we are allowed to treat the emerging Nambu-Goldstone boson, $v\theta$, as a phonon and this fact is fundamental for the approach to the Higgs mechanism that is being presented. We know that fluids and superfluids form a quasi-lattice within Frenkel relaxation time, where phonon modes emerge [9]. Let us consider a current around the first Brillouin zone of vacuum's quasi-lattice. We are then considering a circular phonon in the superfluid vacuum as the event which coincides with the formation of the Goldstone boson (massless Higgs boson's mode) and which then starts the mass acquisition process. The symmetry-induced conserved $U(1)$ current is

$$J_\mu = -v^2\partial_\mu\theta. \tag{1}$$

In our case a topological (chiral) charge, Q, emerges from the current and shifts both the ground state and θ to a new degenerate state, let us say a different vacuum state $\langle\theta\rangle = -\varepsilon$. Then this current connects the original vacuum state with the Nambu-Goldstone boson state $\langle 0|J_0(0)|\theta\rangle \neq 0$. The current must be topologically represented, within this hypothesis, as a circular phonon. Topological states driven by chiral phonons have been observed [10] and other circularly polarized phonons are known, for instance, in Weyl semimetals [11].

The demonstration may start from the Bohr-Sommerfeld relation indicating the circulation in a quantum vortex

$$\oint_C \mathbf{p}\,dx = nh \tag{2}$$

where h is Planck constant and $\mathbf{p}$ is momentum. By considering a single turn, $n = 1$, we see that Planck constant (the quantum of action) expresses a complete turn in a quantized vortex. We know that the abbreviated action for the Lagrangian is $S_0 = \int \mathbf{p}\,d\mathbf{x}$. We can therefore consider Planck constant as the abbreviated (quantum) action for the Lagrangian, defining the path of the phonon. Both are time-independent and have the same units. Finally we use $\mathbf{p} = \hbar\mathbf{k}$ to refer to the phonon. The pseudo-momentum of the phonon accords with the fact the Goldstone boson must be massless. The quantum of action h represented by the turn of a quantized vortex respects the principle of least action in the superfluid vacuum. Also, in Eq. 2, n assumes both positive and negative values representing a vortex and an antivortex and this agrees with a chiral gauge theory. Another hint to the validity of the hydrodynamic approach comes from the definition of vacuum fluctuations, e.g. of the virtual electron-positron pairs populating the quantum vacuum in a continuous creation and annihilation process. It reads

$$\Delta E \Delta t = \frac{h}{2\pi} \tag{3}$$

confirming that vacuum's fluctuations are quantized vortices appearing as a more favorable energy state in the superfluid vacuum with respect to the symmetric "rest "state on the top of the sombrero (spontaneous symmetry breaking occurs). That, taking into account the just discussed meaning of Planck constant and also 2π in Eq. (3), refers to a complete turn. The vacuum is in each moment perturbed by hydrodynamic vortical fluctuations, $\Delta E \Delta t$. Recami and Salesi [12] link Planck constant to spin (and zitterbewegung), finding the relation $|\mathbf{s}| = \hbar/2$: this supports the interpretation of h as a fundamental, internal, quantized motion of a particle, as described here. Moreover in

Sect. 4 spin is described as the combined quantized circulation of vacuum's quanta in the toroidal and poloidal directions of a torus-shaped vortex-particle. Coming back to the reduced action, if we include the parametrization by time we arrive to the standard action. Referred to the Lagrangian density (1), it reads

$$S = \int \left(\int \mathscr{L} \, d^3x \right) dt = \\ = \int \int -\tfrac{1}{2} v^2 (\partial^\mu \theta)(\partial_\mu \theta) + m^2 v^2 \, d^3x \, dt \tag{4}$$

by integrating over all spacetime. Now we can define the radius of the circular phonon which is also the healing length of the arising 3D vortex tube

$$\xi \equiv \sqrt{\frac{V}{8\pi a N}} \tag{5}$$

where V is the volume in the vacuum in which the vortex arises, a is the scattering length and N a normalized number of vacuum's quanta in the volume. We see that, in the Goldstone argument, the (topological) charge operator Q is independent of time

$$\frac{d}{dt} Q = \frac{d}{dt} \int_x J^0(x) = 0 \tag{6}$$

implying zero-frequency vacuum states, exactly as the absence of time parametrization in the abbreviated action $S_0 = h$. Indeed, the circulation path of the phonon does not change in time and refers to a topological (chiral) charge. Here the vacuum is annihilated and we cannot have a mass gap. Thus, the circular phonon produced in the vacuum (the Nambu-Goldstone boson) via spontaneous symmetry breaking is massless and corresponds to a quantized hydrodynamic excitation in the vacuum. This circular phonon moving around the bottom of the Goldstone's sombrero only has kinetic energy. Once these phonons join the massless gauge bosons of the Yang-Mills theory, we know they produce the massive bosons $W^\pm$, Z, completing the Higgs mechanism; and this also holds for fermions but, as we know, via the Yukawa coupling. Let us for instance consider the interaction Lagrangian of leptons with the Higgs

$$\mathscr{L}_{int} = g_e \left(\bar{L} \phi e_R^- + \phi^\dagger \bar{e}_R^- L \right) \tag{7}$$

which is invariant in $SU(2)$ and where g_e is an arbitrary constant, meaning that masses are predicted by the Higgs mechanism but have to be measured. With

$$\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \tag{8}$$

$$\mathscr{L}_{int} = \frac{g_e v}{\sqrt{2}} \left(\bar{e}_L^- e_R^- + \bar{e}_R^- e_L^- \right) + \frac{g_e}{\sqrt{2}} \left(\bar{e}_L^- e_R^- + \bar{e}_R^- e_L^- \right) H, \tag{9}$$

where the mass of the electron is $\frac{g_e v}{\sqrt{2}} = m_e$. Here mass, as an observable, arises from the kinetic energy of vacuum's quanta (the vortex) triggered by the Nambu-Goldstone boson (the phonon) but the presence of a dopant (in this case particle dark matter) in the superfluid vacuum is fundamental (as discussed in Sect. 3 and seq.). The different masses of the Standard Model might depend on different Brillouin zones of vacuum's quasi-lattice involved in the generation of Goldstone bosons and vortices (following the canonical evolution: vortex tube $\to$ vortex torus) and on different Bogoliubov excitation modes. Ultimately, the Higgs boson itself results as an excitation of the Higgs field, probably a hydrodynamic excitation of the doped vacuum.

3 From Goldstone bosons to Bose polarons via vacuum-dark matter interaction

As a further stage in the mass-acquisition process, it is hypothesized that the interaction of the Goldstone boson (phonon) with dark matter's particles, acting as a dopant of the superfluid vacuum, originates a Bose polaron. It is believed that dark matter is concentrated in the galactic discs, where it is responsible for the flat profile of the rotation curve, and in cosmic filaments, but dark matter particles can be also diffused throughout the vacuum. If the transferred momentum-energy is resonant with a Bogoliubov sound mode in the BEC, the resulting interaction can be strong even for weak impurity-boson interaction. The phonon-dark matter polaron represents the subsequent stage toward the formation of a massive free particle. The Bogoliubov spectrum of the superfluid vacuum can be expressed in the form

$$\varepsilon_k = \sqrt{E_k \left(E_k + 2 n_0 g_c \right)}, \tag{10}$$

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where $n_0 g_c = 4\pi\hbar^2 a/m_Q$ is the chemical potential (m_Q is the mass of a vacuum's quantum), a is the scattering length and $E_k = \hbar^2 k^2/2m$. We have a phonon regime for $k\xi \ll 1$, $\varepsilon_k = \hbar ck$, with c speed of sound in the superfluid and critical speed for superfluidity, and a free-particle regime for $k\xi \gg 1$, in which $\varepsilon_k = E_k$. It is interesting to notice that the successful computations in [22] clearly suggest that the speed of sound in the dilatant vacuum corresponds to the speed of light: as vacuum solidification occurs, due to shear stress, only sound can travel through vacuum's quasi-lattice and, after all, dilatant vacuum allows transverse-wave propagation at very high frequencies, compatible with light propagation. This may help in calculating the Bogoliubov excitation spectrum for quantum vacuum. The final passage from the Bose polaron to a massive fermion occurs via the Yukawa interaction. Let us consider mobile impurities (dark matter particles) in a BEC of vacuum's quanta, whose Hamiltonian is

$$\mathscr{H} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}}^{Q} b_{\mathbf{k}}^{\dagger} \hat{b}_{\mathbf{k}} + \tfrac{g_Q}{2V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} b_{\mathbf{k}+\mathbf{q}}^{\dagger} b_{\mathbf{k}'-\mathbf{q}}^{\dagger} b_{\mathbf{k}'} b_{\mathbf{k}} + \\ + \sum_{\mathbf{k}} \xi_{\mathbf{k}} d_{\mathbf{k}}^{\dagger} d_{\mathbf{k}} + \tfrac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} b_{\mathbf{k}+\mathbf{q}}^{\dagger} d_{\mathbf{k}'-\mathbf{q}}^{\dagger} d_{\mathbf{k}'} b_{\mathbf{k}}, \qquad (11)$$

where $b_{\mathbf{k}}^{\dagger}$ and $d_{\mathbf{k}}^{\dagger}$ create a boson and a dark matter particle (impurity) respectively, with momentum $\mathbf{k}$, V is a volume, $\varepsilon_{\mathbf{k}}^{Q}$ is the kinetic energy of a vacuum's quantum and $\xi_k = k^2/2m - \mu$ is the kinetic energy of dark matter particles, with μ chemical potential. In Eq. (11) g_Q and g respectively represent the interaction strength between vacuum's quanta and between a quantum of vacuum and a particle of dark matter, case in which a Bose polaron arises. The vacuum's quanta involved in polarons formation are those partecipating in the phonon dynamics (Sect. 2). In this way the circular phonon starts to acquire mass becoming a free massive particle, in the form of a torus vortex, as discussed below. The Nambu-Goldstone boson's pseudo-momentum changes into a real momentum, thanks to dark matter's contribution. Considering Eq. (10) and the Green's function [13] $G(\mathbf{p},z)^{-1} = G_0(\mathbf{p},z)^{-1} - \sum(\mathbf{p},z)$, where $\mathbf{p}$ is the momentum of a polaron and $G_0(\mathbf{p},z)^{-1} = z - \xi_{\mathbf{p}}$ and $\sum(\mathbf{p},z)$ are the non-interacting Green's function and the self-energy (respectively), the energy of the polaron is found via

$$\varepsilon_{\mathbf{p}} = \xi_{\mathbf{p}} + \mathrm{Re}\sum(\mathbf{p},\varepsilon_{\mathbf{p}}) \qquad (12)$$

where a polaron is well-defined, since a small imaginary part of the self-energy is taken into account and the residue of the polaron,

$$Z_p = \left(1 - \partial \sum(\mathbf{p},z)/\partial z\right)^{-1}\Big|_{z=\varepsilon_{\mathbf{p}}} \qquad (13)$$

is close to unity. To first order in a the energy shift of the polaron is $\mathscr{T} n_Q$, where $\mathscr{T} = 2\pi a/m_r$ (with m_r the reduced mass) is the zero-energy scattering matrix vacuum's quantum-dark matter particle; $\mathscr{T} n_Q$ does not depend on dark matter concentration. To second order in the scattering length, still considering unitary residue and with $\varepsilon_{\mathbf{p}} = \xi_{\mathbf{k}}$, one obtains Landau's effective interaction between two polarons with momenta $\mathbf{p}_1, \mathbf{p}_2$ as

$$f(\mathbf{p}_1,\mathbf{p}_2) = \pm \mathscr{T}^2 \chi(\mathbf{p}_1 - \mathbf{p}_2, \xi_{\mathbf{p}_1} - \xi_{\mathbf{p}_2}), \qquad (14)$$

where $\chi(\mathbf{p},z) = n_0 p^2/\left(m_Q\left(z^2 - E_p^2\right)\right)$ defines the density-density correlation of the BEC at a zero temperature. From Eq. 14 we obtain the Yukawa interaction in real space, given by

$$f(\mathbf{r}) = -\frac{\mathscr{T}^2 n_0 m_Q e^{-\sqrt{2}r/\xi}}{\pi r}, \qquad (15)$$

with r the distance between dark matter particles and ξ the healing length (5) (and coherence length of the condensate). Through the interaction with particle dark matter in the polaron, the initial Goldstone boson (phonon) acquires mass and becomes a vortex tube in the doped vacuum. Then, obeying Helmholtz's second theorem, the tube assumes the form of a torus, in which the dynamics of spin takes form, as discussed below. It is also interesting to reflect on the fact that, if a vortex-particle (e.g. a fundamental fermion) attracts diffused dark matter's particles, this must be true also on macroscopic scale, as far as baryon matter-dark matter interaction is concerned; this could justify the concentration of dark matter where also baryon matter is concentrated, such as in the galactic discs.

4 Spin from vortex dynamics

As here physical vacuum is considered as a doped BEC [3], one can start to describe its hydrodynamic behavior from the Gross-Pitaevskii equation (GPE) [14]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{h^2}{2m}\nabla^2\psi + g\psi|\psi|^2 - \psi\mu_0 \qquad (16)$$

where ψ is the condensate wave function, with m as the mass of a quantum, μ_0 the chemical potential and $g = 4\pi a\hbar^2/m$ a low-energy parameter, where a is again the scattering length between quanta. In the phase representation $\psi = \sqrt{\rho}e^{i\varphi}$, ρ is vacuum density. From (16) we can write the hydrodynamic equations, i.e. the continuity equation and the analogue of the Euler equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho_d \mathbf{v}_S) = 0 \tag{17}$$

$$m\left(\frac{\partial\rho}{\partial t} + \mathbf{v}_S\cdot\nabla\right)\mathbf{v}_S = \nabla\left(\mu_0 - g\rho + \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) \tag{18}$$

where $\mathbf{v}_S = \frac{\hbar}{m}\nabla\varphi$ is the superfluid velocity and

$$\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{19}$$

represents the quantum potential. Since the condensate must be a continuous function in space, its phase is continuous modulo 2π. Let us therefore define again the quantized circulation (Γ), analogously to Eq. (2), through the line integral

$$\oint_C d\mathbf{x}\cdot\mathbf{v}_S = \frac{2\pi\hbar}{m}n \equiv \Gamma \quad (n = 0, \pm1, \pm2, ...) \tag{20}$$

in which C is a close loop in space that must encircle a vortex line, corresponding to the circular path of the Goldstone phonon (Sect. 2) at the origin of the vortex, where $\psi = 0$ and the superfluid's density vanishes. According to Helmholtz's second theorem, the vortex line, if it does not end on a boundary, must form a closed loop and it will indeed form a vortex ring, or we should say a vortex torus.

The problem of ultraviolet divergence and of the radius of the fundamental particles is solved in this approach by the fact that we do not deal with point-particles, as believed in the current theoretical framework, but with toroidal vortices of vacuum's quanta, with radius $r = 2\xi$, with ξ defined in Eq. (5). Here the ratio of the velocities v_1 and v_2 of vacuum's quanta in the vortex (Fig. 1) can satisfactorily represent all spin numbers. On a different basis, a hydrodynamic reformulation of the Barut-Zanghi theory, which includes spin, was proposed by Salesi and Recami [16], who suggest [17] that the quantum potential of a particle

$$Q = -\frac{1}{2}m\vec{v}^2{}_S - \frac{1}{2}\nabla\cdot\vec{v}_S \tag{21}$$

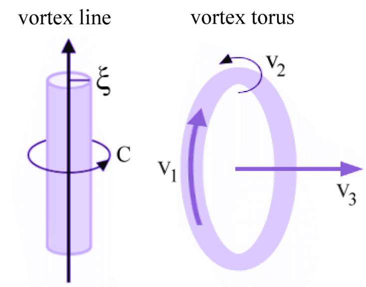

Figure 1: On the left: a vortex line with healing length ξ, actually a vortex tube. On the right: a vortex ring (vortex torus) where v_1 is the toroidal velocity, v_2 the poloidal velocity and v_3 the translational velocity of the vortex.

(using natural units, $\hbar = 1$) may totally arise from its internal motion $\vec{v}_S \times \vec{s}$, where

$$\vec{v}_S = \frac{1}{2m}\rho^{-1}\nabla\rho = \frac{1}{2m}\frac{\nabla R^2}{R^2}, \tag{22}$$

being $\vec{s}$ the direction of spin. By looking at spin as internal motion, the link with the concept of vortex-particle is quite immediate. Any different spin has however to be now explained. In this work, torus-shaped vortices (Fig. 2) are considered. Also Villois, Krstulovic *et al.* analyze vortex tubes evolving into vortex tori in superfluids [18] and demonstrate the emergence of non-trivial topology. Their geometry seems to well account for the spin of fundamental particles. The vortex is triggered by the Nambu-Goldstone boson (phonon) whose chirality is then kept by the quantum vortex (by the resulting massive boson or fermion).

If a vacuum's quantum flowing in the torus-vortex needed the same time the vortex needs to complete two turns in the toroidal direction to return in the same position after having completed one turn in the poloidal direction, then the vortex would have spin-½ (fermion), i.e. the system returns in the same state after a toroidal rotation of 720°, after each quantum forming the vortex has moved along a Möbius-strip path (Fig. 2 on the left). It is interesting to notice that such a bi-component spin can explain in mechanical terms any other type of spin as the ratio of toroidal to poloidal rotations. Defining ω_1, ω_2 as the angular velocities for the cited directions (respectively), the spin angular momentum (S) is determined by the ratio

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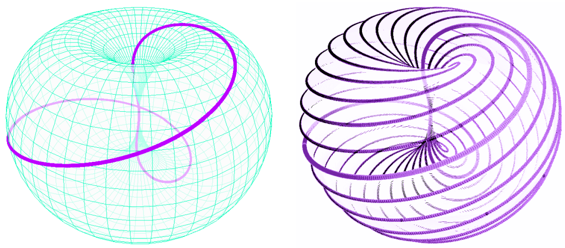

Figure 2: Vortex torus. The ratio of the toroidal angular velocity to the poloidal one may determine the spin of a particle. Each quantum of the superfluid vacuum forming the torus vortex flows along a Möbius-strip trajectory. Given the ratio 1/2 of the poloidal angular velocity to the toroidal angular velocity, each quantum returns in its starting position after a 720° rotation of the particle. These characteristics are for instance typical of fermions spin-½. On the right a simulation with 12 vacuum's quanta.

$\omega_1/\omega_2 = (n\pi/dt)\,/\,(2\pi/dt)$ and after the cancellations

$$\frac{\omega_1}{\omega_2} = \frac{n}{2} = S. \tag{23}$$

One poloidal revolution each two toroidal rotations corresponds to spin-½. The case of spin-0 may be determined by further evolution of the horn-torus into a spheroidal vortex. The parametric equation defining the position of a quantum in the torus-vortex can be expressed as follows

$$\begin{cases} x = (r + \xi\cos(\omega_1 dt + \phi_0))\cos(\omega_2 dt + \theta_0) \\ y = (r + \xi\cos(\omega_1 dt + \phi_0))\sin(\omega_2 dt + \theta_0) \\ z = \xi\sin(\omega_1 dt + \phi_0) \end{cases} \tag{24}$$

where $r \geq 2\xi$ is the distance between the center of the tube and that of the torus, $\omega_1 = d\phi/dt$, $\omega_2 = d\theta/dt$ and ϕ_0 and θ_0 are phases having arbitrary values between 0 and 2π.

The toroidal shape of fundamental particles may also resembles the theorized loops in Loop Quantum Gravity [19, 20]. In our case, however, on a higher scale, to describe particles but it is not excluded that also vacuum's quanta may have a loop-like geometry. At the same time, we can also notice a certain similarity between a vortex line and a string. Could then vacuum's characteristics and dynamics reconcile different approaches? To conclude this section about spin, it is also interesting to consider that an opportune change in the ratio revolutions/rotations in the vortex would leave the particle's mass unvaried but would transform a boson into a fermion or vice versa. Thus, a varying ratio may be important also for understanding the quantum mechanical foundations of supersymmetry. Eventually, it should be noted that chiral vortices are also able to represent matter-antimatter parity.

5 Hydrodynamic interactions

Let us refer to Pshenichnyuk [21], who studied the hydrodynamic interactions between quantum vortices in a doped superfluid, to reflect on the fact that quantum vortices in the superfluid vacuum would probably not behave as we would expect, to be able to justify fundamental interactions, without including a dopant, as indeed done in this work, by resorting to dark matter. In [21] a superfluid is considered in which doping particles change the way the vortices forming in the superfluid interact. The interactions are attractive or repulsive forces and occur via the Bernoulli effect. Interestingly, once the superfluid is doped, the interactions between two vortices match the mathematical behavior of Coulomb's law, or of Gauss's law for gravity. This is a very important clue when we describe the fundamental particles of the Standard Model as quantum vortices in a doped vacuum. Of course, we immediately think of dark matter as the most probable dopant of the superfluid vacuum, that is of dark energy. One therefore comes to the picture of a bi-component vacuum, of which dark energy is the greater, superfluid part. The presence of a dopant in the vacuum is probably also the reason for the dilatant behavior of the vacuum, as demonstrated via the exact solutions of the Pioneer anomaly and of Mercury's perihelion precession in [22]. Not simply a superfluid then but a bi-regime quantum vacuum, which passes to a dilatant (relativistic) regime as shear stress increases. As shown in [23], this is also the reason for the *apparent* relativistic mass increase in special relativity, actually due to the viscous force of dilatant vacuum. Let us come back to the dynamics of forces between quantum vortices. Pshenichnyuk, as well as Huang [7], confirms that attraction or repulsion between vortices (depending on their chirality) is driven by the Bernoulli force. This reads

$$\mathbf{F}_b = \int_S \mathbf{K}(\mathbf{r})\mathbf{n}(\mathbf{r})dS \tag{25}$$

where $K(\mathbf{r}) = \rho v^2/2$ hydrodynamically expresses the density of kinetic energy, which dominates on the vortex surface, while the density of the superfluid drops to zero within the healing distance, and $\mathbf{n}(\mathbf{r})$ is a unit vector normal to the surface S, over which the integral is calculated. Bernoulli force arises in a superfluid as a superposition of the vortices' velocity fields obeying a $1/r$ function. Points where the fields reinforce each other and points where their interaction makes them weaker, generate a pressure-related Bernoulli effect. We notice that Eq. 25 mathematically equals Gauss's law for gravity, which is equivalent to Newton's law of universal gravitation

$$\mathbf{F}_g = \int_S \mathbf{g}(\mathbf{r})\mathbf{n}(\mathbf{r})dS \tag{26}$$

and also Coulomb's law is equivalent to Gauss's law for the electric field, hence also to the Bernoulli force. Indeed, when we consider the force due to a spatially extended body with charge density ρ, the sum of the electric fields of an arbitrary amount of point charges, $\sum_{k=1,n} \mathbf{e}(Q_i)$, becomes an integral over infinitesimal volume elements of the body

$$\mathbf{e} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho}{|\mathbf{r}-\mathbf{r}'|^2} \hat{\mathbf{r}}\, dv \tag{27}$$

which, via the definition of divergence, is equivalent to Gauss's law and, mathematically, also to Bernoulli force. The force is repulsive if the vortices possess the same chirality or attractive if they have different chirality, exactly as positive/negative electric charges in Coulomb's law. The kinetic energy density acting on the dopant's particles, caused by the velocity fields of the superfluid vortices can be written as [21]

$$K^{++} = \frac{\hbar^2}{2m_Q} \frac{\psi_\infty^2 \left(4R^2 + d^2 - 4Rd\cos\alpha\right)}{(R^2+\xi^2)(R^2+d^2+\xi^2-2Rd\cos\alpha)}, \tag{28}$$

$$K^{+-} = \frac{\hbar^2}{2m_Q} \frac{\psi_\infty^2 d^2}{(R^2+\xi^2)(R^2+d^2+\xi^2-2Rd\cos\alpha)}, \tag{29}$$

where $++/+-$ signs refer to same or different topological charges denoting repulsion or attraction, d is the distance between the vortices, R the radius of a doping particle, ξ the healing length and α the azimuthal angle in cylindrical coordinates which is associated with the doping particle. It is important to reflect that Gauss's law describes gravity as a virtual incoming flux, but in [24, 25] it is hypothesized that a real incoming flow exists and corresponds to the gravitational field. If that were demonstrated, quantum gravity would be defined as a hydrodynamic force in a doped vacuum, with no need for elusive gravitons. Interactions between vortices at a distance have been discussed.

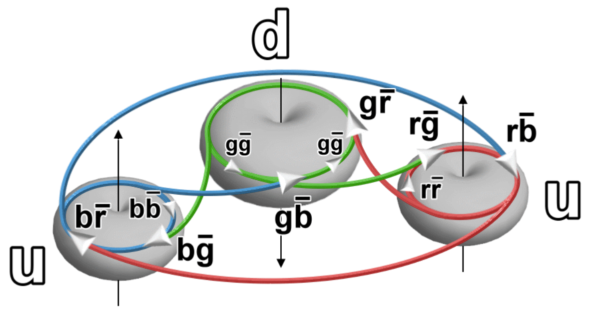

Figure 3: Simplified representation of a proton as a system of three interacting vortex-tori (quarks) in mutual (non-destructive) spin configuration. The gluon flow is represented as the exchange of vacuum's quanta coupled with dark matter particles which directly arises from the interaction of the velocity fields of the vortices due to short distance between them. Animated version available online at [26]

Finally it is interesting to reflect on the hydrodynamics of torus-vortices put at a very short distance, to also analyze the strong interaction. Fig. 3 shows three quarks (with blue, green and red charge, respectively) in a proton, as three torus vortices with mutual spin, i.e. in a non-destructive topology, as regards the sense of toroidal rotations (spin ½, -½, ½). That also agrees with Pauli exclusion principle and let us suppose that two fundamental fermions, e.g. two electrons, would annihilate if they got in contact both with spin ½ or -½ but matter-matter (vortex-vortex) annihilation, albeit possible according to the hydrodynamics of vortices, would be however avoided by electrostatic repulsion and Pauli exclusion principle. Figure 3 also represents the gluon flow as a continuous flow of vacuum's quanta and dark matter particles (grouped in the gluon) which proceed from one vortex to another, according to the gluon exchange of quantum chromodynamics, occurring between quarks, and to the fact that this gluon flow changes the color of the quarks, passing from one to another. Here the wavefunctions are

$$\begin{aligned} \psi_1 &= |r\bar{g}\rangle\,, & \psi_2 &= |r\bar{b}\rangle\,, \\ \psi_3 &= |g\bar{r}\rangle\,, & \psi_4 &= |g\bar{b}\rangle\,, \\ \psi_5 &= |b\bar{r}\rangle\,, & \psi_6 &= |b\bar{g}\rangle\,, \end{aligned} \tag{30}$$
$$\psi_7 = \frac{1}{\sqrt{2}}\,|r\bar{r}\rangle - |b\bar{b}\rangle\,, \quad \psi_8 = \frac{1}{\sqrt{6}}\,|r\bar{r}\rangle + |b\bar{b}\rangle - 2\,|g\bar{g}\rangle\,.$$

Therefore, the structure of a quark would not be static, as for a solid object, but fluid (with spin representing the internal flow) since the constituents of a quark, i.e. the quanta of the doped vacuum, not only circulate in the torus-vortex but also flow from a quark to another within the hydrodynamic mechanism of strong interaction. According to what previously discussed, dark matter — specifically its momentum in the gluon flow driven by vacuum's quanta flowing from a vortex to another — is again responsible for the mass of the particle. In Fig. 3 the flux tubes are evident: here thin for greater simplicity but they are actually much broader flows (see [28, 26]). This visual representation is in some respects similar to the topological model for interacting quarks presented by Jehle [27] and also agrees with the color field dynamics of Cardoso's gauge $SU(3)$ QCD lattice simulations [28], which investigate the structure of the colour fields; the field in Fig. 3 could also assume L-shaped geometry. There, all nine generators of the group $U(3)$ are represented, though for the group $SU(N=3)$ one instead obtains $N^2-1=8$ generators, giving the gluon octet.

Conclusion

As we can see, according to the Ockham razor, vortex-particles in a superfluid, doped vacuum are the simplest solution to explain massive particles, their interactions and spin as quantum hydrodynamic phenomena. Planck constant itself corresponds to one turn of a quantum vortex (a quantum of action). Such an approach probably deserves therefore greater and renewed attention. As regards the Higgs mechanism, circularly polarized phonon in the superfluid vacuum, playing the role of the Nambu-Goldstone bosons, can trigger the mass-acquisition process, becoming Bose polarons by coupling with particle dark matter, and finally torus-vortices, which fully represent spin as the hydrodynamic ratio of revolutions to rotations. The vortices can also produce the fundamental interactions of the Standard Model (which are therefore read not as fundamental but as apparent hydrodynamic forces) via the Bernoulli force acting between them, where electric charges arise from the topological charges of the vortices, depending on their chirality. Forces can be therefore both attractive and repulsive. About the role of particle dark matter as a dopant, when we come, in this model, to the strong interaction and to the gluon flux, the possibility that gluons coincide with dark matter's particles grouped with (or driven by) vacuum's quanta is high. Also the exchange of virtual photons (gauge bosons in both cases) in the electrostatic interaction may be revisited in the light of the exchange of vacuum's quanta and particle dark matter between the vortices.

Within this approach, the reality that we observe and which we are part of, i.e. that $\sim 5\%$ of baryon matter in the universe, would be nothing but the joint product of the hydrodynamics of superfluid dark energy (the vacuum) and particle dark matter (the dopant), which together are dilatant vacuum [22], the remaining $\sim 95\%$ of mass-energy in the universe.

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References

[1] AMENDOLA, L., TSUJIKAWA, S., *Dark Energy. Theory and Observations.* Cambridge University Press, Cambridge (2010)

[2] BERTONE, G., *Particle Dark Matter. Observations, models and searches.* Cambridge University Press, Cambridge (2010)

[3] DAS, S. and BHADURI, Rajat K.: *Dark matter and dark energy from Bose-Einstein condensate*, preprint: arXiv:1411.0753[gr-qc].

[4] SBITNEV, V.I.: *Physical Vacuum is a Special Superfluid Medium*. In: Pahlavani, M.R. (ed.): Selected Topics in Applications of Quantum Mechanics, InTech, Rijeka (2015)

[5] VOLOVIK, G.E.: *The Universe in a helium droplet*, Int. Ser. Monogr. Phys. **117** (2003)

[6] VOLOVIK, G.E.: *The Superfluid Universe*, Int. Ser. Monogr. Phys. **156**, Vol. 1, 570-618 (2013)

[7] HUANG, K.: *A Superfluid Universe*. World Scientific, Singapore (2016)

[8] HUANG, K.: *Dark energy and dark matter in a superfluid universe.* Preprint: arXiv:1309.5707 (2013)

[9] BOLMATOV, D., BRAZHKIN, V. V., TRACHENKO, K., The phonon theory of liquid thermodynamics. *Sci. Rep.* **2**, 421 (2012).

[10] ZHU, H., Yi, J., Li, M., Xiao, J., Zhang, L. et al., Observation of chiral phonons. Science, **359**, Issue 6375, pp. 579-582 (2018).

[11] LIU, D., SHI, J., Phys. Rev. Lett. 119, 075301 (2017)

[12] RECAMI, E., SALESI, G.:*About kinematics and hydrodynamics of spinning particles: Some simple considerations*. Preprint: arXiv:quant-ph/9607025v1 (1996)

[13] CAMACHO-GUARDIAN, A., BRUUN, Georg M. *Landau Effective Interaction between Quasiparticles in a Bose-Einstein Condensate*, Phys. Rev. X **8**, 031042 (2018)

[14] DALFOVO, F. , GIORGINI, S., PITAEVSKII, L.P., STRINGARI, A. : Rev. Mod. Phys. **71**, 463 (1999)

[15] ESPOSITO, S.: *On the Role of Spin in Quantum Mechanics*, arXiv:quant-ph/9902019v1 (1999)

[16] SALESI, G., RECAMI, E.:*Hydrodynamical reformulation and quantum limit of the Barut-Zanghi Theory.* Foundations of Physics Letters, Vol. 10, No. 6 (1997)

[17] RECAMI, E., SALESI, G., Phys. Rev. A57 98, (1998)

[18] VILLOIS, A., KRSTULOVIC, G., PROMENT, D., SALMAN, H.: *A Vortex Filament Tracking Method for the Gross-Pitaevskii Model of a Superfluid*, arXiv:1604.03595v2 (2016)

[19] ASHTEKAR, A., PULLIN, J.:*Loop Quantum Gravity*, World Scientific, 2017

[20] ROVELLI, C., VIDOTTO, F.:*Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory*, Cambridge University Press, 2014

[21] I.A. PSHENICHNYUK: *Pair interactions of heavy vortices in quantum fluids*, arXiv:1705.10072v1 (2017)

[22] FEDI, M. 2018, Physical vacuum as a dilatant fluid yields exact solutions to Pioneer anomaly and Mercury's perihelion precession, Can. J. Phys., doi:10.1139/cjp-2018-0744 (and on zenodo repository: https://zenodo.org/record/2566589)

[23] FEDI, M. 2019, Relativistic mass due to dilatant vacuum and quantum formula for relativistic kinetic energy, doi:10.5281/zenodo.2566526

[24] CAHILL, R.T.: *Gravity as quantum foam in-flow.*, Preprint at: https:// arxiv.org/abs/physics/0307003

[25] FEDI, M.: *Hydrodynamics of the Dark Superfluid: III. Superfluid Quantum Gravity*, 2017, HAL-Archives, CCSD, France, http://hal.archives-ouvertes.fr/hal-01423134

[26] DÄUMLER, W.: animation of strong interaction of Fig. 3, URL: http://www.8128.info/strong-interaction_MFedi.html

[27] JEHLE, H.:*Topological characterization of leptons, quarks and hadrons*, Phys. Lett. B **104**, 3, 207-211 (1981)

[28] CARDOSO, M., CARDOSO, N., BICUDO, P.:*Lattice QCD computation of the colour fields for the static hybrid quark-gluon-antiquark system, and microscopic study of the Casimir scaling*, Phys. Rev. D **81**, 034504 (2010)

[29] FIMMEL, R.O., VAN ALLEN, J., BURGESS, E.: *Pioneer: first to Jupiter, Saturn, and beyond*. NASA Scientific and Technical Information Office, Washington D.C., USA (1980)

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